CS321 Introduction to Theory of Computation
Assignment No. 4, Due: Saturday February 24, 2024

1. Determine whether or not the following languages are regular. If the language is regular then give an NFA or regular expression for the language. Otherwise, use the pumping lemma for regular languages or closure properties to prove the language is not regular.
   (a) $L = \{a^n b^k : k \leq n \leq 2k\}$
   (b) $L = \{b^n a^k : n > 0, k > 0\} \cup \{a^n b^k : k > 0, n > 0\}$
   (c) $L = \{a^n : n = 3k \text{ for some } k \geq 0\}$
   (d) $L = \{a^n : n = k^3 \text{ for some } k \geq 0\}$
   (e) $L = \{w : n_a(w) > n_b(w), w \in \{a, b\}^*\}$

2. Give context-free grammars that generate the following language
   (a) $L_1 = \{w \in \{0, 1\}^* \mid \text{w contains at least three 1's }\}$.
   (b) $L_2 = \{a^i b^j c^k | i, j, k \geq 0, and\ i = j \text{ or } i = k\}$

3. Consider the following grammar G = ({S, A}, {a, b}, S, P} where P is defined below.

$$S \to SS|AAA|\lambda$$
$$A \to aA|Aa|b$$

   (a) Give a left-most derivation for the string abbaba.
   (b) Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some terminal string.

4. Using the CYK algorithm show that the string baabba is in the context free language generated by the following production rules. Here, $V = \{S, A, B, C, D\}$ and $T = \{a, b\}$

$$S \to AB|BA$$

$$A \to AS|a$$
$$B \to BS|b$$
$$A \to BC$$
$$B \to AD$$
$$C \to AA$$
$$D \to BB$$

Show at least twol derivation trees.

1. Determine whether or not the following languages are regular. If the language is regular then give an NFA or regular expression for the language. Otherwise, use the pumping lemma for regular languages or closure properties to prove the language is not regular.

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   (e) $L = \{w : n_a(w) > n_b(w), w \in \{a,b\}^*\}$

(a)

Assume that $m = 5$ and $w = a^3b^2$, So $|w| \ge 5$

$w = \underbrace{aaa}_{x}\underbrace{b}_{Y}\underbrace{b}_{z}$

$|xy| \le 5$ and $|Y| \ge 1$

$w_i = xy^iz$

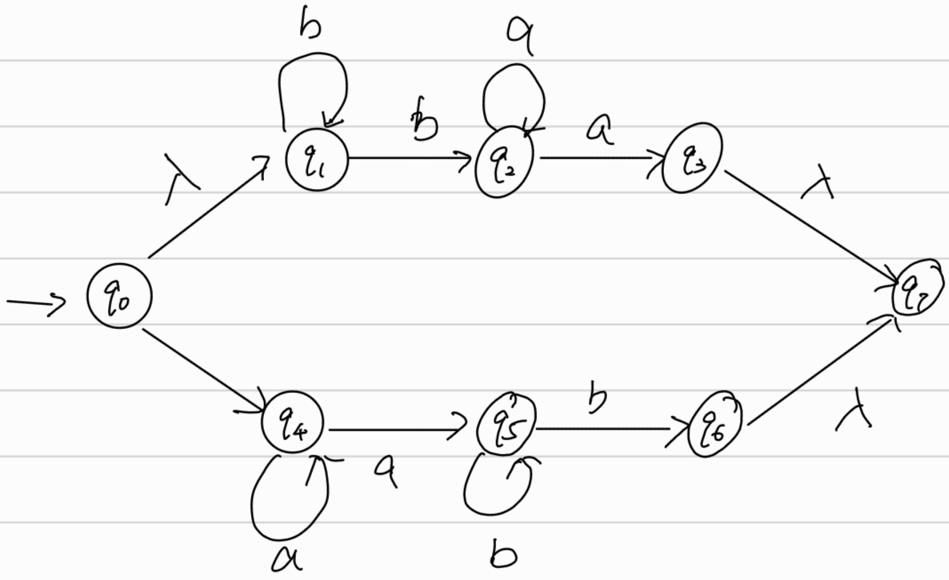
$w_2 = xy^2z = aaab^2b = a^3b^3$ $\quad w_2 \in L$

$w_3 = xy^3z = aaab^3b = a^3b^4$ $\quad w_3 \notin L$

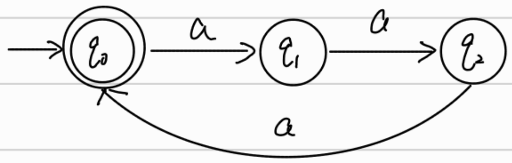
So for all i, $w_i \notin L$

So it's proven that it is not a regular language.

