竹北嘸校念

Group F

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Table of contents

Introduction

Due to the flourishing of the Hsinchu Science Park ten years ago, many people have settled in Zhubei and had children. Now, a significant number of school-age children are facing a shortage of schools.

Data

Number of Students:
    Zhubei City Household Registration Office

Districts:
    Village of Zhubei City

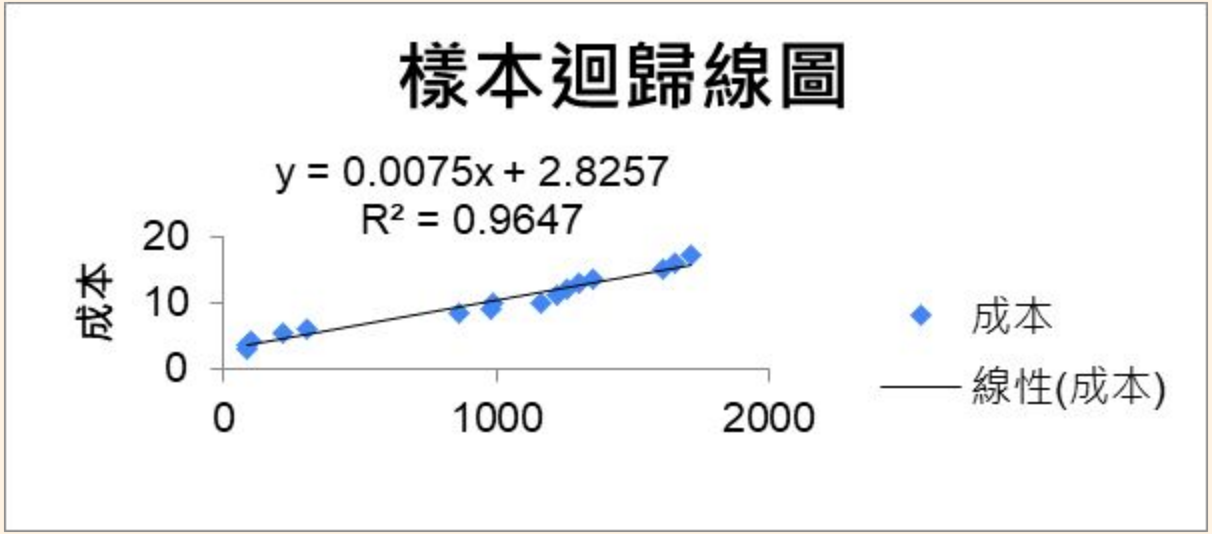
Information of School:
    Education Bureau of HsinChu County Government

