WARMUP (Free Response)

1. 6

2. 18

3. 45

4. 49pi

INSCRIBED SHAPES PRACTICE (Free Response)

1. 1089/2

2. 75

3. 16pi

ANGLE SUM PRACTICE (Free Response)

1. 128

2. 720

PRACTICE PROBLEMS

Easy Difficulty

1. 13 (A)

2. 144 (D)

3. 64pi

4. 25 (C)

Medium Difficulty

5. 144 - 36pi (B)

6. 80 (E)

7. 32sqrt3 (NOT IN CHOICES ❌)

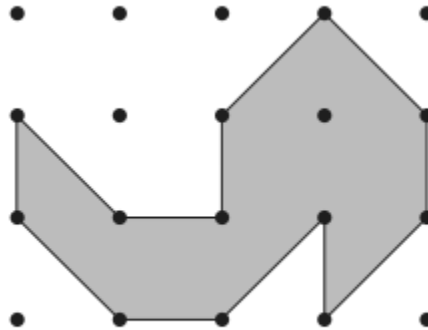8. (50pi/3) - (25sqrt3/2)

MOCK 1 (15 Questions)

Mock 1 (Mixed: some MCQ, mostly Free Response)

1. 55

2. 9

3. 600

4. 1

5. 28 (A)

6. 22

7. 48

8. 400

9. 70

10. 1/pi

11. 150

12. 149

13. 144

14. 40/3

15. 5

SOLUTIONS

Warmup (10 minutes)

1. Triangle ABC has side lengths 3, 4, and 5. What is its area?
   3*4/2 = 6.

2. In the figure below, each box represents a 1x1 cm² square. What is the area of the figure, in square centimeters? (Source: MATHCOUNTS)

   6 cm^2

3. A triangle has angles n, *n*, and 90, in degrees. What is the degree measure of angle n?

45 deg

4. A circle is inscribed in a square with area 196 cm². What is the area of the circle?

Answer: 49pi

Explanation:
Square area 196 means side length is 14.
Circle is inscribed, so diameter = 14, radius = 7.
Area = pi × 7^2 = 49pi

Class Problems

Practice
1. A square is inscribed in a circle with radius 16.5 centimeters. What is the area of the square in square centimeters?

Answer: 1089/2

Explanation:
Circle radius 16.5 means diameter is 33.
That diameter is the diagonal of the square.
Square area = (diagonal^2)/2 = 33^2 / 2 = 1089/2

2. A square is inscribed in a circle with circumference 150π. If the side length of the square is n*sqrt(2), what is n?

Answer: 75

Explanation:
Circumference = 2pi r = 150pi → r = 75
Diameter = 150 (this is square diagonal)
Diagonal of square = (side)*sqrt(2)*
*But side is n*sqrt(2)
So diagonal = n*sqrt(2)*sqrt(2) = 2n
2n = 150 → n = 75

3. A hexagon is inscribed in a circle. The hexagon has area 24sqrt3 m². What is the area of the circle in square meters? (The area of a hexagon is 3sqrt(3)/2 * a², where a is the side length of the hexagon)

Answer: 16pi

Explanation:
Hexagon area formula: (3sqrt3/2) * a^2 = 24sqrt3
Cancel sqrt3:
(3/2)a^2 = 24
a^2 = 16
a = 4
For a regular hexagon inscribed in a circle, radius = side length.
So radius = 4
Circle area = pi * 4^2 = 16pi

Practice
1. A hexagon's smallest angle is 80, and the other 5 angles have the same angle measure. What is the angle measure of the other 5 angles?

Answer: 128

Explanation:
Interior sum for hexagon = (6-2)*180 = 720
80 + 5x = 720
5x = 640
x = 128

2. Rectangle + triangle forms hexagon HIJKLM. Find sum of angles of HIJKLM.
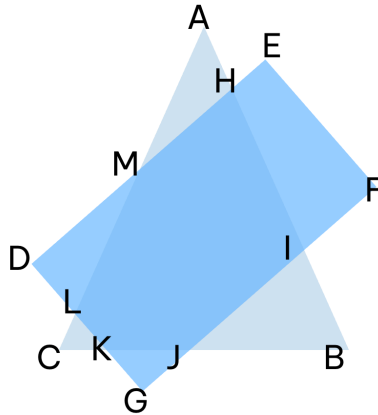
Answer: 720

Explanation:
 HIJKLM is a hexagon.
 Sum of interior angles = 720.
 The picture is a distraction.