(b)



(c)



(d)

Assume that $m = 8$, $w = a^8 \in L$

$$w = \underbrace{aaaa}_{x}\ \underbrace{a}_{y}\ \underbrace{aaa}_{z}$$

$|xy| \leq 8 \qquad |y| \geq 1$

$w_i = xy^i z$

$w_2 = xy^2 z = aaaa\ aa\ aaa$

$= a^9 \notin L$, so $w_2 \in L$

For all i, $w_i \in L$

So it's proven that it is not a regular language

(e)

$L = \{a, aab, aaabb, \dots\}$

Assume $m = 4$, $w = aaabb \in L$

$|w| \geq 4$

$$w = \underbrace{aaa}_{x}\ \underbrace{b}_{y}\ \underbrace{b}_{z}$$

$w_i = xy^i z$

$w_2 = xy^2 z = aaa\,b^2 b = a^3 b^3 \qquad w_2 \notin L$

So for all i, $w_i \notin L$

So it's proven that it is not a regular language.

2. Give context-free grammars that generate the following language

   (a) $L_1 = \{w \in \{0,1\}^* \mid \text{w contains at least three 1's}\}$.

   (b) $L_2 = \{a^i b^j c^k | i, j, k \geq 0, and\ i = j \text{ or } i = k\}$

(a)

$$S \to A1A1A1A$$
$$A \to 0A$$
$$A \to 1A$$
$$A \to \lambda$$

$\Rightarrow$

$$S \to A1A1A1A$$
$$A \to 0A \mid 1A \mid \lambda$$

(b)

$$S \to AB$$
$$A \to aAb$$
$$A \to \lambda$$
$$B \to cB$$
$$B \to \lambda$$

$$S \to C$$
$$C \to aCc$$
$$C \to D$$
$$D \to bD$$
$$D \to \lambda$$

$\Rightarrow$

$$S \to AB \mid C$$
$$A \to aAb \mid \lambda$$
$$B \to cB \mid \lambda$$
$$C \to aCc \mid D$$
$$D \to bD \mid \lambda$$

3. Consider the following grammar G = ({S, A}, {a, b}, S, P} where P is defined below.

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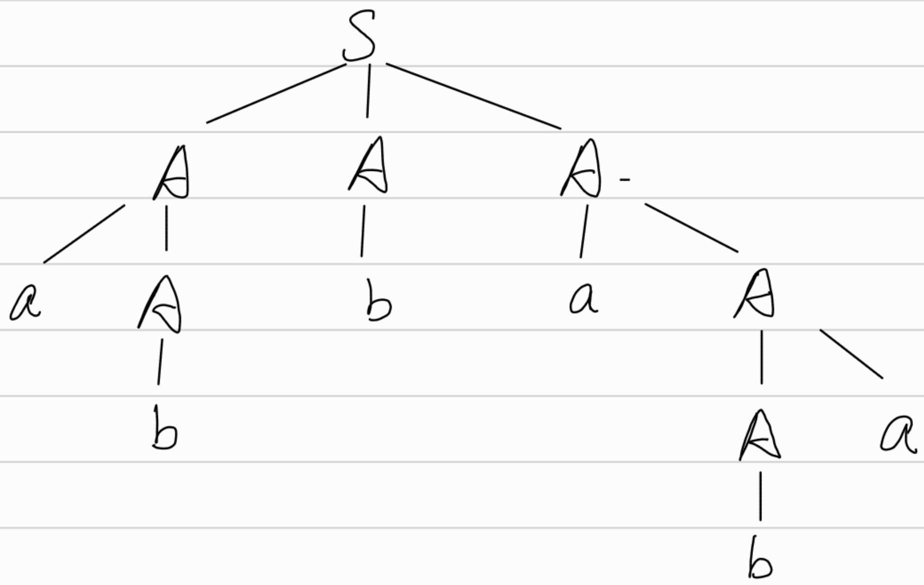
(a) Give a left-most derivation for the string abbaba.

(b) Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some terminal string.

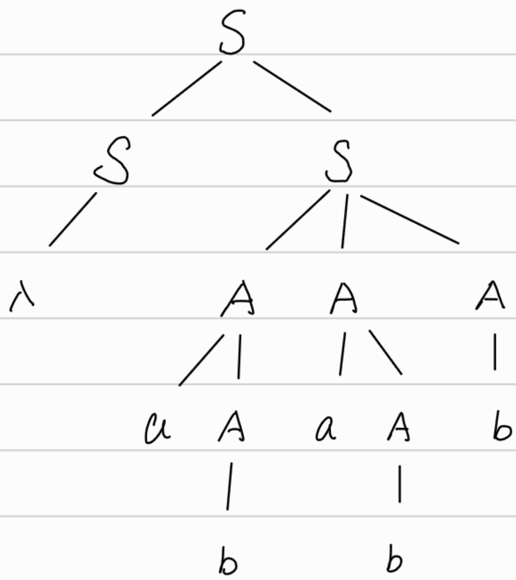
a. Left-most derivation for the string "abbaba"

