Digital Communication Circuits Laboratory

Lab. 4

Interpolator

I. Purpose

In this lab., we will learn the method to implement a digital interpolator and to realize how to generate samples with fractional delay.

II. Principle

Digital interpolator is a module that is commonly seen in digital baseband signal processing blocks of communication systems. In wirelined or wireless communication systems, the analog signal will be converted to digital signal by analog-to-digital converter (ADC) driven by sampling frequency generated from an oscillator. Because the oscillators at the transmitter and the receiver may have tiny frequency and phase deviation, there exists sampling clock offset as shown in Fig. 1.

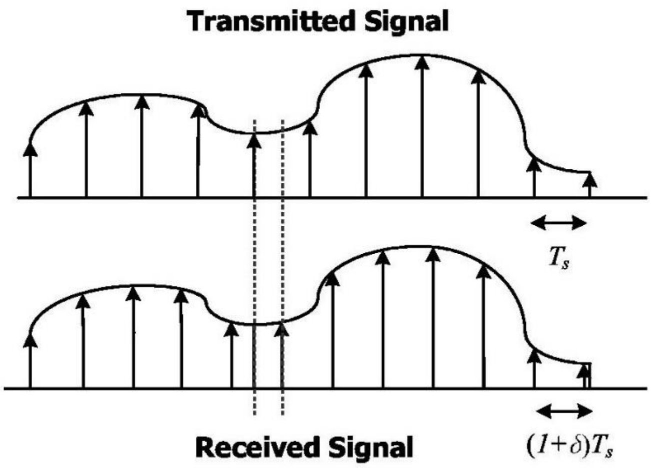

Fig. 1 Sampling clock offset

To solve the problem of sampling clock offset, we need an interpolator to generate samples with fractional delay to restore the samples that we want. From course "Signals and Systems, " we know that an ideal interpolator has impulse response described by $\mathrm{sinc}(t) = \dfrac{\sin(\pi t / T_s)}{\pi t / T_s}$ in the time domain as shown in Fig. 2. In the frequency domain, the frequency response has a rectangular shape.

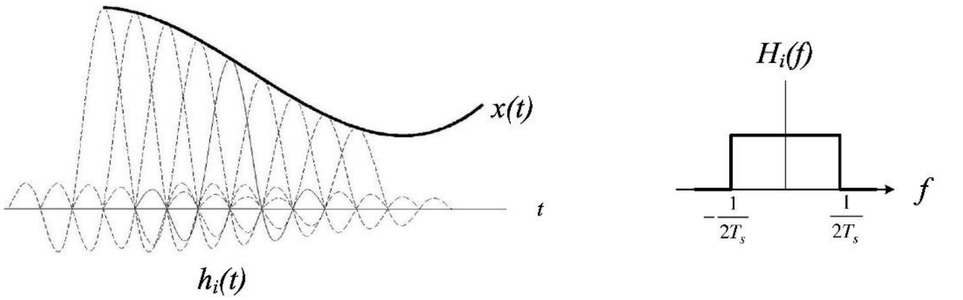

Fig. 2 Ideal interpolator.

However, it is feasible to implement an ideal interpolator. One of the reasons is that it has an infinite length. In addition, it is not a causal filter because it contains impulse response in the left half plane. Even so, the ideal interpolator reveals the properties of an interpolator.

1. The frequency response must keep constant magnitude in the range regarding to signal bandwidth.
2. The frequency response must have linear phase n the range regarding to signal bandwidth.

There are various kinds of digital interpolators which can be seen as an alternative of ideal interpolator. Polynomial-based digital interpolators are widely used because of the following advantages.

1. The polynomial-based interpolators have close form to describe the required interpolator coefficients.
2. The polynomial-based interpolators have good frequency response in the frequency domain.
3. The polynomial-based interpolators are easily to be implemented.

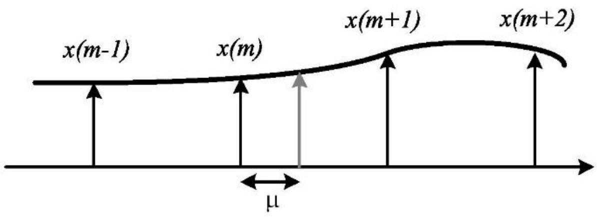

Fig. 3 Definition of sample point with fractional delay.

