CS321 Introduction to Theory of Computation
Assignment No. 1, Due: Friday January 19, 2024

1. Prove that $\overline{S_! \cup S_2} = \overline{S}_1 \cap \overline{S}_2$ where S_1 and S_2 are sets and $\overline{S}$ is the complement of the set S.

2. A tree is a graph with no cycle. Show by induction that a tree with n nodes contains $n-1$ edges.

3. Prove by induction that the sum of the first k odd integers is equal to k^2. For example, $1 = 1^2, 1+3 = 4 = 2^2, 1+3+5 = 9 = 3^2, 1+3+5+7 = 16 = 4^2$, and so on. (Hint: The kth odd integer is $2k-1$).

4. A rational number is of the form m/n where m and n are integers. For example, $2/3, 3/4, 2/5, 4/7, 3/8, 5/9, 11/18, 9/25$ are some rational numbers. Show by contradiction that $\sqrt{2}$ is not a rational number.

5. Let the input symbols in a finite automata be $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Design a DFA that accepts all integers which are divisible by 3. (Hint: An integer is divisible by 3 if the sum of the digits is divisible by 3).

6. For this problem assume that the input symbols are $\{0, 1\}$. Design a DFA that accepts the binary string if it is divisible by 3.