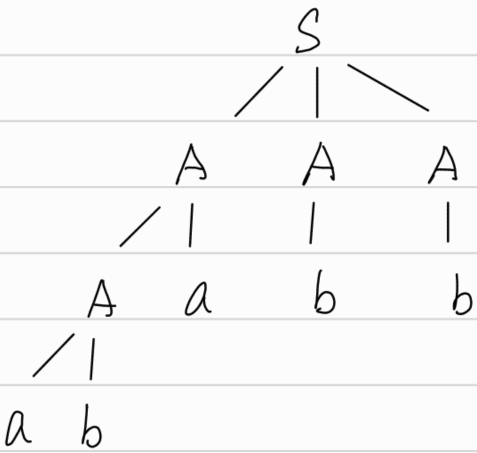
S	
$\Rightarrow AAA$	$[S \to AAA]$
$\Rightarrow aAAA$	$[A \to aA]$
$\Rightarrow abAA$	$[A \to b]$
$\Rightarrow abbA$	$[A \to b]$
$\Rightarrow abbaA$	$[A \to aA]$
$\Rightarrow abbaAa$	$[A \to Aa]$
$\Rightarrow abbaba$	$[A \to b]$



$$S \xrightarrow{2} AAA \xrightarrow{4} aAAA \xrightarrow{6} abAA \xrightarrow{5} abAaA$$
$$\xrightarrow{6} abbaA \xrightarrow{5} abbaAa \xrightarrow{6} abbaba$$

b.





4. Using the CYK algorithm show that the string baabba is in the context free language generated by the following production rules. Here, $V = \{S, A, B, C, D\}$ and $T = \{a, b\}$

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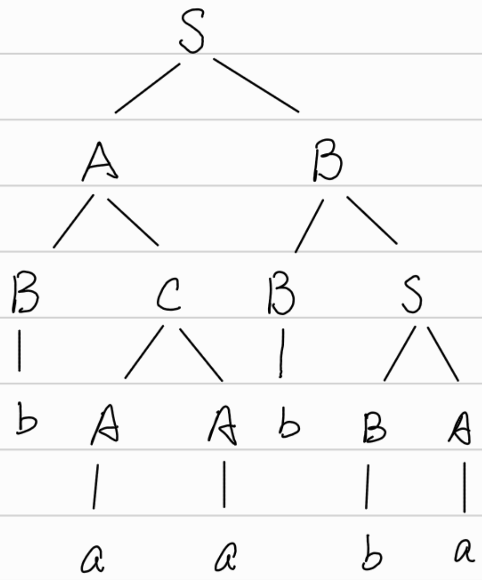
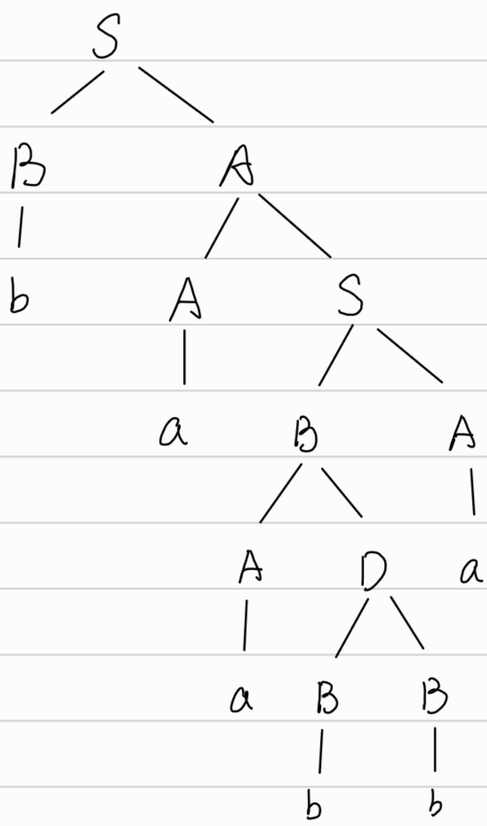
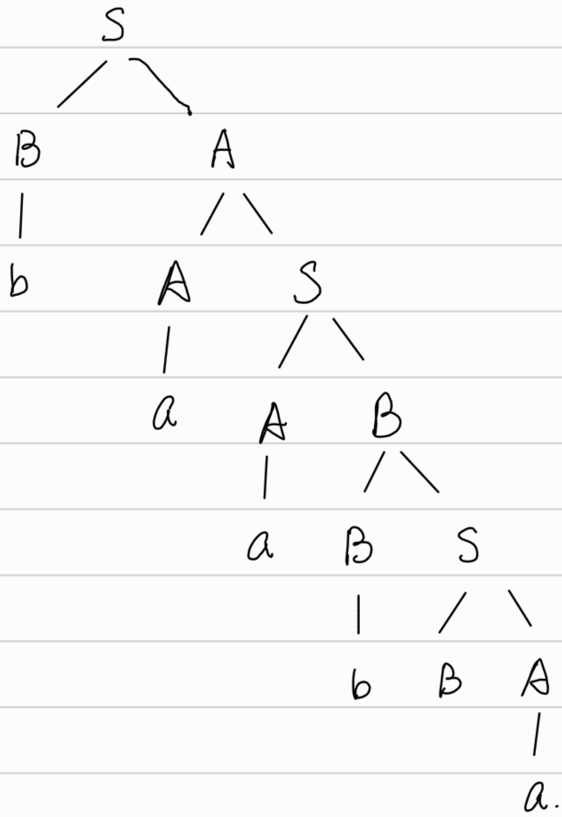
Show at least twol derivation trees.



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