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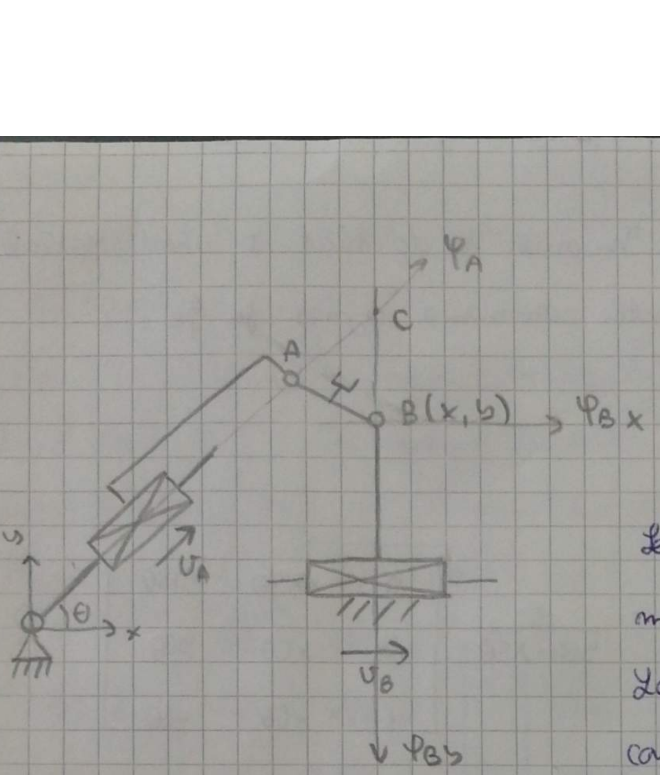


Mechanism with two active joints and three passive joints => we'll have 3 actuated constraints ⇔ 3 I/O equations.

Leg A has 3 joints that allow a planar motion => no structural constraint.

Leg B has 2 joints: 2 dof and the structural constraint $\varphi_{By}$ through B and $\perp v_B$

**1]** System of linear equations relating $\xi$ with $v_A$, $v_B$

To express everything in terms of $v_A, v_B$ (velocities of the actuated joints) I looked for forces reciprocal to the passive joints: $\varphi_A$ through O and A and $\varphi_{Bx}$ through B and parallel to $\vec{v}_B$. I also consider $\varphi_{By}$ as previously defined.

Having $\varphi_A$, $\varphi_{Bx}$, $\varphi_{Bx}$ I can define $\Phi \cdot \xi = \Lambda \dot{q}$ as:

$$\underset{(3\times3)}{\begin{bmatrix} \varphi_A \cdot \\ \varphi_{Bx} \cdot \\ \varphi_{By} \cdot \end{bmatrix}} \underset{(3\times1)}{\xi} = \underset{(3\times3)}{\begin{bmatrix} \varphi_A \tau_A & 0 & 0 \\ 0 & \varphi_B \tau_B & 0 \\ 0 & 0 & f_{By} \end{bmatrix}} \underset{(3\times1)}{\begin{bmatrix} v_A \\ v_B \\ 0 \end{bmatrix}}$$

$\varphi_A$, $\varphi_{Bx}$: actuated constraints

$\varphi_{By}$: structural constraint

Where $\varphi_A = [f_{ax}, f_{ay}, m_{ao}] = [f_a c\theta,\ f_a s\theta,\ 0]$

$\tau_A = [v_{Ax}, v_{Ay}, w] = [c\theta, s\theta, 0]$

$\Rightarrow \varphi_A \cdot \tau_A = [f_a c\theta \;\; f_a s\theta \;\; 0] \begin{bmatrix} c\theta \\ s\theta \\ 0 \end{bmatrix} = f_a$

$\varphi_B = [f_{Bx},\ 0,\ \vec{OB} \wedge f_B \vec{i}]$

$\tau_B = [1, 0, 0]$

$\Rightarrow \varphi_B\ \tau_B = [f_{Bx} \;\; 0 \;\; \vec{OB} \wedge f_B \vec{i}] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = f_{Bx}$

So I can substitute in the previous formula and obtain:

$$\begin{bmatrix} \varphi_{A} \cdot \\ \varphi_{Bx} \cdot \\ \varphi_{By} \cdot \end{bmatrix} \xi = \begin{bmatrix} f_a & 0 & 0 \\ 0 & f_{Bx} & 0 \\ 0 & 0 & f_{By} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ 0 \end{bmatrix}$$

By also substituting $[\phi \cdot]$:

$$\underbrace{\begin{bmatrix} f_a C\theta & f_a S\theta & 0 \\ f_{Bx} & 0 & -y f_{Bx} \\ 0 & f_{By} & x f_{By} \end{bmatrix}}_{\text{in the form } [\, f_x \;\; f_y \;\; m_z \,]} \xi = \begin{bmatrix} f_a & 0 & 0 \\ 0 & f_{Bx} & 0 \\ 0 & 0 & f_{By} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ 0 \end{bmatrix}$$

