MTH 464/564: Homework # 2, due 11/02/2022

To receive full credit, present complete answers that show all work.

Task 1 (25 points) Write an optimization program for the steepest descent iteration

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \nabla f(\mathbf{x}_k) \tag{1}$$

and for the Newton iteration

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \left[\nabla^2 f(\mathbf{x}_k)\right]^{-1} \nabla f(\mathbf{x}_k) \tag{2}$$

using the backtracking line search procedure below to select a proper step length α_k,

$\alpha = \alpha_0$ % initialization
% check Armijo condition and reduce step size, if necessary
while $f(\mathbf{x}_k + \alpha \mathbf{p}_k) > f(\mathbf{x}_k) + c\alpha \mathbf{p}_k^t \cdot \nabla f(\mathbf{x}_k)$
    $\alpha = \rho\alpha$
end
$\alpha_k = \alpha$

Set the initial step length $\alpha_0 = 1, \rho = 0.5, c = 0.01$. Provide a printout of your code(s).

Task 2 (25 points) Test your codes on the Rosenbrock function

$$f(\mathbf{x}) = A(x_2 - x_1^2)^2 + (1 - x_1)^2, \quad \mathbf{x} = (x_1, x_2)$$

Perform two sets of experiments corresponding to the parameter value $A = 1$ and $A = 100$. Use the initial guess point $\mathbf{x}_0 = (-1.2, 1)$ and a convergence criteria $\|\nabla f(\mathbf{x}_k)\| < 10^{-3}$. For each set of experiments and each algorithm provide

- The approximate solution, the number of iterations and the number of function evaluations
- Graphs showing the evolution of the cost function and the norm of the gradient during the optimization process.
- *MTH 564 students*: Provide a plot of the optimization "path" $\mathbf{x}_0, \mathbf{x}_1, \ldots$ produced by each algorithm.