A. Linear interpolator

Linear interpolator is the simplest interpolator that is described by the first-order polynomial. Consider the point $x(m+\mu)$ to be interpolated in Fig. 3, where $0 \le \mu < 1$. By linear interpolation,

$$x(m+\mu) = \mu x(m+1) + (1-\mu)x(m). \tag{1}$$

B. Second-order polynomial interpolator

Linear interpolator may be too simple in some situation. To improve the quality after interpolation, we can raise the order of polynomail and the second-order interpolation can be used. Three adjacent sample points are adopted for interpolation. It uses the following equation.

$$x(m+\mu) = C_0 x(m) + C_{-1}(m+1) + C_{-2}(m+2) \tag{2}$$

and

$$\begin{cases} C_0 = (1-\mu)(2-\mu)/2 \\ C_{-1} = \mu(2-\mu) \\ C_{-2} = -\mu(1-\mu)/2 \end{cases} \tag{3}$$

C. Piecewise parabolic interpolator

We can also use piecewise parabolic interpolator to generate the the interpolation results. Its equation is given as follows. Four sample points are used for interpolation.

$$x(m+\mu) = C_1 x(m-1) + C_0 x(m) + C_{-1}(m+1) + C_{-2}(m+2), \tag{4}$$

where

$$, \quad \begin{cases} C_1 = -\alpha\mu + \alpha\mu^2 \\ C_0 = 1 + (\alpha-1)\mu - \alpha\mu^2 \\ C_{-1} = (\alpha+1)\mu - \alpha\mu^2 \\ C_{-2} = -\alpha\mu + \alpha\mu^2 \end{cases} \tag{5}$$

and α is a value between 0 and 1. In this lab., we set $\alpha = 0.5$.

D. Implementation of Interpolator based on Farrow Structure

Farrow structure is an efficient implementation method to realize the interpolator because of hardware sharing. The equation of interpolation can be rewritten as follows.

$$\begin{aligned} x(m+\mu) &= \sum_{i=I_1}^{I_2} x(m-i)C_i = \sum_{i=I_1}^{I_2} x(m-i) \sum_{l=0}^{N} b_l(i)\mu^l \\ &= \sum_{l=0}^{N} \mu^l \sum_{i=I_1}^{I_2} b_l(i)x(m-i) = \sum_{l=0}^{N} \mu^l\, v(l) \end{aligned} \tag{6}$$

where

$$v(l) = \sum_{i=I_1}^{I_2} b_l(i)x(m-i).$$

The piece-wise parabolic polynomial interpolator of Farrow structure is shown in Fig. 4.

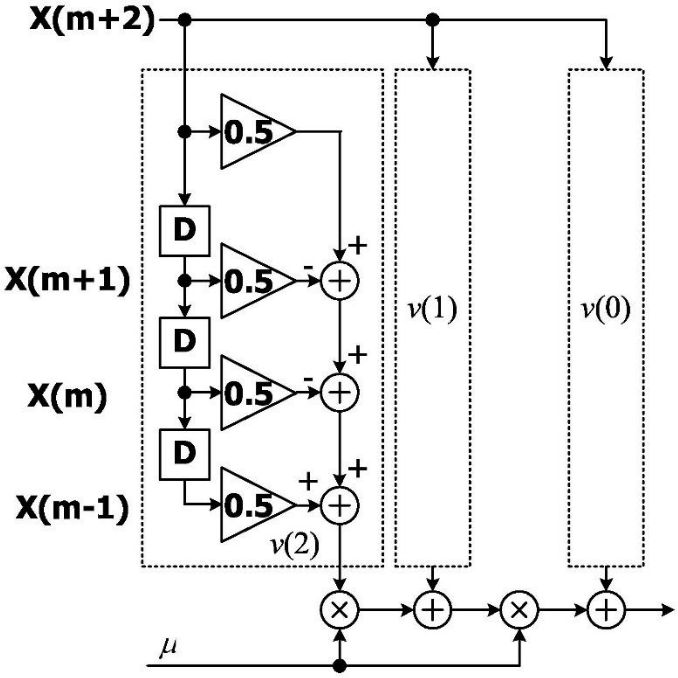

Fig. 4 Farrow structure of the piece-wise parabolic interpolator.

III. Procedures

1. The time-domain impulse response of interpolator can be expressed as

$$h(t)|_{t=(i+\mu)T_s} = h\big((i+\mu)T_s\big) = \sum_{l=0}^{N} b_l(i)\mu^l.$$

   Try to draw the time-domain impulse response of the linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator.

