Decontamination from Black Virus Using Parallel Strategy

by

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Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
In partial fulfillment of the requirements
For the Master degree in
Computer Science

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Abstract

In this thesis, the problem of decontaminating networks from *black virus* (BVs) using parallel strategy with a team of system mobile agents (the BVD problem) is studied. The BV is a harmful process whose initial location is unknown a priori. It destroys any agent arriving at the network site where it resides, and once triggered, it spreads to all the neighboring sites, i.e, its clones, thus increasing its presence in the network. In order to permanently remove any presence of the BV with as less execution time as possible and minimum number of site infections (and thus casualties), we propose parallel strategy to decontaminate the BVs: instead of exploring the network step by step we employ a group of agents who follow the same protocol to explore the network at the same time, thus dramatically reducing the time needed in the exploration phase and minimizing the casualties. Different protocols are proposed in meshes, tori, and chordal rings following the monotonicity principle. Then we analyze the cost of all our solutions and compare to the asynchronous BV decontamination. Finally conclusion marks are presented and future researches are proposed.

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Chapter 1

Introduction

A distributed system is a group of computational entities cooperating with each other to achieve one or more tasks. This thesis deals with distributed computing by mobile agents in networks. More specifically, we deal with the problem of deploying a group of mobile agents who follow the same protocol, to explore the network and decontaminate a dangerous virus (called Black Virus) present on some of the the network nodes. In this Chapter, the motivations of the problem are provided, following is a brief summary of the contributions. Finally, an overview of the organization of the thesis is presented.

1.1 Problem and Motivation

Mobile agents are widely used in distributed systems and networks. Their employ can cause some security issues, thus threatening to the network. For example, a contaminated or infected host can destroy working agents for various malicious purposes; A malicious agent can contaminate or infect other computer nodes so they become malfunctional or crashed. These static harmful hosts, often called *Black Holes* (BH) trigger the problem called *Black Hole Search* (BHS), the focus of which is to locate their positions. This problem has been studied in many variants. For example, different topologies and different settings (syn-

chronous and asynchronous). Harmful agents trigger instead the problem called *Intruder Capture* (IC). The main focus of the Intruder Capture is to deploy a group of mobile agents to capture an extraneous mobile agent (the intruder) who moves arbitrarily fast through the network and infects the visiting sites. Also this problem has been investigated in a variety of topologies. More detailed literature review will be provided in Chapter 2. Note that a black hole is static and damages only the agents reaching it without leaving any detectable trace. The intruder, instead, is mobile and harmful to the network nodes but does not cause any harm to other system agents. A new harmful presence called *black virus* BV has been initially introduced by Cai et al. in[**?**]. It is a dangerous process that resides at an unknown site in a network and destroys any upcoming agents, but unlike the BH, the node where the original BV resides becomes clean when an agent reaches it. At the same time, the original BV multiplies, creating clones of itself, and spreads to all neighbouring nodes, thus increasing its presence and damage in the network. A BV is destroyed when it moves to a site where there is already an agent. Based on this harmful presence, a new problem called *Black Virus Decontamination* (BVD) is presented by Cai et al., the main focus of which is to use a group of system agents to permanently remove any presence of the BV from the network. A protocol defining the actions of the agents solves the BVD problem if at least one agent survives and the network is free of BVs. Also, a desirable property of a decontamination protocol is that the nodes which have been explored or cleaned by mobile agents are not be recontaminated by the BV spreading. A solution protocol with such a property is called *monotone* (see[**?**]). Some important cost measures are: the number of node infections by the BVs (casualties); s the size of the team, i.e, the number of agents employed by the solution, the time needed by the solution. Solutions in which the agents explore the network's nodes sequentially have been proposed in [**?**], [**?**] and [**?**] in various topologies. The size of the team is minimizes in [**?** **?**] and the number of site infections is also minimized in such case. However, in all those solutions, the time cost is quite high and not scalable, as it is linear in the size of the network. In

the thesis, we are interested in faster solutions using parallel strategies, where we deploy a larger number of mobile agents following the same protocol to decontaminate the network concurrently, with the goal to decrease time. For our strategies, we also introduce a new measure, the *total working time* (TWT), which is obtained as a combination of all three classical costs (number of agents, execution time, casualties), and we compare it with the TWT of the existing algorithms.

1.2 Our Contribution

1. In this thesis, we propose parallel strategies to solve the BVD problem. This is the first attempt to deal with this issue in a parallel way. Like in previous work, agents are not allowed to communicate with each other unless they are in the same network node so the protocol should enable the agents in different nodes to move independently but in a synchronized fashion so to achieve the global goal. We give simple but efficient solution to deal with this problem with acceptable costs. We also give the size of the minimum exploring team to guarantee both the TWT and the casualties we reach.

2. The BVD problem is investigated for three important topologies: *meshes*, *tori*, *chordal rings*. All the protocols are asymptotically optimal both in term of TWT and casualties. We compare our solution with [**?**] and [**?**] in which the exploring route is sequential, and the result is that our solution is better than theirs in terms of TWT and casualties. We should be point out that in chordal rings especially, the more complicated the chordal ring becomes, the higher are the improvements of our algorithms in terms of in TWT compared to [**?**].

1.3 Thesis Organization

The thesis is organized as follows:

Chapter 2 contains a literature review on related problems. We begin by reviewing the Black Hole Search and Intruder Capture problem, we then focus on the solution of BVD problem where the mobile agents explore the network sequentially. The problem has been studied in different topologies: two-dimensional grids, three-dimensional grids, tori, chordal rings, hypercubes and arbitrary network. Also, the variant of this problem, which is decontamination of an arbitrary network from multiple black virus is reviewed.

Chapter 3 introduces terminology, definitions and model for the BVD problem used in the rest of the thesis. Also we describe the high level ideas that serve as the basis of all our solutions. Since monontonicity is a necessary condition for spread optimality, we explain this concept and draw some conclusions.

Chapter 4 focuses on the BVD problem for the mesh topology. In this chapter, an optimal algorithm in terms of casualties and TWT is developed. Complexity analysis in terms of casualty and TWT are performed and results obtained. Some comparisons are also made between our solution and [**?**] and the result shows that our solution is better.

Chapter 5 presents the BVD problem for the chordal ring topology. In this chapter, we introduce the *Three Jump Notifying Technique* (TJNT) to manipulate each mobile agent to efficiently move along their route during the exploration , and to avoid the spread of clone BV after the original BV is triggered. Based on this technique, we develop the parallel strategy for the mobile agents to decontaminate the chordal ring. Complexity analysis in terms of casualty and TWT are performed. Finally some comparisons are made between our solution and [**?**] and the result shows that our solution performs better.

Chapter 6 summaries the main conclusion of our work and present some open problems and future work.

Chapter 2

Literature Review

Mobile agents have been widely used in distributed computing due to their features, especially mobility, which allow them to migrate between computers at any time during their execution. A group of agents can be used to perform a variety of tasks, for example, network exploration, monitoring, maintenance, etc. However, the introduction of mobile agent tend to cause security problems, possibly threatening the network. Various security issues and solution algorithms have been proposed by Flocchini and Santoro in [**?**]. Generally, the threat that the mobile agents cause are divided into two categories: in first case, the malicious agents can cause network nodes malfunction or crash by contaminating or infecting them (harmful agents); in second case, the contaminated or infected hosts can destroy working agent for various malicious purposes (harmful hosts). These two threats trigger two problems: Black Hole Search (BHS) and Intruder Capture (IC) which will be introduced in the following Sections. We then review the BVD problem, which deals with the decontamination of a harmful presence that cause the network node malfunction, leaves the network node clean when it is triggered, and spreads to all its neighbouring nodes increasing its presences. In the section introducing BVD problem, we describe the solutions that have been proposed to decontaminate the networks, and review different decontamination algorithms based on different strategies.

2.1 Black Hole Search, BHS

The BHS problem assumes there is a BH or multiple static BHs residing at certain network nodes. The BHs will destroy any upcoming agents without leaving any detectable trace.The task is to use a team of agents to locate the black hole(s). The task is completed when at least one agent survives and reports the location(s) of the black hole(s). Note that this task is dangerous for the agent; the solution is based on graph exploration and the goal can be reached totally depending on the sacrifice of some agents and on the observation and deduction by surviving agents **???**. In [**?**], Das et al. considered a model for unknown environment with dispersed agents under the weakest possible setting, many exploration models and works were included in this article. **???** Their research includes different ways to mark the node and to communicate among agents (pebble, marker, and whiteboard), one or multiple agents, asynchronous agents, different topologies (ring, tree, directed or undirected) and different agent memory size limitation and etc.

