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On the Synthesis of Discrete Controllers for Timed Systems

An Extended Abstract

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Abstract

This paper presents algorithms for the automatic synthesis of the real time controllers by finding a winning strategy for certain games defined by the timed automata of Alur and Dill.

Introduction

Consider a dynamical system P whose presentation describes all its possible behaviours. A subset of the plant's behaviours, satisfying some criterion is defined as good or acceptable.

A controller C is another system which can interact with P in a certain manner by observing the state of P and by issuing control actions that influence the behaviour of P.

The synthesis problem is then, to find out whether, for a given P, there exists a realizable controller C such that their interaction will produce only good behaviours.

Discrete case

Definition 1 (*Plant*)

A plant automaton is a tuple $\mathcal{P} = (Q, \Sigma_c, \delta, q_0)$ where Q is a finite set of states, Σ_c is a set of controller commands, $\delta : Q \times \Sigma_c \longmapsto 2^Q$ is the transition function and $q_0 \in Q$ is an initial state.

Definition 2 (*Controllers*)

A controller (strategy) for a plant specified by $\mathcal{P} = (Q, \Sigma_c, \delta, q_0)$ is a function $C : Q^+ \longmapsto \Sigma_c$. A simple controller is a controller that can be written as a function $C : Q \longmapsto \Sigma_c$.

We are interested in the simpler cases of controllers that base their decisions on a finite memory.

Definition 3 (*Trajectories*)

Let $\mathcal{P}$ be a plant and let $C : Q^+ \longmapsto \Sigma_c$ be a controller. An infinite sequence of states $\alpha : q[0], q[1], \ldots$ such that $q[0] = q_0$ is called a trajectory of $\mathcal{P}$ if

$$q[i+1] \in \bigcup_{\sigma \in \Sigma_c} \delta(q[i], \sigma)$$

and a C-trajectory if $q[i+1] \in \delta(q[i], C[\alpha[0..i]])$ for every $i \geq 0$. The corresponding sets of trajectories are denoted by $L(\mathcal{P})$ and $L_C(\mathcal{P})$.

For every infinite trajectory $\alpha \in L(\mathcal{P})$:

- $\mathit{Vis}(\alpha)$ denote the set of all states appearing in α
- $\mathit{Inf}(\alpha)$ denote the set of all states appearing in α infinitely many times

Definition 4 (*Acceptance Condition*)

Let $\mathcal{P} = (Q, \Sigma_c, \delta, q_0)$ *be a plant. An acceptance condition for* $\mathcal{P}$ *is*

$$\Omega \in \{(F, \Box), (F, \Diamond), (F, \Diamond\Box), (F, \Box\Diamond), (\mathcal{F}, \mathcal{R}_n)\}$$

where $\mathcal{F} = \{(F_i, G_i)\}_{i=1}^{n}$ *and* F, F_i *and* G_i *are certain subsets of* Q *referred as the good states. The set of sequences of* $\mathcal{P}$ *that are accepted accordig to* Ω *is defined as follows:*

$L(\mathcal{P}, F, \Box)$	$\{\alpha \in L(\mathcal{P}) : Vis(\alpha) \subseteq F\}$	α always remains in F
$L(\mathcal{P}, F, \Diamond)$	$\{\alpha \in L(\mathcal{P}) : Vis(\alpha) \cap F \neq \emptyset\}$	α eventually visits F
$L(\mathcal{P}, F, \Diamond\Box)$	$\{\alpha \in L(\mathcal{P}) : Inf(\alpha) \subseteq F\}$	α eventually remains in F
$L(\mathcal{P}, F, \Box\Diamond)$	$\{\alpha \in L(\mathcal{P}) : Inf(\alpha) \cap F \neq \emptyset\}$	α visits F infinitely often
$L(\mathcal{P}, \mathcal{F}, \mathcal{R}_n)$	$\{\alpha \in L(\mathcal{P}) : \exists i \alpha \in L(\mathcal{P}, F_i, \Box\Diamond) \cap L(\mathcal{P}, G_i, \Diamond\Box)\}$	α visits F_i infinitely often and eventually stays in G_i

Discrete case

Definition 5 (*Controller Synthesis Problem*)

*For a plant $\mathcal{P}$ and an acceptance condition Ω, the problem **Synth**$(\mathcal{P}, \Omega)$ is: Find a controller C such that $L_C(\mathcal{P}) \subseteq L(\mathcal{P}, \Omega)$ ot otherwise show that such a controller does not exists.*

