

1. Draw a random scalar to simulate the user characteristics.

$$d_i \sim U(0, 1)$$

2. Draw T-length random vector to simulate the brand price indices in each time period.

$$r_b \sim \text{LogNormal}(0, 0.25)$$

3. Draw a T-length random vector for each i, t, b to simulate the user i's ad-exposures at day t and brand b, denoted as $x_{i,t,b}$.

$$x_{i,0,b} = \vec{0}$$

$$x_{i,t,b} \sim \text{Poisson}(a)$$

4. Simulate the influence of user i's history ad-exposures before day t to the conversion of day t, denoted by $h_{i,0,b}$, as a function of some constants, $x_{i,t-1}$ and $h_{i,t-1,b}$.

$$h_{i,0,b} \sim N(0, 1)$$

$$h_{i,t,b} = 0.5 * \text{sigmoid}(1.0 + \alpha_1 x_{i,t-1} + \alpha_2) + 0.5 * h_{i,t-1,b}, \alpha_1 \sim N(0, 1)$$

5. Simulate the influence of user i's ad-exposures at and before day t to the conversion of day t, denoted by $u_{i,t,b}$, as a function of some constants, $x_{i,t,b}$, d_i , $h_{i,t,b}$ and r_b .

$$u_{i,t,b} = 0.4 * d_i + \beta_1 x_{i,t,b} + 0.01 * h_{i,t,b} + \beta_3 r_b, \beta_1 \sim U(0.1, 0.9)$$

6. Simulate the user i's attempt to make a conversion at day t as a sigmoid function of $u_{i,t,b}$, denoted as $p_{i,t,b}$.

$$p_{i,t,b} = \text{sigmoid}(u_{i,t,b})$$

7. Simulate the user i's actual decision whether to make a conversion by draw a Bernoulli random value with parameter = $p_{i,t,b}$.

$$y_{i,t,b} \sim \text{Bernoulli}(p_{i,t,b})$$