2. Rectangle DEFG is layered on top of triangle ABC, forming hexagon HIJKLM. A pentagon is drawn. What is the sum of the angles of HIJKLM?

Answer: 720

Explanation:
 HIJKLM is a hexagon.
 Sum of interior angles = 720.
 The picture is a distraction.

Practice Problems

Estimated time: 60 minutes
Check on chick1n.github.io/EJAcademy.
(Bootcamp Portal, password is "ejboot")
(case sensitive)

Easy Difficulty
1. Triangle ABC is right angled at C. If AC = 5 and BC = 12, find AB. [1 coin]
    A. 13   B. 7   C. 14   D. 15   E. 17

Answer: 13 (choice A)

Explanation:
5-12-13 right triangle.
AB = sqrt(5^2 + 12^2) = sqrt(25 + 144) = 13

2. What is the degree measure of each interior angle of a regular decagon (10 sided polygon)? [1 coin]
    A. 120   B. 130   C. 140   D. 144   E. 148

Answer: 144 (choice D)

Explanation:
Interior angle = $(n-2)180/n$
= $8180/10$
= 144

3. A circle has a circumference of 16π centimeters. What is its area, expressed in terms of π? [1 coin]

Answer: 64pi

Explanation:
2pi r = 16pi $\rightarrow$ r = 8
Area = pi r^2 = 64pi

4. Amelia wants to walk to her local library to study biology. If her house is located at the point (3, 4) and the library's location is (6, 8) on the coordinate plane, how many minutes will it take her to walk straight to the library, granted she walks 0.2 kilometers/minute? [1 coin]
    A. 15   B. 20   C. 25   D. 30
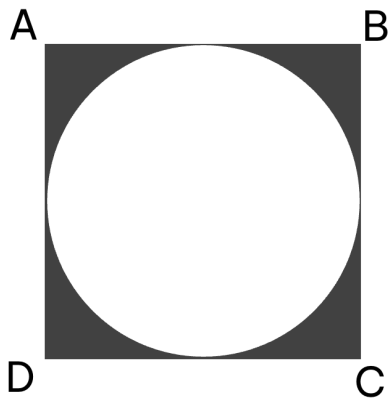   E. 60

Answer: 25 minutes (choice C)

Explanation:
Distance = sqrt((6-3)^2 + (8-4)^2) = sqrt(9 + 16) = 5
Time = distance/speed = 5 / 0.2 = 25

Medium Difficulty
5. A square has side length 12 cm, and a circle is inscribed inside of the square. What is the area inside of the square but outside the circle? (Gray region) [2 coins]

A. 144-144π   B. 144-36π   C. 36-6π   D. 36-36π
E. 144-18π

Answer: 144 - 36pi (choice B)

Explanation:
Square area = 12^2 = 144
Inscribed circle radius = 6
Circle area = 36pi
Gray = 144 - 36pi

6. The angles of a triangle are 2n deg, n+10 deg, and 3n-10 deg. What is the largest angle of this triangle?
A. 40   B. 50   C. 60   D. 70   E.80

Answer: 80 (choice E)

Explanation:
Sum is 180:
2n + (n+10) + (3n-10) = 180
6n = 180
n = 30
Angles: 60, 40, 80
Largest = 80

7. A circle is inscribed inside of a hexagon. If the diameter of the circle is 8 units, what is the area of the hexagon?
A. 12sqrt3   B. 24sqrt3   C. 36sqrt3   D. 48sqrt3   E. 64sqrt3

My computed answer: 32sqrt3
But this is NOT in the answer choices.

Explanation (why I got 32sqrt3):
Inscribed circle radius = 4 (inradius/apothem)
For a regular hexagon, apothem = (sqrt3/2)*side

4 = (sqrt3/2)*s
s = 8/sqrt3
Hexagon area = (3sqrt3/2)*s^2
= (3sqrt3/2) * (64/3)
= 32sqrt3

<u>Hard Difficulty</u>
8. The below diagram shows two overlapping circles, with each of their respective centers on each other's edges. The radius of each circle is 5 centimeters. What is the area of R2, the overlapping region between the circles?

