# Fundamental of Discrete-time Signal

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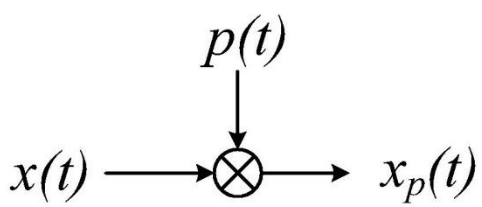
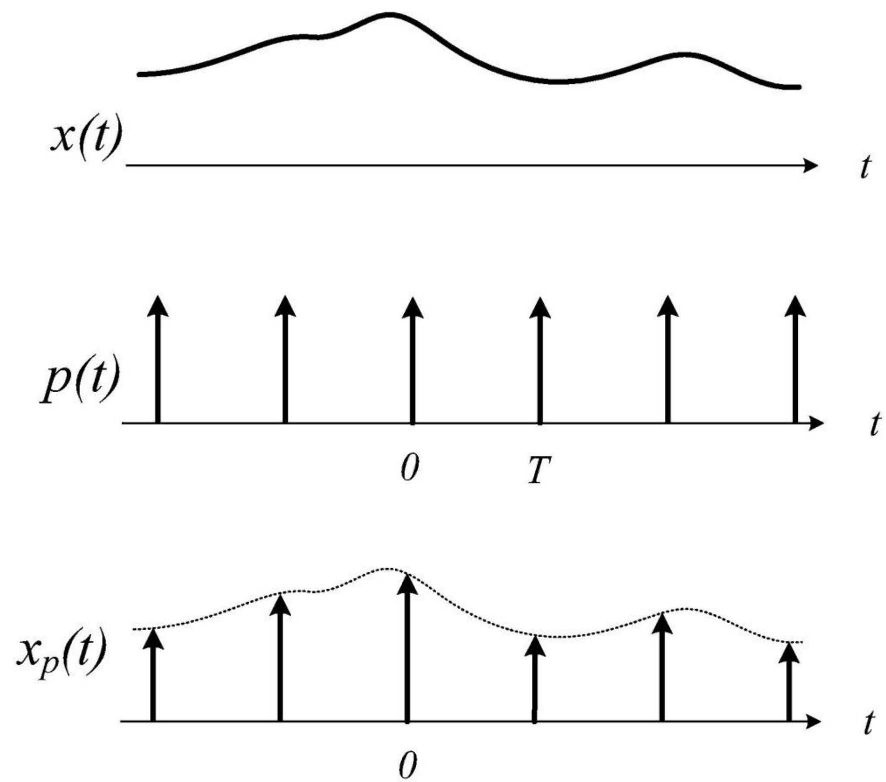
# Sampling (1/2)

