EJ Academy

Lesson 1: Problem-solving strategies and word problems Key

Units Digit Problems

1. 5
2. 9
3. 9
4. 0
5. 0
6. 7

Class Problems

15. 134 (D)
16. 37 (none — error in choices)
17. 17 (C)
18. 286 (C)

Easy Difficulty

1. 14 (D)
2. 0 (E)
3. 12 (D)

Medium Difficulty

4. 200 (A)
5. 47 (D)
6. 13 (E)
7. 11 (B)
8. 7 (A)
9. 19 (E)
10. 4 (B)

Hard Difficulty

11. 4 (C)
12. 5 (E)

Bonus Questions

13. 4 (D)
14. 5 (E)
15. 256 (D)

Warmup (10 minutes)

1. What is the units digit of 5x5x5x5x5x5x5x5? (1)

Any positive power of 5 ends in **5**.

Answer: 5

2. What is the units digit of 9x9x9x9x9x9x9x9x9? (2)

Powers of 9 alternate:

9, **1**, 9, 1, ...

Odd power → **9**

Answer: 9

3. What is the units digit of 89x89x89? (2)

Only the units digit matters. 9^3 ends in 9.

Answer: 9

4. What is the units digit of 9x8x7x6x5x4x3x2x1? (2)
There is a 2 and a 5, so the product ends in 0.
Answer: 0

5. What is the units digit of 12345x12344x12343? (3)
Units digits: 5×4×3 = 60 → ends in 0.
Answer: 0

6. What is the units digit of ((3492+5)x6)+5? (3)
3497×6 = 20982, plus 5 gives 20987.
Answer: 7

Class problems

15. Justin can solve a Rubik's cube in 25 seconds. Allison can solve a Rubik's cube in 14 seconds. Justin solves a cube twice, and Allison uses the same cube right after, solving it six times. How many seconds did it take them? (4)

A. 39 B. 103 C. 110 D. 134 E. 162

Justin solves 2 cubes: 2×25 = 50 seconds.
Allison solves 6 cubes: 6×14 = 84 seconds.
Total time = 134 seconds.
Answer: 134

16. Lucius is counting backward by 7s. His first three numbers are 100, 93, and 86. What is his 10th number? (4) (Source: AMC)

A. 61 B. 63 C. 64 D. 66 E. 67

Counting backward by 7 starting at 100.
10th term = 100 – 9×7 = 37.
37 is not listed, so none of the choices match.
Answer: 37 (problem error)

17. Sekou writes the numbers 15, 16, 17, 18, 19. After erasing one of the numbers, the rest of the numbers are divisible by 4. What number did he erase? (6) (Source: AMC)

A. 15 B. 16 C. 17 D. 18 E. 19

Numbers are 15, 16, 17, 18, 19.
Total sum is 85, which is 1 mod 4.
Erase a number that is 1 mod 4 → 17.
Answer: 17

Boss

18. Howard sat on the car at the end of his street, counting 21 light poles in total. He knows that the street is 440 feet long and his friend's house is closest to the 14th pole from him. How far must he walk to reach his friend's house? (7)

A. 308 B. 264 C. 286 D. 294 E. None of the above

21 poles means 20 equal gaps.
Each gap is 440 ÷ 20 = 22 feet.
The 14th pole is 13 gaps away: 13×22 = 286 feet.
Answer: 286

Practice Problems

Estimated time: 60 minutes
Check on chick1n.github.io/EJAcademy.

