

Includes triangles, coordinate counting, angles, and equations.

Credits: Written and edited by Justin Liu

NOTE: This lesson is half length, since the other 60 minutes is a mock AMC/Mathcounts style exam.

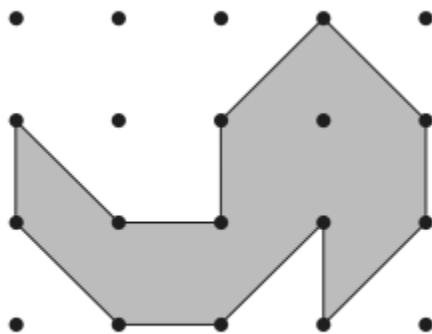
Website: <https://chick1n.github.io/EJAcademy/> Bootcamp portal pass: "ejboot"

Name: \_\_\_\_\_

### Warmup (10 minutes)

1. Triangle ABC has side lengths 3, 4, and 5. What is its area?

2. In the figure below, each box represents a  $1 \times 1$  cm $^2$  square. What is the area of the figure, in square centimeters? (Source: MATHCOUNTS)



3. A triangle has angles  $n$ ,  $n$ , and 90, in degrees. What is the degree measure of angle  $n$ ?

4. A circle is inscribed in a square with area  $196 \text{ cm}^2$ . What is the area of the circle?

## Class Problems

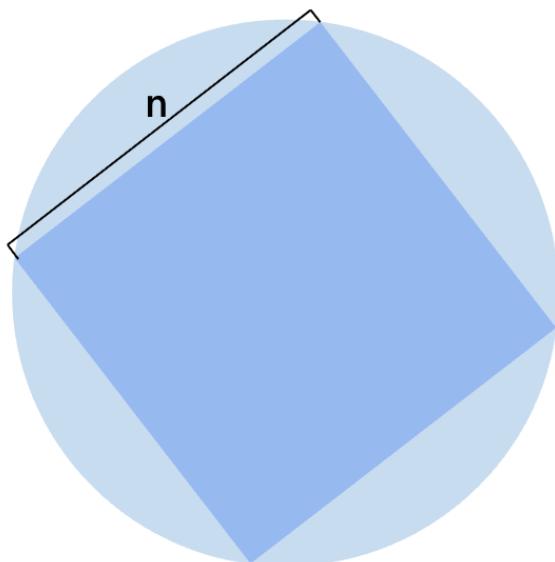
### Inscribed shapes

Inscribed circles, squares, hexagons, etc appear on nearly every major math contest. Let's dive into a few examples and how to find side lengths from inscribed shapes.

Example 1: A square is inscribed in a circle of diameter 6 inches. What is the area of the square, in square inches?

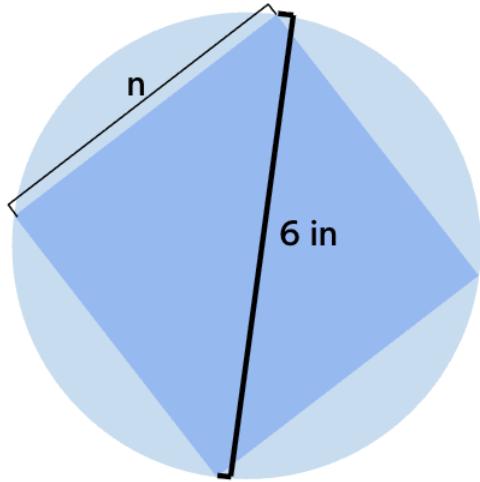
To solve this, let's draw a diagram. The side length is key to finding the area of a square, because the area is just the side length squared.

**Diameter = 6 inches**



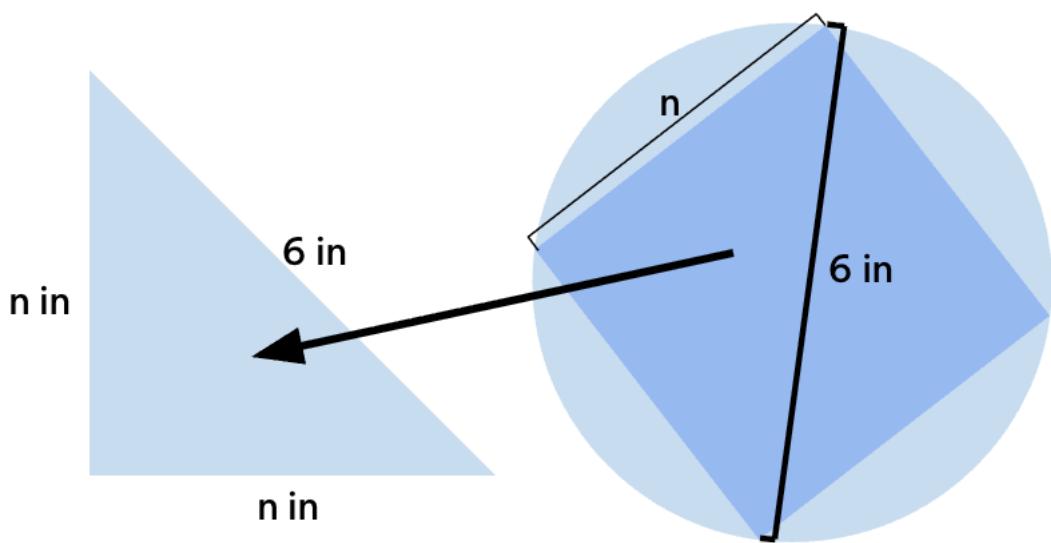
Next, we realize that the opposite corners of the square are the diameter length. Let's draw a line between those.

