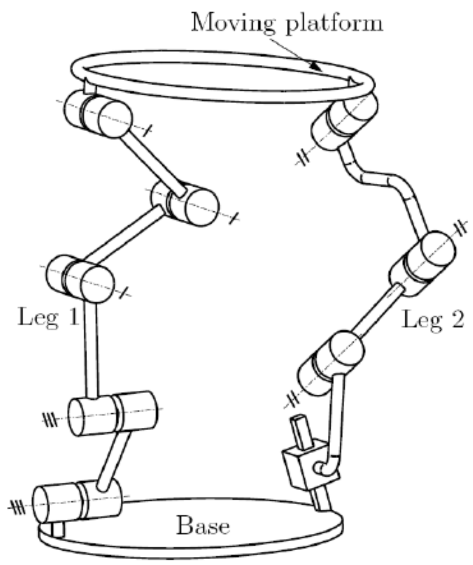
MMM — Assignment 3

Due Date: Wednesday, 27 November 2019, 17:00

For each parallel mechanism, PMi, shown below, do the following:

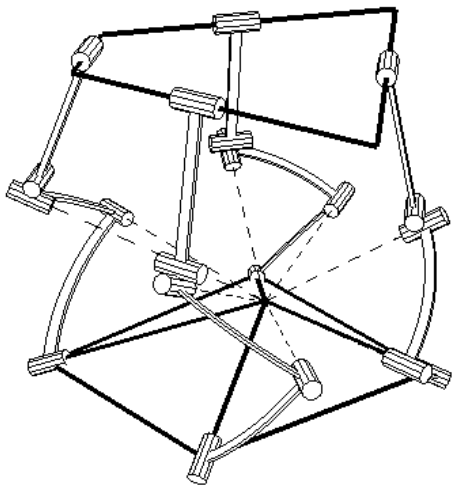
PMi.1 Identify the system of wrench constraints (of the platform) for each leg.
PMi.2 Identify the total wrench constraint system of the platform.
PMi.3 Identify the system of platform freedoms.
PMi.4 Answer additional questions, if any.
PMi.5 Answer additional questions, if any.
PMi.6 …

When describing each wrench or twist system: (a) provide a basis giving the vector components of the basis twist/wrenches in a chosen reference frame; (b) give an adequate geometric description of the system by specifying screws of what pitches underlie the system and what are the locations of their axes (for finite-pitch screws) or their directions (for infinite-pitch screws).



PM1. In The 5R Leg 1, joint axes 1 and 2 are parallel, and so are 4, 5, and 6, while 2 and 3 are skew. In the PRRR Leg 2, the axes of joints 2, 3, and 4 are parallel, and the direction of joint 1 (the slider) is *not* perpendicular to the revolute-joint axes. In the configuration shown: all joint axes in Leg 1 are horizontal, while those in Leg 2 are not.

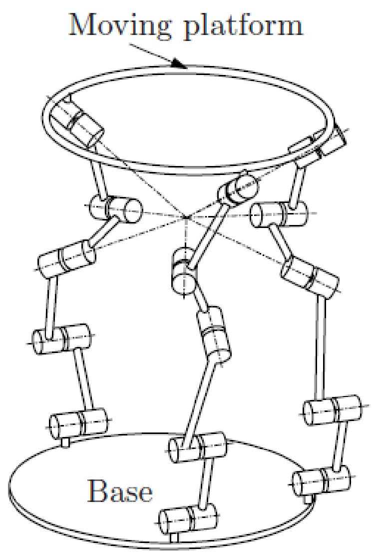
PM1.4. Repeat steps PM1.1-3 for the mechanism obtained by removing the first two joints in leg 1 and the P joint in leg 2. Can these three joints be used to actuate the original mechanism?



PM2. In each 5R leg, joints 1, 2, and 3 intersect in a point in the base, common for the four legs. Joint axes 4 and 5 are parallel to each other and to a plane in the platform common for the four legs.