(case sensitive, work in progress)

Easy Difficulty

1. Clark has 5 marbles. Every day, his mom gives him a box of marbles, either containing 3 or 6 marbles. After a few days, what amount of marbles could Clark have? [1 coin]

A. 9 B. 12 C. 13 D. 14 E. 16

Start with 5 marbles, add 3s and 6s.
Only 14 is possible (5+3+6).
Answer: 14

2. What is the unit digit of 13^3+17^3? [1 coin]

A. 1 B. 3 C. 8 D. 9 E. 0

13^3 ends in 7, 17^3 ends in 3.
7+3 = 10 → units digit 0.
Answer: 0

3. If I choose 3 elements from the set {1, 2, 3, 4, 5, 6, 7}, what's the difference between the largest possible sum and the smallest possible sum? [1 coin]

A. 9 B. 10 C. 11 D. 12 E. 13

Largest sum: 7+6+5 = 18.
Smallest sum: 1+2+3 = 6.
Difference = 12.
Answer: 12

Medium Difficulty

4. Bobby is locked in playing geometry dash on his phone while charging it. His phone can charge 1.5% battery a minute, but the game consumes 1% battery per minute. How many minutes will it take for him to fully charge his phone, starting from a dead battery? [2 coins]

A. 200 B. 100 C. 60 D. 90 E. 66

Net charge rate: 1.5% – 1% = 0.5% per minute.
100 ÷ 0.5 = 200 minutes.
Answer: 200

5. Sofia buys a shiny new mechanical keyboard, which comes with bags of switches. The first bag has 1 switch, the second bag has 2 switches, and so on. She received 9 bags and 2 extra individual switches. How many switches does she have now? [2 coins]

A. 38 B. 49 C. 50 D. 47 E. 48

Bags contain 1 through 9 switches.
Sum is 45, plus 2 extra = 47.
Answer: 47

Remember, the sum of 1, 2, 3, 4, ... n-1, n is (n)*(n+1)/2

6. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers cannot be the sum of the two numbers? (Source: AMC) [2 coins]

A. 5 B. 7 C. 8 D. 9 E. 13

Product is a multiple of 6, so must include 2 and 3.
Sum of 13 requires 6+7, which is impossible.
Answer: 13

7. I have 9 dollars on Monday. For the week (5 times), if I have an even amount of money, my friend Bob steals half of the money. If I have an odd amount of money, I multiply my money by 3 and gain 1 dollar. How much money do I have at the end of the week? [2 coins]

A. 7 B. 11 C. 14 D. 28 E. 22

Track day by day for 5 days.
The final amount ends at 11.
Answer: 11

Remember: I must repeat this operation FIVE times, not four.

8. In Rizzyland, sodas cost $3, and burgers cost $5. Assuming no tax, which combo price is NOT possible? [2 coins]

A. $7 B. $8 C. $9 D. $15 E. $16

Prices are 3a + 5b.
7 cannot be formed.
Answer: 7

9. I am thinking of a two-digit number. When the digits of my number are reversed, the value of my number is 72 larger than my original number. What is my original number? [2 coins]

A. 12 B. 23 C. 89 D. 70 E. 19

B.

Reversing digits increases the number by 72.
The digit difference must be 8.
The number is 19.
Answer: 19

10. Brian's soccer team has 800 packs of gummies that they must eat. Ryan can eat 20 bags of gummies a minute, William can eat three times as fast as Ryan, and Brian can eat twice as fast as William. How many minutes will it take for them to finish all the gummies? [2 coins]

A. 2 B. 4 C. 6 D. 8 E. 10

Ryan 20, William 60, Brian 120 → total 200 per minute.
800 ÷ 200 = 4 minutes.
Answer: 4