Diameter = 6 inches



When we draw the line, we notice that our line splits the square into two isosceles right triangles, with each having two sides length  $n$  and one side length 6.

Diameter = 6 inches



$$n^2 + n^2 = 6^2$$

By the pythagorean theorem,  $n^2 + n^2 = 6^2$ .

Therefore,  $2n^2 = 36$

$n^2 = 18$ .

We can then find  $n$ , but from here we realize the area of a square with length  $n$  is just  $n^2$ . So, without needing to simplify, we know our answer is  $n^2 = \mathbf{18}$ .

### Practice

1. A square is inscribed in a circle with radius 16.5 centimeters. What is the area of the square in square centimeters?

2. A square is inscribed in a circle with circumference  $150\pi$ . If the side length of the square is  $n\sqrt{2}$ , what is  $n$ ?

3. A hexagon is inscribed in a circle. The hexagon has area  $24\sqrt{3} \text{ m}^2$ . What is the area of the circle in square meters? (The area of a hexagon is  $3\sqrt{3}/2 * a^2$ , where  $a$  is the side length of the hexagon)

### Sum of angles and angle chasing

One of the best ways to solve problems where it asks for the angle of multiple polygons is by finding the total sum of angles. The sum of the degree measures of interior angles for any

polygon with n sides is  $(n-2)*180^\circ$ . So, the sum of a triangle is  $180^\circ$ , while the sum of a quadrilateral is  $360^\circ$ .

Example: The angles of a pentagon are  $60^\circ$ ,  $70^\circ$ ,  $n^\circ$ ,  $(n+2)^\circ$ ,  $(n+3)^\circ$ . What is n?

Since a pentagon has 5 sides, the sum of the angles is  $(5-2)*180^\circ$ .  $3*180 = 540$ .

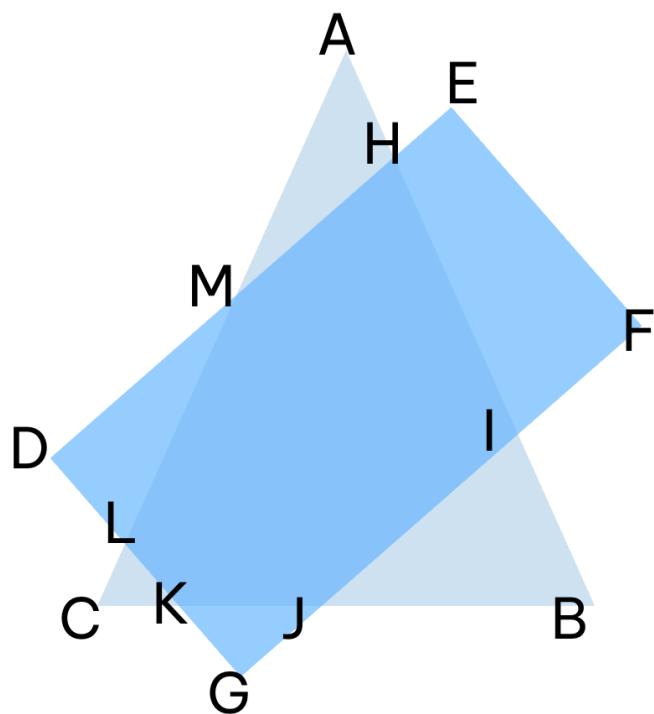
So,  $60+70+n+n+2+n+3 = 540$ . Subtracting 135 from both sides,  $3n = 405$ .

Dividing by 3,  $n = 135$ .

### Practice

1. A hexagon's smallest angle is  $80^\circ$ , and the other 5 angles have the same angle measure. What is the angle measure of the other 5 angles?

2. Rectangle DEFG is layered on top of triangle ABC, forming hexagon HIJKLM. A pentagon is drawn. What is the sum of the angles of HIJKLM?