The BHS problem has been widely studied in various topologies and settings: by Chalopin [**?** **?**] in asynchronous rings and tori, Czyzowicz et al. [**?**] in directed graphs, Dobrev et al. [**?**] in arbitrary graph, [**?**] in anonymous ring and [**?**] in common interconnection networks, .Cooper et. al. [**?**] using multiple agents, and Glaus [**?**] locating a blacking hole without using the knowledge of incoming link. What is more, Czyzowicz et al. [**?**] studies the complexity of searching for a black hole, Klasing et. al. [**?**] investigated the approximation results and hardness results about the black hole search in arbitrary networks, and [**?**]investigated the approximation bounds for black hole search problems.

The main difference made in the literature is that whether the system is synchronous or asynchronous. In synchronous model, one agent is sent to explore a node, and other agents can know whether the node is safe or not by waiting until a bounded timer expires. Synchronous model allows detection of unknown number of multiple BH's. In asynchronous

model, there is not bounded agent's travelling time on link, so there is no way to distinguish between a slow link and black hole. The "cautious walk" is the main exploring method to check if the node is safe or not. If one agent is sent to explore to a node, before the node is confirmed safe, no more agents are sent to the node. Asynchronous model only allows detection of known number of BH's with an additional assumption that the number of nodes in the topology is known and if the sum of the explored nodes and the number of the BH's is equal to the number of nodes in the network. In [**?** **?** **?**], they study a variation of model where communication among agents is achieved by placing tokens on the nodes. In [**?** **?**], they investigate the case of black lines in arbitrary networks for anonymous and non-anonymous nodes. In synchronous setting, Czyzowicz et al.[**?**] located a black hole in a synchronous tree network for a given starting node and the minimum number of agents to locate a black hole is two. Cooper et al.[**?**] consider locating and repairing faults in network. The agent would die after repairing the fault. They present that the number of step sufficient to complete this task is $\Theta(n/k + D)$, where n is the number of nodes in the network and D is the diameter of the network. Finally, some recent studies dealt with scattered agents searching for a black hole in rings and toris [**?** **?** **?**]. Also, Hohl, Ng, Sander et. al in [**?** **?** **?** **?**] discussed various methods to protect mobile agent against malicious host. Note that the BH is static which means that it does not propagate in the network and thus not harmful to other sites. The number of BHs does not increase or decrease.

What is worth pointing out is that the number of BHs remains the same as it of the beginning, thus not causing harm to other sites of the network. **A bit more...**

2.2 Intruder Capture

The IC problem assumes that there is an intruder moving with an arbitrary speed from node to node in the network and contaminating the sites it visits. The goal is to deploy a

group of mobile agents to capture the intruder; the intruder is captured when it comes in contact with an agent. Note that the intruder does not cause any harm to the upcoming agents. This problem is equivalent to the problem of decontaminating a network contaminated by a virus while avoiding any recontamination which is also called connected graph searching and has been extensively studied (see [**?**]). The Intruder Capture problem is first introduced in [**?**] and has been widely investigated in a variety network topologies: trees [**?** **?** **?**], hypercubes[**?**], multi-dimensional grids[**?**], pyramids[**?**],chordal rings[**?**], tori[**?**], outerplanar graphs[**?**], etc. The studies of arbitrary graph has been started in [**?** **?**]. **?????** Note that monotone is a critical principle in the solutions of IC problems. **A bit more...**

2.3 Agent Capabilities

Different capacities granted to the mobile agents have an impart on solving the BHS problem, IC problem and also the BVD problem.We discuss these capabilities in the following section.

Communication Mechanisms Mobile agents can communicate with each other only when they are in the same node in a network. Various communication methods have been studied in literature: whiteboard, tokens and time-out.

In [**?** **?** **?** **?**], the whiteboard model is used, which is a storage space located at each node and agents arriving there are able to read and write. In the token model, (see [**?** **?**]), tokens are like pebbles that the agents can drop off and pick up at nodes or edges. Time-out mechanisms achieve implicit communication (or synchronization) among the agents but can only be used in synchronous settings. (see [**?** **?** **?**]).

Knowledge of the topology Different assumptions on the agents' knowledge of the topology have an impact on solutions of some of the problems mentioned above, for example, the BHS problem. In [**?**], Dobrev et al. present three types of topological knowledge in an asynchronous arbitrary network and show the results of the BHS problem based on different setting of the topology knowledge.

Other capabilities In some studies, agents are endowed with visibility, which means that they can see whether or not their neighbouring nodes are clean or contaminated (see [**?** **?**]).It is observed that the visibility assumption allows to drastically decrease the time and move complexities in tori, chordal rings and hypercubes when dealing with IC problem. For example, in chordal ring $C_n\{d_1 = 1, d_2, ..., d_k\}$, the number of agents, the time and the moves required in local model are $(2d_k + 1)$, $3n - 4d_k - 1$, $4n - 6d_k - 1$ respectively, while in the visibility model, they are $2d_k$, $\left\lceil \frac{n-2d_k}{2(d_k - d_{k-1})} \right\rceil$, $n - 2d_k$. In tori, the number of agents, time and moves required are, respectively, $2h + 1$, $hk - 2h$, $2hk - 4h - 1$, while in the visibility model they are $2h$, $\lceil \frac{k-2}{2} \rceil$, $hk - 2h$, respectively. The authors also compare the complexity of both models in hypercubes and propose an algorithm that requires $\Theta\ (\frac{n}{\sqrt{logn}})$ agents and $O(n \log n)$ moves, and another algorithm in the visibility model that requires $\frac{n}{2}$ agents and $O(n \log n)$ moves.

In [**?**], the concept of *k-hop visibility* is presented. The agents have *full topology* knowledge if each of them have a map in their memory of the entire network including the identities of the node and the labels of the edges. An agent has *k-hop visibility*, when at a node v the agent can see the k-neighbourhood $N^{(v)}$ of v, including the node identities and the edge labels. Note that *Diam-hop* visibility is equivalent to full topological knowledge.

Another interesting capability of agents is cloning, which is introduced in [**?**]. Cloning is the capacity for an agent to create copies of itself. In [**?**] it is also discussed how the combination of different capacities allows different optimal strategies for the IC problem in the hypercube. For example, a time and move optimal algorithm is devised when

visibility and cloning are assumed or when cloning, and synchronicity are assumed and the model is local. However, the time and move-optimal strategy is obtained at the expense of increasing the number of agents. The last capability of agents that has been discussed is immunity, where a node is immune from recontamination after an agent departs under some conditions. Two kinds of immunity have been proposed: local and temporal. In local immunity, (see [**?** **?**], the immunity of a node depends on the state of its neighbouring nodes. More specifically, a node remains clean after the departure of an agent until more than half of its neighbours are contaminated. In the temporal immunity, a node is immune for a specific amount of time. The node remains clean until this time expires and it becomes recontaminated if at least one of its neighbours are contaminated. In models without immunity assumption, a node becomes recontaminated if it has at least one contaminated neighbours.

2.4 Black Virus Decontamination

2.4.1 Overview

The BVD problem is first introduced by Cai at al. in citeCai: A black virus is a extraneous harmful process endowed with capabilities for destruction and spreading. The location of the initial BV(s) is known a priori. Like a BH, a BV destroys any agent arriving at the network where it resides. When that happen, the clones of the original BV spread to all it neighbouring nodes and remain inactive until an agent arrives. The BVD problem is to permanently remove any BVs in the network using a team of mobile agents. In [**?**] the strategy to decontaminate a BV is to surround all its neighbouring nodes and send an agent to each of them. In this case, the node where the original BV resides is clean and all its clones are destroyed by the guarding agents in its neighbouring nodes.The authors have presented different protocols in various topologies: q-grid, q-torus, hypercudes

([**?**]), and arbitrary graph [**?**]. A basic idea of implementing the decontamination has also been proposed by assuming that the timing is asynchronous which divides the whole decontamination process into two part: "shadowed exploration"and "surround and eliminate". In order to minimize the spread of the virus, they use a "safe-exploration" technique which is executed by at least two agents: the "Explorer Agent" and the "Leader Explorer Agent". Both agents initially resides at a safe node $u$called, the *homebase*. The Explorer Agent moves to a node v to explore it and it needs to return to node u to report the node v is safe. The "Leader Explorer Agent" determines if the node v is safe or not by the arrival of the "Explorer Agent". If node v is safe, both agents move to node v. For the purpose of insuring monotonicity, at any point in time the already explored nodes must be protected so that they are not be recontaminated again. After the BV is detected, the "surround and eliminate" begins. In this phase, some agents are employed to surround the new-formed BVs (the clones of the original BV) then some agents are sent to the clones to permanently destroy them. This is called the "Four-step Cautious Walk" and is widely used in BVD problem with synchronous setting. Also, BVD problem in chordal ring has been discussed in [**?**].