2. Please draw the frequency response of the linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator. Observe their properties in the frequency domain. Note that proper resolution may be required for the observation.

3. Assume that $x[m] = \cos(2\pi\left(\frac{mT_s}{8T_s} + \frac{\phi}{10}\right))$, where is the last digit of your student ID.

It means that the sampling frequency is 8 times the frequency of the sinusoidal wave. Please use linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator to interpolate the sampled waveform in the region of $24 \leq m \leq 64$ with $\mu = 0, \frac{1}{6}, \frac{2}{6}, \ldots \frac{5}{6}$. Compare the error between the interpolated outputs and the floating-point results.

4. Assume that $x[m] = \cos(2\pi\left(\frac{mT_s}{4T_s} + \frac{\phi}{10}\right))$, where is the last digit of your student ID. It means that the sampling frequency is 4 times the frequency of the sinusoidal wave. Please use linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator to interpolate the sampled waveform in the region of $12 \leq m \leq 32$ with $\mu = 0, \frac{1}{6}, \frac{2}{6}, \ldots \frac{5}{6}$. Compare the error between the interpolated output and the true value.
5. Please determine the word-length of the linear interpolator so that the error between quantized outputs and floating point output in Q3 can be smaller than 2^{-a}, where $a = -10$ for even-numbered students and $a = -9$ for odd-numbered students.
   a. Wordlength of input
   b. Wordlength of μ

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6. Please determine the word-length of the piece-wise parabolic interpolator so that the error between quantized outputs and floating point output in Q3 can be smaller than 2^{-a}, where $a = -9$ for even-numbered students and $a = -10$ or odd-numbered students.
   a. Wordlength of input
   b. Wordlength of μ
   c. Wordlength of multiplier (by μ)
   d. Wordlength of adder
7. Please write Verilog to describe your piece-wise parabolic interpolator of Farrow structure. Please insert D flip-flops of your inputs and outputs. Check the results of the hardware.
8. Synthesis your design and check the timing report.
9. Please measurement your design output on FPGA board and check the error.

IV. Results

1. Please draw the time-domain impulse response of the linear interpolator, second-

order polynomial interpolator, and piecewise parabolic interpolator. Show the correct labels of x axis and y axis. (20%)

2. Please draw the frequency response, including magnitude and phase, of the linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator. Observe their properties in the frequency domain. Note that proper resolution may be required for the observation. Show the correct labels of x axis and y axis. (20%)
3. Show the error between the floating-point results and the interpolated outputs by linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator in the region of $24 \leq m \leq 64$ with $\mu = 0, \frac{1}{6}, \frac{2}{6}, \ldots \frac{5}{6}$. (15%) Write your comments about comparison of interpolators. (5%)
4. Show the error between the floating-point results and the interpolated outputs by linear interpolator, second-order polynomial interpolator, and piecewise parabolic interpolator in the region of $12 \leq m \leq 32$ with $\mu = 0, \frac{1}{6}, \frac{2}{6}, \ldots \frac{5}{6}$. (15%) Write your comments about comparison of interpolators (5%)
5. Please depict the final architecture of the linear interpolator (6%) and show the results of different word-length settings versus the root mean squared error for
   a. Wordlength of input. (7%)
   b. Wordlength of μ. (7%)

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6. Please depict the final architecture of the piece-wise parabolic interpolator (10%) and show the results of different word-length settings versus the root mean squared error for
   a. Wordlength of input (5%)
   b. Wordlength of μ (5%)
   c. Wordlength of multiplier (by μ) (5%)
   d. Wordlength of adder (5%)

7. Design your piece-wise parabolic interpolator of Farrow structure. Please note that your input will change every six clock cycles and your μ value will change every clock cycle. Show the timing diagram of behavior simulation and post-route simulation results. Also depict the error between the Verilog outputs and Matlab floating-point outputs (30%)
8. Show your timing report and critical path. Check if the critical path is reasonable. (10%)
9. **Compare to the post-route simulation results (by Matlab figure.) (10%) and**

show your measurement results. (10%) **(Demo to TA until 11/22. (10%))**

V. Reference

[1] L. Erup, F. M. Gardner and R. A. Harris, "Interpolation in digital modems. II. Implementation and performance," IEEE transactions on Communications, Vol. 41, Jun. 1993, PP. 998-1008.