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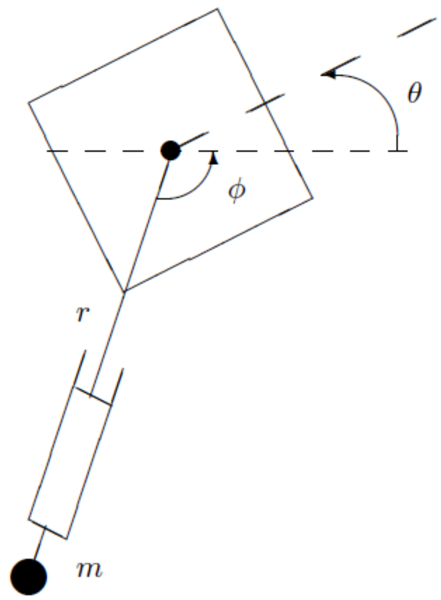
Homework 5

The system

We consider the case of a hopping robot. It has a body and a single leg. The orientation of the main body with reference to the leg is given by the action of a motor that gives a torque u1. The leg has a translation piston that controls it via u2.



If we consider:

- m as the mass of the leg
- J as the inertia momentum of the body
- X1 as the (variable) length of the leg
- X3 as the angular position of the body
- X5 as the angular position of the leg

and if we neglect the action of gravity, then the equations that describe our system in matrix form are:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 x_6^2 \\ x_4 \\ 0 \\ x_6 \\ -2\dfrac{x_2 x_6}{x_1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{1}{m} \\ 0 & 0 \\ \dfrac{1}{J} & 0 \\ 0 & 0 \\ -\dfrac{1}{m x_1^2} & 0 \end{pmatrix}$$

Computation of H2

The previous system can also be simplified in the generic system

$$\dot{x} = f(x)\,x + g(x)\,u$$

So we can write g(x) as:

$$g(x) = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{m} \\ 0 & 0 \\ \frac{1}{J} & 0 \\ 0 & 0 \\ -\frac{1}{mx_1^2} & 0 \end{pmatrix}$$

We can now look for H2. We know that, since it must be $g(x)^{\perp}$, the dimension of H2 must be equal to:

$$\dim(H_2) = n - k = 6 - 2 = 4$$

This means that there are 4 basis for H2:

$$H_2 = span\{\omega_1, \omega_2, \omega_3, \omega_4\}$$

In order to find them we can take the definition of orthonormal annihilator and look for the elements that will compose the omegas:

$$(h_1\ h_2\ h_3\ h_4\ h_5\ h_6)\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{m} \\ 0 & 0 \\ \frac{1}{J} & 0 \\ 0 & 0 \\ -\frac{1}{mx_1^2} & 0 \end{pmatrix} = 0$$

By calculating we obtain:

$$\begin{cases} 0\,h_1 + 0\,h_2 + 0\,h_3 + \frac{1}{J}h_4 + 0\,h_5 - \frac{1}{mx_1^2}\,h_6 = 0 \\ 0\,h_1 + \frac{1}{m}\,h_2 + 0\,h_3 + 0\,h_4 + 0\,h_5 + 0\,h_6 = 0 \end{cases}$$

$$\begin{cases} \frac{1}{J}h_4 - \frac{1}{mx_1^2}\,h_6 = 0 \\ \frac{1}{m}\,h_2 = 0 \end{cases}$$

$h_1,\ h_3,\ h_5$ values don't matter, so we can neglect them for a while.

We have constraints on the other values:

$$h_2 = 0$$

If

$$h_4 = J$$

then

$$h_6 = m\, x_1^2$$

Now that we know a few of the elements with which we will build the omegas, we can define the general form of an exact basis. In particular, ω_i is exact if there exists a function φ belonging to k such that

$$\omega_i = \left[\frac{\partial\varphi}{\partial x_1}\ \frac{\partial\varphi}{\partial x_2}\ \frac{\partial\varphi}{\partial x_3}\ \frac{\partial\varphi}{\partial x_4}\ \frac{\partial\varphi}{\partial x_5}\ \frac{\partial\varphi}{\partial x_6}\right]$$

From the values that we know:

$$\varphi = \sim x_1 + \sim x_2 + \sim x_3 + J\, x_4 + \sim x_5 + m x_1^2\, x_6$$

Where $\sim$ indicates that the value can be chosen. And

$$d\varphi = \sim dx_1 + \sim dx_2 + \sim dx_3 + J\, dx_4 + \sim dx_5 + [m x_1^2\, dx_6 + 2 m x_1 x_6 dx_1] =$$

$$= (\sim + 2 m x_1 x_6)\, \boldsymbol{dx_1} + \sim \boldsymbol{dx_2} + \sim \boldsymbol{dx_3} + J\, \boldsymbol{dx_4} + \sim \boldsymbol{dx_5} + m x_1^2\, \boldsymbol{dx_6}$$

So, for example, one set of bases could be:

$$\omega_1 = 2 m x_1 x_6 dx_1 + J\, dx_4 + m x_1^2\, dx_6$$

$$\omega_2 = 2 m x_1 x_6 dx_1 + dx_3 + J\, dx_4 + m x_1^2\, dx_6$$

$$\omega_3 = 2 m x_1 x_6 dx_1 + 2 dx_3 + J\, dx_4 + m x_1^2\, dx_6$$

$$\omega_4 = 2 m x_1 x_6 dx_1 + dx_5 + J\, dx_4 + m x_1^2\, dx_6$$

Integrability of H2

In order to understand if H2 is fully integrable, we recall that: let V = $span\{\omega_1, \omega_2, \dots, \omega_r\}$ be a subspace of E. V is closed iff it has a basis made of closed one-forms only. In this case we built the bases in such a way, so we can say that H2 is integrable.