2.4.2 BVD in different topologies

Protocols to solve the BVD problems in grid are BVD-2G and BVD-qG which deal with BVD problems in 2-dimensional grid (meshes) and q-dimensional grid respectively. BVD-2G performs a BV decontamination of a 2-dimensional grid of size n using $k = 7$ agents and 3 casualties (i.e., losses of agents), within at most $9n + O(1)$ moves and at most $3n$ time. While protocol BVD-qG performs a decontamination of a q-dimensional grid of size $d_1 \times d_2 ... \times d_q$ using $3q + 1$agents and at most $q + 1$ casualties, within at most $O(qn)$ moves and at most $\Theta(n)$ time. Algorithm to decontaminate the BV in a q-dimensional torus,called BVD-qT uses $4q$ agents with 2 casualties with at most $O(qn)$ moves and $\Theta(n)$. Protocol BVD-qH is to perform a BV decontamination of a q-hypercube using $2q$

agents and q casualties with at most $O(n \log n)$ moves and $\Theta(n)$. In arbitrary graphs (see [**?**]), two protocols are presented: GREEDY EXPLORATION and THRESHOLD EXPLORATION. In these two protocols, $\Delta+1$ agents are needed and both of the protocols are worst-case optimal with respect to the team size, where Δ represents the maximum degree in G. Though the protocols are described for a synchronous setting, they easily adapt to asynchronous ones with an additional $O(n)$ moves for the coordinating activities. An advantage of these protocols is that the agents can use only local information to execute the protocol. Another interesting fact based on these two protocols is that both GREEDY ROOTED ORIENTATION and THRESHOLD ROOTED ORIENTATION produce an optimal acyclic orientation rooted in the homebase.

In [**?**], solutions for BVD in chordal ring are discussed. The solutions are designed for different kinds of chordal rings: double loops, triple loops, consecutive-chords rings and finally general chordal ring. In double loops, three strategies are proposed with an upper bound of moves is $4n-7$ and a maximum of 12 agents employed. In triple loops, wo classes of chordal rings are discussed: $C_n(1, p, k)$ and $C_n(1, k-1, k)$. In any triple loop $C_n(1, p, k)$, a maximum of $5n-6k+22$ moves and 24 agents are needed for the decontamination while in any triple loop $C_n(1, k-1, k)$, a maximum of $5n - 7k + 22$ moves and 19 agents are needed. Finally in the consecutive-chords ring, a maximum of $(k + 2)n - 2k - 3$ moves and $4k + 1$ agents are needed. Decontamination strategies are described in synchronous settings, but with a cost of $O(n)$ moves the strategies can be used in asynchronous settings as well.

Chapter 3

Definitions and Terminology

3.1 Model

3.1.1 Network, Agent, Black Virus

Network The environment in which mobile agents operate is a network modelled as simple undirected connected graph with $n = |V|$ nodes (or sites) and $m = |E|$ edges (or links). We denote by $E(v) \subseteq E$ the set of edges incident on $v \in V$, by $d(v) = |E(v)|$ its degree, and by$\Delta(G)$ (or simply Δ) the maximum degree of G. Each node v in the graph has a distinct $id(v)$. The links incident to a node are labelled with distinct port numbers. The labelling mechanism could be totally arbitrary among different nodes; without loss of generality, we assume the link labelling for node v is represented by set $l_v = 1, 2, 3, ..., d(v)$.

Agent A group of mobile agents are employed to decontaminate the network. The agent is modelled as a computational entity moving from a node to neighbouring node. More than one agents can be at the same node at the same time. Communication among agents occurs at this time; there are no a priori restrictions on the amount of exchanged information.When the agent arrives at a node, it can leave message on that node and read massage on that node. Information on the nodes can be set (writing the information on

a white board) at the beginning of the exploration, so when the agent reach the node, it can update the information on the white board or update its own memory by learning the information on the white board. We assume that all the agent' s moves follow the same clock. Also, without specially pointing out, agents are endowed with 1-hop visibility. When we say k-hop visibility, it means at a node v, an agent can see the labels of the edges incident to it and the identities of all its 1-hop neighbours. Also, it can also see whether or not there are agents there but cannot communicate with each other.

Black Virus In G there is a node infected by a black virus (BV) which is a harmful process endowed with reactive capabilities for destruction and spreading. The location of the BV is not known at the beginning. It is not only harmful to the node where it resides but also to any agent arriving at that node. In fact, a BV destroy any agent arriving at the network site where it resides, just like the black hole. Instead of remaining static as the black hole, the BV will spread to all the neighbouring sites leaving the current node clean. The clones can have the same harmful capabilities of the original BVs (fertile) or unable to produce further clones(sterile). A BV will be destroyed if and only if the BV arrive at a node where there is already an agent. Thus, the only way to eliminate the BV is to surround it completely and let an agent attack it. In such situation, the attacking agent will be destroyed while the clones of the original BV will be permanently eliminated by the agents residing the neighbouring nodes of the original node. We assume that at the same node, multiple BVs (clone or original) are merged. More precisely, at any time, there is at most one BV at each node. Another important assumption is that when a BV and an agent arrive at an empty at the same time, the BV dies and the agent survive remaining unharmed.

Also, in this thesis, we assume that when a BV is triggered, it take negative time for its clones to spread to all its neighbour. More specifically, when a BV is triggered by an agent at $\mathrm{T}(t_i)$, then in $\mathrm{T}(t_i)$, all the neighbours of this agents(if exist) receive the clones of it. The reason we make this assumption is that when we try to eliminate the new formed BVs

parallelly in the chordal ring and assume that it takes one unit of time for both the agents and the BV to move from one node to another, then we are faced with a tricky situation: if the sites of two(or more) agents are connected, after these two clones are triggered(we send an agent to each of them to permanently destroy them), one of their clones spread to another site and since the agents sent to contaminate them die, these two sites are empty when the second round clones arrive, which make the decontamination invalid. While with the assumption, the tricky situation can be easily solved: after the two clones are triggered, one of their clones spread to another site and both the original clone and the second round clone are destroyed by the agent.

Summarizing, there are five possible situations when an agent arrive at a node v:

- agents arrive at a node which is empty or contains other agents, they can communicate with each other and the node v is clean.
- agents arrive at a node which contains a BV,the clones of the BV (BVC) spread to all the neighbours of v and the agent dies, leaving node v clean.
- A BVC arrives at a node which is empty or there is already a BV: the node becomes/stays contaminated; it merges with other BVs.
- BVCs arrive at a node v which contains one or more agents, the BVCs are destroyed but the agents are unharmed.
- A BVC and an agent arrive at an empty node at the same time, the BVC dies while the agent remains unharmed.

3.1.2 Problem, Cost

The BLACK VIRUS DECONTAMINATION (BVD) problem is to permanently remove the BV, and its clones from the network using a team of mobile agents starting from a

given node, called home base(HB). The solutions where the agents explore the network sequentially have been proposed in some classes of topologies. chordal rings, hypercubes and arbitrary graph. In this thesis, we are interested in parallel strategies in BVD problem: instead of exploring the network in sequence, we explore it in parallel; in chordal ring, we also propose a parallel solution to surround the clones of the original BV. In this thesis, the efficiency measurements we have are: *spread* of BV (also measures the number of agents *casualties*; the *size* of the team, i.e, the number of agents employed by the solution; total working time(TWT) (calculated by multiplying the *size* of the team and the time cost by the solution. Note that TWT does not contains any practical meaning but exist only as a measurement. We propose TWT to compare more fairly the time of two protocols when the number of agents is different.

3.1.3 Monotone, Synchrony

A desirable property of a decontamination protocol is to prevent the nodes which been explored or cleaned by mobile agent from being recontaminated which will occur if the clones of the BV are able to move to a explored node in absence of agent. A BVD protocol with such property is called monotone. Monotone property is the necessary condition for spread optimality.

Asynchrony refers to the execution timing of agent movement and computations. The timing can be *Synchronous* or*Asynchronous*. When the timing is synchronous, there is a global clock indicating discrete time unit; it takes one unit for each movement (by agent or BV); computing and processing is negligible. When we have asynchronous agents, there is no global clock, and the duration of any activity (e.g., processing, communication, moving) by the agents, the BV, and its clones is finite but unpredictable. In this thesis, all our protocols work in synchronous setting.