Answer: (50pi/3) - (25sqrt3/2)

Explanation:
Distance between centers is 5.
You get a 120-degree sector overlap shape on each side.
Overlap area = 2*(sector area - triangle area)

Sector area:
120 degrees is 1/3 of a circle
So sector = (1/3)*pi*25 = 25pi/3

Triangle area:
Triangle has sides 5 and 5 with included angle 120.
Area = (1/2)*5*5*sin(120)
sin(120) = sqrt3/2
Triangle area = 25sqrt3/4

One cap area = 25pi/3 - 25sqrt3/4
Two caps = 50pi/3 - 25sqrt3/2

Mock 1

60 minutes, 15 questions

1. Suppose the operator σ returns (a+b)+(a*b). What is the value of 6 σ 7?

Answer: 55

Explanation:
(6+7) + (6*7) = 13 + 42 = 55

2. Three consecutive positive integers sum to 24. What is the largest of these three integers?

Answer: 9

Explanation:
 Middle is 24/3 = 8
 So numbers are 7, 8, 9

3. Every time Henry clicks his mechanical pencil, the lead comes out 2 mm. His box of premium Japanese lead contains 20 pieces, and each has 60 mm of usable length. How many times will he click his pencil before he runs out of all his lead?
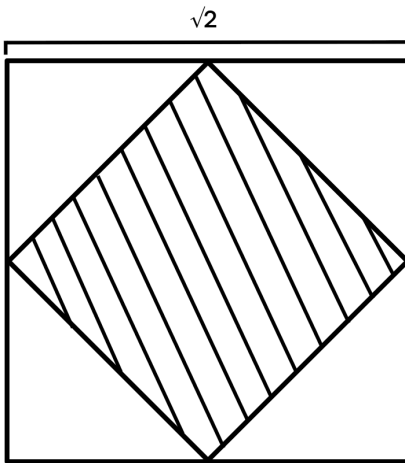
Answer: 600

Explanation:
 Total lead = 20*60 = 1200 mm
 Clicks = 1200/2 = 600

4. A smaller square is inscribed in a larger square, with all its vertices on the midpoints of the sides of the larger square. What is the area of the shaded region, if the side length of the larger square is $\sqrt{2}$?

Answer: 1

Explanation:
 Large square area = (sqrt2)^2 = 2
 Midpoint square area is always half the big square
 So small square area = 1
 Shaded = 2 - 1 = 1

5. Reeve's Pet Adoption Center has 8 dogs. If I want to adopt 2 of the 8 dogs, and the order I adopt them doesn't matter, how many ways can I do this?

A. 28   B. 30   C. 32   D. 34   E. 56

Answer: 28 (choice A)

Explanation:
 Combination 8 choose 2 = 8*7/2 = 28

6. How many times does the digit 1 appear from the integer range 110 to 120, inclusive?

Answer: 22

Explanation:
 110 has 2 ones
 111 has 3 ones
 112–119 each has 2 ones (8 numbers) so 16 ones
 120 has 1 one
 Total = 2 + 3 + 16 + 1 = 22

7. Gavin has 245 fish in 5 tanks, with one more fish in Tank B compared to Tank A, two more fish in Tank C compared to Tank B, 3 more fish in Tank D compared to Tank C, and 4 more fish in Tank E compared to Tank D. How many fish are in tank C?

B = A+1
 C = B+2
 D = C+3
 E = D+4
 Find C.

Answer: 48

Explanation:
 Let A = x
 B = x+1
 C = x+3
 D = x+6
 E = x+10
 Sum = 5x + 20 = 245
 5x = 225
 x = 45
 C = 48

8. Bambi the bamboo stalk grows 1 foot on the first day, 3 feet on the second day, 5 feet on the third day, etc. Bambi just sprouted above the soil yesterday, and today he was just planted outside in the sun. How tall will Bambi be after 20 days have passed? (hint: find a pattern)

Answer: 400

Explanation:
 This is sum of first 20 odd numbers.
 That always equals $20^2 = 400$

9. Sally the snail is climbing a mountain that requires 60 feet of climbing. Every 5 seconds, she suddenly climbs up 7 feet, and every 5 seconds, a wind gust blows her down 3 feet. The first gust of wind comes 3 seconds after the first climb. After how many seconds will she cross the finish line?

