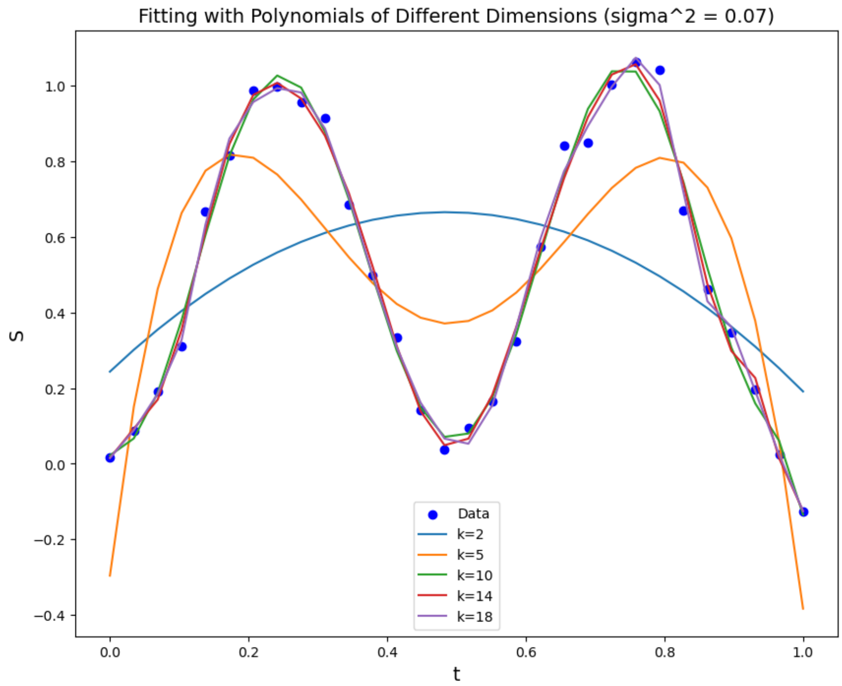
HW3 r12631001 許喬淇

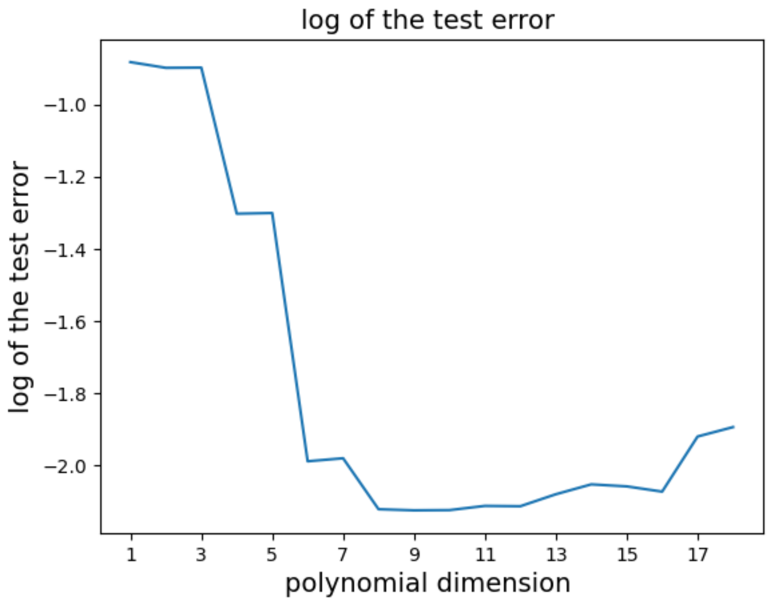
Problem 1

a) Fit the signal with a polynomial bases of dimensions k = [2 5 10 14 18]. Plot the 5 curves superimposed over a plot of the data points.