3.2 General strategy

Following the solution in sequential case, we decomposed the BVD process into two separate phase: *Shadowed Exploration* and *Surrounding and Elimination*. The task of the first phase is to locate the BV and the second phase is to decontaminate the BV and its clones. Apart from these two basic phase, we have initialization which is to deploy the agents properly at the beginning of executing the protocol because we explore the network in parallel, and the arrangement of the agents is crucial to successfully execute the protocol.

Phase1: Shadowed Exploring Agents employed are divided into two group: shadowing group and exploring group, and the number of agents in two group is the same. For convenience, we call the agent in the shadowing group the shadow agent (SA and those in the exploring group the exploring agent EA (one EA is accompanied by one SA. As the name indicated, agents in exploring group explore the network and the agents in shadowing group follow the agents protecting the node which have been explored. More precisely, at T_1, EA moves to node v; at T_2, SA moves to node v and EA moves to node u supposing that node v is clean.

Phase2: Surrounding and Elimination In this phase, since we already know the position of the BV, we employ agents to surround all the neighbours of the BVCs. Once all the agent arrive the proper positions(all the neighbours of the BVCs are guarded), we employ another group of agents (the number of them is equal to the number of BVCs) to move to the BVCs, thus permanently destroy them. Usually, some of the agents moving to the neighbours of the BVCs are from the shadowing group and exploring group because in this way we can save the number of agents used in the whole protocol. Note that not all the agents are informed when the BV is detected. More specifically, only the agents who receives the clones know the existence of the BV, and other agents keep moving in the network. In some simple topologies, such as meshes and torus, the second phase begins when the BV is detected since the number of agents are enough to proceed the second

phase. In some more complicated topologies, for example, the chordal ring which we discuss later, we take some other measures to call back enough number of agents to finish the second phase.

3.3 Conclusion

In this Chapter, we presented the model of our problem and also some important terminologies. Also we described a general strategy for our problem depending on particular setting: synchronous timing, parallel strategy... In the next chapter, we discuss the parallel strategies in BVD problem in two simple topologies: meshes and torus.

Chapter 4

Parallel Black Virus Decontamination in Meshes

4.1 Introduction

In this chapter, we discuss parallel strategy on BVD problem in grid and tori. In the sequential strategy (*BVD-2G*) for 2-dimensional grids which are meshes, an "explorer agent" and a "leader explorer agent" are sent to explore the graph and locate the BV. They traverse the mesh in a snake-like fashion, column by column, following "cautious walk". In the sequential protocol (*BVD-qG*) for q-dimensional grids, the grid is partitioned into $d_1 \times \ldots \times d_{q-2}$ 2-dimensional grids of size $d_{q-1} \times d_q$, and each 2-dimensional grid is explored using the shadowed traversal technique as described in the 2-dimensional grids. Similarly, in the protocol ($BVD - qT$) for q-dimensional torus, the torus is partitioned into $d_1 \times \ldots \times d_{q-1}$ ring of size d_q. The exploration procedure traverses a ring and, when back to the starting point, proceeds to another ring, with a neighbouring starting point. After locating the BV, agents surround the new formed BVs sequentially and eliminate them. These strategies are simple to follow, but at the same time they are not time-efficient. We now consider the situation when more than two agents are allowed to participate in the

exploring phase and we focus on decreasing the time cost in the exploring phase and the number of casualties; that is, we focus on how to design a new strategy so that the we are able to reach the destination faster, but with acceptable cost in terms of number of agents. The general idea is simple, we will employ a group of agents and place them in a specific array at the beginning. Informally, in the shadowed exploration of our strategy in 2-dimensional grid(*PBVD-2G*), q-dimensional grid(*PBVD-qG*)and tori(*PBVD-qT*), the agents who are employed to explore the graph stay in that array and that "agent array" traverse that graph in the shadowed exploration. Note that after the BV is triggered, not all the agents automatically enter the elimination phase but only the agents who know the existence of BV. However, in some cases, the number of agents who know the existence of BV is not sufficient for surrounding and eliminating the BVs, so in the elimination phase, our strategies employ some agents who know the existence of the BV (because they receive the clones of the original BV) to notify some other agents to participate in the elimination phase. We study the number of agents, the time cost, the number of movements and casualty, comparing them with the ones of the corresponding sequential strategy.

4.2 Parallel BV Decontamination of Grids

4.2.1 Base Case: 2-Dimensional Grid

A 2-dimensional grid (which is a mesh) of size $d_1 \times d_2$ has $n = d_1 \times d_2 (d_1 > 2, d_2 > 2)$ nodes. Without loss of generality, let $d_1 < d_2$ and let the nodes of M be denoted by their column and row coordinate (x_1,x_2), $1 \leq x_1 \leq d_1$, $1 \leq x_2 \leq d_2$. Observe that in a mesh, we have three types of nodes: *corner* (entities with only two neighbours), *border*(entities with three neighbours), and *interior*(with four neighbours). Our strategy follows two phases: shadowed exploration and elimination. In the first phase, the network is traversed until the location of the BV is determined. That location is found after the visit, at which time all

the unprotected neighbours have become BVs. Note that in $PBVD-2G$, there is only one new formed BV. In the second phase, the new formed BV is surrounded and permanently eliminated. Note that when we say that the second phase starts, we actually mean that those agents knowing the existence of BV start to surround the BV, or notify some other agents, and then eliminate the BV, but not that all the agents enter this phase. There are two significant differences between $PBVD-2G$ and the sequential strategy: mainly the number of agents employed in the exploration phase and the route of agents in the exploration phase. We also describe the routes of agents in the elimination phase.

Shadowed Exploration Phase

As we mentioned above, we should place the agents in a specific array at the beginning and then let them explore the graph. Now let us consider how to arrange them at the beginning and how to design the routes for them to explore the graph. We prefer to place the agents at the borders (or the corners) of the mesh because in this way we can reduce the casualties. For the same purpose of reducing the casualties, we prefer to arrange all the agents in an array so that when one of the exploring agent triggers the BV, the exploring agents and shadowed agents guide as many neighbours of the BV as possible. In another word, we want all the agents explore the graph in one direction rather than from different directions. With these two principles, our strategy in the shadowed exploration is that given specific number of agents, we place them in one border of the mesh and if there are more agents, we place them on the row which is parallel to the border and so on. Then we design routes for them so that at any time they move to one direction to explore the graph.

Monotonicity is a principle that we should obey in the whole process, which means one exploring agent should be followed by at least one shadowed agent, so the number of agents in the exploration phase should be at least twice the number of the exploring agents. To

guarantee the monotonicity and the two principles, we should employ $2a\,(a \in \mathbb{Z}^+)$ agents in this phase and place a of them in one of the border and the others in the second line paralleling to the border. Let us now consider the number of agents we should employ. When $a = 1$, the arrangement is actually the sequential case. Since we want to explore the graph in parallel, we start with $a = 2$, in which case, the initial arrangement would be as Fig4.1:

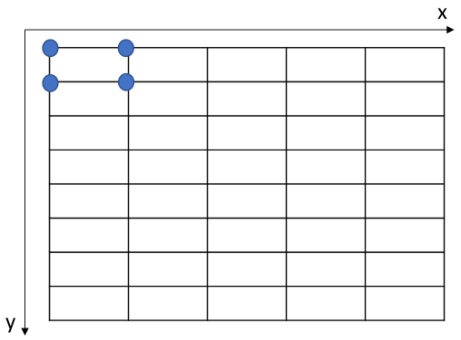


Figure 4.1: Arrangement of agents at the beginning(when $a = 2$)

