MMM Course— Assignment 5

Due date and time: Friday, 20 December2019, 8:00AM

Question 1. Points A and B are constrained to move in the plane on lines intersecting at angle θ, Figure 1. Describe precisely the fixed and moving centrodes realizing the planar motion of body AB starting from a configuration like the one showed in the figure. Provide a rigorous mathematical proof that the instantaneous center traces the curves described by you.

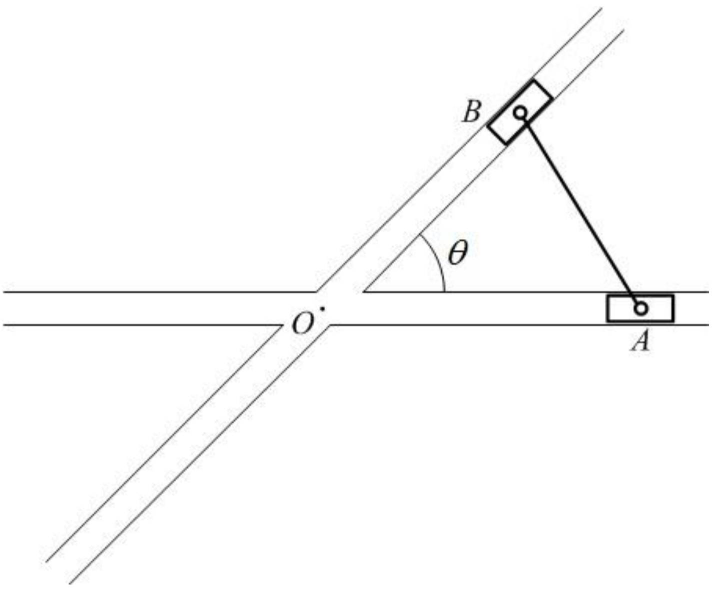


Figure 1.

Question 2. Points A and B are constrained to move in the plane on circles with centers O_A and O_B with $|O_AA| = |O_BB|$, $|O_AO_B| = |AB|$, and $|O_AA| > |AB|$, Figure 2. Describe precisely the fixed and moving centrodes realizing the planar motion of body AB starting from a configuration like the one showed in the figure. Provide a rigorous mathematical proof that the instantaneous center traces the curves described by you. Plot the centrodes for the link-length parameters using a table of value given below.

Question 3. In the conditions of Question 2, let points O_A and A be fixed, while points O_B and B move on circles. Describe precisely the fixed and moving centrodes realizing the planar motion of the body O_BB starting from a configuration like the one showed in the figure. Provide a rigorous mathematical proof that the instantaneous center traces the curves described by you. Make a drawing (or a computer simulation) illustrating and explaining the rolling of the centrodes. Plot the centrodes for the link-length parameters using a table of value given below.

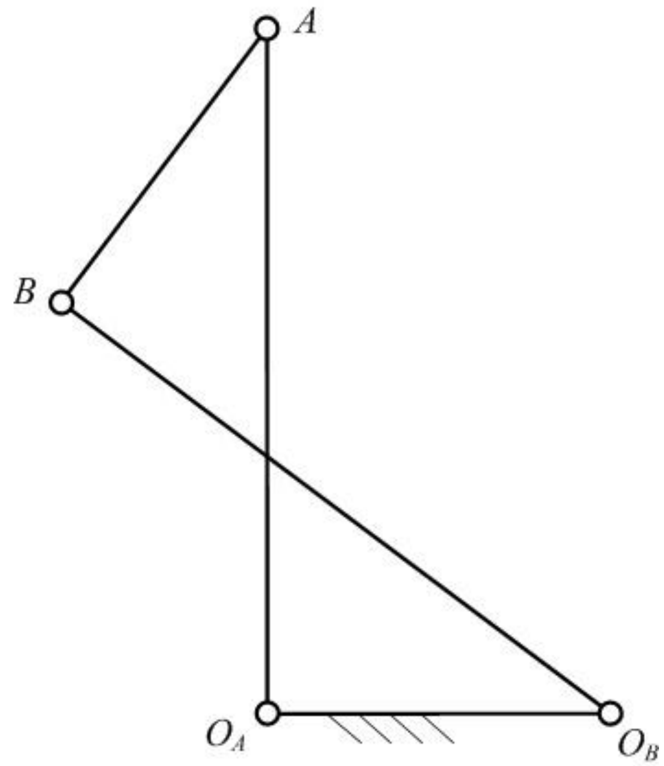


Figure 2.

