MATHEMATICAL AND NUMERICAL METHODS IN ENGINEERING: COURSE PROJECT

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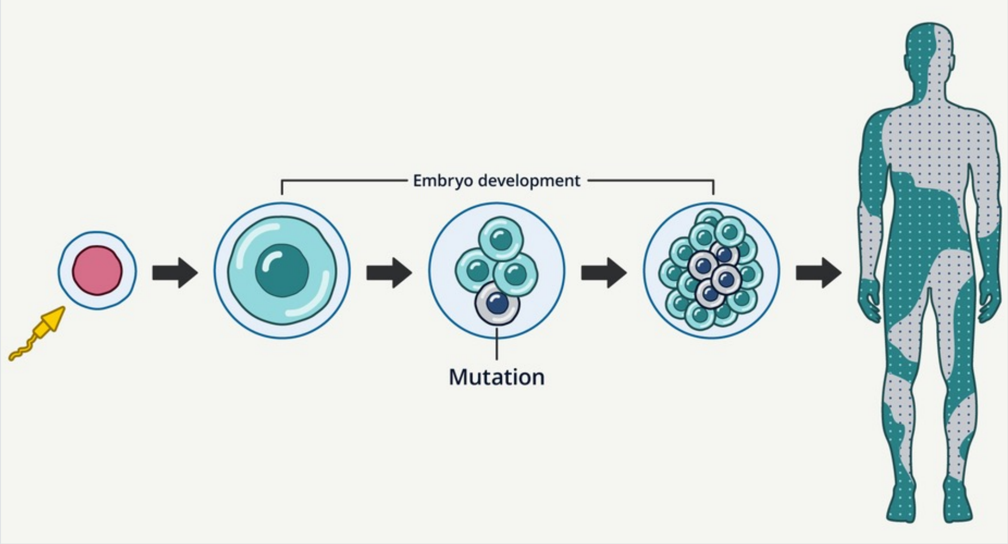
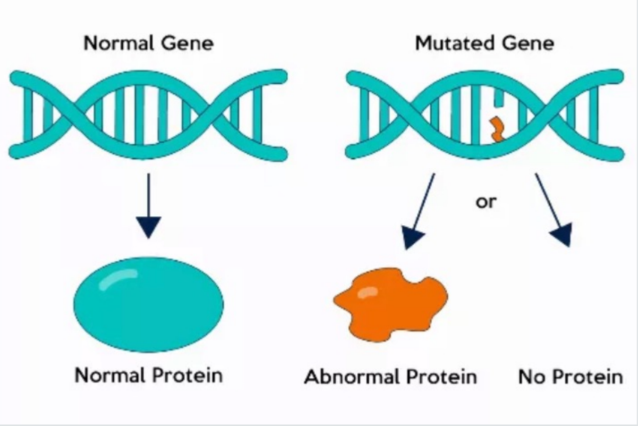
1ST ACTIVITY: GENE MUTATIONS

Problem: spreading of a gene → **GENETIC MUTATIONS**

- Pathogenic
- Probably pathogenic
- Uncertain significance
- Probably benign
- Benign