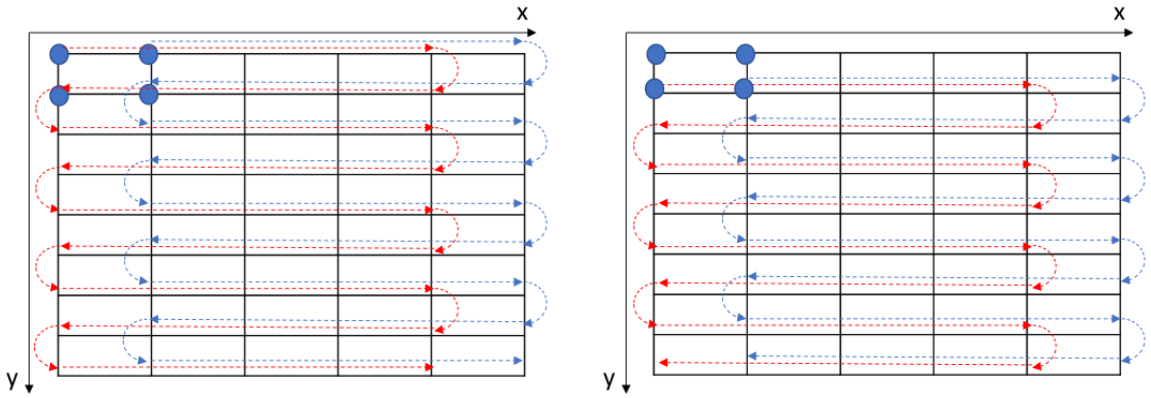
Let us now consider the routes for the agents. For convenience, we assume that all the agents move at time $t_i\,(i \in \mathbb{Z}^*)$ because some time should be reserved for the coordination after the BV is triggered. More detail about the moving cycle would be discussed after we decide the number of agents and the routes for them. Let $v = (x, y)$ be the node under exploration, with $1 \leq x_i \leq d_1$, $1 \leq i \leq d_2$. Additionally, we define "Vertical Moving Mark" (VMM) for every agent: VMM can only change between 0 and 1; every time when the agent moves SOUTH, that value changes. For example, if one agent continues to move SOUTH at t_i and its $Vertical_D$ is originally 0, then its $Vertical_D$ changes into 1 at t_1, 0 at t_2 and so on. Every agent hold two VMMs in its memory and let's say VMM_1 and VMM_2. The original value of VMM_1 of the agents is 0; the original value of VMM_2 of agents residing at node $(1, y)$ and node $(2, y)$ $(1 \leq y \leq a)$ is 0 and 1 respectively.

We now define the action of the $2a$ agents:

- Let $b = a - 1$. When all agents' VMM_1 are "0", then those agents with VMM_2 equal to "0" move EAST when $x \neq d_1 - 1$ and move SOUTH for b steps when $x = d_1 - 1$; those agents with VMM_2 equal to "1" move EAST when $x \neq d_1$ and move SOUTH for b steps when $x = d_1$. When all agents' VMM_1 are "0", then those agents with VMM_2 equal to "0" move WEST when $x \neq 2$ and move SOUTH for b steps when $x = 2$; those agents with VMM_2 equal to "1" move WEST when $x \neq 1$ and move SOUTH for b steps when $x = 1$.

- An agent moves only to a node that it has not explored yet. (Note that when residing at a node, an agent is able to know whether it has explored the neighbours of that node or not.)

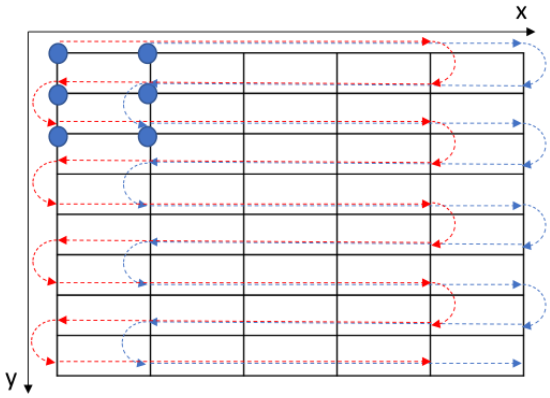
Informally, the resulting routes of agents are snakelike routes. When $a = 2$, the routes of agents are shown as Fig4.2. In order to show the routes more clearly, we present the routes of agents in different line respectively.



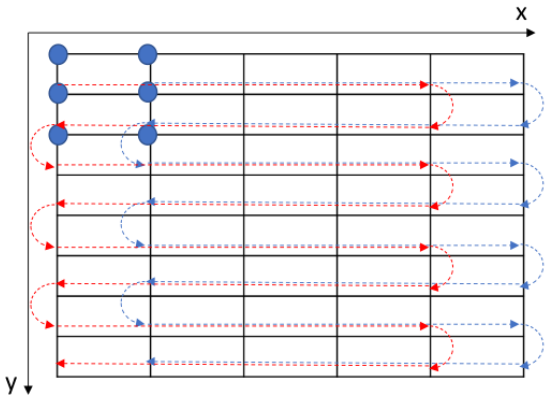
(a) Routes of agents in the first line when $a = 2$ (b) Routes of agents in the second line when $a = 2$

Figure 4.2: Routes of agents when $a = 2$

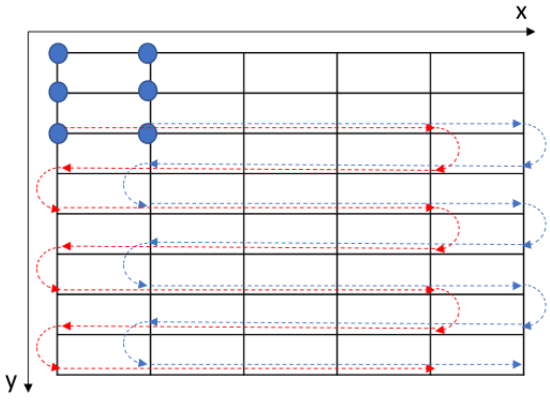
Fig4.3 shows the routes of agents when $a = 3$.



(a) Routes of agents in the first line when $a = 3$



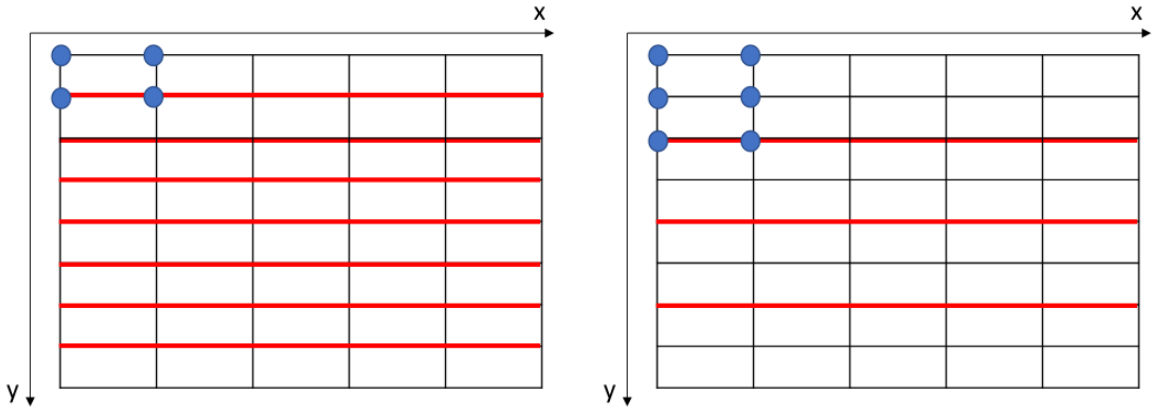
(b) Routes of agents in the second line when $a = 3$



(c) Routes of agents in the second line when $a = 3$

Figure 4.3: Routes of agents when $a = 3$

We can easily observe that some rows of the grid are traversed twice for the purpose of avoiding the explored nodes being contaminated (as shown in Fig 4.4)



(a) Rows that are double traversed when $a = 2$ (b) Rows that are double traversed when $a = 3$

Figure 4.4: Rows that are double traversed (marked with red rows)

We can see that when the grid is fixed (so d_1 is fixed), the number of double traversed rows decrease as a increases, which is easy to image. Let us denote by r the number of rows that are double traversed, then $r = \lceil \frac{d_1 - a}{a-1} \rceil$ where d_1 is the number of rows of the grid. Informally, r indicates the time that is spent in the exploration phase, and we can reduce this time by employing more agents. As we can see from the equation, when $a = d_1$, then $r = 0$, which means if we employ $2d_1$ agents to explore the graph, none of the rows are traversed twice.

We now discuss the strategy when we employ $2d_1$ agents, and this strategy can be easily modified to fit the situation where we employ less than $2d_1$ agents.

Initially, $2d_1$ agents are placed at the first two columns at t_0 and we place another agent at the top and bottom of the first column (we shall describe their roles in the following part). More specifically, their coordinates are (1, x_i) and (2,x_i) where $1 \leq x_i \leq d_1$. The agents residing in the first column are in the shadowing group while the agents residing in the second column are in the exploring group. If the BV resides in a node of the first column, then all of its clones are destroyed. If the BV resides in a node in the second column, then the elimination phase begins. It is obvious that if the BV does not reside in any node in the first column, then an agent in the exploring group should be destroyed when the BV

is exposed. Let us assume that the we starts at t_0. Agents residing in nodes of the second column move EAST at the beginning of t_i, $i = 0, 1, \ldots, d_2 - 1$. More precisely, the agent located in (x, y) moves to $(x + 1, y)$ at the beginning of t_i, $i = 0, 1, \ldots, d_2 - 1$. Agents residing in the first column simply follow the node in the second column (see Fig.4.5). When one of the nodes in the second column is destroyed by a BV, the second phase starts.