Answer: 70 seconds

Explanation:
 Every 5 seconds, she gains net +4 (up 7 then later down 3).
 But she crosses right AFTER a climb (before the next gust).
 After n climbs, height right after the nth climb is:
 $7n - 3(n-1) = 4n + 3$
 We want $4n + 3 >= 60$
 $4n >= 57$
 $n = 15$
 15th climb happens at time 0,5,10,... so:
 $(15-1)*5 = 70$ seconds

10. A square is inscribed in a circle, and a circle is inscribed inside of that square. Another square is then inscribed in that circle. What is the ratio of the area of the smallest square to the area of the largest circle?

Answer: 1/pi

Explanation:
 Let largest circle radius be R.
 First square has side R*sqrt2*.
 *Circle inside that has radius R/sqrt2.
 Square inside that circle ends up with side R.
 Smallest square area = R^2
 Largest circle area = piR^2
 Ratio = 1/pi

11. I have a secret number. If my number is n, and n+3 is divisible by 11, what is the sum of all possible numbers less than 60?

Answer: 150

Explanation:
n = 11k – 3
k=1 gives 8
k=2 gives 19
k=3 gives 30
k=4 gives 41
k=5 gives 52
Next would be 63 too big
Sum = 8+19+30+41+52 = 150

12. It is known the difference of two perfect squares is 51. What is the smallest possible sum of the two squares?

Answer: 149

Explanation:
$a^2 - b^2 = 51$
$(a-b)(a+b) = 51$
Factor pairs: (1,51) and (3,17)

(1,51):
a=26, b=25 → sum squares = 1301

(3,17):
a=10, b=7 → sum squares = 100+49 = 149
Smallest is 149

13. I have 6 different textbooks: 3 French, 1 English, 1 Spanish, and 1 Chinese. How many ways are there to arrange these textbooks on my bookshelf, if all three french textbooks must be next to each other?

Answer: 144

Explanation:
Treat the 3 French as 1 block.
Now 4 objects: (French-block, Eng, Span, Chinese) → 4! = 24 ways
French block internal order: 3! = 6 ways
Total = 24*6 = 144

14. The diamond shaped region is bounded by the graph $|4x|+|ay| \leq 20$. What is the value of a such that the region has an area equal to 15?

Answer: 40/3

Explanation:
When y=0:
$|4x| <= 20 \rightarrow |x| <= 5$
Horizontal diagonal length = 10

When x=0:
$|a y| <= 20 \rightarrow |y| <= 20/a$
Vertical diagonal length = 40/a

Diamond area = (1/2)*(diag1)(diag2)*
Area = (1/2)*10*(40/a) = 200/a
Set 200/a = 15
a = 200/15 = 40/3

15. Let f(x) equal $f(x) = -x^2+2ax$. For how many ordered pairs of numbers (a,b) where a and b are integers from 1 to 3 is sqrt(f(a)-f(ab)) a perfect square?

Answer: 5

Explanation:
Compute f(a):
$f(a) = -a^2 + 2a*a = a^2$

Compute f(ab):
$f(ab) = -(ab)^2 + 2a*(ab)$
$= -a^2 b^2 + 2a^2 b$
$= a^2(-b^2 + 2b)$

Difference:
f(a) - f(ab)
$= a^2 - a^2(-b^2 + 2b)$
$= a^2(1 + b^2 - 2b)$
$= a^2(b-1)^2$

Square root:
$sqrt(a^2(b-1)^2) = a*|b-1|$

Now test b:

b=1 → *a0 = 0 perfect square for all a (3 pairs)*
b=2 → a1 = a perfect square only when a=1 (1 pair)
b=3 → a*2 = 2a, perfect square only when a=2 (since 4 is square) (1 pair)

Total = 3 + 1 + 1 = 5