To obtain $\xi$ we must compute $[\Phi\cdot]^{-1}$. In order to do that I used Matlab:
I defined the variables as symbolic with command "syms $\rho_A$ $\rho_B$ ..."
I defined the matrix $\Phi$
I defined the inverse of $\Phi$
In this way I obtained:

$$[\Phi\cdot]^{-1} = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} \frac{y}{\rho_A} & \frac{-xS\theta}{\rho_{Bx}} & \frac{-yS\theta}{\rho_{By}} \\ \frac{-x}{\rho_A} & \frac{xC\theta}{\rho_{Bx}} & \frac{yC\theta}{\rho_{By}} \\ \frac{1}{\rho_A} & \frac{-C\theta}{\rho_{Bx}} & \frac{-S\theta}{\rho_{By}} \end{bmatrix}$$

By substituting in $\xi = [\Phi\cdot]^{-1}\Lambda\dot{q}$:

$$\xi = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} \frac{y}{\rho_A} & \frac{-xS\theta}{\rho_{Bx}} & \frac{-yS\theta}{\rho_{By}} \\ \frac{-x}{\rho_A} & \frac{xC\theta}{\rho_{Bx}} & \frac{yC\theta}{\rho_{By}} \\ \frac{1}{\rho_A} & \frac{-C\theta}{\rho_{Bx}} & \frac{-S\theta}{\rho_{By}} \end{bmatrix}\begin{bmatrix} \rho_A & 0 & 0 \\ 0 & \rho_{Bx} & 0 \\ 0 & 0 & \rho_{By} \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ 0 \end{bmatrix} =$$

$$= \frac{1}{yC\theta - xS\theta}\begin{bmatrix} y & -xS\theta & -yS\theta \\ -x & xC\theta & yC\theta \\ 1 & -C\theta & -S\theta \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ 0 \end{bmatrix}$$

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2] Space of possible e.e. twists when:

a. all joints are free

$\dim(T_A) = 3 \quad T_A = span(\tau_x, \tau_y, \rho_z) \quad W_A = \emptyset$ (planar movement)

$\dim(T_B) = 2 \quad T_B = span(\tau_x, \rho_z) \quad W_B = span(\varphi_y)$

$W_{rot} = span(\varphi_y) \Rightarrow T = span(\tau_x, \rho_z)$

From the previous formula:

$$\xi = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} yv_A - xv_B S\theta \\ -xv_A + xv_B C\theta \\ v_A - v_B C\theta \end{bmatrix} = \begin{bmatrix} v_{xee} \\ v_{yee} \\ w_{zee} \end{bmatrix} \quad (2\ dof)$$

It is clear that $w_{zee} = -x v_{yee}$, so the two are linearly dependent: we obtain the same result as before: the ee can translate along x and rotate around z

b. slider A is locked

$\dim(T_A) = 2 \quad T_A = span(\rho_{Oz}, \rho_{Az}) \quad W_A = span(\varphi_A)$

B as before

$W_{rot} = span(\varphi_A, \varphi_y) \Rightarrow T = span(\rho_z)$

From the previous formula:

$$\xi = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} -xv_B S\theta \\ +xv_B C\theta \\ -v_B C\theta \end{bmatrix} = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} -xS\theta \\ +xC\theta \\ C\theta \end{bmatrix} v_B = \begin{bmatrix} v_{xee} \\ v_{yee} \\ w_{zee} \end{bmatrix}$$

Everything is function of $v_B \vec{i}$ so I can write:

$$\xi = \begin{bmatrix} 1 \\ -\frac{1}{tg\theta} \\ \frac{1}{x\,tg\theta} \end{bmatrix} v_{ox}$$

This result was obtained by dividing all the variables for $v_{xee}$
(1 dof)

e. slider B is locked

A as before

$dim(\mathcal{T}_B) = 1 \qquad \mathcal{T}_B = span(\rho_z^B) \qquad W_B = span(\varphi_x, \varphi_y)$

$W_{rot} = span(\varphi_x, \varphi_y) \Rightarrow \mathcal{T} = span(\rho_z)$

From the previous formula:

$$\xi = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} yv_A \\ -xv_A \\ v_A \end{bmatrix} = \frac{1}{yC\theta - xS\theta}\begin{bmatrix} y \\ -x \\ 1 \end{bmatrix} v_A = \begin{bmatrix} v_{xee} \\ v_{yee} \\ w_{zee} \end{bmatrix} \quad (1 \text{ dof})$$

3] Let a wrench $\Psi_{ext}$ be the only force acting on the links. Find $(f_A, f_B)$ for which the system is in equilibrium.