Cost:
    Government Budget

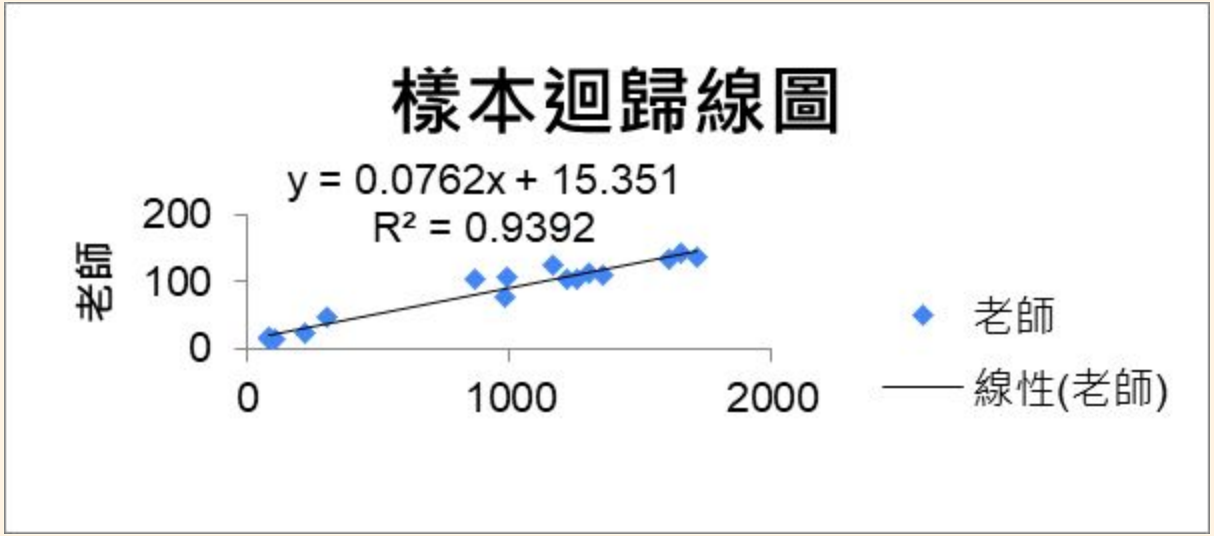
Data

OBJECTID_1	VILLAGE	TOWN	COUNTY	X	Y
992	鹿場里	竹北市	新竹縣	251937.856697	2745356.90408
996	斗崙里	竹北市	新竹縣	250814.18704	2746092.91465
1001	東興里	竹北市	新竹縣	253684.773	2745732.66394
1002	北崙里	竹北市	新竹縣	250946.901454	2746894.43367
1003	新崙里	竹北市	新竹縣	249563.64554	2746821.54778

Data



Data



Situation I

Problem Description

Our goal is to choose places for new schools and distribute students from each district such that total cost can be minimized and all limits will be satisfied.

The main costs arise from building school and commuting. There are limitations for school capacity and teachers needed.

We set three situations to achieve the goal.

Problem Description

Situation 1: two sizes

School Size	Big school	Small School
Student Capacity	1250	400
Number of Teachers	110	40
Building Cost	1.2B	500M

Formulation - Variable

Variable	Description
L	The set of districts in Zhubei, which also serves as candidate locations for a new school, $L = \{1, ..., 30\}$
K = 2400	The fixed number of teachers in Zhubei city
S_i^B	The number of existing big schools in district i , $\forall\ i \in L$
S_i^S	The number of existing small schools in district i , $\forall\ i \in L$
P_i	The number of students in districts i , $\forall\ i \in L$

Formulation - Variable

T_B	= 110	The number of teachers for big school
T_S	= 40	The number of teachers for small school
N_B	= 1250	The number of students in big school
N_S	= 400	The number of students in small school
d_{ij}		The distance between district i where the students are and district j where the school is located , $\forall\ i, j \in L$

Formulation - Variable

x_j^B	The number of big schools built in district j, $\forall\ j \in L$ (decision variable)
x_j^S	The number of small schools built in district j, $\forall\ j \in L$ (decision variable)
C_B = 1.2B	The building cost of a big school
C_S = 500M	The building cost of a small school
z_{ij}	The percentage of students in district i who study at school built in district j, $z_{ij} \in [0,1]\ \ \forall i \in L,\ j \in L$ (decision variable)

Formulation - Objective function

Cost of Building schools + Commuting cost

$$\min \quad \sum_{i \in L} \left[(x_i^B - S_i^B) C_B + (x_i^S - S_i^S) C_S \right] + 50 \sum_{i \in L} \sum_{j \in L} d_{ij} z_{ij} P_i$$

Formulation - Constraints

$$\sum_{i \in L} z_{ij} P_i \leq x_j^B N_B + x_j^S N_S, \ \forall j \in L,$$

(The total schools in district j has a capacity limitation.)

$$\sum_{j \in L} z_{ij} = 1, \ \forall i \in L,$$

(The students in district i must attend school.)

$$\sum_{i \in L} \left(x_i^B T_B + x_i^S T_S \right) \leq K,$$

(The demand for teachers will not exceed.)

Formulation - Constraints

$$\sum_{i \in L} P_i \leq \sum_{i \in L} (x_i^B N_B + x_i^S N_S),$$

(The number of students will not exceed the total capacity of schools.)

$$x_i^S \geq S_i^S, \forall i \in L,$$

(The number of small schools in district i is not less than the original number.)

$$x_i^B \geq S_i^B, \forall i \in L,$$

(The number of big schools in district i is not less than the original number.)