Question 4 (bonus). For each of Questions 1, 2 and 3: Does the rolling of the identified centrodes yield *all* possible configurations of the moving body allowed by the linkage? Prove your answer.

Hint. In all cases, the centrodes are relatively simple, familiar curves. The proofs required in Questions 1-3 can be made using simple planar geometry.

Note. You can get up to 100% of the mark for this homework by answering the first there questions, and up to 110% by answering all four.
Simulations can be performed in Matlab/Mathematica/Maple or CAD.
For **Question 2 and 3**,

$$|AB| = l_a$$

The value of l_a is given by first digit of your student ID/Matricola.

$$|OA_A| = l_b$$

$$k = 0.1 + (0.012)n$$

$$l_b = 1/k$$

Value of *n* is given below:

Nome	Cognome	n
Akshith	Sirigiri	1
Aliya	Arystanbek	2
Andrea	Pitto	3
Andrea	Tiranti	4
Aurora	Bertino	5
Azay	Karimli	6
Chetan Chand	Chilakapati	7
Chiara	Terrile	8
chiara	saporetti	9
Cristina	Naso Rappis	10
Daniel	Nieto Rodriguez	11
Daulet	Babakhan	12
Davide	Piccinini	13
Ege Doruk	Sayin	14
Elena	Merlo	15
Emanuele-riccardo	rosi	16
Filip	Hesse	17
Francesco	Porta	18
Francesco	Testa	19
Gabriele	Reverberi	20
Gerald	Xhaferaj	21
Geraldo	Margjini	22
Giulia	Scorza Azzarà	23
isabella-sole	bisio	24
JIHAD	KHALIL	25
Josep	Rueda I Collell	26
Justin Lawrence	Lee	27
Kamali	Babu	28
Laiba	Zahid	29
Latif	Xeka	30
Luca	Tarasi	31
Luca	Covizzi	32
manoj	kunapalli	33
Marco	Demutti	34
Matteo	Dicenzi	35
Matteo	Palmas	36
Mohamed Emad Lotfy Fahmy	Qaoud	37
Mohan Krishna	Dasari	38
Muhammad	Tahir	39
Muhammad Raza	Rizvi	40

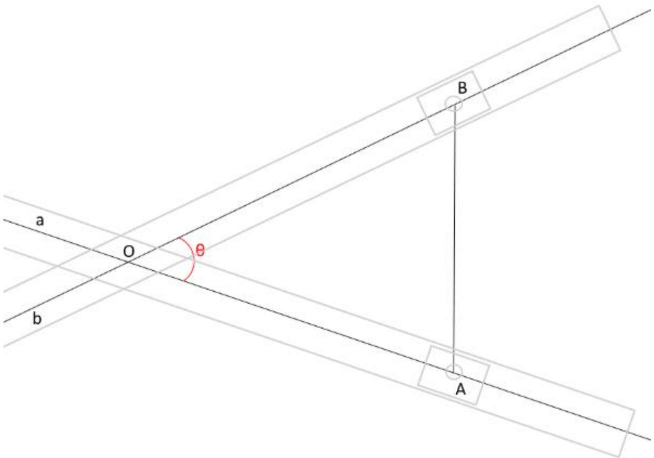
Muhammad Sayum	Ahmed	41
Muhammad Talha	Siddiqui	42
MUHAMMAD USMAN	ASHRAF	43
Paul Toussaint	Ndjomo Ngah	44
riccardo	lastrico	45
roberta	alessi	46
Roberto	Albanese	47
Roberto	Canale	48
saivinay	manda	49
Sandeep	Soleti	50
Sara	Romano	51
Sathish kumar	subramani	52
Sebastiano	Viarengo	53
Serena	Roncagliolo	54
Silvana Andrea	Civiletto	55
Simone	Voto	56
soundarya	pallanti	57
Srikanth Gopichand	Koppula	58
srinivasan	anbarasan	59
STANLEY	MUGISHA	60
Steven	Palma Morera	61
Surishoba surendra	Reddy polaka	62
Syed Hani Hussain	Kazmi	63
Vincenzo	Di Pentima	64
Vishruth		65
Vivek Vijaykumar	Ingle	66
Zaid	Zaid	67
Yasmin	El Sayed	68
Zere	Gumar	69
muhamed	irfan	70

