

CS321 Introduction to Theory of Computation  
Assignment No. 2, Due: Friday February 2, 2024

1. Suppose that a bank only permits passwords that are strings from the alphabets  $\Sigma = \{a, b, c, d, 1, 2, 3, 4, \#, \$, \&\}$ . The passwords follow the rules
  - (a) The length can be 5, 6 or 7.
  - (b) The first alphabet must be from  $\{a, b, c, d\}$ .
  - (c) The last two alphabets must be from  $\{1, 2, 3, 4\}$
  - (d) Exactly one alphabet should be from  $\{\#, \$, \&\}$ .

The set of legal passwords forms a regular language  $L$ . Construct a NFA for  $L$ .

2. Design an NFA with no more than five states for the set  $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$
3. Find an NFA with four states for  $L = \{a^n : n \geq 0\} \cup \{b^m a : m \geq 1\}$ .
4. Convert the NFA defined by

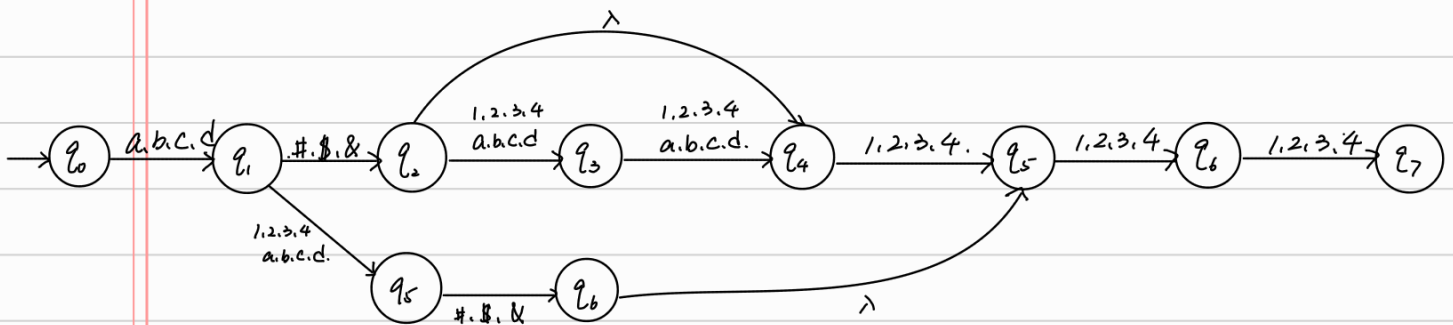
$$\begin{aligned}\delta(q_0, a) &= \{q_0, q_1\}. \\ \delta(q_1, b) &= \{q_1, q_2\} \\ \delta(q_2, a) &= \{q_2\} \\ \delta(q_0, \lambda) &= \{q_2\}\end{aligned}$$

with initial state  $q_0$  and final state  $q_2$  into an equivalent DFA.

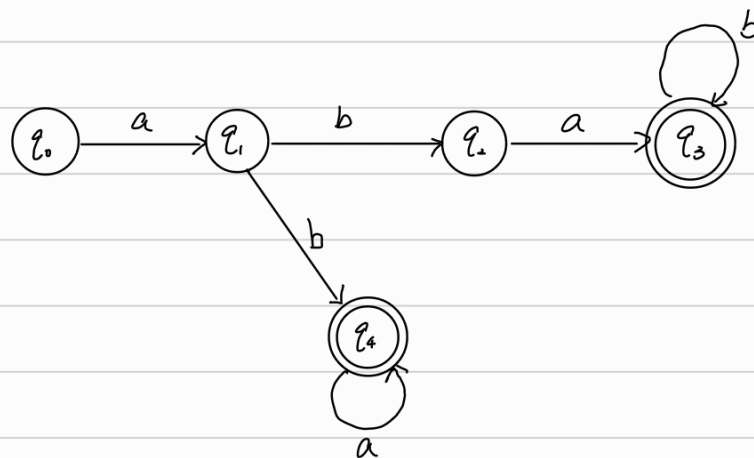
5. Show that if  $L$  is regular, so is  $L^R$ .

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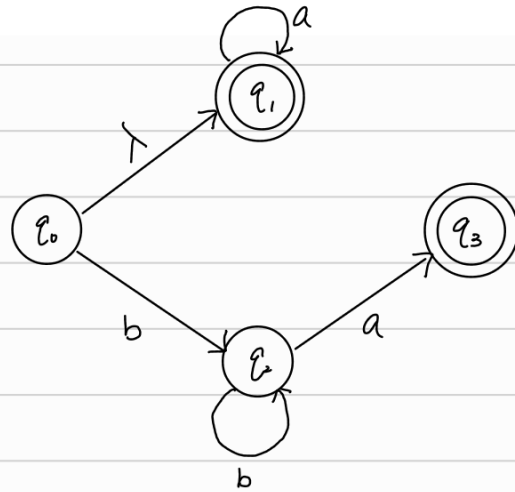
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3. Find an NFA with four states for  $L = \{a^n : n \geq 0\} \cup \{b^m a : m \geq 1\}$ .



4. Convert the NFA defined by

$$\delta(q_0, a) = \{q_0, q_1\}.$$

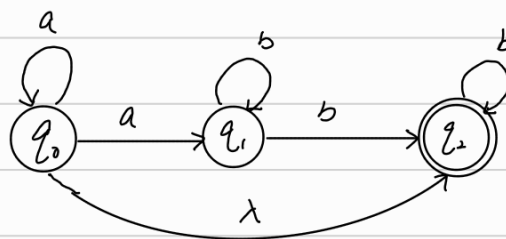
$$\delta(q_1, b) = \{q_1, q_2\}$$

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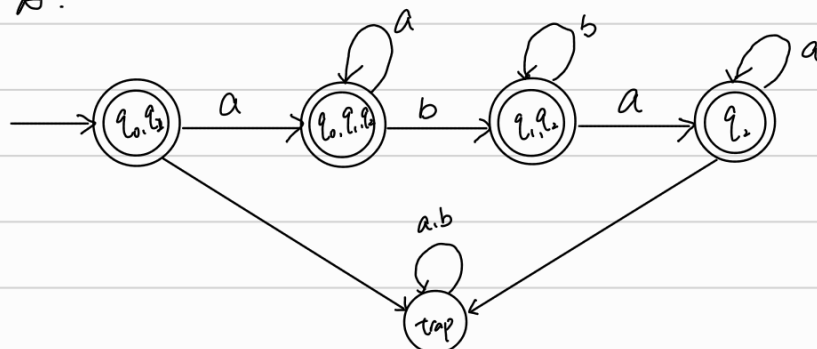
$$\delta(q_0, \lambda) = \{q_2\}$$

with initial state  $q_0$  and final state  $q_2$  into an equivalent DFA.

Given NFA.:



DFA.



5. Show that if  $L$  is regular, so is  $L^R$ .

- ① Express  $L$  as a regular expression: Since  $L$  is regular, there exists a regular expression  $r$
- ② Reverse the order of the symbols and operations in  $r$  to get a new regular expression  $r'$
- ③ Show that  $r'$  generates  $L^R$ , it can be proven that for any regular expression  $r$ ,  $L(r') = (L(r))^R$ . The reversed regular expression  $r'$  generates the reversal of the language generated by  $r$
- ④ Since  $r'$  generates  $L^R$ , and regular expressions are closed under reversal, so we can calculate that  $L^R$  is also regular.