Homework1 - On a nonlinear infection model

Reference: *Can the COVID-19 epidemic be managed on the basis of daily data?* by Francesco Casella, Dipartimento di Elettronica, Informazione e Bioingegneria Politecnico di Milano, Italy.

The standard and basic SIR model is used to model the population dynamics of some infection as COVID-19. It considers a population of N individuals, consisting in three compartments:

- S is the amount of Safe individuals, not yet infected and Susceptible to become infected. Their population will decrease if the Safe patients have the opportunity to encounter Infected patients. This sub-population does not include patients which were infected but have Recovered from the infection.
- I represents the amount of Infected patients. Their population will increase when some "S" individuals get infected, and it decreases when Infected people Recover from the disease.
- R is the amount of population which was infected, but has Recovered from the infection and is healthy again.

Note that by definition $N = S + I + R$.
The dynamics is thus described by the differential equations

$$\dot{S} = -\frac{\beta IS}{N} \tag{1}$$

$$\dot{I} = \frac{\beta IS}{N} - \gamma I \tag{2}$$

$$\dot{R} = \gamma I \tag{3}$$

The amount of newly infected patient is proportional to both S and I. The parameter β is a virulence factor which depends on drugs (if any), on vaccine and eventually on political regulations on the mobility of the population. Thus, β is considered to be the control input. γ is the inverse of the constant duration of the disease for an individual.
QUESTION 1: Among the three variables S, I and R how many are independent, or transcendent with respect to the field of real numbers ?
ANSWER 1 : Only two variables, say S and I. In fact, we got the algebraic relation $R = N-S-I$.
Furthermore, there is no way to get an algebraic relation (over real number) involving the two remaining variables S and I. The latter are thus independent, or transcendent with respect to the field of real numbers.
We may drop equation (3) above to get the well-posed nonlinear state space representation:

$$\dot{S} = -\frac{\beta IS}{N} \tag{4}$$

$$\dot{I} = \frac{\beta IS}{N} - \gamma I \tag{5}$$

Define further the two output equations

$$Y_1 = S \tag{6}$$
$$Y_2 = I \tag{7}$$

QUESTION 2: Check whether or not the two outputs Y_1 an Y_2 are differentially algebraically dependent or transcendent with respect to the field of real numbers. I.e., does there exist a (possibly nonlinear) differential equation $F(Y_1, Y_2, \dot{Y}_1, \dot{Y}_2, ...) = 0$ which does not involve the control input β ?
Recall that N and γ are constant real numbers and my be involve in such a differential equation.
ANSWER 2:.....

Further explanations (in french) on the SIR model:
https://www.youtube.com/watch?v=-2tI3MQFqkI