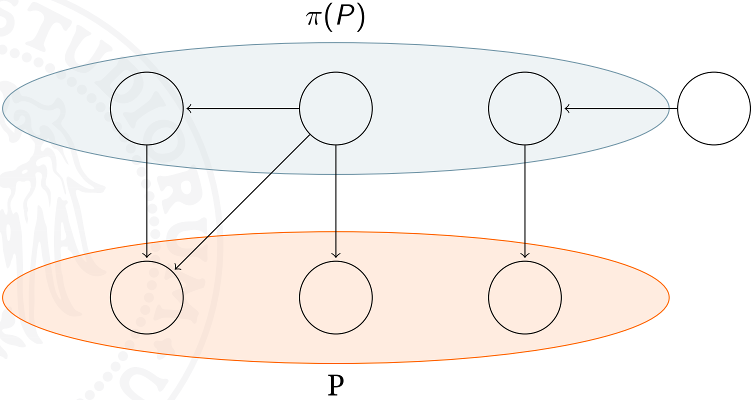
Definition 6 (*Controllable Predecessors*)

Let $\mathcal{P} = (Q, \Sigma_c, \delta, q)$ be a plant and a set of states $P \subseteq Q$. The controllable predecessors of P is the set of states from which the controller can "force" the plant into P in one step:

$$\{q : \exists \sigma \in \Sigma_c \,.\, \delta(q, \sigma) \subseteq P\}$$

We define a function $\pi : 2^Q \longmapsto 2^Q$, mapping a set of states $P \subseteq Q$ into the set of its Controllable predecessors:

$$\pi(P) = \{q : \exists \sigma \in \Sigma_c \,.\, \delta(q, \sigma) \subseteq P\}$$


$\pi(P)$
P

Discrete case

Theorem 1

For every $\Omega \in \{(F, \Box), (F, \Diamond), (F, \Diamond\Box), (F, \Box\Diamond), (\mathcal{F}, \mathcal{R}_n)\}$ *the problem* ***Synth***$(\mathcal{P}, \Omega)$ *is solvable. Moreover, if* $(\mathcal{P}, \Omega)$ *is controllable then it is controllable by a simple controller.*

Definition 7 (*Winning states*)

For a plant $\mathcal{P} = (Q, \Sigma_c, \delta, q_0)$ and an acceptance condition Ω, we denote $W \subseteq Q$ as the set of winning states, namely, the set of states from which a controller can enforce good behaviors according to Ω.

Discrete case

We can characterize this states by the following fixed-point expressions:

- $\Box$ $\nu W\big(F \cap \pi(W)\big)$
- $\Diamond$ $\nu W\big(F \cup \pi(W)\big)$
- $\Diamond\Box$ $\mu W \nu H\Big(\pi(H) \cap \big(F \cup \pi(W)\big)\Big)$
- $\Box\Diamond$ $\nu W \mu H\Big(\pi(H) \cup \big(F \cap \pi(W)\big)\Big)$
- $\mathcal{R}_1$ $\mu W\Big\{\pi(W) \cap \nu Y \mu H.W \cup G \cap \Big(\pi(H) \cup \big(F \cap \pi(Y)\big)\Big)\Big\}$

Then the plant is controllable iff $q_0 \in W$

Discrete case

Let see in more details how this works. Consider the case $\Diamond$:

$$\begin{array}{l} W_0 := \emptyset \\ \textbf{for } i := 0, 1, \ldots \textbf{ repeat} \\ \quad W_{i+1} := F \cup \pi(W_i) \\ \textbf{until } W_{i+1} = W_i \end{array}$$

$$\begin{array}{l} W_0 := \emptyset \\ W_1 := F \cup \pi(W_0) = F \cup \pi(W_0) = F \\ W_2 := F \cup \pi(W_1) = F \cup \pi(F) \\ \ldots \end{array}$$

finally: $W_n := F \cup \pi(W_{n-1}) = F \cup \pi(F \cup \pi(\ldots(F \cup \pi(F))))$

Discrete case

In the process of calculating $W_i + 1$, whenever we add a state q to W_i, there must be at least one action $\sigma \in \Sigma_c$ such that $\delta(q, \sigma) \subseteq W_i$.

So we define the controller at q as $C(q) = \sigma$.

When the process terminates, the controller is synthesized for all the winning states.

It can be seen that if the process fails, that is $q_0 \notin W$, then for every controller command there is a possibly bad consequence that will put the system outside F, and no controller, even an infinite state one, can prevent this.