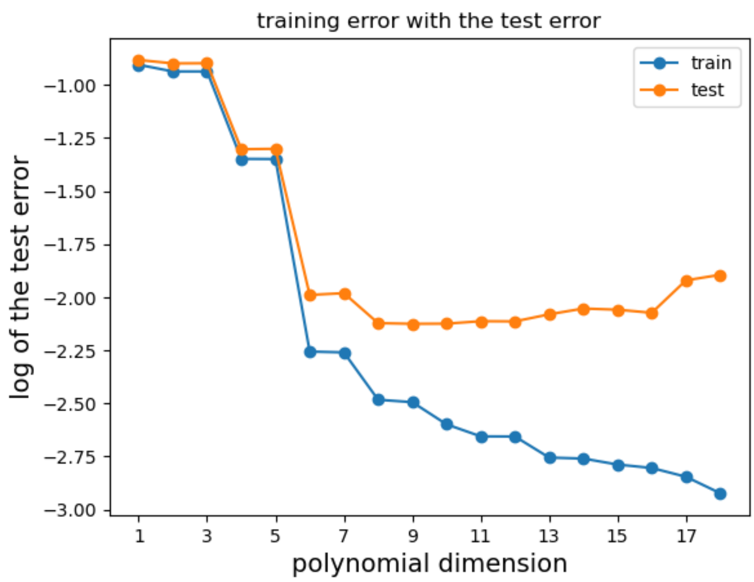


b) Let the $e_k(S)$ denote the mean squared training error of the fitting of the data set S with polynomial basis of dimension k. Plot the log of the training error $e_k(S)$versus the polynomial dimension $k = 1,\ldots,18$.

c) Let $e_k(S,T)$ denote the mean squared "test" error of the test signal T on the polynomial of dimension k fitted from training set S. Plot the log of the test error versus the polynomial dimension $k = 1,\ldots,18$.



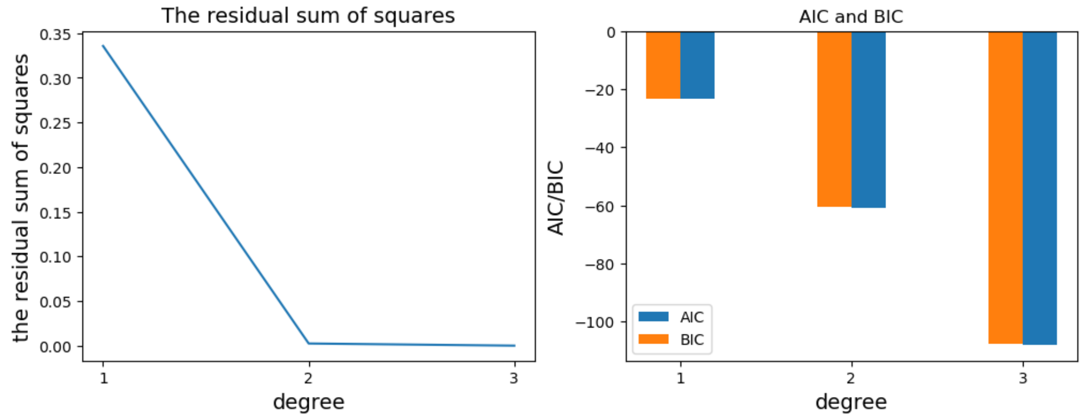
d) Compare the training error with the test error. Describe what you observe. When polynomial dimension k=8, the test error has minimum. When k>8, overfitting is happened because the fitted model is too complex.



Problem 2

Construct the least squares polynomial model of degree 1, compute the residual sum of squares, and AIC and BIC value for the model. Repeat this for the polynomial of degree 2 and 3. Plot the data with the models. What model should be chosen based on AIC and BIC?

The model of degree 3 should be chosen with a smaller AIC or BIC value.



