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Homework 4

In some special coordinates, the kinematic model of the N-trailer system is described by the following system:

$$\dot{x} = \begin{pmatrix} 1 & 0 \\ x_3 & 0 \\ x_4 & 0 \\ \vdots & \vdots \\ x_n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \text{ Where } \dot{x} = \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \\ \vdots \\ \dot{x_n} \end{pmatrix}$$

What is given is also H_2, of $dimH_2 = n - 2$:

$$H_2 = span\{\omega_1, \omega_2, \dots, \omega_{n-2}\} = span\{-x_3dx_1 + dx_2, -x_4dx_1 + dx_3, \dots\}$$

Where the basis 1 and 2 can be written as:

$$\omega_1 = [-x_3 \ 1 \ 0 \ 0 \dots 0] \begin{pmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_n \end{pmatrix}; \qquad \omega_2 = [-x_4 \ 0 \ 1 \ 0 \ 0 \dots 0] \begin{pmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_n \end{pmatrix}$$

Exercise 1:

Compute the *n* - 4 remaining basis vectors of *H*2 in the form of row vectors. Write the corresponding 1-forms ω_3 and ω_{n-2}.

In general, a basis has the form:

$$\omega_i = [-x_{i+2} \dots 0 \dots \ 1_{i+1} \ 0 \ 0 \dots 0] \begin{pmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_n \end{pmatrix} \rightarrow \ in\ one\ form\colon \omega_i = -x_{i+2}dx_1 + dx_{i+1}$$

Where 1_{i+1} means it has a 1 at the (i+1)-th position, preceded and followed by zeros.

One forms of bases 3 and n-2:

$$\omega_3 = -x_5dx_1 + dx_4$$

$$\omega_{n-2} = -x_ndx_1 + dx_{n-1}$$

Exercise 2:

$$\omega_1 = -x_3 dx_1 + dx_2$$

$$\dot{\omega_1} = -\dot{x_3} dx_1 - x_3 d\dot{x}_1 + d\dot{x}_2 =$$

$$= -(x_4 u_1) dx_1 - x_3 d(\dot{x_1}) + d(\dot{x_2}) =$$

$$= -x_4 u_1 dx_1 - x_3 d(u_1) + d(x_3 u_1) =$$

$$= -x_4 u_1 dx_1 - x_3 du_1 + (dx_3 u_1 + x_3 du_1) =$$

$$= -x_4 u_1 dx_1 + dx_3 u_1 =$$

$$= (-x_4 dx_1 + dx_3) u_1 =$$

$$= \omega_2 u_1$$

So $\dot{\omega}_1 \in \ H_2$

Exercise 3:

$$\omega_2 = -x_4 dx_1 + dx_3$$

$$\dot{\omega_2} = -\dot{x_4} dx_1 - x_4 d\dot{x}_1 + d\dot{x_3} =$$

$$= -(x_5 u_1) dx_1 - x_4 d(u_1) + d(x_4 u_1) =$$

$$= -x_5 u_1 dx_1 - x_4 du_1 + (dx_4 u_1 + x_4 du_1) =$$

$$= -x_5 u_1 dx_1 + dx_4 u_1 =$$

$$= (-x_5 dx_1 + dx_4) u_1 =$$

$$= \omega_3 u_1$$

So $\dot{\omega}_2 \in \ H_2$

Exercise 4:

$$\omega_{n-2} = \ -x_n dx_1 + dx_{n-1}$$

$$\dot{\omega}_{n-2} = -\ \dot{x_n} dx_1 - x_n d\dot{x}_1 + d\dot{x}_{n-1} =$$

$$= -(u_2) dx_1 - x_n d(u_1) + d(x_n u_1) =$$

$$= -u_2 dx_1 - x_n du_1 + dx_n u_1 + x_n du_1 =$$

$$= -dx_1 u_2 + dx_n u_1$$

So $\dot{\omega}_{n-2} \notin \ H_2$

Exercise 5:

$$H_3 = span\{\omega_1, \dots, \omega_{n-3}\}$$