

1. Draw a random scalar to simulate user characteristics, d_i .

$$d_i \sim U(0, 1)$$

2. Draw T-length random vector to simulate the brand price indices $r_{b,t}$ in each time period.

$$r_{b,t} \sim \text{LogNormal}(0, 0.25)$$

3. Draw a T-length random vector for each i, t, b and ad-position to simulate user i 's ad-exposures on day t to brand b 's ads, $x_{i,t,b}$.

$$x_{i,0,b} = \vec{0}$$

$$x_{i,t,b} \sim \text{Poisson}(a)$$

4. Simulate user i 's initial conditions with respect to each brand, $h_{i,0,b}$.

$$h_{i,0,b} \sim N(0, 1)$$

5. Given $h_{i,0,b}$ and parameters, forward simulate user i 's recurrent term for each b in each t , denoted $h_{i,t,b}$, as a function $x_{i,t-1}$ and $h_{i,t-1,b}$. The recurrence generates time-dependence in user behavior.

$$h_{i,t,b} = 0.5 * \text{sigmoid}(1.0 + \alpha_1 x_{i,t-1,b} + \alpha_2) + 0.5 * h_{i,t-1,b}, \alpha_1 \sim N(0, 1)$$

6. Compute $u_{i,t,b}$, the "attractiveness" of brand b for user i in time t as a linear function of $x_{i,t,b}$, d_i , $h_{i,t,b}$ and $r_{b,t}$.

$$u_{i,t,b} = 0.4 * d_i + \beta_1 x_{i,t,b} + 0.01 * h_{i,t,b} + \beta_3 r_{b,t}, \beta_1 \sim U(0.1, 0.9), \beta_3 \sim \text{LogNormal}(0, 1)$$

7. Transform $u_{i,t,b}$ via a sigmoid function to obtain the probability of conversion by user i of brand b in time t , $p_{i,t,b}$.

$$p_{i,t,b} = \text{sigmoid}(u_{i,t,b})$$

8. Simulate conversion by user i of brand b in time t $y_{i,t,b}$ by drawing a Bernoulli random variate with mean $p_{i,t,b}$.

$$y_{i,t,b} \sim \text{Bernoulli}(p_{i,t,b})$$