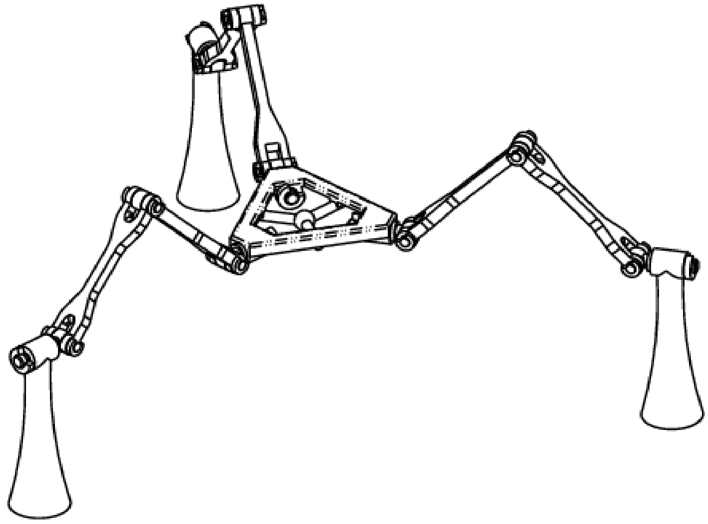
PM2.4. Describe and draw a serial chain with the same full-cycle mobility of the end-effector as the PM.

PM2.5. Repeat steps PM2.1-3 for the mechanism obtained from PM2 by removing (blocking) joint 4 in one of the legs. Would the result change if a joint 5 is removed instead? What if all 4^{th} and 5^{th} joints are removed?



PM3. In each leg: joint axes 1 and 2 are parallel to a common direction fixed in the base. Joint axes 3, 4, 5 intersect in a point in the platform, common for all legs. Assume that the five joint twists in a leg are independent.

PM3.4. Consider one of the legs as a separate serial chain. Can its joint twists become dependent? If yes, describe and illustrate all such leg postures. How would the wrench and twist systems of the PM change if *one* of its legs is in such a singular posture?

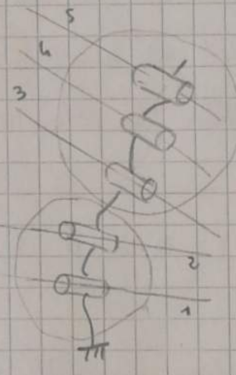


PM4. In each 5R leg, joint axes 2, 3, 4, are parallel (but not coplanar) and perpendicular to both joint axes 1 and 5. The three base-joint axes, as well as the three platform-joint axes, are concurrent and coplanar. In this configuration, joints axes 1 and 5 coincide for each leg and all joint 3 axes are horizontal (i.e., all joint axes are horizontal).

PM4.4. Repeat steps PM1.1-3 for the mechanism obtained from PM1 by removing (i.e. blocking) the first joints in all legs.

CHIARA SAPORETTI
S4798994

PM1. 1



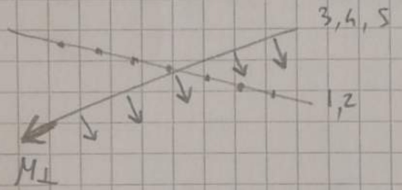
LEG 1: system 1,2 describes all rotations parallel and coplanar to ρ_1 and ρ_2, while system 3, 4, 5 describes a planar motion with rotation parallel to the $\rho_{3,4,5}$ and translation perpendicular to the rotation:

$\dim(T1) = \dim(1,2) + \dim(3,4,5) - \dim(1,2 \cap 3,4,5) =$

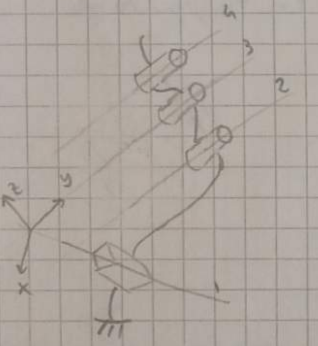
$= 2 + 3 - 0 = 5$

$\Rightarrow \dim(W1) = 1$

$W1 = \text{span}(M_\perp)$, were $M_\perp$ is the only couple perpendicular to all rotations:



LEG 2: system 2,3,4 describes a planar motion.