Chiara Saporetti s4798994

Assignment 5

Exericise 1:

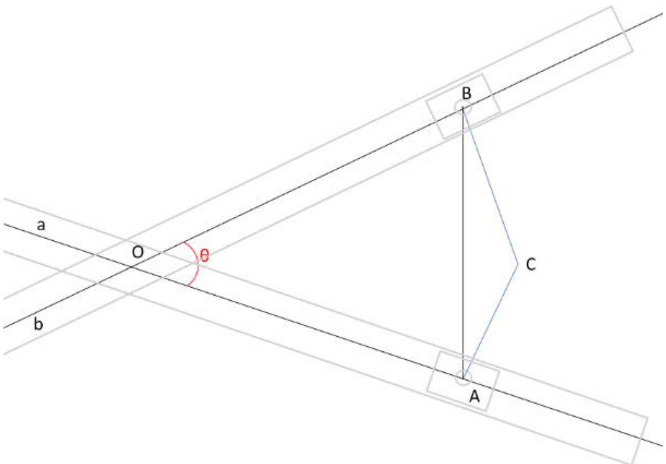
In the figure there are the bar AB and the rails on which it moves. Theta is the angle that the rails form when they intersect. The lines a and b are the ones passing through A and B respectively and parallel to the rails.



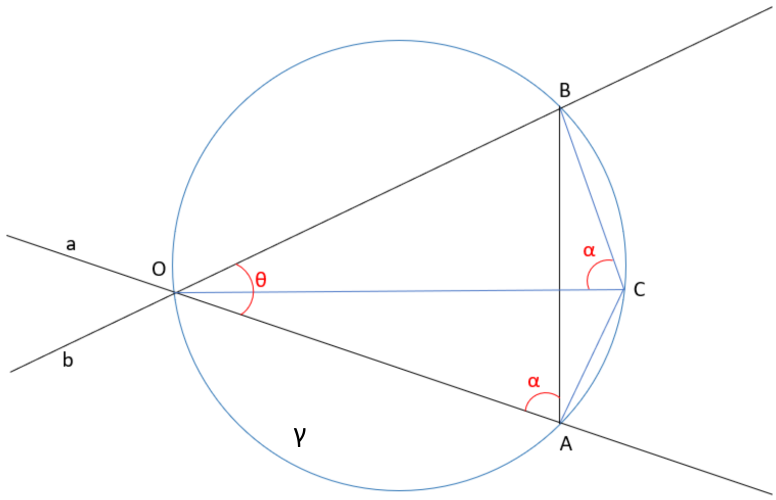
To find the IC:

$$IC \equiv C \begin{cases} \in \textit{line through } B, \textit{perpendicular to } b \\ \in \textit{line through } A, \textit{perpendicular to } a \end{cases}$$

The point is represented in the figure below:



We need to understand how C moves wrt AB. To do so we analyze the relationship between O, A, B,C.

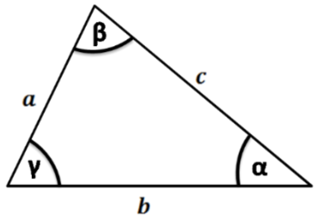


Firstly we consider the congruence between some angles:

$O\hat{C}B \equiv O\hat{A}B = \alpha$ because they insist on the same chord of circumpherence γ.