# Practice Problems

Estimated time: 60 minutes

Check on [chick1n.github.io/EJAcademy](https://chick1n.github.io/EJAcademy).

(Bootcamp Portal, password is “ejboot”)

(case sensitive)

## Easy Difficulty

1. Triangle ABC is right angled at C. If AC = 5 and BC = 12, find AB. [1 coin]

A. 13

B. 7

C. 14

D. 15

E. 17

2. What is the degree measure of each interior angle of a regular decagon (10 sided polygon)? [1 coin]

A. 120

B. 130

C. 140

D. 144

E. 148

3. A circle has a circumference of  $16\pi$  centimeters. What is its area, expressed in terms of  $\pi$ ? [1 coin]

4. Amelia wants to walk to her local library to study biology. If her house is located at the point (3, 4) and the library’s location is (6, 8) on the coordinate plane, how many minutes will it take her to walk straight to the library, granted she walks 0.2 kilometers/minute? [1 coin]

- A. 15  
E. 60

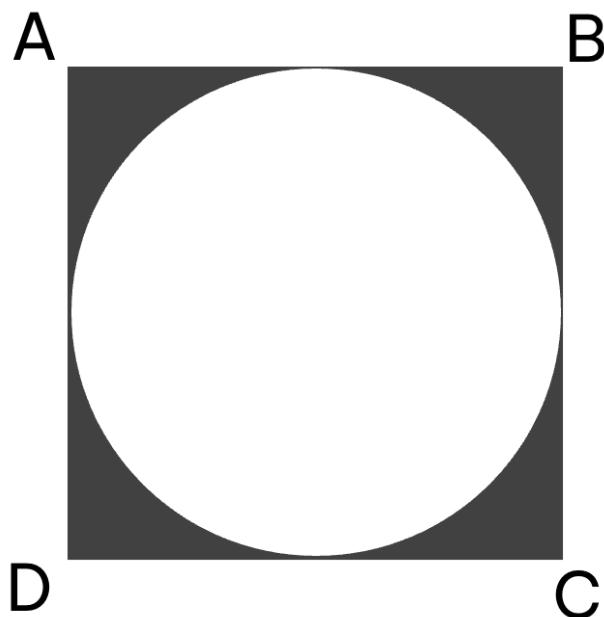
B. 20

C. 25

D. 30

Medium Difficulty

5. A square has side length 12 cm, and a circle is inscribed inside of the square. What is the area inside of the square but outside the circle? (Gray region) [2 coins]



- A.  $144 - 144\pi$   
B.  $144 - 36\pi$   
C.  $36 - 6\pi$   
D.  $36 - 36\pi$   
E.  $144 - 18\pi$