$\dim(T2) = \dim(1) + \dim(2,3,4) - \dim(1 \cap 2,3,4) =$

$= 1 + 3 - 0 = 4$

$\Rightarrow \dim(W2) = 2$

$W_2 = \text{span}(M_x, M_z)$ where M_x and M_z are the two couples that are perpendicular to the rotations, since I considered them along direction y.

PM1. 2

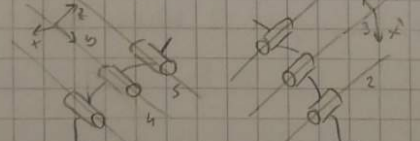
$W_{TOT} = \text{span}(M_x, M_y, M_z) \Rightarrow \dim(W_{TOT}) = 3 \Rightarrow \dim(T_{TOT}) = 3$

PM1. 3

$T_{TOT} = \text{span}(\tau_x, \tau_y, \tau_z)$

PM1. 4

By removing the joints 1,2 from leg 1 and the joint 1 from leg 2 we obtain the mechanism in the drawing.



LEG 1 LEG 2

If I consider leg 1:

$\dim(W1) = 3$

$W1 = \text{span}(M_x, M_z, f_y)$

Leg 2 has same W_i, it just has different directions of the elements: $W2 = \text{span}(M_{x'}, M_{z'}, f_{y'})$

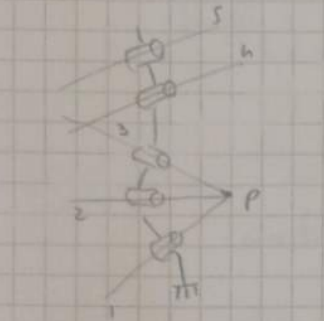
$W_{TOT} = \text{span}(M_x, M_y, M_z, f_y, f_{y'})$ since $M_x, M_y, M_{x'}, M_{y'}$ are four l.d. couples that can be described with M_x, M_y, M_z.

$\dim(W_{TOT}) = 5 \Rightarrow \dim(T_{TOT}) = 1$

So I obtain:

$T_{TOT} = \text{span}(\tau_\perp)$ where $\tau_\perp$ is the translation along a direction perpendicular to the plane described by $\varphi_y, \varphi_{y'}$. Since we have this translation it means that the mechanism moves even if we remove the three joints. This means we can't use them to actuate it.

PM2.1

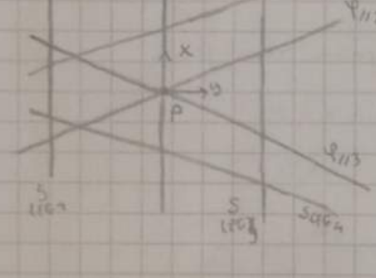


$\dim(T_i) = \dim(1,2,3) + \dim(4,5) - \dim(1,2,3 \cap 4,5) = 3 + 2 - 0 = 5 \quad \forall i = 1,2,3,4$

$\Rightarrow \dim(W_i) = 1$

$W_i = \text{span}(\varphi_{//})$ where $\varphi_{//}$ is the force which has direction parallel to 4,5 and passes through P.

PM2.2



$W_{TOT} = \text{span}(\varphi_x, \varphi_y)$

in fact: $\varphi_{//1}, \varphi_{//2}, \varphi_{//3}, \varphi_{//4}$ all belong to the same plane, and can therefore be substituted with $\varphi_x, \varphi_y \in$ plane

