Machine learning Assignment2

Chiara Saporetti

3 November 2020

Abstract This assignment aims to implement in MATLAB three different linear regression models and to also calculate their mean square error objective. Results are tested on two different datasets and on different percentages of the data.

1 Introduction

1.1 Linear regression problem

Linear regression is a machine learning tool whose purpose is to model the relationship between a dependent variable t (target) and a set of one or more dependent variables X (data). To do so, it looks for a linear model y(x) that predicts t given X:

$$y = wX \tag{1}$$

Where the w vector represents the weights that must be calculated. In matrix form:

$$\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix} \simeq \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 1 & ... & x_1 d \\ x_2 1 & ... & x_2 d \\ \vdots & \ddots & \vdots \\ x_N 1 & ... & x_N d \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \tag{2}$$

Since the problem above would have no closed form solution, we look for a solution based on optimization: we try to minimize an objective function J which is the expectation of a chosen loss function (the mean square error function). So we calculate the mean square error (loss function):

$$\lambda_S E = (t - y)^2 \tag{3}$$

We then calculate the function that minimizes the mean value of the loss over the whole dataset (objective function):

$$J = \frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2 \tag{4}$$

And we look for the minimum of this function, which is a quadratic function. [1]

1.2 The datasets

The datasets are:
- Motor Trend Car Road Tests set, made of 32 observations and 4 variables.
- Turkish stock exchange dataset, made of 536 observations and 2 variables.

2 Task 1 and 2

The main loads the two datasets through two different functions. For the turkish dataset it uses the MATLAB load() function, while for the car dataset it uses the MATLAB readtable() function, since it also contains string data together with numeric data.
Then it calls three functions to solve the linear regression problem with different models and plots the results.
Firstly it calls oneDimLinReg() to compute the slope for the linear model y of the first dataset.
Secondly it does the same but on different subsets created by randomly picking 10% of the original dataset.
Thirdly it calls oneDimLinReg_intercept() to compute the slope and intercept of the model of the second dataset by considering two of its variables only.
Lastly it normalizes the values of the Car data set, calls multiDimLinReg() on the entire second dataset and shows a table of the de-normalized results.

2.1 oneDimLinReg.m

Input:

- x: data
- t: target

Output:

- w: weights

This function computes the weights for the 1D linear model (i.e. the slope of y = w1 x). In formula:

$$w = \frac{\sum_{i=1}^{N}(x_i t_i)^2}{\sum_{i=1}^{N}(x_i)^2} \tag{5}$$

2.2 oneDimLinReg_intercept.m

Input:

- x: data

- t: target

Output:

- w1: slope
- w0: intercept

his function computes the weights for the 1D linear model (i.e. the slope and intercept of y = w1 x + w0). In formula:

$$w_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(t_i - \bar{t})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{6}$$

$$w_0 = (t_i - w1\bar{x}) \tag{7}$$

2.3 multiDimLinReg.m

Input:

- X: data matrix
- t: target vector

Output:

- W: weights vector

This function computes the weights vector for the multi dimensional linear regression problem. In formula:

$$w = (X^T X)^{-1} X^T t \tag{8}$$

3 Task 3

The main divides the datasets in two parts: 5% is used to build a training set, while 95% is used to build a test set. In this particular case, the training set of the Turkish set is made of 27 lines, while the one of the car set is made of 2 lines only. The code then enters a loop and re-computes the 1-D problem and multi-D problems using the training sets. It subsequently tests the model on the test sets and computes the objective functions J for all cases. Finally it averages the results of all the J and shows them.

3.1 meanSquareError.m

Input:

- x: data
- t: target
- w1: slope
- w0: intercept
- problem: type of linear regression problem. can be 1 for 1-D, 2 for multi-D.

Output:

- objective: objective function

This function computes the mean square error objective of a linear regression problem. In particular, based on the *problem* variable it computes the error for either the one or the multi dimensional problem.
The 1D problem's objective is calculated as:

$$J = \frac{1}{N}\sum_{i=1}^{N}(t_i - y_i)^2 \tag{9}$$

While the multi dimensional problem's one is calculated through the MATLAB function immse().