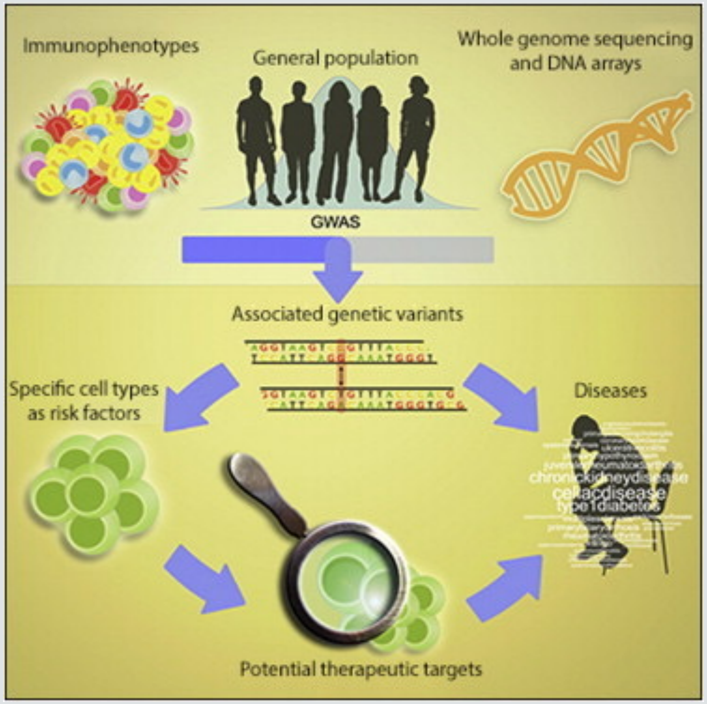


1ST ACTIVITY: GENE MUTATIONS

GENETIC MUTATIONS

- DIRECTLY cause diseases → Mendelian, genetic diseases
- increase RISK of DEVELOPING diseases → non-congenital diseases
- DECREASE risk of developing diseases or MITIGATE their severity → protective genetic variants

IMPORTANT: studying the spread of these genes into a population



FISHER MODEL

- linear habitat → $\Omega_0 := (-2, 2)$ → enlarged domain $\Omega := (-4, 4)$
- mutant gene appears → spreading
- $u(x, t)$: frequency of the mutant gene at a given location $x \in \Omega_0$ at the specified time $t \geq 0$
- PDE model:

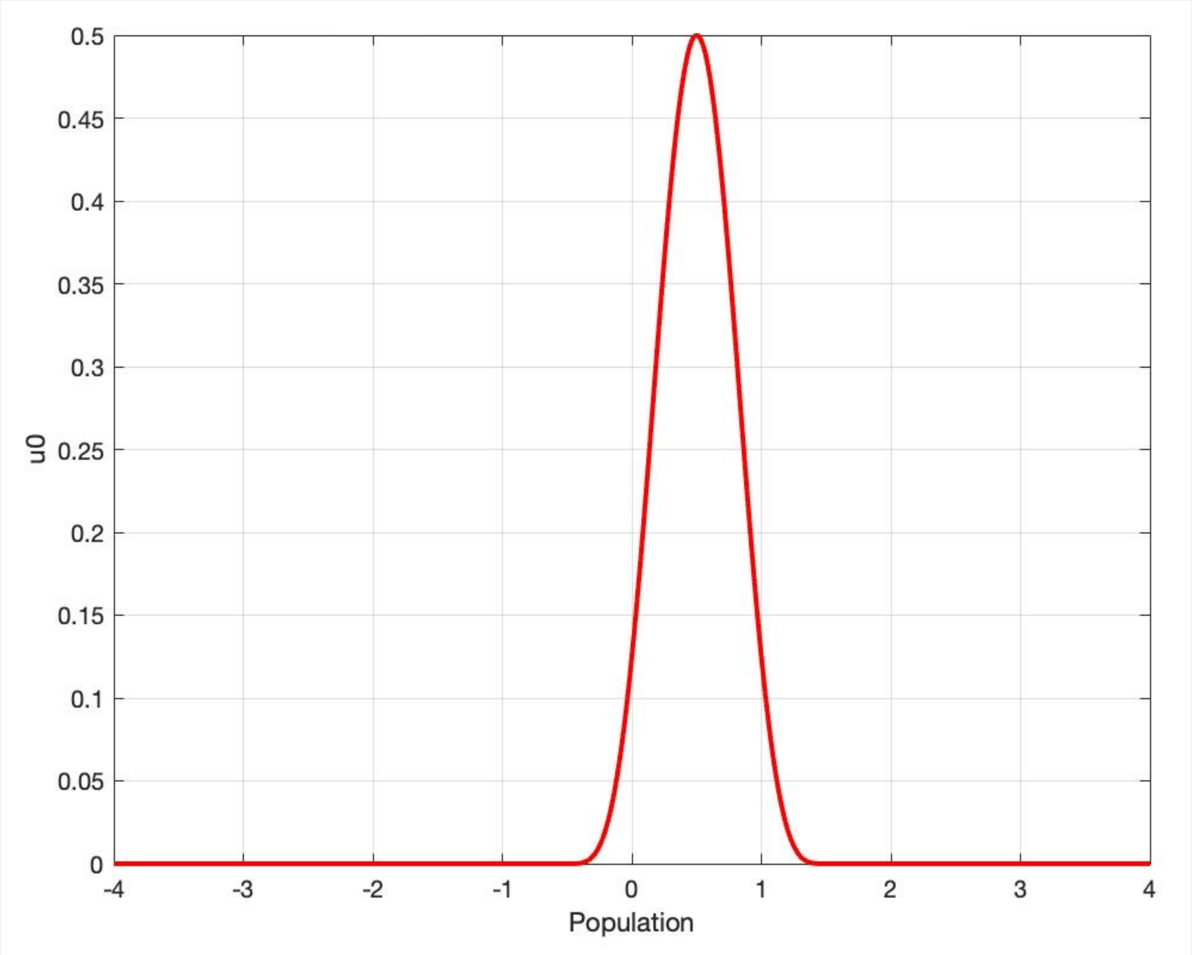
$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = \lambda u(1-u) & x \in (-4,4),\ t \in (0,T] \\ u(\pm 4, t) = 0 & t \in (0,T] \\ u(x,0) = u_0(x) & x \in (-4,4) \end{cases}$$

