Recall: for a set S, the <u>power set</u> denoted by $\mathcal{P}(S)$ is the set of all subsets of S.

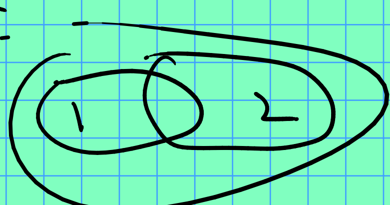
Say $S = \{0, 1, 2\}$.
Then $\mathcal{P}(S) = \{\{\}, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$.

(If S is say `set<int>`, $\mathcal{P}(S)$ would be of type `set<set<int>>`).

S: ~~0~~ 1 2

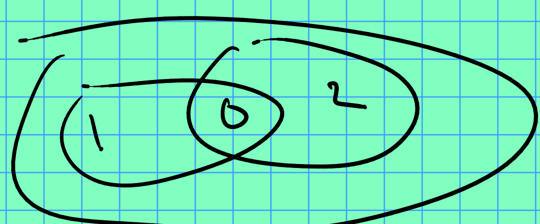
Remove say 0 to get a smaller set $\{1,2\}$ for which we can make a recursive call. Since recursion is magic, we'll get $\mathcal{P}(\{1,2\})$.

$\mathcal{P}(\{1,2\})$



All of $\mathcal{P}(\{1,2\})$ are also in $\mathcal{P}(S)$, but we're missing all subsets that have 0 as an element. How to add them back?

$\cup = \mathcal{P}(S)$



Let's trace it:

(0 1 2) $= \{\{\}, \{2\}, \{1\}, \{1,2\}, \{0\}, \{0,2\}, \{0,1\}, \{0,1,2\}\}$.

(1 2) $= \{\{\}, \{2\}, \{1\}, \{1,2\}\}$

$2 = \{\{1\}, \{2\}\}$

$0 = \{\{\}\}$

$K \subseteq S \Rightarrow \mathcal{P}(K) \subseteq \mathcal{P}(S)$