Problem 3

a) Find the minimum value of the function $f(x,y,z) = (x + y + z)^2$, subject to the constraint $x^2 + 2y^2 + 3z^2 = 1$.

$$F(x,y,z,\lambda) = f(x,y,z) - \lambda g(x,y,z)$$
$$= (x+y+z)^2 - \lambda(x^2+2y+3z^2)$$
$$= x^2+y^2+z^2+2xy+2yz+2xz - \lambda(x^2+2y+3z^2-1)$$

$$F_x = 2x+2y+2z-\lambda\cdot 2x = 0 \quad \text{— ①}$$
$$F_y = 2y+2x+2z-2\lambda = 0 \quad \text{— ②}$$
$$F_z = 2z+2x+2y-\lambda\cdot 6z = 0 \quad \text{— ③}$$
$$F_\lambda = x^2+2y+3z^2-1 = 0 \quad \text{— ④}$$

① $\Rightarrow x+y+z-\lambda = 0$

② $\Rightarrow y+x+z-\lambda = 0$, $\lambda = x+y+z$

③ $\Rightarrow z+x+y-3z\lambda = 0 \Rightarrow \lambda - 3z\lambda = 0$

$\lambda(1-3z) = 0$

④ $\Rightarrow x^2+2y+3z^2 = 1$

$$\begin{cases} \lambda = 0 \\ z = \frac{1}{3} \end{cases}$$

$$\begin{cases} x+y+\frac{1}{3} = 0, \; y = -\frac{1}{3}-x \\ x^2+2y+\frac{1}{3} = 0 \end{cases}$$

$$x^2 - \frac{2}{3} - 2x + \frac{1}{3} = 0$$

$$3x^2 - 6x - 1 = 0$$

$$x = \frac{6 \pm \sqrt{36+12}}{6} = \frac{6 \pm 4\sqrt{3}}{6} = 1 \pm \frac{2}{3}\sqrt{3}$$

$$\fallingdotseq -0.1547 \text{ or } 2.1547$$

if $x = -0.1547$, 代入①

$\Rightarrow -0.1547 + y + \frac{1}{3} = 0$, $y \doteq -0.17863$

$f(x,y,z) = (-0.1547 - 0.17863 + \frac{1}{3})^2 = 1.111e^{-11}$

if $x = 2.1547$

$\Rightarrow 2.1547 + y + \frac{1}{3} = 0$, $y \doteq -2.488$

$f(x,y,z) = (2.1547 - 2.488 + \frac{1}{3})^2 = 1.111e^{-9}$

$\Rightarrow$ 最小值 $f(x,y,z) = 1.111e^{-11}$

b) Find the minimum value of the function $f(x,y,z) = xy + z^2$, subject to the constraint $x^2 + y^2 + z^2 - 1 = 0$.

$f(x,y,z) = xy + z^2$

$g(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$

$F(x,y,z,\lambda) = xy + z^2 + \lambda(x^2 + y^2 + z^2 - 1)$

$F_x = y + 2x\lambda = 0 \Rightarrow y = -2x\lambda$

$F_y = x + 2y\lambda = 0 \Rightarrow x = -2y\lambda$

$F_z = 2z + 2z\lambda = 0 \Rightarrow 2z(1+\lambda) = 0 \begin{cases} z = 0 \\ \lambda = -1 \end{cases}$

$F_\lambda = x^2 + y^2 + z^2 - 1 = 0$

<case 1> $z = 0$

$\begin{cases} y = -2x\lambda \\ x = -2y\lambda \\ x^2 + y^2 = 1 \end{cases}$ $\Rightarrow$ circle in xy-plane, radius = 1, center at origin.

$f(x,y,0) = xy$

當 x, y 一正一負 $f(x,y,0)$ 有最小值

<case 2> $\lambda = -1$

$\begin{cases} y = 2x \\ x = 2y \\ x^2 + y^2 + z^2 = 1 \end{cases}$ $\Rightarrow$ sphere with radius 1, center at origin

$f(x,y,z) = xy + z^2$

$x^2 + y^2 + z^2 \le 1 \Rightarrow z^2 \le 1$, z^2 is minimized when $z = 0$,

and xy is minimized as discussed in case 1.

the minimium value of $f(x,y,z) = xy + z^2$ subject to the constraint $x^2 + y^2 + z^2 - 1 = 0$ occurs at the point on the circle, where xy is minimized $(xy = -1)$.

$\Rightarrow$ minimum value of $f(x,y,z) = xy + z^2$ occurs at

the point $(x,y,z) = (-1,1,0)$ and $(x,y,z) = (1,-1,0)$,

$f(x,y,z) = f(-1,1,0) = -1$