HW2 r12631001 許喬淇

Part 1

Problems 2.5

You are preparing a report and have to insert in it an image of size 2048×2048 pixels.

(a) Assuming no limitations on the printer, what would the resolution in line pairs per mm have to be for the image to fit in a space of size 5*5 cm?

<solution>

$$(\frac{2048\ lines}{50mm})/2\ \cong\ 20\ lines/min$$

(b) What would the resolution have to be in dpi for the image to fit in 2*2 inches?

<solution>

$$(2048\ dots)/(2\ inch)\ =1024\ dots\ per\ inch$$

Problem 2.12

Suppose that a flat area with center at (x_0,y_0) is illuminated by a light source with intensity distribution

$$i(x,y)=Ke^{-[(x-x_0)^2+(y-y_0)^2]}$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0, and let K=255. If the intensity of the resulting image is quantized using k bits, and the eye can detect an abrupt change of eight intensity levels between adjacent pixels, what is the highest value of k that will cause visible false contouring?

<solution>

i: intensity is quantized using i bits

$8=\frac{256}{2^i}$, $i=5$

The highest value of $k(=5)$ will cause visible false contouring.

Problem 2.18

Consider the image segment shown in the figure that follows.

(a) As in Section 2.5, let V={0, 1 } be the set of intensity values used to define adjacency. Compute the lengths of the shortest 4-, 8-, and m-path between p and q in the following image. If a particular path does not exist between these two points, explain why.

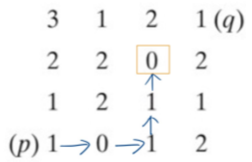
$$\begin{array}{ccccc} & 3 & 1 & 2 & 1\,(q) \\ & 2 & 2 & 0 & 2 \\ & 1 & 2 & 1 & 1 \\ (p) & 1 & 0 & 1 & 2 \end{array}$$

<solution>

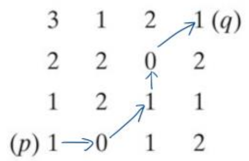
4-path doesn't exist.

當 p 為橘色矩形之位置時，不可能到達 q，因 p 之 4-adjacent 非 V 集

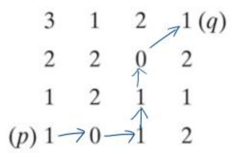
合內之值(即 1、2)。



The length of the shortest 8-path is 4.



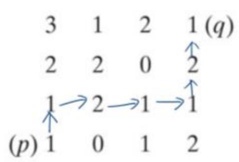
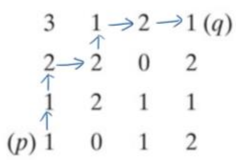
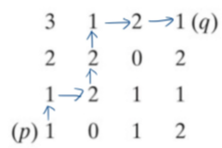
The length of the shortest m-path is 5.



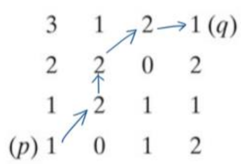
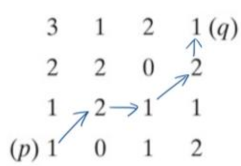
(b) Repeat (a) but using V={1, 2}.

<solution>

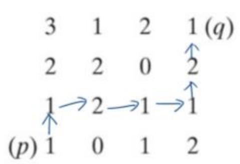
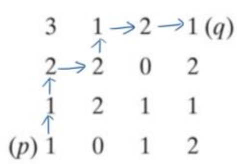
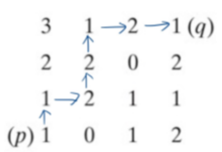
The length of the shortest 4-path is 6.

The length of the shortest 8-path is 4.

The length of the shortest m-path is 5.

Problem 2.36

With reference to Table 2.3, provide single, composite transformation functions for performing the following operations:

(a) Scaling and translation.

<solution>

$x' = c_x x + t_x$

$y' = c_y y + t_y$

(b) Scaling, translation, and rotation.
<solution>
$x' = c_x(x\ cos\ cos\theta - y\ sin\ sin\theta) + t_x$
$y' = c_y(x\ sin\ sin\ \theta\ + y\ cos\ cos\ \theta\ ) + t_y$

(c) Vertical shear, scaling, translation, and rotation.
<solution>
$x' = c_x\big((x + s_v y)\ cos\ cos\theta - y\ sin\ sin\theta\big) + t_x$
$y' = c_y(x\ sin\ sin\ \theta\ + y\ cos\ cos\ \theta\ ) + t_y$

(d) Does the order of multiplication of the individual matrices to produce a single transformations make a difference? Give an example based on a scaling/translation transformation to support your answer.
<solution>
Yes, it makes a difference.