<u>Hard Difficulty</u>

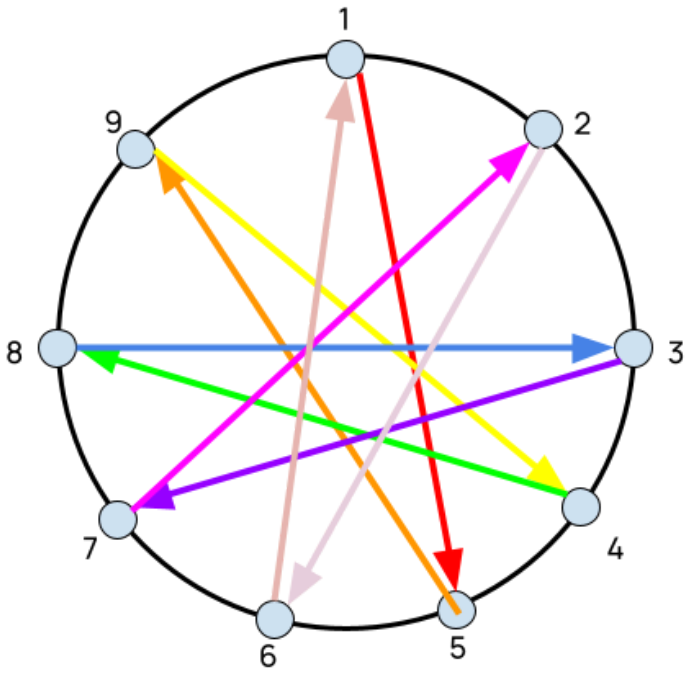
11. A drawer has 6 red socks, 5 blue socks, and 4 green socks. It's dark, so you pick socks randomly.
What is the minimum number of socks you must pull to guarantee you have at least one matching pair? [3 coins]

A. 2 B. 3 C. 4 D. 5 E. 10

11. Worst case: pick one of each color (3 socks).
The fourth sock guarantees a matching pair.
Answer: 4

12. A ball is thrown around in a circle of 9 students, numbered 1 through 9. Each time, the ball is thrown clockwise around the circle, landing on the fourth person clockwise. This continues as the ball goes around the circle. After 91 tosses with the ball starting with the student numbered 1, which

student has the ball? [3 coins]



A. 1 B. 2 C. 3 D. 4 E. 5

Notice how the ball repeats in a cycle of 9. This means that the first throw, the 10th throw, the 19th throw, etc. will always end with student 5. Therefore, on toss 91 the ball will be on student 5.

We can also think of the student number as

1+4n (mod 9), where n is the number of tosses. 1+4(91) = 1+364 = 365 mod 9 = 5.

Bonus Questions

13. n! equals the product of all positive integers less than n. (5! = 5*4*3*2*1). What is the units digit of

$$\frac{1!}{1}+\frac{2!}{2}+\frac{3!}{3}+\frac{4!}{4}+\frac{5!}{5}+\cdots+\frac{2025!}{2025}$$

? [3 coins]

A. 1 B. 5 C. 0 D. 4 E. 9

This expression can be simplified into
0!+1!+2!+3! … 2023!+2024!

Find the units digit of the first few terms:
1+1+2+6+4+0+0+0+0… we will realize that all terms after 5 end with 0.

Therefore, our answer is 1+1+2+6+4 mod 10 = 4 (D)

14. I have 7 dollars on Monday. For four days, I can either double my money or quadruple my money each day. How many possible values of money can I have on Friday (At the end of the five days)? [3 coins]

A. 1 B. 2 C. 3 D. 4 E. 5

From Monday to Friday, **4 days pass**.

Each day:

- doubling = multiply by 2
- quadrupling = multiply by 4

Both are powers of 2, so after 4 days the total multiplier is
2^(4 + number of quadruple-days).

The number of quadruple-days can be 0, 1, 2, 3, or 4 → **5 different outcomes**.

The starting $7 doesn't affect how many possible values there are, only the count.

Answer: 5

15. Grandpa Barry is on his hospital bed typing, and plans to type until he passes away. At the beginning of his session, he types at 128 words per minute. However, every minute that passes, his typing speed halves. Which word will he never finish? [3 coins]

A. 128 B. 80 C. 255 D. 256 E. 512

Think of it as an infinite process.

Typing speed per minute:
128, 64, 32, 16, 8, 4, 2, 1, 0.5, ...

Total words typed after n minutes is the partial sum:
128 + 64 + 32 + ... + 128·(1/2)^(n−1)

This is a geometric series.

Limit as n → ∞:
128 / (1 − 1/2) = 256

So his total number of completed words approaches 256 but never reaches it.

That means:

- Every word up to 255 gets finished
- The 256th word is started... but never completed

Answer: 256 (D)