- $\lambda > 0$: intensity of selection of the mutant gene or index of the mutation survival advantage
- initial profile:

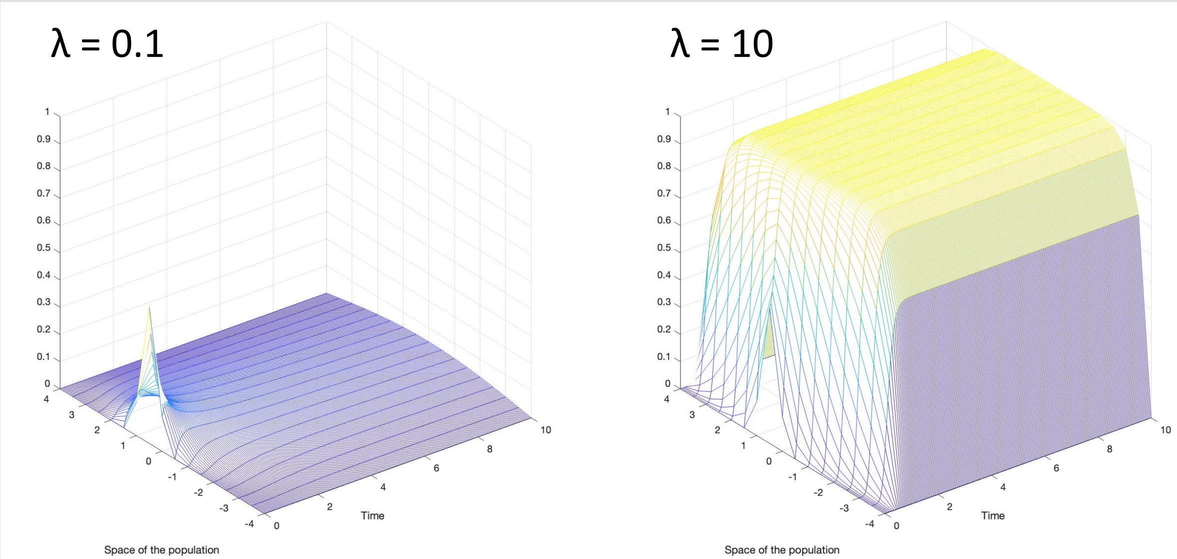
$$u_0(x) = \frac{1}{2}\cos^4\left(\pi\left(\frac{x}{2} + \frac{3}{4}\right)\right), \text{ if } -0.5 \leq x \leq 1.5$$