To have equilibrium it must be valid that:

$\tau^T \dot{q} = \Psi_{out} \cdot \xi$ (PVW) Where $\Psi_{out} = -\Psi_{ext}$ and $\tau$: input forces/torques

In this specific case we can write:

$$f^T \dot{q} = -\Psi_{ext}\, \xi \rightarrow \overset{(1\times3)\,(3\times1)}{\Psi_{ext}\, \xi} = -\overset{(1\times3)\,(3\times1)}{f^T \dot{q}} \longrightarrow \Psi\left(\Phi^{-1}\Lambda \dot{q}\right) = -f^T \cancel{\dot{q}} \rightarrow -f^T = \overset{(1\times3)\,(3\times3)(3\times3)(3\times3)}{\Psi\, \Phi^{-1}\Lambda}$$

$$\rightarrow \underbrace{[\Psi\cdot]\, \frac{1}{yC\theta - xS\theta} \begin{bmatrix} y & -xS\theta & -yS\theta \\ -x & xC\theta & yC\theta \\ 1 & -C\theta & -S\theta \end{bmatrix}}\ = -f^T$$

where $\Psi\cdot$ is, generically: $[f_x\ f_y\ m_z]$

$$[f_x\ f_y\ m_z]\ \frac{1}{yC\theta - xS\theta} \begin{bmatrix} y & -xS\theta & -yS\theta \\ -x & xC\theta & yC\theta \\ 1 & -C\theta & -S\theta \end{bmatrix}$$

$$= \left[\left(\frac{yf_x - xf_y + m_z}{yC\theta - xS\theta}\right), \left(\frac{-xf_xS\theta + xf_yC\theta - m_zC\theta}{yC\theta - xS\theta}\right), \left(\frac{-yf_xS\theta + yf_yC\theta - m_zS\theta}{yC\theta - xS\theta}\right)\right]$$

For the equivalence to be satisfied we set:

$$-\frac{yf_x - xf_y + m_z}{yC\theta - xS\theta} = f_A$$

$$-\frac{yf_xS\theta + yf_yC\theta - m_zS\theta}{yC\theta - xS\theta} = 0 \rightarrow yf_xS\theta + yf_yC\theta - m_zS\theta = 0$$

$$-\frac{-xf_xS\theta + xf_yC\theta - m_zC\theta}{yC\theta - xS\theta} = f_B$$

If $\Psi$ is a couple with moment // $O_z$ and of 1Nm: $\Psi = [0\ 0\ 1]$

$$\frac{1\vec{k}}{yC\theta - xS\theta} = f_A$$

$$\frac{-C\theta\vec{k}}{yC\theta - xS\theta} = f_B$$

$-S\theta = 0 \rightarrow \theta = k\pi \quad \forall k = 1,2,3.$

The $\theta = k\pi$ is the condition for which we can have $\Psi$ as given and be in equilibrium

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4] If $f_A = 10$, $f_B = 0$, find all wrenches $\psi$ applied to the ee. for the system to be in equilibrium

By considering $f_B = 0$:

$x f_y C\theta - x f_x S\theta - m_z C\theta = 0 \rightarrow m_z = x f_y - x f_x tg\theta$

I substitute $m_z$ in $f_A$' formula:

$y f_x - \cancel{x f_y} + (\cancel{x f_y} - x f_x tg\theta) = -(yC\theta - xS\theta) f_A$

$(y - x tg\theta) f_x = -(yC\theta - xS\theta) f_A$

$$f_x = -\frac{(yC\theta - xS\theta)}{(y - x\, tg\theta)} f_A = -C\theta \frac{(yC\theta - xS\theta)}{(yC\theta - xS\theta)} f_A = -C\theta f_A$$

since $t$ is $\frac{S\theta}{C\theta}$

I substitute $f_x$ in $m_z$:

$m_z = x f_y + x\, tg\theta\, C\theta f_A = x f_y + x f_A S\theta$

$$\rightarrow f_y = \frac{m_z - x f_A S\theta}{x} = \frac{m_z}{x} - f_A S\theta$$

So we can define $\psi$ to be:

$$\psi = \begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} = \begin{bmatrix} -C\theta f_A \\ \frac{m_z}{x} - f_A S\theta \\ m_z \end{bmatrix} = \underbrace{-\begin{bmatrix} C\theta \\ S\theta \\ 0 \end{bmatrix} f_A}_{\psi_P} + \underbrace{\begin{bmatrix} 0 \\ 1/x \\ 1 \end{bmatrix} m_z}_{\psi_0}$$

general equation for $\psi$

$\psi_P = (-f_A C\theta \vec{i}, -f_A S\theta \vec{j}, 0)$ in this case: $(-10 C\theta \vec{i}, -10 S\theta \vec{j}, 0)$

$\psi_0 = (0, 1/x\, m_z \vec{j}, m_z \vec{k})$

So $\psi_P$ is a force in the xy plane

$\psi_0$ is a wrench made by a force along y at distance x from o and a moment along z. But this is the description of $\psi_{Bs}$: as a matter of fact this is a structural contraint: it's the wrench balanced by a null input $t \Leftrightarrow f_A = f_B = 0$, as in this case