$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$, $T = \begin{bmatrix} 1 & T_x \\ 0 & T_y \end{bmatrix}$

① 先 scaling 再 translate

$TS = \begin{bmatrix} 1 & T_x \\ 0 & T_y \end{bmatrix}\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & T_x \\ 0 & T_y \end{bmatrix}\begin{bmatrix} S_x \\ S_y \end{bmatrix} = \begin{bmatrix} S_x + T_x S_y \\ T_y S_y \end{bmatrix}$

different

② 先 translate 再 scaling

$ST = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}\begin{bmatrix} 1 & T_x \\ 0 & T_y \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}\begin{bmatrix} 1 + T_x \\ T_y \end{bmatrix} = \begin{bmatrix} S_x + T_x S_x \\ S_y T_y \end{bmatrix}$

Problem 3.12
An image with intensities in the range [0, 1] has the PDF, $p_r(r)$, shown in the following figure. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown in the figure. Assume continuous quantities, and find the transformation (expressed in terms of r and z) that will accomplish this.

<solution>
$s = T(r) = \int_0^r p_r(w)\,dw = \int_o^r(-2w + 2)\,dw = -r^2 + 2r$
$v = G(z) = \int_0^z p_z(w)\,dw = \int_0^z p_z(2w)\,dw = z^2$
$z = G^{-1}(v) = \pm\sqrt{v}$（負不合）
replace v as s
$z = \sqrt{-r^2 + 2r}$

Problem 3.21
Given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Give the convolution of the two.
<solution>

$$\text{Convolution result} = \begin{bmatrix} 9 & 12 & 12 & 12 & 9 \\ 12 & 16 & 16 & 16 & 12 \\ 12 & 16 & 16 & 16 & 12 \\ 12 & 16 & 16 & 16 & 12 \\ 9 & 12 & 12 & 12 & 9 \end{bmatrix}$$

(b) Does your result have a bias?
<solution>
Yes, normally.

Part 1

1. Read a color BMP or JPEG image file and display it on the screen. You may use the functions provided by Qt, OpenCV, or MATLAB to read and display an image file. (10%)

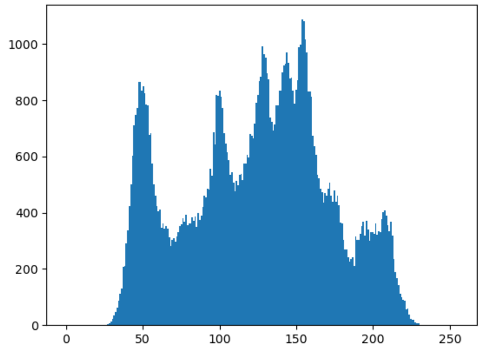
   cv2.imread()讀取照片、cv2.cvtColor()轉換格式
   使用 Lenna.jpg 作為本作業之測試目標影像

2. Convert a color image into a grayscale image using the following equations:
   A. GRAY = (R+G+B)/3.0
   B. GRAY = 0.299*R + 0.587*G + 0.114*B
   Compare the grayscale images obtained from the above equations. One way to compare the difference between two images is by image subtraction (5%)

   將圖像相減觀察差異，若計算後的值為負的，則取其絕對值。
   觀察兩張圖片差異：發現使用 A.算式計算出之影像亮暗邊緣明顯，使用 B.算式計算出之影像則較平滑，兩張影像相減後發現影像亮部之差異較明顯。

3. Determine and display the histogram of a grayscale image. (10%)

呼叫 RGB2GRAY_B()得到第二題 B.方法計算之灰階圖，接著計算分布密度儲存於已初始化之陣列 his_arr，將 his_arr 以線性函數歸一化，再一一繪製長條。發現 Lenna.jpg 之灰階分布較集中於 50 至 160，分布於 0、255 的值則沒有。



4. Implement a manual threshold function to convert a grayscale image into a binary image. (10%)

可自訂閥值，若影像之灰階值大於閥值則為 255，小於閥值則為 0。

5. Implement a function to adjust the spatial resolution (enlarge or shrink) and grayscale levels of an image. Use an interpolation method on enlarging an image. (10%)

可和給定影像尺寸大小和灰階影像 bit 數目於 Function nearest_neighbour，原始圖像大小為(316, 316, 3)經過鄰近差值法放大。將 bit 分別設為 2、3 比較差異，發現 bits 影像中確實由 4 個灰階所組成，而 3bits 之灰階影像相較 2 bit 圓滑。

6. Implement a function to adjust the brightness and constrast of an image. (10%)

Function adjust_brightness_contrast()可給定亮度和對比度 ratio，當亮度、對比度皆為 1 時與原圖相同；將對比設為 2 時，發現亮暗部對比增強，圖像呈現的顏色也更暖色；將亮度設為 2(為原圖亮度的兩倍)，發現圖像變白了。

7. Implement a histogram equalization function for automatic constrast adjustment.

先計算像素值的直方圖，計算分布密度，再利用原圖之分度密度建立映射表，下圖為 Lenna.jpg 均值化之前後直方圖，均值化前可以看到 0~20、230~240 之分布均為 0，均值化後雖然密度分布沒有調整為一定值，但灰階有分布於 0~255(0~20、230~240 之分布不為 0)。