$O\hat{B}C \equiv O\hat{A}C = \frac{\pi}{2}$ for construction

We can now consider the theorem of sinus:



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

In our case:

From triangle OBC: $\frac{OB}{sin\alpha} = \frac{OC}{\sin\left(\frac{\pi}{2}\right)}$

From triangle OBA: $\frac{OB}{sin\alpha} = \frac{AB}{\sin\theta}$

So: $\frac{OB}{sin\alpha} = \frac{OC}{\sin\left(\frac{\pi}{2}\right)} = \frac{AB}{\sin\theta}$

$$\rightarrow \frac{OC}{1} = \frac{AB}{\sin\theta}$$

Where: AB is constant (it's the length of the bar) and theta is constant too, since it's the physical angle that the rails form when intersecting in O.

⇨ OC=const: the **fixed centrode** is a circle with center in O and radius equal to OC. So it has equation

$$x^2 + y^2 = \frac{AB^2}{\sin^2(\theta)} \begin{cases} if\ \theta = 0 + k\pi \\ if\ \theta = \frac{\pi}{2} + k\pi \end{cases}$$

In the first case we'd have a pure translation (superimposed rails). In the second case we'd have a circle with diameter coincident with the lenght of the bar: OC=AB.

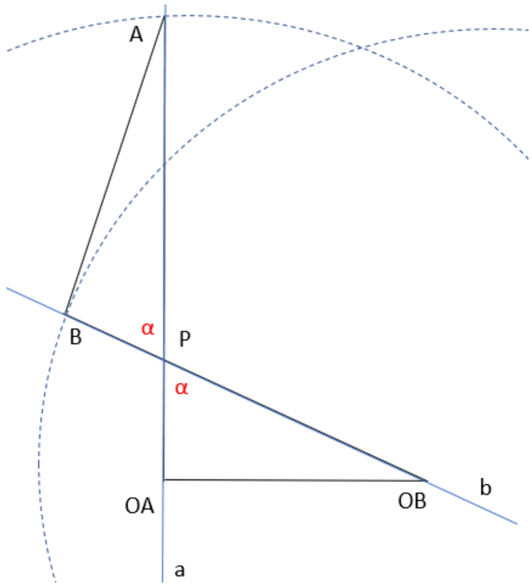
We can also see this circle as a locus of points:

$$F = \{P \in R^2 \;\; s.t. \;\; PO = const\}$$

- ⇨ For any configuration of AB, given a reference frame fixed on AB, C must move as previously described, with OC=const. This means that A, B, C, O always describe a circle with diameter OC and passing through the four points. This circle, which represents the **moving centrode**, is γ.

Exercise 2:

In the figure we can see the antiparallelogram ABO_AO_B, with O_AO_B fixed.



$$|OAA| = |OBB|$$

$$|OAOB| = |AB|$$

$$|OAA| > |AB|$$

We look for the relations between the points like before.

The triangles ABO_B and O_AO_BA are congruent:

$$AB = O_AO_B \text{ for construction}$$

$$O_BB = AO_A \text{ for construction}$$

$$AO_B \text{ in common}$$

⇨ $A\hat{B}P = P\widehat{O_A}O_B$

The triangles APB and O_AO_BP are congruent:

$$A\hat{P}B = O_A\hat{P}O_B = \alpha \text{ because they are opposite angles wrt P}$$

$$AB = O_AO_B \text{ for contruction}$$

$$A\hat{B}P = P\widehat{O_A}O_B \text{ because of the previous demostrated congruence}$$

Since A rotates around O_A and B rotates around O_B, IC must be in the intersection between line b and line a

