胡杨和樟子树申请量$D_i, i=1,2$符合分布$F_i(\mu_i,\sigma_i^2)$，即$D_i \sim F_i(\mu_i,\sigma_i^2), i=1,2$，两种树预分配量为$Q_i, i=1,2$，假设$\mu_1+\mu_2 \le M$， D_1取值为ξ的概率为$F_1(\xi)$， $\mu_1=\int_0^\infty \xi F_1(\xi)d\xi$，同理既有

$$\mu_1+\mu_2=\int_0^\infty \xi F_1(\xi)d\xi+\int_0^\infty \omega F_2(\omega)d\omega \le M,$$

且Q_i满足条件$Q_1,Q_2>0, Q_1+Q_2<M$。每棵树成本为c，超出预分配量每棵树成本为m，已给出条件$(m-c)/c>(\sigma_1/\mu_1)^2>(\sigma_2/\mu_2)^2$。根据预设分布优化分配量量,最小化预分配成本及超出预分配后的增补成本之和,给出最优分配量$Q_i, i=1,2$显示表达式。订购成本

$$O_i=cQ_i+m[D_i-Q_i]^+, i=1,2, \quad [Q]^+=\max\{Q,0\}.$$

由于$D_i \le Q_i$时$[D_i-Q_i]^+=0$,所以只需考虑$D_i>Q_i$即可,且$D_i \sim F_i(\mu_i,\sigma_i^2), i=1,2$,而此时$[D_i-Q_i]^+=D_i-Q_i$,增补量为$[D_i-Q_i]^+$的概率为$F_i(D_i)$，故增补树苗量期望为

$$\begin{aligned}
&\int_{Q_i}^\infty (\xi-Q_i)F_i(\xi)d\xi=\int_{Q_i}^\infty \xi F_i(\xi)d\xi-Q_i\int_{Q_i}^\infty F_i(\xi)d\xi \\
&=\left(\int_0^\infty \xi F_i(\xi)d\xi-\int_0^{Q_i}\xi F_i(\xi)d\xi\right)-Q_i\left(\int_0^\infty F_i(\xi)d\xi-\int_0^{Q_i}F_i(\xi)d\xi\right) \\
&=\left(\mu_i-\int_0^{Q_i}\xi F_i(\xi)d\xi\right)-Q_i\left(1-\int_0^{Q_i}F_i(\xi)d\xi\right) \\
&=(\mu_i-Q_i)-\left(\int_0^{Q_i}\xi F_i(\xi)d\xi-Q_i\int_0^{Q_i}F_i(\xi)d\xi\right) \\
&=(\mu_i-Q_i)-\int_0^{Q_i}(\xi-Q_i)F_i(\xi)d\xi
\end{aligned}$$

而由于订购成本为

$$O_i=cQ_i+m\int_{Q_i}^\infty (\xi-Q_i)F_i(\xi)d\xi, i=1,2,$$

所以总订购成本$O=O_1+O_2$为

$$O=c(Q_1+Q_2)+m\left(\int_{Q_1}^\infty (\xi-Q_1)F_1(\xi)d\xi+\int_{Q_2}^\infty (\xi-Q_2)F_2(\omega)d\omega\right),$$

结合上述增补树苗量期望即有

$$\begin{aligned}
O&=c(Q_1+Q_2)+m\left(\int_{Q_1}^\infty (\xi-Q_1)F_1(\xi)d\xi+\int_{Q_2}^\infty (\xi-Q_2)F_2(\omega)d\omega\right) \\
&=c(Q_1+Q_2)+m\left((\mu_1-Q_1)-\int_0^{Q_1}(\xi-Q_1)F_1(\xi)d\xi+(\mu_2-Q_2)-\int_0^{Q_2}(\omega-Q_2)F_2(\omega)d\omega\right) \\
&=c(Q_1+Q_2)+m(\mu_1-Q_1)+m(\mu_2-Q_2)-m\left(\int_0^{Q_1}(\xi-Q_1)F_1(\xi)d\xi+\int_0^{Q_2}(\omega-Q_2)F_2(\omega)d\omega\right) \\
&=m(\mu_1+\mu_2)-(m-c)(Q_1+Q_2)-m\left(\int_0^{Q_1}(\xi-Q_1)F_1(\xi)d\xi+\int_0^{Q_2}(\omega-Q_2)F_2(\omega)d\omega\right) \\
&=m\left(\mu_1+\mu_2+\int_0^{Q_1}(Q_1-\xi)F_1(\xi)d\xi+\int_0^{Q_2}(Q_2-\omega)F_2(\omega)d\omega\right)-(m-c)(Q_1+Q_2)
\end{aligned}$$

设$F(Q_1)=\int_0^{Q_1}(Q_1-\xi)F_1(\xi)d\xi=\int_0^{Q_1}Q_1F_1(\xi)d\xi-\int_0^{Q_1}\xi F_1(\xi)d\xi$，则

$$\frac{\partial F}{\partial Q_1}=\int_0^{Q_1}F_1(\xi)d\xi+Q_1F_1(Q_1)-Q_1F_1(Q_1)=\int_0^{Q_1}F_1(\xi)d\xi$$

同理，O对Q_1,Q_2求偏导，并令其等于0，则有

$$\begin{cases}\dfrac{\partial O}{\partial Q_1}=m\int_0^{Q_1}F_1(\xi)d\xi-(m-c)=0 \\ \dfrac{\partial O}{\partial Q_2}=m\int_0^{Q_2}F_2(\omega)d\omega-(m-c)=0\end{cases} \Rightarrow \begin{cases}\int_0^{Q_1}F_1(\xi)d\xi=\dfrac{m-c}{m} \\ \int_0^{Q_2}F_2(\omega)d\omega=\dfrac{m-c}{m}\end{cases}$$

在不考虑$Q_1+Q_2<M$情况下，可粗略解得Q_1为$F_1(\xi)$的$\dfrac{m-c}{m}$分位数，Q_2为$F_2(\omega)$的$\dfrac{m-c}{m}$分位数