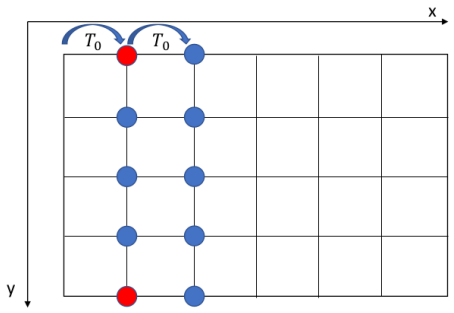


Figure 4.5: Agents move at time t_0. (The red node indicates that there are two agents residing there

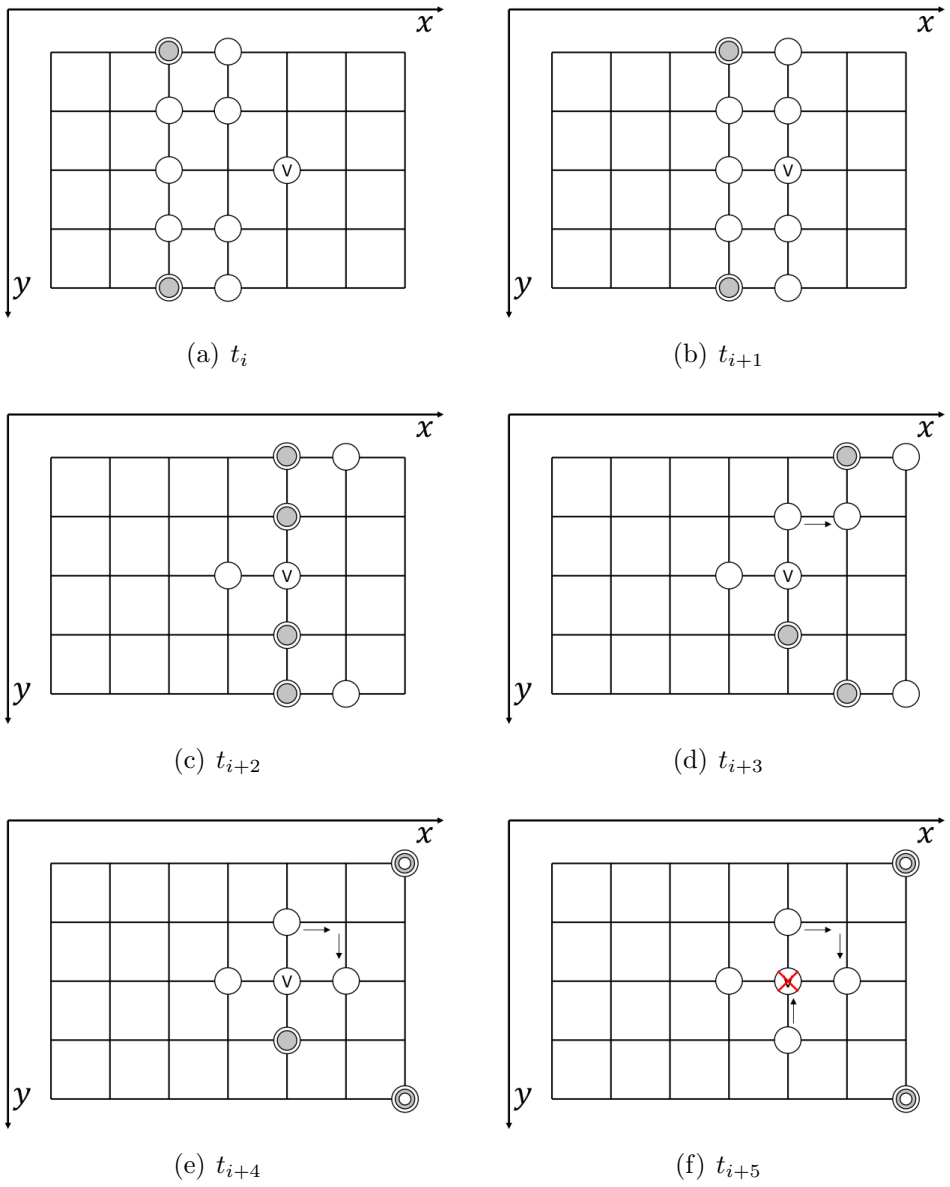
Elimination Phase

The elimination begins when one of the nodes in the second column is destroyed by a BV (let us say, at time t_i). No matter where the BV is, there are always three agents residing on its north (if the BV is not in the first row), west and south (if the BV is not in the last row), so only one BV clone survives. In another words, only one node becomes BV after the BV has been triggered. Observe that in the parallel strategy, not all agents participate in the elimination phase automatically when the BV is explored because only the ones that receive the clones of the original BV and those that are notified by other agents can participate in the elimination phase. So, in some situations, agents that receive the BV clone should notify other agents to participate in the elimination (case 1,2 and 3). In one particular situation (case 4), we instead use the agent that we take along the way (called following agent) to complete the elimination phase. Let the node where the surviving clone reside be (x, y). We have four different situations depending on the location of the

new formed BV, each situation corresponding to a different route taken in the elimination phase.

- Case 1: When $2 < x < d_1$, $1 < y < d_2 - 1$ (an interior node becomes a new formed BV), then the agents residing in node $(x-1, y+1)$, $(x-1, y-1)$ and $(x-2, y)$ (say a, b, c) receive a BV clone at time t_i, and they know the location of the original BV and also the new formed BVs. After they receive the BV clone, these agents move EAST at t_{i+1} for one step (for example, to node $(x, y+1)$, $(x, y-1)$ and $(x-1, y)$) and stop. Note that other agents including the ones residing in node $(x-2, y+1)$ and $(x-2, y-1)$ (say agent d and e) at t_i do not know the existence of the BV so they keep moving EAST and arrive at nodes $(x, y+1)$, $(x, y-1)$ at the end of t_{i+2} when they meet agent a and b respectively. Agent a and b inform them of the location of the new formed BV and the routes of agents d and e are as follow:
  route of d: $(x, y+1)(at\ t_{i+2}) \rightarrow (x+1, y+1)(at\ t_{i+3}) \rightarrow (x+1, y)(at\ t_{i+4})$.
  route of e: $(x, y-1)(at\ t_{i+2}) \rightarrow (x, y)(at\ t_{i+5})$.

  The routes of agents are showed in Fig.4.6 where "one circle" indicates that there is one agent residing here; "two circle" indicates that there are two agents residing here; "three circle" indicates that there are three agents residing here.



(a) t_i (b) t_{i+1}

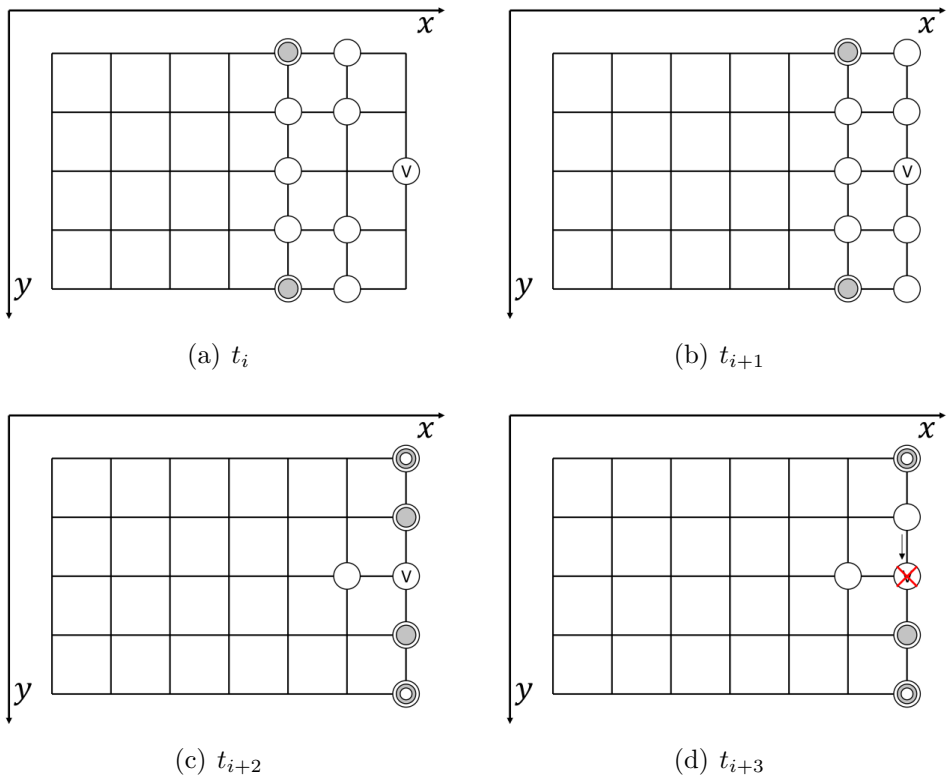
(c) t_{i+2} (d) t_{i+3}

(e) t_{i+4} (f) t_{i+5}

Figure 4.6: Arrangement of the agents in elimination phase when the new formed BV resides in a interior node