⇨ $IC \equiv P$

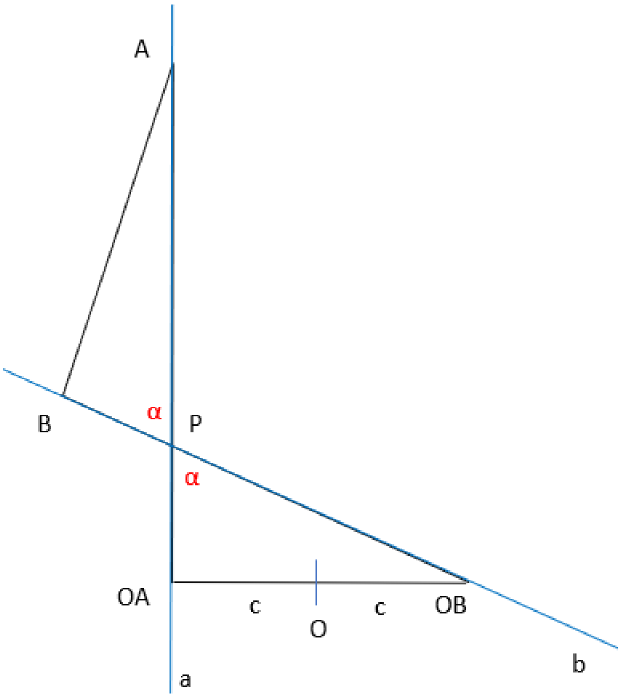
To find the **fixed centrode** we must consider P moving around O_AO_B: to understand how it moves we then need to analyze $O_AP \text{ and } PO_B$, using the relations found before.

$$O_AP + PO_B = BP + PO_B = BO_B = const \rightarrow O_AP + PO_B = const$$

So we need to have a trajectory curve that respects this relation: it is an ellipse with foci O_A *and* O_B and center O, middle point of segment O_AO_B. We can then describe the moving centrode as a locus of points:

$$F = \{P \in R^2 \ \ s.t. \ \ |PO_A| + |PO_B| = 2a\}$$

Demostration of fixed centrode (similar for the moving one):



$$O_AP + PO_A = L$$

Where L is the length of the longest side of the antiparallelogram: BO_B.

If I consider a reference frame in O, I can write the previous equivalence as:

$$\sqrt{y^2 + (x+c)^2} + \sqrt{y^2 + (x-c)^2} = L$$

Where (x,y) are the coordinates of P, while c = $\frac{O_AO_B}{2}$

By elevating to square the equivalence I obtain:

$$\sqrt{y^2 + (x+c)^2} = -\sqrt{y^2 + (x-c)^2} + L$$

$$\left(\sqrt{y^2 + (x+c)^2}\right)^2 = \left(-\sqrt{y^2 + (x-c)^2} + L\right)^2$$

$$y^2 + x^2 + c^2 + 2xc = y^2 + x^2 + c^2 - 2xc + L^2 - 2L\sqrt{y^2 + (x-c)^2}$$

$$4xc = -2L\sqrt{y^2 + (x-c)^2} + L^2$$

$$\frac{L^2 - 4xc}{2L} = \sqrt{y^2 + (x-c)^2}$$

By again squaring:

$$\left(\frac{4x^2c^2}{l^2}+\frac{L^2}{4}-2xc\right)=y^2+x^2+c^2-2xc$$

$$x^2\left(\frac{L^2-4c^2}{L^2}\right)+y^2=\frac{L^2-4c^2}{4}$$

Keeping into account that 2c = a:

$$\frac{x^2}{\frac{L^2}{4}}+\frac{y^2}{\frac{L^2}{4}-\frac{a^2}{4}}=1$$

I obtained the equation of an ellipse with semiaxis:

$$a1=\frac{L^2}{4};\quad b1=\frac{L^2}{4}-\frac{a^2}{4}$$

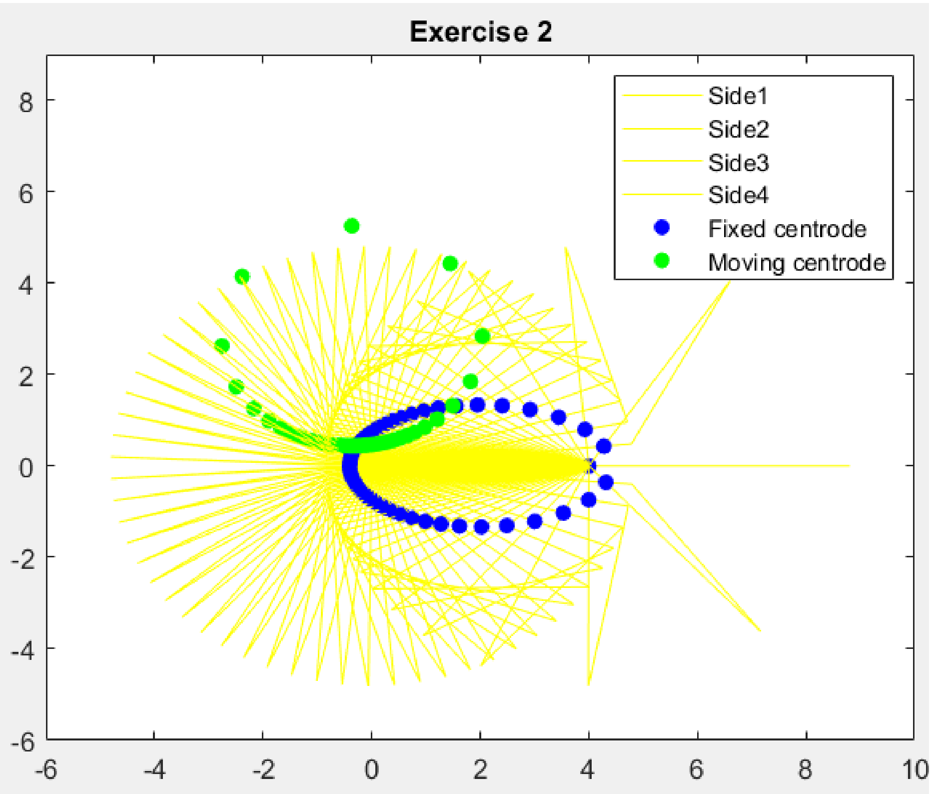
To find the **moving centrode** we fix AB: P must move around it. To understand how it moves we then need to analyze AP, PB, using the relations found before.

$$AP+PB=BP+PO_B=BO_B=const\rightarrow AP+PB=const$$

The curve that respects this relation is an ellipse with foci A, B and center in the middle point of AB. We can then describe the moving centrode as a locus of points:

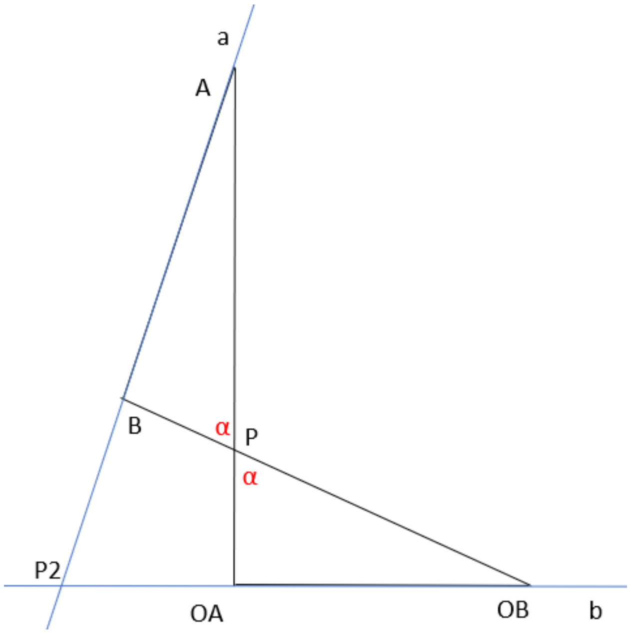
$M=\{P\in R^2\ \ s.t.\ \ |PA|+|PB|=2a\}$ where 2a is the major axis of the ellipse.

With the values given (long side=4.8077, short side=4):



Because of the type of mechanism, we can see that the IC are not equally spaced, but the shape of the curve is clearly an ellipse.

Exercise 3:



We have the same mechanism as before, but in this case O_A and A are fixed, while O_B and B can move. IC is in the intersection between the lines a and b, so it coincides with P2. We analyze the distances P2O_A and P2A.

