Canonic Signed Digit (CSD) Representation

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Hardware Reduction

- Constant multiplication
  - Can be replaced by shift-and-add
  - In binary representation, $C = \sum_{k=0}^{N-1} a_k 2^k \quad a_k \in \{0,1\}$
    - The more “1”s, the more shifters and adders
  - CSD provides method to reduce shifters and adders

$$C = \sum_{k=0}^{N-1} c_k 2^k \quad c_k \in \{-1,0,1\}$$

Examples

Binary representation

00111010111 $\quad 471_{dec}$

CSD

00111010111

$0011101100\bar{1}$

$0011110\bar{1}00\bar{1}$

$01000\bar{1}0\bar{1}00\bar{1}$

512-1-8-32=471

Binary representation

11001011101 $\quad -419_{dec}$

CSD

$\bar{1}1001100\bar{1}01$

$\bar{1}1010\bar{1}00\bar{1}01$

$0\bar{1}010\bar{1}00\bar{1}01$

-512+128-32-4+1=-419

Properties of CSD

- No 2 consecutive bits in a CSD number are non-zero.
- The CSD representation of a number contains the minimum possible number of non-zero bits.
- The CSD representation of a number is unique.
- CSD numbers cover the range (-4/3,4/3), out of which the values in the range [-1,1) are of greatest interest.

Systematic Approach (1/2)

$B=(b_{N-1},b_{N-2},\ldots,b_1,b_0)$

$c=(c_{N-1},c_{N-2},\ldots,c_1,c_0)$

Initialization

$b_{-1}=0$

$g_{-1}=0$

$b_N=b_{N-1}$

Loop

for i=0 to N-1

{

$s_i=b_i \oplus b_{i-1}$

$g_i=\overline{g_{i-1}}\cdot s_i$

$c_i=(1-2b_{i+1})g_i$

}

Start and End

Disable when Consecutive

Systematic Approach (2/2)

i	N	N-1								0	-1
b_i	1	1	0	1	1	1	0	0	1	1	
s_i		1	1	0	0	1	0	1	0	1	
g_i		0	1	0	0	1	0	1	0	1	0
$1-2b_{i+1}$		-1	-1	0	-1	-1	-1	1	1	-1	
C_i		0	-1	0	0	-1	0	1	0	-1	