- Case 2: When $x = d_1$, $2 < y < d_2 - 1$ (a border node becomes a new formed BV), then the agents residing in node $(x-1, y+1)$, $(x-1, y-1)$ and $(x-2, y)$ (say a, b, c) receive a BV clone at time t_i. As above, they move EAST for one step and stop. The agents residing in nodes $(x-2, y+1)$ and $(x-2, y-1)$ (say a, b) at t_i have no knowledge of the BV, so they keep moving and arrive at nodes $(x, y+1)$

and $(x, y-1)$ at t_{i+2} when they are informed of the location of the new formed BV. One of the agents should move to the new formed BV to decontaminate it while the other stops moving at t_{i+3}. In order to avoid conflict, we always employ the agent who observes that the BV is in its SOUTH (say agent a) to move to the new formed BV (see Fig 4.7).



(a) t_i (b) t_{i+1}

(c) t_{i+2} (d) t_{i+3}

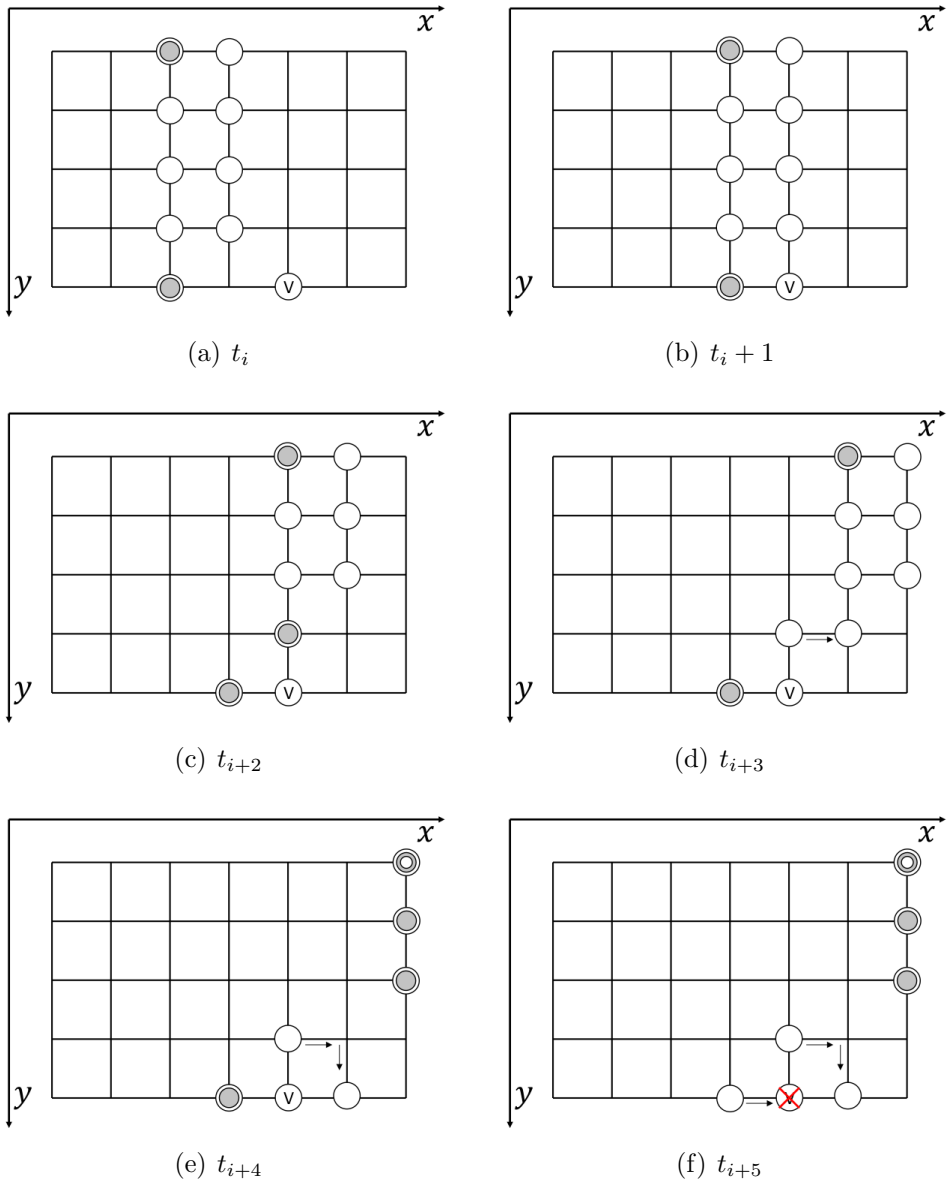
Figure 4.7: Arrangement of the agents in the elimination phase when the new formed BV resides in a border node (when $x = d_1$)

- Case 3: When $2 < x < d_1$, $y = 1$ or $y = d_2 - 1$ (a border node becomes a new formed BV). For convenience, we only discuss the situation when $y = 1$(the solution can be easily modified to fit the scenario when $y = d_2 - 1$). In this case, agents residing in nodes $(x-1, y+1)$ and $(x-2, y)$ (say a, b, c, where c is the following agent) receive a BV clone at time t_i. Agents a, b and c move EAST for one step and arrive at node $(x, y+1)$ and node $(x-1, y)$ at t_{i+1}. The agent in node $(x-2, y+1)$ (say, d) does not know the existence of the BV, so it keeps moving arriving at node $(x, y+1)$ at

$t_i + 2$. After that, the routes of the agents c (the following agent) and d are described below:

route of d: $(x, y+1)(at\ t_{i+2} \rightarrow (x+1, y+1)(at\ t_{i+3} \rightarrow (x+1, y)(at\ t_{i+4})$.

route of c: $(x-1, y)(at\ t_{i+2} \rightarrow (x, y)(at\ t_{i+5})$.



(a) t_i (b) $t_i + 1$

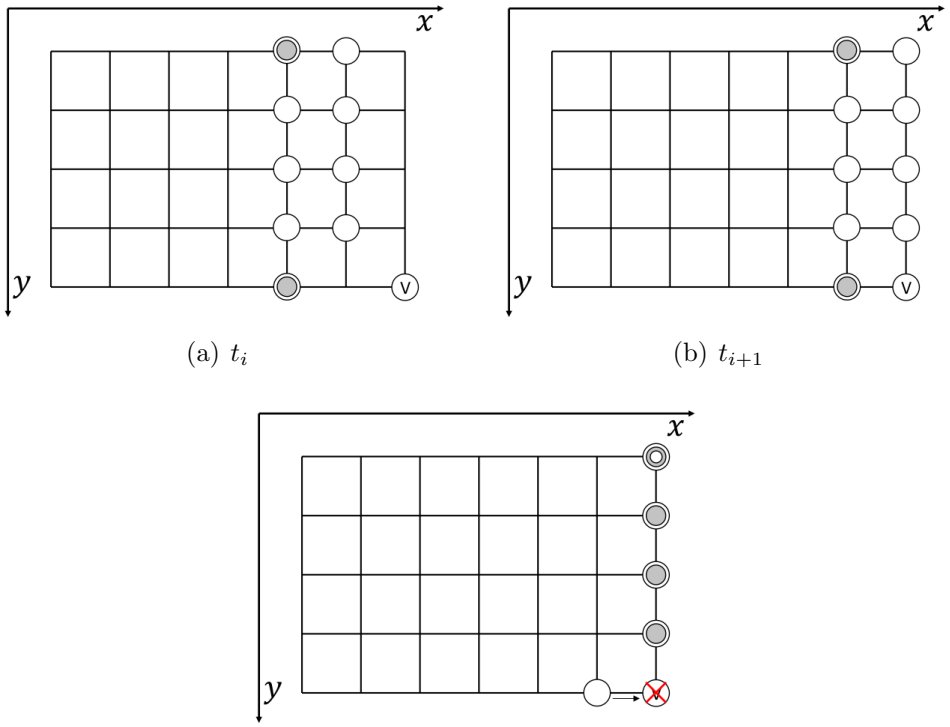
(c) t_{i+2} (d) t_{i+3}

(e) t_{i+4} (f) t_{i+5}

Figure 4.8: Arrangement of the agents in the elimination phase when the new formed BV resides in a border node(when $y = 1$ or $y = d_2 - 1$)