Triangles AP2O_A and B$O_B P2$ are congruent:

$$AO_A = BO_B \text{for construction}$$

$$B\hat{A}P = P\widehat{O_B}O_A \text{ demonstrated before}$$

$$A\widehat{O_A}P2 = P2\hat{B}O_B \text{ because they are supplementary to equivalent angles } (P\widehat{O_A}O_B = A\hat{B}P)$$

- ⇨ From this equivalence we obtain $P2B = P2O_A$

This last equivalence is useful because:

$$\begin{cases} P2A - P2B = AB \\ P2A - P2B = P2A - P2O_A \; from\ the\ equivalence \end{cases}$$

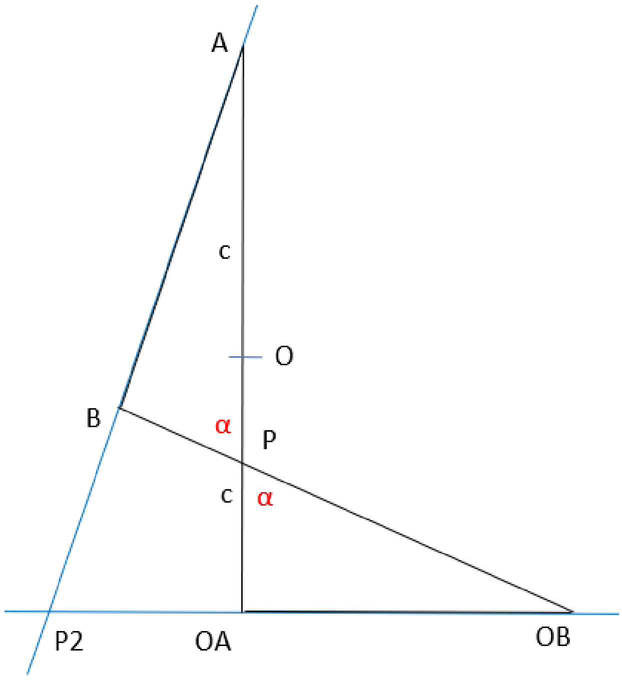
- ⇨ $P2A - P2O_A = const$

The curve that respects this relation is the hyperbola with foci A, O_A. So the **fixed centrode** can be described as a locus of points s.t.:

$$F = \{P \in R^2 \;\; s.t. \;\; |P2A| - |P2O_A| = 2a\,\}$$

Demonstration of fixed centrode (similar for the moving one):

Similarly as before, if we consider a reference frame fixed in O, middle point of O_AA:



$$P2A - P2OA = \sqrt{x^2 + (y-c)^2} - \sqrt{x^2 + (y+c)^2} = l$$

Where, in this case: l is the length of AB, and $c = \frac{O_A A}{2}$

$$\left(\sqrt{y^2 + (x-c)^2}\right)^2 = \left(\sqrt{y^2 + (x+c)^2} + l\right)^2$$