6. The angles of a triangle are  $2n$  deg,  $n+10$  deg, and  $3n-10$  deg. What is the largest angle of this triangle?

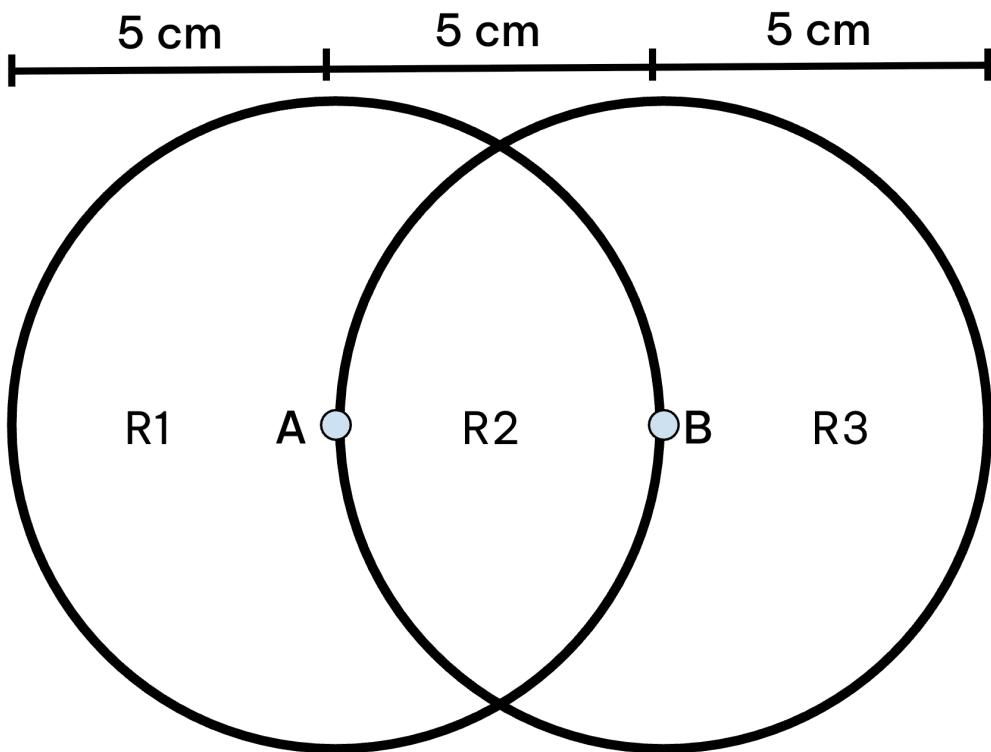
- A. 40  
B. 50  
C. 60  
D. 70  
E. 80

7. A circle is inscribed inside of a hexagon. If the diameter of the circle is 8 units, what is the area of the hexagon?

- A.  $12\sqrt{3}$       B.  $24\sqrt{3}$       C.  $36\sqrt{3}$       D.  $48\sqrt{3}$       E.  
 $64\sqrt{3}$

Hard Difficulty

8. The below diagram shows two overlapping circles, with each of their respective centers on each other's edges. The radius of each circle is 5 centimeters. What is the area of R<sub>2</sub>, the overlapping region between the circles?



# Mock 1

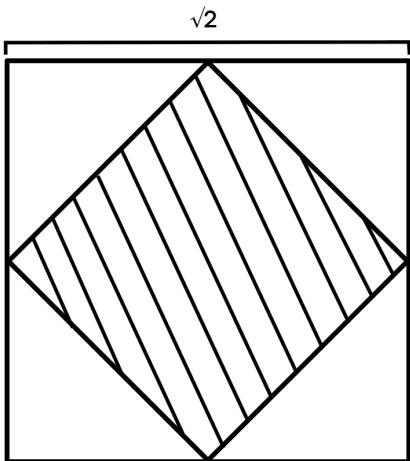
60 minutes, 15 questions

1. Suppose the operator  $\sigma$  returns  $(a+b)+(a*b)$ . What is the value of  $6 \sigma 7$ ?

2. Three consecutive positive integers sum to 24. What is the largest of these three integers?

3. Every time Henry clicks his mechanical pencil, the lead comes out 2 mm. His box of premium Japanese lead contains 20 pieces, and each has 60 mm of usable length. How many times will he click his pencil before he runs out of all his lead?

4. A smaller square is inscribed in a larger square, with all its vertices on the midpoints of the sides of the larger square. What is the area of the shaded region, if the side length of the larger square is  $\sqrt{2}$ ?



5. Reeve's Pet Adoption Center has 8 dogs. If I want to adopt 2 of the 8 dogs, and the order I adopt them doesn't matter, how many ways can I do this?

- A. 28      B. 30      C. 32      D. 34      E. 56

6. How many times does the digit 1 appear from the integer range 110 to 120, inclusive?

7. Gavin has 245 fish in 5 tanks, with one more fish in Tank B compared to Tank A, two more fish in Tank C compared to Tank B, 3 more fish in Tank D compared to Tank C, and 4 more fish in Tank E compared to Tank D. How many fish are in tank C?

8. Bambi the bamboo stalk grows 1 foot on the first day, 3 feet on the second day, 5 feet on the third day, etc. Bambi just sprouted above the soil yesterday, and today he was just planted outside in the sun. How tall will Bambi be after 20 days have passed? (hint: find a pattern)

9. Sally the snail is climbing a mountain that requires 60 feet of climbing. Every 5 seconds, she suddenly climbs up 7 feet, and every 5 seconds, a wind gust blows her down 3 feet. The first gust of wind comes 3 seconds after the first climb. After how many seconds will she cross the finish line?

10. A square is inscribed in a circle, and a circle is inscribed inside of that square. Another square is then inscribed in that circle. What is the ratio of the area of the smallest square to the area of the largest circle?

11. I have a secret number. If my number is  $n$ , and  $n+3$  is divisible by 11, what is the sum of all possible numbers less than 60?

12. It is known the difference of two perfect squares is 51. What is the smallest possible sum of the two squares?

13. I have 6 different textbooks: 3 French, 1 English, 1 Spanish, and 1 Chinese. How many ways are there to arrange these textbooks on my bookshelf, if all three french textbooks must be next to each other?

14. The diamond shaped region is bounded by the graph  $|4x| + |ay| \leq 20$ . What is the value of  $a$  such that the region has an area equal to 15?

15. Let  $f(x)$  equal  $f(x) = -x^2 + 2ax$ . For how many ordered pairs of numbers  $(a,b)$  where  $a$  and  $b$  are integers from 1 to 3 is  $\sqrt{f(a) - f(ab)}$  a perfect square?