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$



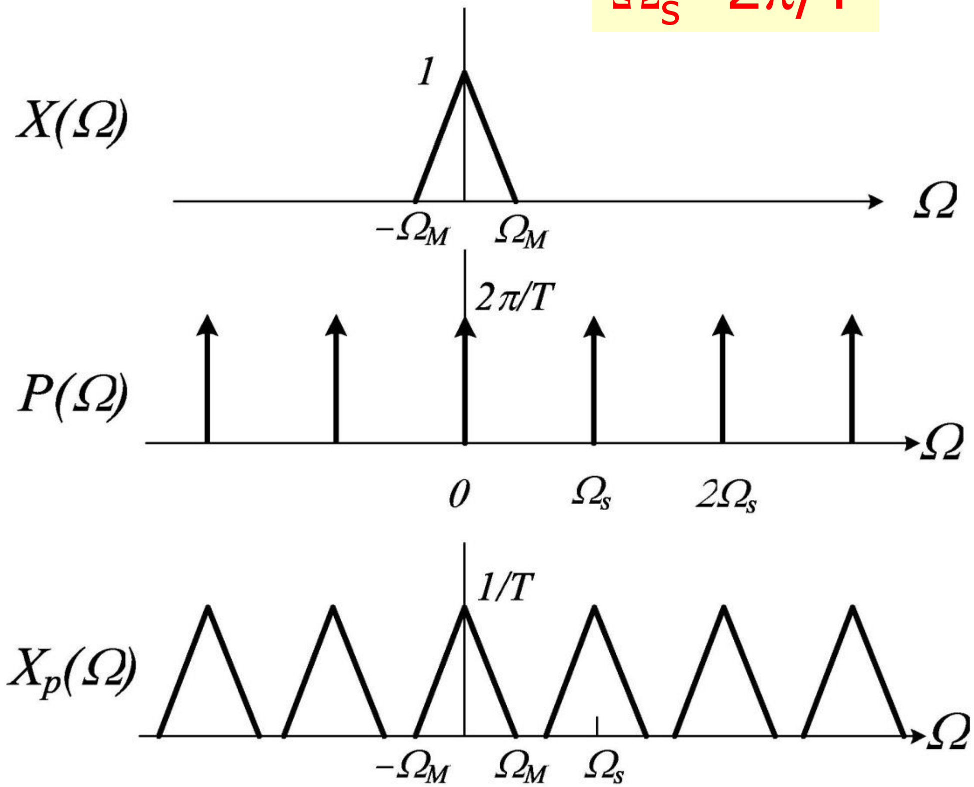



# Sampling (2/2)

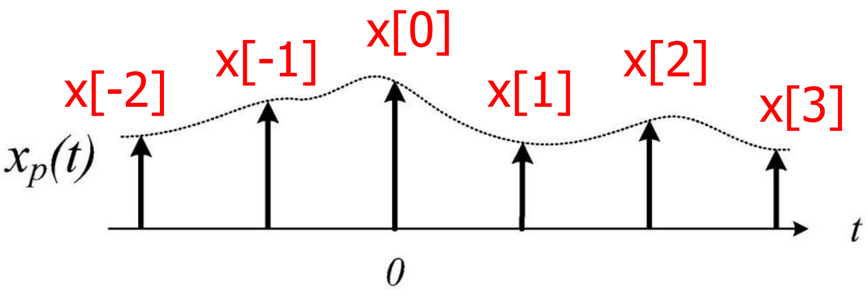
$\Omega_s = 2\pi/T$

$$P(\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$$

$$X_p(\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\Omega - k\Omega_s)$$



# Discrete-Time Signal




$$x[n] = x_p(nT)$$

# Discrete-Time Fourier Transform

- Discrete-time Fourier transform of a discrete time-domain signal *x[n]*

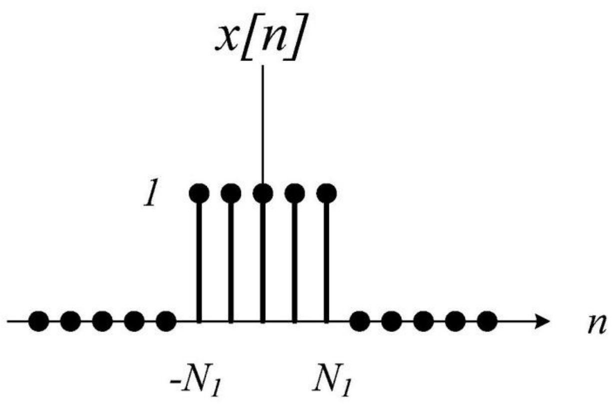
$$X(e^{j\omega}) = \mathrm{F}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Inverse discrete-time Fourier transform

$$x[n] = \mathrm{F}^{-1}\{X(e^{j\omega})\} = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

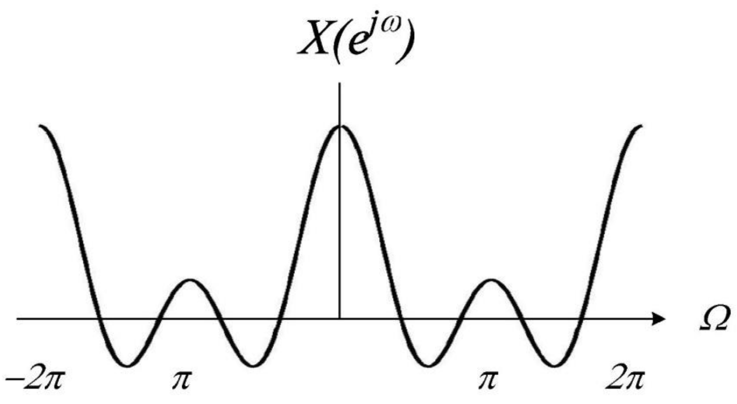
# Discrete Fourier Transform Example

- Time domain




$$x[n] = \begin{cases} 1, & |n| \le N_1 \\ 0, & |n| > N_1 \end{cases}$$

- Frequency domain




$$X(e^{j\omega}) = \frac{\sin(\omega(N_1 + \frac{1}{2}))}{\sin(\omega/2)}$$

# Properties of Fourier Transform Pair

- Time domain
- Frequency domain

Periodic $\Longleftrightarrow$ Discrete

Discrete $\Longleftrightarrow$ Periodic

# Connection

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$$

$$X_p(\Omega) = \sum_{k=-\infty}^{\infty} x(nT)e^{j\Omega nT}$$

$$= \sum_{k=-\infty}^{\infty} x[n]e^{j\Omega nT}$$

$$= X(e^{j\omega})\big|_{\omega=\Omega T} = X(e^{j\Omega T})$$

Fourier Transform

$$X(\Omega) = \mathrm{F}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt$$

Discrete-Time Fourier Transform

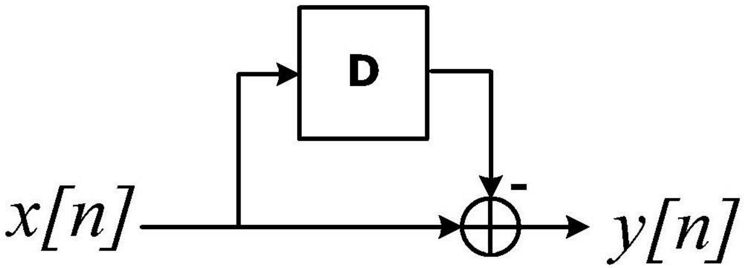
$$X(e^{j\omega}) = \mathrm{F}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

# Filtering (1/2)

- Consider simple difference equation

$$y[n] = \frac{x[n] - x[n-1]}{2}$$

$$Y(\Omega) = \frac{1}{2}(1 - e^{-j\Omega})X(\Omega)$$




$$H(\Omega) = \frac{1}{2}(1 - e^{-j\Omega})$$

# Filtering (2/2)