$$y^2 + x^2 + c^2 - 2xc = y^2 + x^2 + c^2 + 2xc + l^2 + 2l\sqrt{y^2 + (x+c)^2}$$

$$\sqrt{y^2 + (x+c)^2} = \frac{-4xc - l^2}{2l}$$

$$y^2 + x^2 + c^2 + 2xc = \frac{4x^2c^2}{l^2} + \frac{l^2}{4} + 2xc$$

$$x^2\left(\frac{4c^2 - l^2}{l^2}\right) - y^2 = \frac{4c^2 - l^2}{4}$$

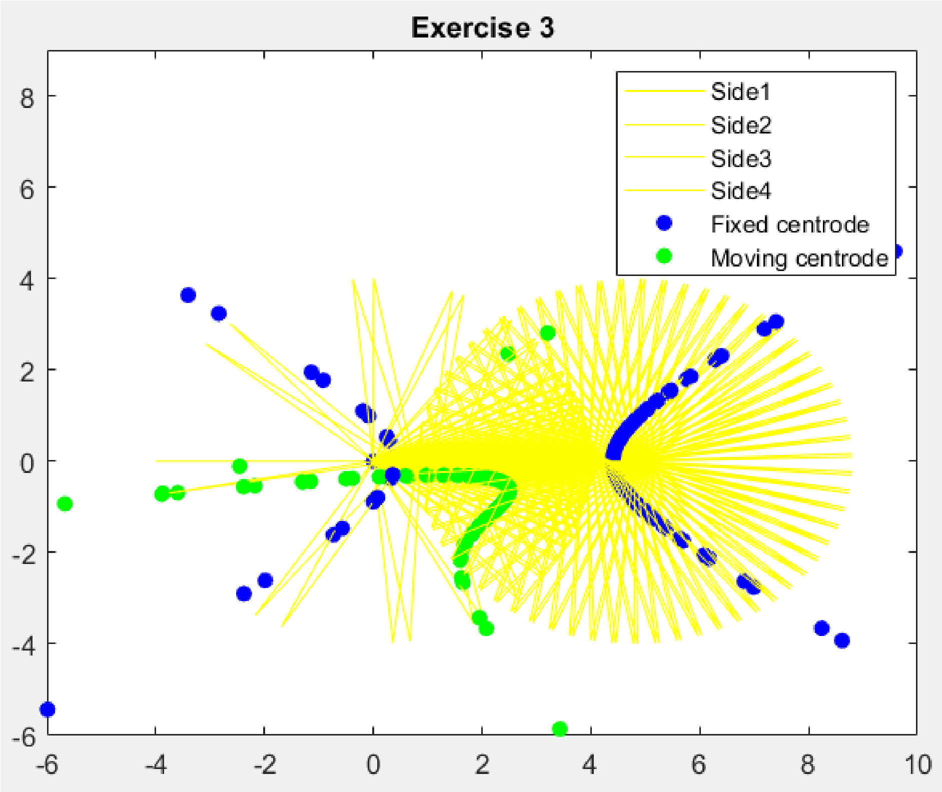
Since $2c=AO_A=L$:

$$\frac{x^2}{\frac{l^2}{4}} - \frac{y^2}{\frac{L^2}{4} - \frac{l^2}{4}} = 1$$

And the **moving centrode** can be described as a locus of points s.t.:

$$M = \{P \in R^2 \;\; s.t. \;\; |P2B| - |P2O_B| = 2a\}$$

With the given values(long side=4.8077, short side=4):

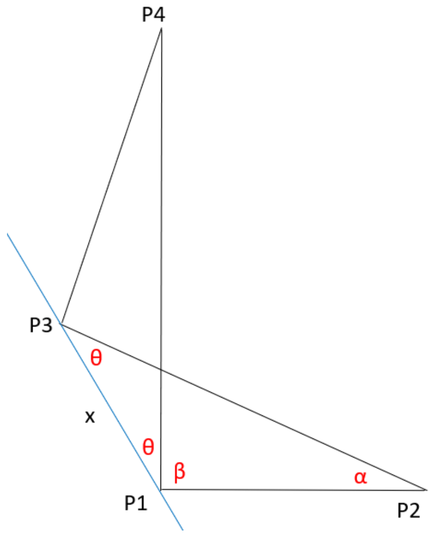


Exercise 4:

As a definition of centrodes, according to the book Theory of Machines and Mechanisms of Shingley, we read: “The plane motion of one rigid body relative to another is completely equivalent to the rolling motion of one centrode on the other. The instantaneous point of rolling contact is the instantaneous center”. This means that the rolling of the fixed and moving centrodes yields all possible configurations of the moving body allowed by the linkage.

Note on the matlab code:

As reference for the construction of the mechanism I created this model, where the long sides are equal to b and the short sides are equal to a:



For the cosine theorem:

$$x = a^2 + b^2 + 2abcos\theta$$

For the sine theorem:

$$\frac{sin\alpha}{x} = \frac{sin\theta}{a}$$

For the sum of the internal angles of a triangle:

$$\beta = \pi - \alpha - 2\theta$$

These are the values used for the angles in the matlab code. The points P1, P2, P3, P4 are calculated with reference to them.