4 Results

Hereby are shown the results that were obtained.
Figure 1 shows the solution to the linear regression problem applied to the entire Turkish dataset.
Figure 2 shows the different results obtained by applying the same problem to subsets created by randomly picking 10% of the original dataset. The red line is the model computed on the entire dataset, while the green lines are the ones computed on the subsets. In the second case the results are less accurate then the first case, since the number of observations is lower (54 observations each).
Figure 3 shows the solution to the linear regression problem applied to the car dataset considering just the mpg as data and the weight as target.
Figure 4 shows the t and y of the multi-D problem applied to the car set. Results are denormalized.

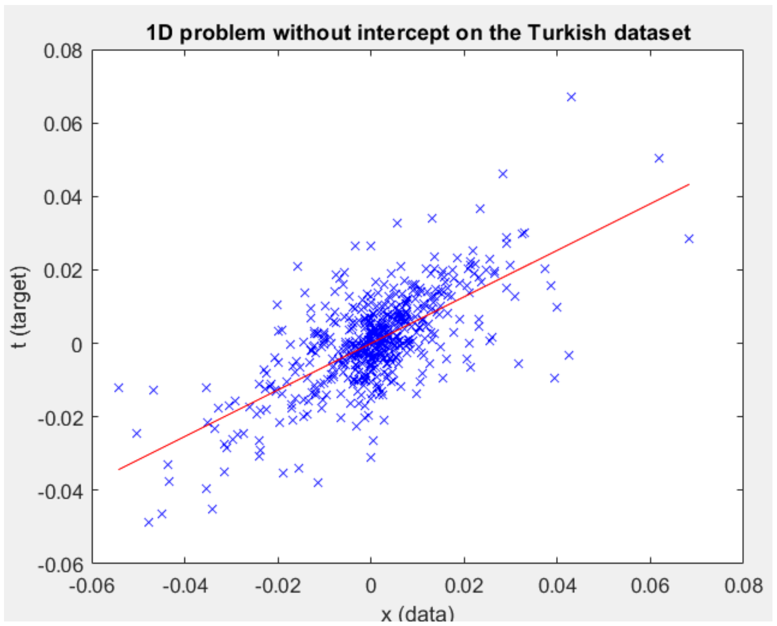


Figure 1: 1D problem without intercept

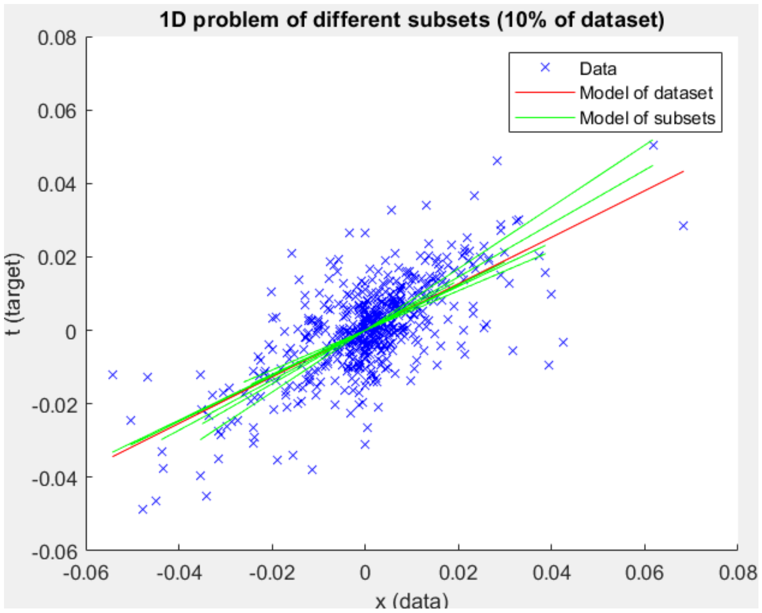


Figure 2: 1D problem without intercept on different subsets

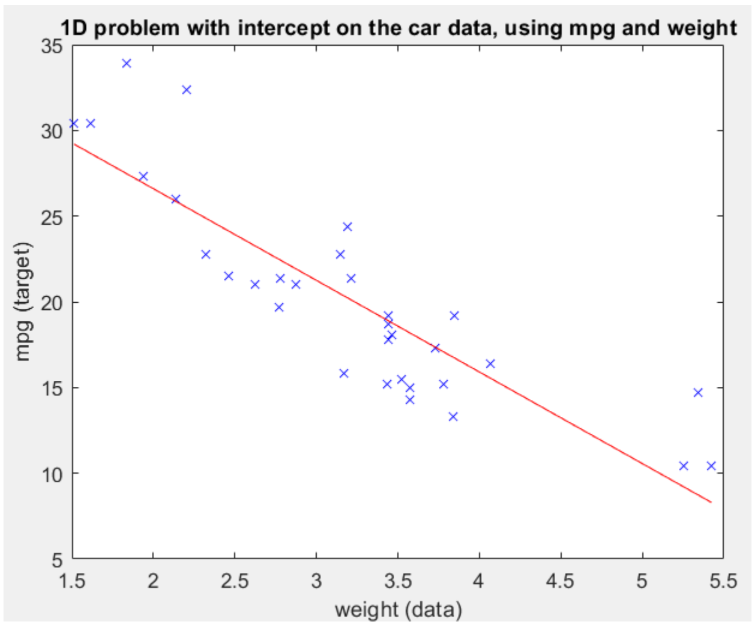
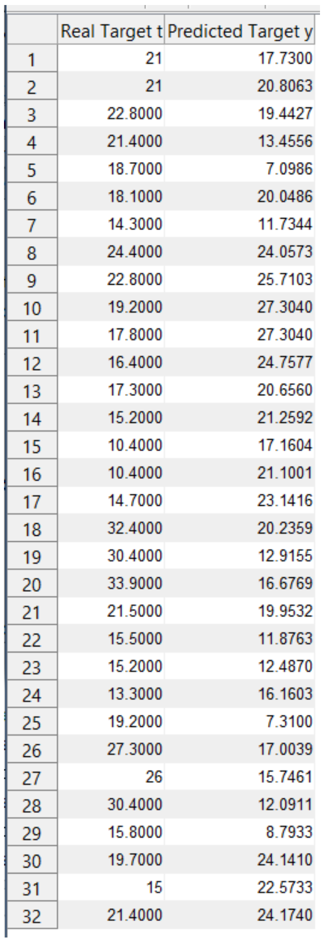


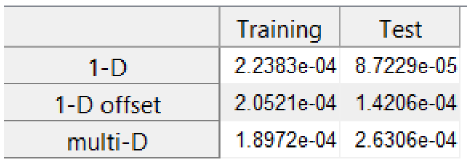
Figure 3: 1D problem with intercept

Figure 5 shows the results on the training set and on the test set, which have in general the same magnitude. The 1-D model is computed with more observations with respect to the other two, so it should have lower