$\varphi_{//i} = \alpha_i \varphi_x + \beta_i \varphi_y$

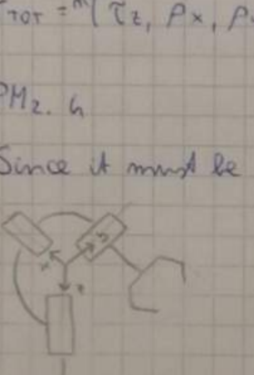
PM2.3

$\dim(T) = 4$

$T_{TOT} = \text{span}(\tau_z, P_x, P_y, P_z)$

PM2.4

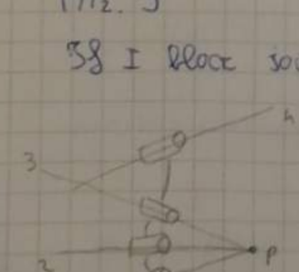
Since it must be $T = \text{span}(\tau_z, P_x, P_y, P_z)$, the serial chain that has that T could be:



which represents a spherical joint in series with a prismatic one

PM2.5

If I block joint 4 in leg 1:



$\dim(T_1) = 4 \Rightarrow \dim(W_1) = 2$

$W_1 = \text{span}(\varphi_{//}, \varphi_P)$ where $\varphi_{//}$ is the same as before, while φ_P is a force passing through P and intersecting axis of joint 4. Apart from these constraints, the direction of φ_P is free.

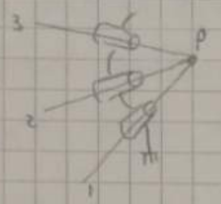
CHIARA SAPORETTI
S4798994

$W_{TOT} \overset{span}{=} (\varphi_x, \varphi_y, \varphi_z)$

$T_{TOT} \overset{span}{=} (\rho_x, \rho_y, \rho_z)$

If I had removed joint 5 instead of 4 nothing would have changed in terms of platform freedom, since the analisys would have been the same.

If all 4^{TH} and 5^{TH} joints were removed: $\dim(W_i) = 3$



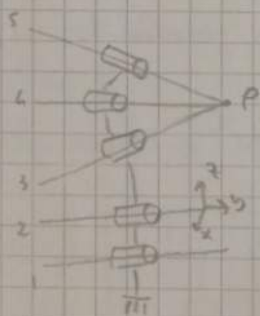
$W_i \overset{span}{=} (\varphi_x, \varphi_y, \varphi_z) \quad \forall i = 1,2,3,4$

where $\varphi_{x,y,z}$ all pass through P

$W_{TOT} \overset{span}{=} (\varphi_x, \varphi_y, \varphi_z)$

$T_{TOT} \overset{span}{=} (\rho_x, \rho_y, \rho_z)$ the mechanism would lose the capability to translate along the z direction, like in the case of removing just one 4^{TH} or 5^{TH} joint.

PM3. 1



$W_i \overset{span}{=} (\varphi_{//}) \overset{span}{=} (\varphi_y) \quad \forall i = 1,2,3$

where $\varphi_{//}$ is a force with direction parallel to 1,2 and that passes through point P

PM3. 2

$W_{TOT} \overset{span}{=} (\varphi_y) \Rightarrow \dim(W_{TOT}) = 1 \Rightarrow \dim(T_{TOT}) = 5$

the force is the same for all the legs since all joints 1 are parallel and all joints 3,4,5 intersect in P.

PM3. 3

$T_{TOT} \overset{span}{=} (\tau_x, \tau_z, \rho_x, \rho_y, \rho_z)$

PM3. 4

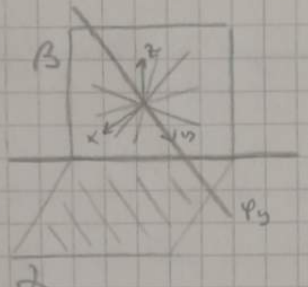
To be in a singular configuration the leg could be, for example, with joints 1,2 on a plane (as before) and 3,4,5 on another. In this case the whole system would be spanned by $\rho_1, \rho_2, \rho_3, \rho_4$, as ρ_5 can be written as a linear combination of ρ_3 and ρ_4: $\rho_5 = \rho_3 \alpha + \rho_4 \beta$.

$\dim(T_i) = 4 \Rightarrow \dim(W_i) = 2$

As in the previous case it can't be constrained by a couple, so I look for two forces.