RESULTS: INITIAL PROFILE

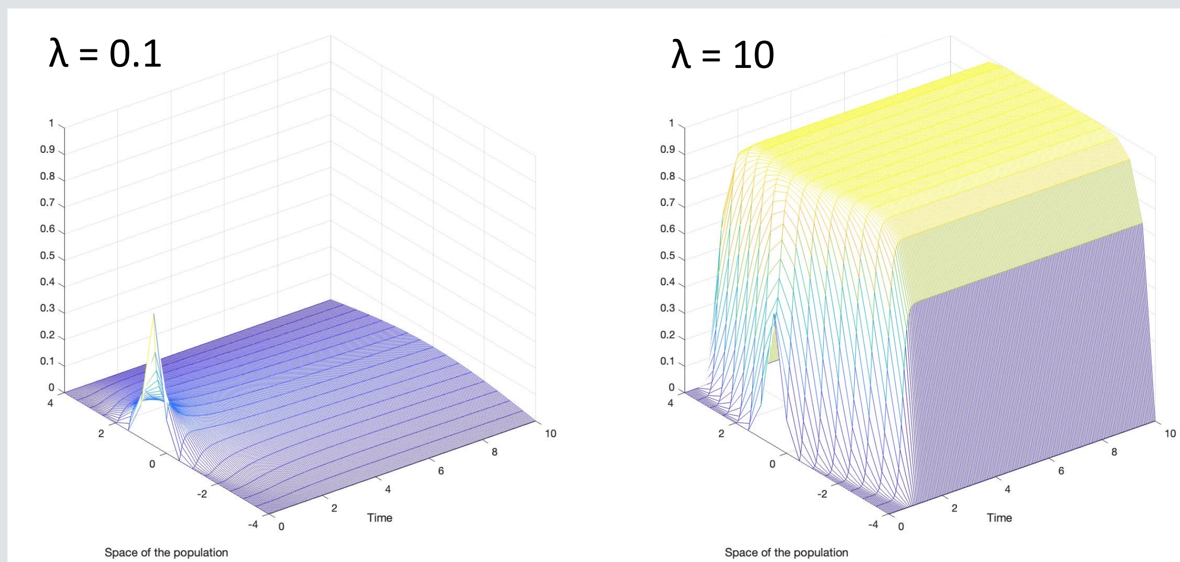
$$u_0(x) = \frac{1}{2}\cos^4\left(\pi\left(\frac{x}{2}+\frac{3}{4}\right)\right),\ \text{if } -0.5 \le x \le 1.5$$



RESULTS: u(x, t)



numerical approximation:
BEC with Δt = 0.05,
h = 0.5

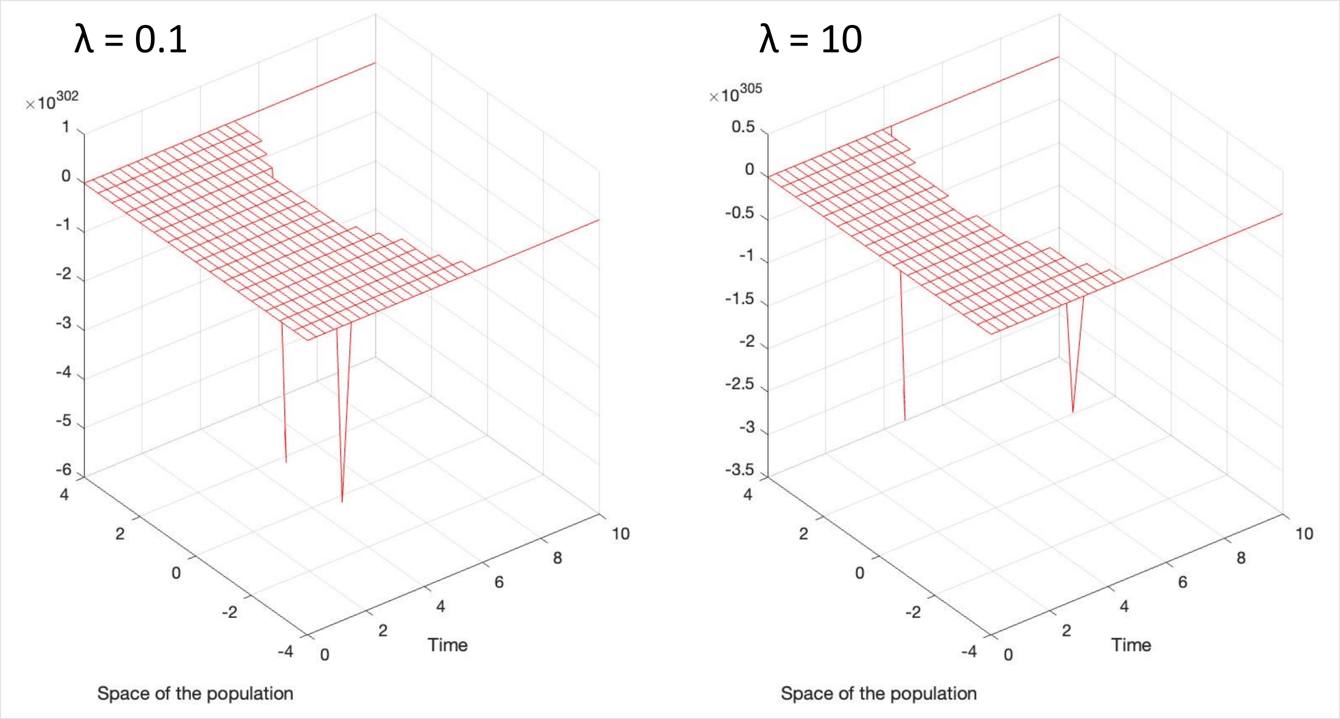


numerical approximation:
FEC with Δt = 0.05,
h = 0.5

RESULTS: $u(x, t)$

numerical approximation:

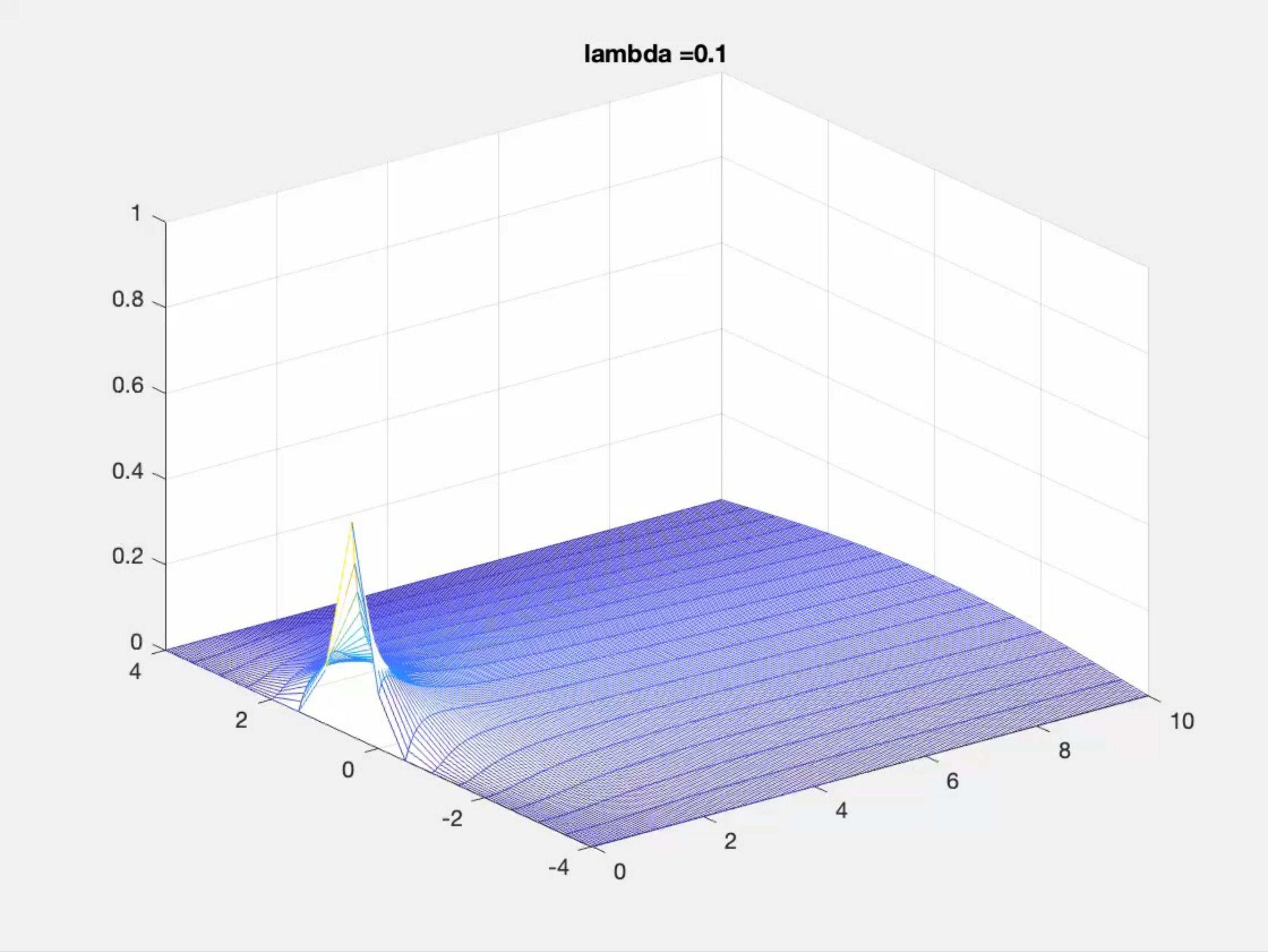
FEC with $\Delta t = 0.3$



CONDITIONALLY STABLE method: $\Delta t \leq \dfrac{h^2}{2}$