errors. Here, this only happens in the test set. The multi-D model has higher errors since the W is computed through the Moore-Penrose inverse which can bring to numerical errors. To solve this problem it would be useful to make an iterative computation by successive approximations. Finally, results on the training sets are mostly lower then results on the training set, which is reasonable since the model was computed on the training sets.



	Real Target t	Predicted Target y
1	21	17.7300
2	21	20.8063
3	22.8000	19.4427
4	21.4000	13.4556
5	18.7000	7.0986
6	18.1000	20.0486
7	14.3000	11.7344
8	24.4000	24.0573
9	22.8000	25.7103
10	19.2000	27.3040
11	17.8000	27.3040
12	16.4000	24.7577
13	17.3000	20.6560
14	15.2000	21.2592
15	10.4000	17.1604
16	10.4000	21.1001
17	14.7000	23.1416
18	32.4000	20.2359
19	30.4000	12.9155
20	33.9000	16.6769
21	21.5000	19.9532
22	15.5000	11.8763
23	15.2000	12.4870
24	13.3000	16.1603
25	19.2000	7.3100
26	27.3000	17.0039
27	26	15.7461
28	30.4000	12.0911
29	15.8000	8.7933
30	19.7000	24.1410
31	15	22.5733
32	21.4000	24.1740

Figure 4: multi-D problem intercept



	Training	Test
1-D	2.2383e-04	8.7229e-05
1-D offset	2.0521e-04	1.4206e-04
multi-D	1.8972e-04	2.6306e-04

Figure 5: Training-Test results

References

[1] *https://2020.aulaweb.unige.it/pluginfile.php/276344/mod*$_f$*older/content/0/ml* $-$ $2020 - 21 - -03.pdf?forcedownload = 1.$