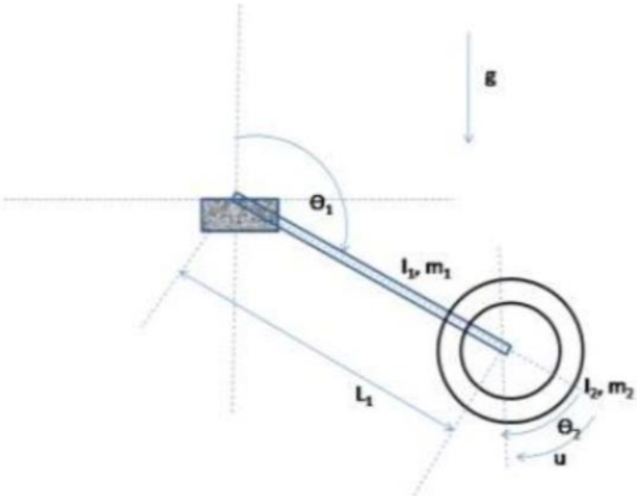
Saporetti Chiara s4798994

Homework 6

Inertia pendulum



This system has 2 degrees of freedom in the vertical plane and 1 input u, the torque.

It can be described by the following equations:

$$\begin{cases} \dot{q}_1 = p_1 \\ \dot{p}_1 = -\dfrac{m_{22}}{|M|} m_0 g \; sinq_1 - \dfrac{m_{12}}{|M|} u \\ \dot{q}_2 = p_2 \\ \dot{p}_2 = \dfrac{m_{21}}{|M|} m_0 g \; sinq_1 + \dfrac{m_{11}}{|M|} u \end{cases}$$

Where g is the constant gravity, m0 is a mass and M is a constant inertia matrix.

State space model

Given the previous equations:

$$\begin{cases} \dot{q}_1 = p_1 \\ \dot{p}_1 = -\dfrac{m_{22}}{|M|} m_0 g \; sinq_1 - \dfrac{m_{12}}{|M|} u \\ \dot{q}_2 = p_2 \\ \dot{p}_2 = \dfrac{m_{21}}{|M|} m_0 g \; sinq_1 + \dfrac{m_{11}}{|M|} u \end{cases}$$

We rewrite the system as:

$$\begin{pmatrix} \dot{q}_1 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ -\dfrac{m_{22}}{|M|} m_0 g \; sinq_1 \\ p_2 \\ \dfrac{m_{21}}{|M|} m_0 f \; sinq_1 \end{pmatrix} + \begin{pmatrix} 0 \\ -\dfrac{m_{12}}{|M|} \\ 0 \\ \dfrac{m_{11}}{|M|} \end{pmatrix} u$$

Which can be simplified in the generic system

$$\dot{x} = f(x)\, x + g(x)\, u$$

So we can write g(x) as:

$$g(x) = \begin{pmatrix} 0 \\ -\frac{m_{12}}{|M|} \\ 0 \\ \frac{m_{11}}{|M|} \end{pmatrix}$$

H2

We can now look for H2. We know that, since it must be $g(x)^{\perp}$, the dimension of H2 must be equal to:

$$\dim(H_2) = 4 - 1 = 3$$

This means that there are 3 bases for H2:

$$H_2 = span\{\omega_1, \omega_2, \omega_3\}$$

In order to find them we can take the definition of orthonormal annihilator and look for the elements that will compose the omegas:

$$(h_1\ h_2\ h_3\ h_4) \begin{pmatrix} 0 \\ -\frac{m_{12}}{|M|} \\ 0 \\ \frac{m_{11}}{|M|} \end{pmatrix} = 0$$

$$0\, h_1 - \frac{m_{12}}{|M|}\, h_2 + 0\, h_3 + \frac{m_{11}}{|M|}\, h_4 = 0$$

$$\frac{m_{12}}{|M|}\, h_2 = \frac{m_{11}}{|M|}\, h_4$$

So:

$$\omega_1 = [1 \quad 0 \quad 0 \quad 0] = dq_1$$

$$\omega_2 = [0 \quad 1 \quad 0 \quad 0] = dq_2$$

$$\omega_3 = [0 \quad m_{11} \quad 0 \quad m_{12}\,] = m_{11} dp_1 + \ m_{12}\, dp_2$$

H3

We want now to understand how H3 will appear:

$$H_3 = span\{\ \dots ? \dots \}$$

$$\dot{\omega}_1 = \frac{d}{dt}(dq_1) = d\dot{q}_1 = dp_1 \notin \ H_2$$

$$\dot{\omega}_2 = \frac{d}{dt}(dq_2) = d\dot{q}_2 = dp_2 \notin H_2$$

$$\dot{\omega}_3 = \frac{d}{dt}(m_{11}dp_1 + m_{12}\,dp_2) =$$

$$= \frac{d}{dt}(m_{11}dp_1) + \frac{d}{dt}(\,m_{12}dp_2) =$$

$$= m_{11}d\dot{p}_1 + m_{12}d\dot{p}_2 =$$

$$= m_{11}d\left(-\frac{m_{22}}{|M|}m_0 g\ sinq_1 - m_{11}u\right) + m_{12}d\left(\frac{m_{21}}{|M|}m_0 g\ sinq_1 + m_{12}u\right) =$$

$$= -m_{11}\frac{m_{22}}{|M|}m_0 g\ dq_1\ cosq_1 + m_{12}\frac{m_{21}}{|M|}m_0 g\ dq_1\ cosq_1 =$$

$$= const \cdot dq_1 \in H_2$$

So H3 will be spanned by this last omega, since its derivative belongs to H2. Then, since

$$\dim(H_3) = 2$$

We need to look for another basis:

$$\dot{\omega} = \alpha_1 dp_1 + \alpha_2 dp_2 + \alpha_3 const \cdot dq_1 = 0$$

By setting $\alpha_1 = 0;\ \alpha_2 = 0;\ \alpha_3 = 1$ we find the first basis, ω_3.

By choosing $\alpha_1 = m_{12};\ \alpha_2 = m_{12};\ \alpha_3 = 0$ we can find the second one, and define:

$$H_3 = span\{\ \omega_3, m_{11}dq_1 + m_{12}dq_2\ \}$$