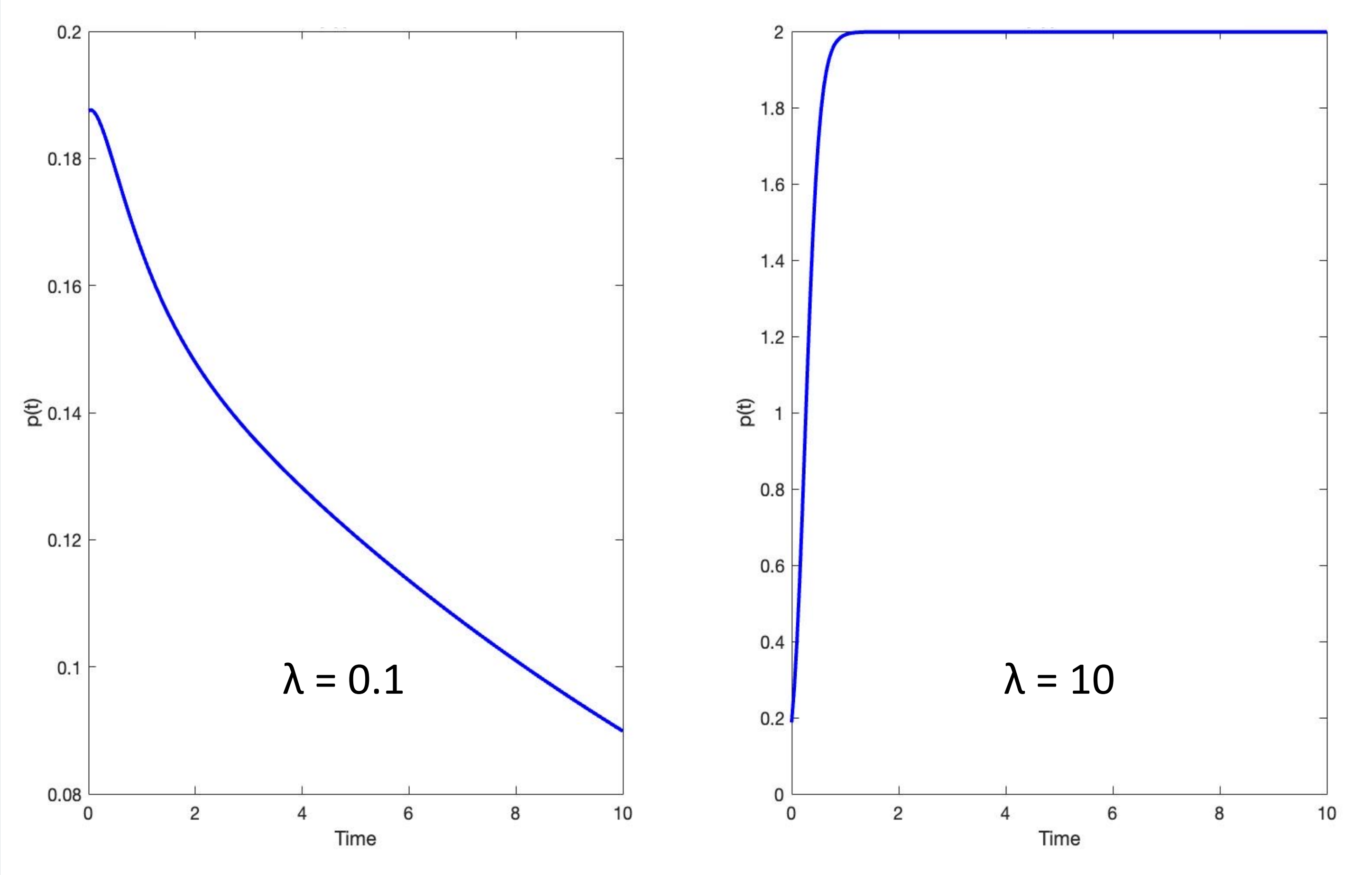
RESULTS: THE MEANING OF λ

lambda_vect = [0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10];