Note: I could have calculated $\dim(W_i) = 2$ also by noticing that in this case we have $\dim(W_i) = \dim(1,2) + \dim(3,4,5) - \dim(1,2 \cap 3,4,5) = 3 + 2 - 1 = 4$

The problem can be represented by plane α and plane β intersecting. On plane A there are parallel rotations, on plane β rotations all intersecting in one point.



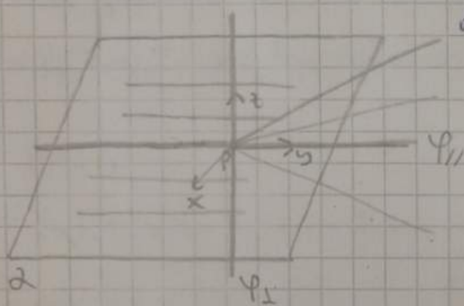
$w_i = \text{span}(\varphi_y, \varphi_{INT})$

$W_{ROT} = \text{span}(\varphi_y, \varphi_{INT})$

$T_{ROT} = \text{span}(\tau_\perp, \rho_x, \rho_y, \rho_z)$

where $\tau_\perp$ is the translation perpendicular to both φ_y and φ_{INT}: direction of common normal.

Another singular configuration is given when $P \in$ plane α:



also in this case we have:

$\dim(w_i) = \dim(1,2) + \dim(3,4,5) - \dim(1,2 \cap 3,4,5) =$

$= 2 + 3 - 1 = 4$

$w_i = \text{span}(\varphi_{\parallel}, \varphi_\perp) = \text{span}(\varphi_x, \varphi_y)$

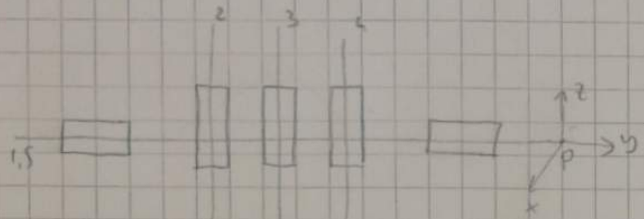
where $w_\perp$ passes through P and is parallel to 1,2, while $\varphi_{\parallel}$ is the same as before.

$W_{ROT} = \text{span}(\varphi_x, \varphi_y)$

$T_{ROT} = \text{span}(\tau_z, \rho_x, \rho_y, \rho_z)$

CHIARA SAPORETTI
S4798994

PM4.1



P is the point in which all axes 1,5 intersect

$\dim(T_i) = 3+1-0 = 4 \Rightarrow \dim(W_i) = 2 \quad \forall i = 1,2,3$

$W_i = \mathrm{span}(\mu_x, \varphi_z)$

Where μ_x is the couple reciprocal to all the rotations, and φ_z is a force with direction parallel to 2,3,4 and passing through P. In general it must intersect axis 1,5, but it is convenient to choose this particular position so that it will be the same for all legs

PM4.2

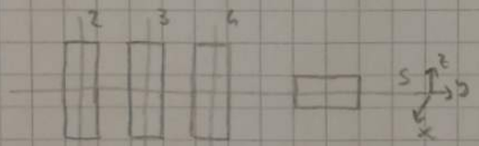
$W_{TOT} = \mathrm{span}(\mu_x, \varphi_x, \varphi_y)$

PM4.3

$T_{TOT} = \mathrm{span}(\rho_y, \rho_z, \tau_z)$

PM4.4

By removing all joints 1 I obtain:



$\dim(T_i) = 3+1-0 = 4 \Rightarrow \dim(W_i) = 2 \quad \forall i = 1,2,3$

$W_i = \mathrm{span}(\mu_x, \varphi_z)$

$W_{TOT} = \mathrm{span}(\mu_x, \varphi_x, \varphi_y)$

$T_{TOT} = \mathrm{span}(\rho_y, \rho_z, \tau_z)$

So the mechanism guarantees the same platform freedoms as before. In fact 1 and 5 were linearly dependent in the previous mechanism.