where $\mathrm{SP}(V_i, R_{ia}, R_{jb}, V_j) = \mathrm{SP}(V_i, R_{ia}) + \mathrm{SP}(R_{ia}, R_{jb}) + \mathrm{SP}(R_{jb}, V_j)$. Here V_i and R_{ia} are in the same line-segment, so we can use Lemma 1 to calculate $\mathrm{SP}(V_i, R_{ia})$, so does $\mathrm{SP}(R_{jb}, V_j)$. The $SP(R_{ia}, R_{jb})$ is the only part we need to calculate using all-pair shortest path algorithms. We should notice that R_{ia} and R_{jb} could be the same vertex, but it does not make any difference to the lemma.

5.1.2. Vertex Reduction

VRA iteratively removes ignorable vertices from the graph until only reserved vertices remain. In each iteration, we remove all ignorable vertices from the graph but keep the edges. If there is more than one route between a pair of reserved vertices, we keep the shortest one and remove others.

5.1.2.1. Remove Ignorable Vertices

Since the degree of ignorable vertices is no more than 2, an ignorable vertex has only 0, 1 or 2 neighbours. In the case of 0 or 1 neighbour, we simply delete the ignorable vertex; if it has two neighbours, we connect its two neighbours before it is removed, as shown in Figure 5.1. When we remove an ignorable vertex, the weight of the line which connects its two neighbours is equal to the sum of its recent two lines' weights (i.e., $W_l + W_r$). After we remove all ignorable vertices in a line-segment, the two reserved vertices at the end of the line-segment are connected with a line directly, whose weight is the sum of all the intermediate ones' (i.e., $W_1 + W_2 + W_3 + W_4$).

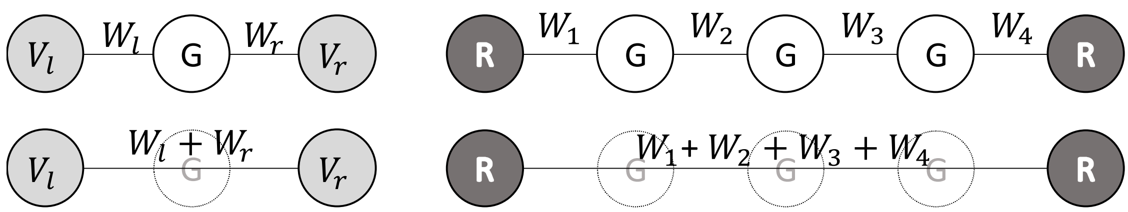

Figure 5.1: Remove Ignorable Vertices