Result and Observation

	Our Solution	Reality
Total Cost	3.6 B	2.58 B

Guess: Only two sizes of school make it not flexible.

Improvement: Let some variables be linearly decided.

Situation II

Problem Description

Situation 2: multiple sizes

School Size	Multiple Sizes
Student Capacity	Continuous variables determined by the regression model.
Number of Teachers	
Building Cost	

Formulation - Variables

Variable	Description
L	The set of districts in Zhubei, which also serves as candidate district for a new school
Q	The set of position of newly built school in a district, $Q = \{1, 2\}$
K	The fixed number of teachers in Zhubei city
P_i	The number of students in district i, $\forall i \in L$,
y_{ij}	The number of students in school at position j within district i, $\forall i \in L$, $\forall j \in Q$

Formulation - Variables

T_{ij}	The number of teachers in school at position j within district i, $\forall i \in L, \forall j \in Q$
S_{ij}	The original number of students in school at position j within district i, $\forall i \in L, \forall j \in Q$
d_{ij}	The distance between district i where the students are and district j where the school is located, $\forall i, j \in L$
C_{ij}	The building cost of the school at position j within district i, $\forall i \in L, \forall j \in Q$

Formulation - Variables

F_{ij}	If there is a school at position j within district i, $F_{ij} = 1$; otherwise, $F_{ij} = 0$, $\forall i \in L, \forall j \in Q$ (decision variable)
B_{ij}	If there is a school newly built at position j within district i, $B_{ij} = 1$; otherwise, $B_{ij} = 0, \forall i \in L, \forall j \in Q$ (decision variable)
z_{ij}	The percentage of students in districts i who study at school built in district j, $z_{ij} \in [0, 1], \forall i \in L,\ j \in L$ (decision variable)

Formulation - Objective function

Cost of Building schools + Commuting cost

$$\min \quad \sum_{i \in L} \sum_{j \in Q} C_{ij} B_{ij} + 50 \sum_{i \in L} \sum_{j \in L} d_{ij} z_{ij} P_i$$

Formulation - Constraints

$$\text{s.t.} \quad \sum_{i \in L} \sum_{j \in Q} y_{ij} = 17410,$$

$$C_{ij} = 10^8(0.0075 y_{ij} + 2.8257 F_{ij}) \quad \forall i \in L,$$

(the building cost of school at location j in district i)

$$T_{ij} = 0.0762 y_{ij} + 15.351 F_{ij} \quad \forall i \in L,$$

(the number of teachers for school j in district i)

Formulation - Constraints

$$\sum_{j \in L} z_{ij} = 1, \forall i \in L, \tag{15}$$

(All of the students in district i must attend school.)

$$\sum_{j \in L} z_{ji} P_j \leq \sum_{j \in Q} y_{ij}, \forall i \in L, \tag{16}$$

(The number of students attend school cannot exceed the total capacity of schools in district i.)

$$\sum_{i \in L} T_{ij} \leq K, \tag{17}$$

(The demand for teachers will not exceed.)

Formulation - Constraints

$$\sum_{i \in L} P_i \leq \sum_{i \in L} \sum_{j \in Q} y_{ij},$$

(The number of students will not exceed the total capacity of schools.)

$$C_{ij} \leq 10^{11} y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, C_{ij} = 0),$$

$$T_{ij} \leq 10^{4} y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, T_{ij} = 0),$$

$$y_{ij} \geq S_{ij}, \forall i \in L, j \in Q,$$

Formulation - Constraints

$$y_{ij} \leq 10^5 F_{ij}, \forall i \in L, j \in Q,$$

$$y_{ij} - S_{ij} \leq 1800 B_{ij}, \forall i \in L, j \in Q,$$

(The maximum capacity is 1800.)

$$y_{ij} \geq 300 B_{ij}, \forall i \in L, j \in Q.$$

(The minimum capacity is 300.)