RESULTS: p(t)

$$\boxed{p(t) := \frac{1}{4}\int_{-2}^{2} u(x,t)dx,}$$ $\longrightarrow$ fraction of individuals in (−2, 2) that have the mutant gene

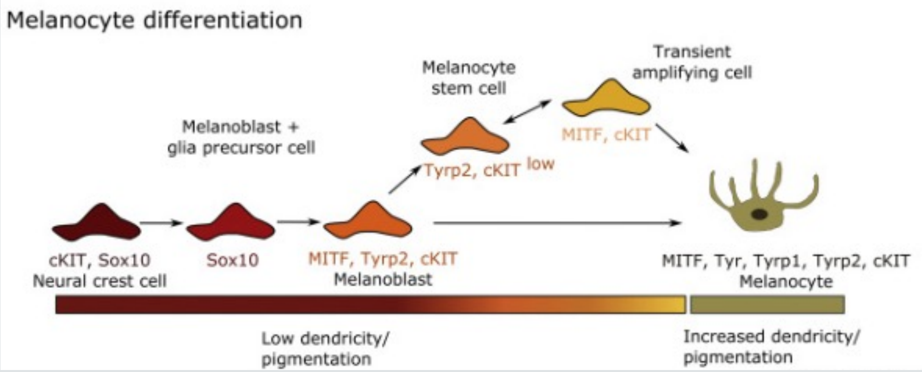


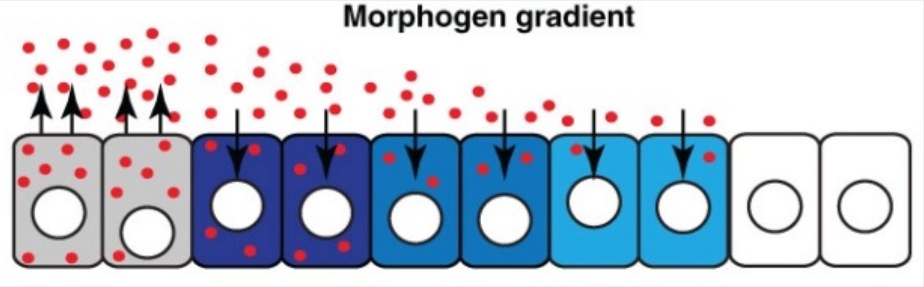
2^{ND} ACTIVITY: PATTERNS

Problem: spreading of some chemical components = STABLE concentration distributions, but NON-HOMOGENEOUS in space

PATTERNS, COLORATION of COATS

Key role: MELANIN and MORPHOGENES





TURING MODEL

- For every $t \in (0,T]$, find $u(x,y,t)$, $v(x,y,t)$ such that:

$$\begin{cases} u_t - \Delta u = f(u,v) & (x,y,t) \in (0,L)\times(0,L)\times(0,T] \\ v_t - d\Delta v = g(u,v) & (x,y,t) \in (0,L)\times(0,L)\times(0,T] \\ u(x,y,0) = u_0(x,y) & (x,y) \in (0,L)\times(0,L) \\ v(x,y,0) = v_0(x,y) & (x,y) \in (0,L)\times(0,L) \\ \text{+periodic B.C.} & \text{on } \partial\Omega_T \times (0,T] \end{cases}$$

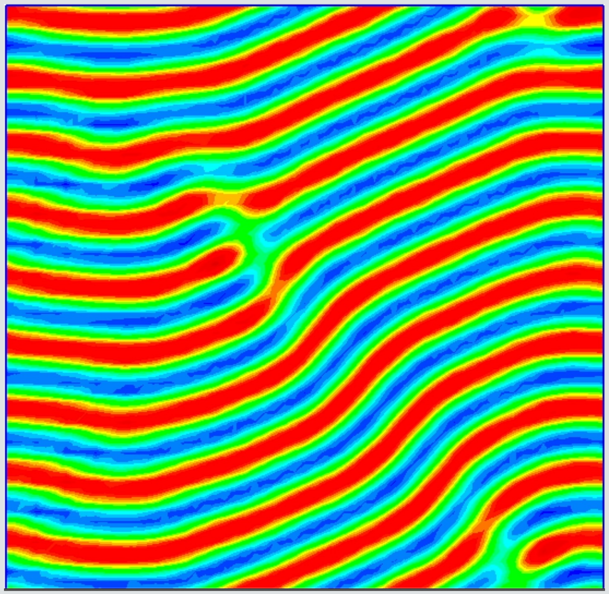
- where:

$$\boxed{f(u,v) = u - \alpha v + \gamma u v - u^3, \quad g(u,v) = u - \beta v}$$

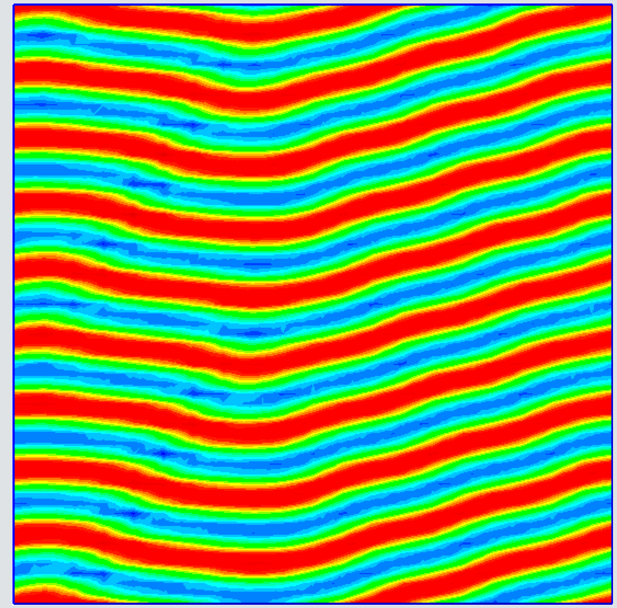
- $\beta = 5$
- $u_0(x, y)$, $v_0(x, y)$ are randomly initialized functions $\forall (x, y) \in \Omega_T$

RESULTS: DIFFERENT PATTERNS

$\alpha = 7.45, \gamma = 1.0$



$\alpha = 7.45, \gamma = 0.0$



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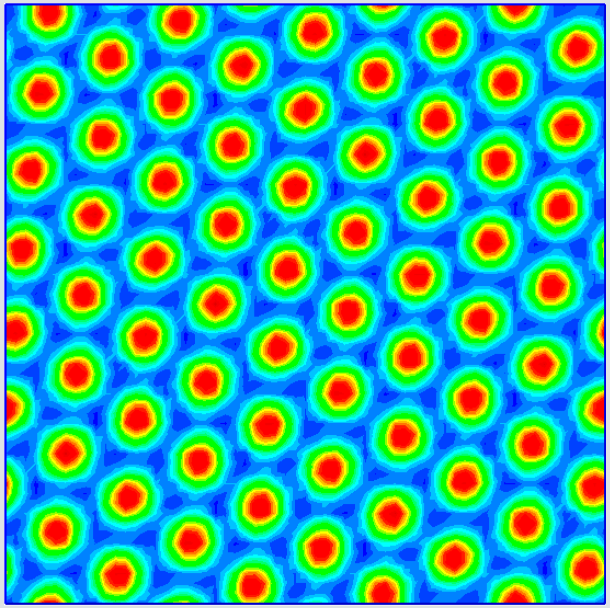

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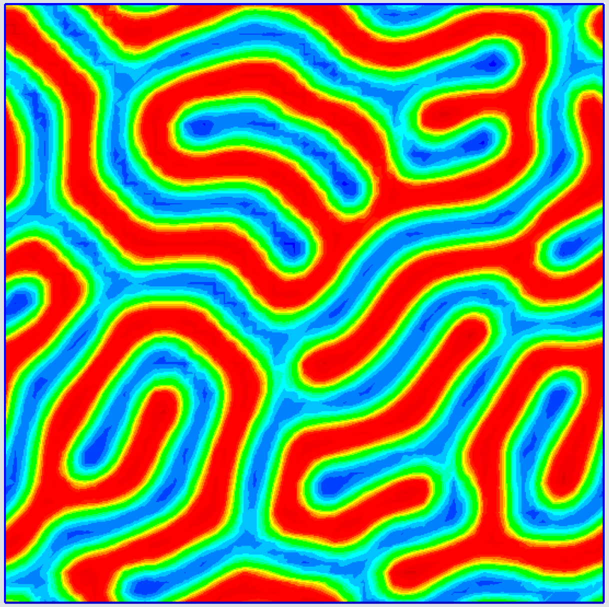

THANK YOU FOR YOUR TIME!

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