Homework 4 - Accessibility of the N-trailer system

Due on Tuesday 21st april 2020, 11:59 PM

In some special coordinates, the kinematic model of the N-trailer system is described by

$$\dot{x} = \begin{pmatrix} 1 & 0 \\ x_3 & 0 \\ x_4 & 0 \\ \vdots & \vdots \\ x_n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

We computed $\dim \mathcal{H}_2 = n-2$ so that $\mathcal{H}_2 = span\{\omega_1, \omega_2, \cdots, \omega_{n-2}\}$. The two first basis vectors were computed as well as

$$\omega_1 = -x_3 dx_1 + dx_2$$

and

$$\omega_2 = -x_4 dx_1 + dx_3$$

1. Compute the $n-4$ remaining basis vectors of $\mathcal{H}_2$ in the form of row vectors $\begin{bmatrix} * & 0 & 0 & * & 0 & 0 & \cdots & 0 \end{bmatrix}$, ... , $\begin{bmatrix} * & 0 & 0 & 0 & \cdots & 0 & * & 0 \end{bmatrix}$. Write the corresponding 1-forms ω_3 and ω_{n-2}.
2. Check whether $\dot{\omega}_1 \in \mathcal{H}_2$
3. Check whether $\dot{\omega}_2 \in \mathcal{H}_2$
4. Check whether $\dot{\omega}_{n-2} \in \mathcal{H}_2$
5. Write a basis for $\mathcal{H}_3$