Result and Observation

	without weight	with weight	Reality
Total Cost	2.44 B	2.94 B	2.58 B
Districts of New School	15, 17	15, 17	8, 29

Observation: Using values regressed by linear model truly reduces the total cost even without the weight of 50.

Situation III

Problem Description

Situation 3: multiple sizes with over-enrollment

<table>
<tr><th>School Size</th><th>Multiple Sizes</th></tr>
<tr><td>Student Capacity</td><td rowspan="3">Continuous variables determined by regression model.</td></tr>
<tr><td>Number of Teachers</td></tr>
<tr><td>Building Cost</td></tr>
</table>

Formulation - Variables

We add a decision variable x_{ij}, the number of students in school at position j within district i, $\forall i \in L$, $\forall j \in Q$.

The remains are the same as situation 2.

Formulation - Objective function

Cost of Building schools + Commuting + Over-enrollment

$$\min \quad \sum_{i \in L} \sum_{j \in Q} C_{ij} B_{ij} + 50 \sum_{i \in L} \sum_{j \in L} d_{ij} z_{ij} P_i + 10^5 \sum_{i \in L} \sum_{j \in Q} (x_{ij} - y_{ij})$$

Formulation - Constraints

$$\sum_{i \in L} \sum_{j \in Q} x_{ij} = 17410,$$

$$C_{ij} = 10^8 (0.0075 y_{ij} + 2.8257 F_{ij}), \forall i \in L,$$

$$T_{ij} = 0.0762 y_{ij} + 15.351 F_{ij}, \forall i \in L,$$

$$\sum_{j \in L} z_{ij} = 1, \forall i \in L,$$

$$\sum_{j \in L} z_{ji} P_j \leq \sum_{j \in Q} x_{ij}, \forall i \in L,$$

(The number of students will not exceed the total capacity of schools.)

Formulation - Constraints

$$\sum_{i \in L} T_{ij} \leq K,$$

$$\sum_{i \in L} P_i \leq \sum_{i \in L} \sum_{j \in Q} x_{ij},$$

$$C_{ij} \leq 10^{11} y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, c_{ij} = 0),$$

$$T_{ij} \leq 10^{4} y_{ij}, \forall i \in L, j \in Q \text{ (if } y_{ij} = 0, c_{ij} = 0),$$

$$y_{ij} \geq S_{ij}, \forall i \in L, j \in Q,$$

Formulation - Constraints

$$y_{ij} \leq 10^5 F_{ij}, \forall i \in L, j \in Q,$$

$$y_{ij} - S_{ij} \leq 1800 B_{ij}, \forall i \in L, j \in Q,$$

$$y_{ij} \geq 300 B_{ij}, \forall i \in L, j \in Q,$$

$$x_{ij} \leq 1.2 y_{ij}, \forall i \in L, j \in Q.$$

(The number of students of a school can not over enroll 120% of the capacity.)

Result and Observation

	Situation 3 without weight	Situation 3 with weight	Situation 2	Reality
Total Cost	2.442 B	2.842 B	2.444 B	2.580 B
Districts of New School	24, 29	24, 29	15, 17	8, 29

Observation: Firstly, weight barely affects the result, too. In addition, over-enrollment is effective since the cost of situation 3 is the lowest among the 3 cases.

Conclusion

Conclusion

Situation 1:

Limited number of school size is the reason of vacancies, which in turn lead to imprecise prediction.

Conclusion

Situation 2:

The linearization of school size and teacher’s number successfully solves the problem in situation 1. Also, resulting in a better decision than reality.

Conclusion

Situation 3:

Considering the issue of over-enrollment in real life, we relax the school capacity, getting the best solution in this problem.

Conclusion

Compared to the decision made by the government, we also build two schools, but in different districts and at a lower cost.

Conclusion

We expect to obtain more detailed data (e.g., the real distance from student's home to school, the actual building cost of schools) for more accurate analysis to assist the government in making decisions.

Thank you