- Case 4: When $x = d_1$ and $y = 1$ or $y = d_2 - 1$ (a corner node becomes a new formed

BV). For convenience, we only discuss the situation when $y = 1$ and with some simple modification, the strategy can fit the scenario when $y = d_2 - 1$. In this case, agents residing in node $(x-1, y+1)$ and $(x-2, y)$ (say, a, b) receive a BV clone at t_i. Both of them keep moving for one step arriving at nodes $(x, y+1)$ and $(x-1, y)$ at t_{i+1}. Then agent b moves to the BV to destroy it at t_{i+2}



(a) t_i (b) t_{i+1}

(c) t_{i+2}

Figure 4.9: Arrangement of the agents in the elimination phase when the new formed BV resides in a corner node(when $x = d_1$ and $y = 1$ or $y = d_2 - 1$)

Analysis and Comparisons

Theorem 1. *Algorithm $PBVD - 2G$ performs a decontamination of a mesh (size of $n = d_1 \times d_2$) using $k = 2(d_1 + 1)\,(d_1 = min(d_1, d_2))$ agents, $(2(\sqrt{n} + 1)$ agents in the worst case) and at most 1 casualties.*

Proof. Let $v = (x, y)$ be the node containing the BV. When one of the agent in the exploring group moves to v, it will be destroyed and the BV will move to all neighbours

of v. If $x = 1$, then the neighbours $(x, y+1)$, $(x, y-1)$ and $(x+1, y)$ are protected by agents and neighbour $(x-1, y)$ actually does not exist; if $x > 1$, then neighbours $(x, y+1)$, $(x, y-1)$ and $(x-1, y)$ are protected by agents. So when the clones of BV moves to the neighbours of v, those contain an agent will not be infected by the BV clone; this means that the BV can safely move only to the unexplored neighbours of v, of which are at most one. In other words, after v is explored, at most one BV node is formed. According to our elimination strategy, the new formed BV node can be surrounded and destroyed using at most five agents: one to enter a BV and four to protect the neighbours. Since we have one new formed BV, the number of agents participating in the elimination phase is at most five. In addition to the agent destroyed by the original BV, the number of agent needed to complete the elimination phase is at least six. Since we employ $k = 2d_1 + 2$ $(d_1 \geq 3)$ which means at the beginning we have at least eight agents, so $2d_1 + 2$ agents are enough for the decontamination algorithm. In the worst case (which is the case of a square mesh where $d_1 = d_2$) the number of agent is equal to $2\sqrt{n} + 2$. □

Let us now consider the number of movements.

Theorem 2. *Algorithm $PBVD-2G$ performs a BV decontamination of a mesh of size n with at most $2n - \sqrt{n} + O(1)$ movements and at most $\sqrt{n} + 11$ in time.*

Proof. Let $v = (x, y)$ be the BV node, and let the size of the grid be $n = d_1 \times d_2$. Let us first consider the number of movements performed during the shadowed exploration. Since all the agents simply move EAST at the beginning of T(2n) $(n = 0, 1, \ldots, d_2 - 1)$, the travelling distance is x for agents in the exploring group (EA) and $x - 1$ for agents in the shadowing group(SA). We have d_1 EAs and $d_1 + 2$ SAs, then we have an overall cost of at most $2x(d_1 + 1) - (d_1 + 2)$ movements for this phase. Consider now the number of movements performed for Surrounding and Elimination. In this part, we only compute the movements of the agents that participate in the Surrounding and Elimination. More specifically, we ignore the movements of the agents who do not know the existence of the

BV in the whole process. As we discussed in the Elimination phase, when the new formed BV is located in a interior node, eight movements are needed in this phase; when the new formed BV is located in a border node (say (a, b)), then six movements are needed when $a = d_1, 2 < b < d_2 - 1$ and eight movements are needed when $2 < a < d_1, b = 1 \, or \, b = d_2 - 1$; when the new formed BV is located in a corner node, then four movements are needed in this phase. Hence, $O(1)$ movements are performed in this phase. In total we have that the number of movements is at most $2x(d_1+1)-(d_1+2)+O(1) \leq 2\sqrt{n}(\sqrt{n}+1)-(\sqrt{n}+2)+O(1)$, which is $2n + \sqrt{n} + O(1)$.

As for the time complexity. The time required for the exploration phase is equal to the number of movements of each EA, which is d_1; the time required for the surrounding and elimination phase is at most eleven. So in total the parallel mesh decontamination algorithm terminate in time at most $\sqrt{n} + 11$. □

Table 4.1 shows a comparison between our strategy and the sequential strategy.

Table 4.1: Comparision between PBVD-2G and BVD-2G

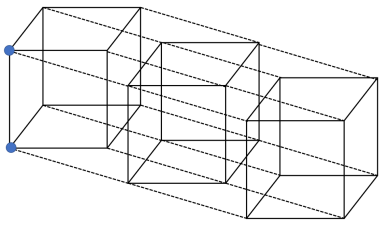
	agents	time	movements	casualties
PBVD-2G	$2(d_1 + 1)$ $(d_1 = min(d_1, d_2))$	$\sqrt{n} + 11$	$2n + \sqrt{n} + O(1)$	1
BVD-2G	7	$3n$	$9n + O(1)$	3

4.2.2 Multi-Dimensional Grid

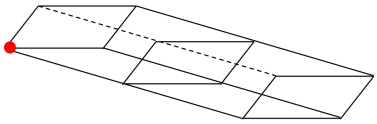
Let M be a q-dimensional grid of size $d_1 \times \ldots \times d_q$ and let each node of M be denoted by its coordinates $(x_1, \ldots, x_q)$, $1 \leq x_i \leq d_q$. The algorithm, called *PBVD-qG*, follows a general strategy similar to the one described in Section 4.2.1: a safe exploration with shadowing, followed by a surrounding and elimination phase. Our general idea is as follows: (1)

Transform the PBVD-qG problem into a BVD-qG problem (2) Use the BVD-qG strategy to solve the transformed problem.

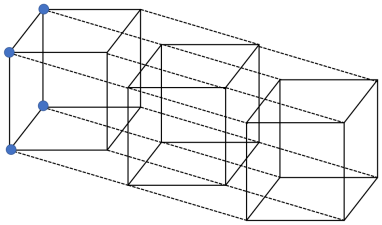
In [**?**], the multi-dimensional grid is partitioned into $d_1 \times \ldots d_{q-2}$ 2-dimensional grids of size $d_{q-1} \times d_q$. Each of these grids is explored using the BVD-2G: after traversing a grid in a snake-like fashion column by column, the agent returns to the starting point and from that starting point, it proceeds to another grid with a neighbouring starting point. In our strategy PBVD-qG, we use the similar exploring routes as that in BVD-G, which is the snake-like route. Additionally, we use the idea of dimensionality reduction. In our parallel strategy we use the term "p-dimensional agent group" to refer to a group of $d_1 \times \ldots \times d_p$ agents ($p < q$) organized in a $d_1 \times \ldots \times d_p$ grid (i.e., fully occupying a $d_1 \times \ldots \times d_p$ sub-grid of the original q-dimensional grid). Informally, a "p-dimensional agent group" can be viewed as one "large agent" and the q-dimensional grid as a "virtual" $(q - p)$-dimensional grid. Clearly, the larger p, the smaller the size of the virtual grid to be explored (see Fig4.10 for examples).



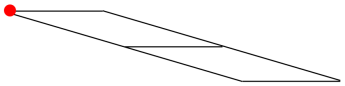
(a) The arrangement of agents on a 4-dimensional grid



(b) When we view the "one dimensional" agent as a large agent and the grid can be view as a three dimensional grid



(c) The arrangement of agents on a 4-dimensional grid

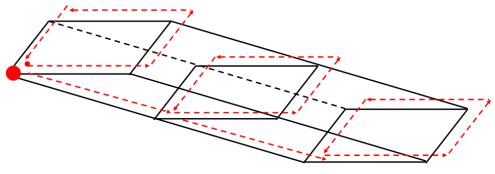


(d) When we view the "two dimensional" agent as a large agent and the grid can be view as a two dimensional grid

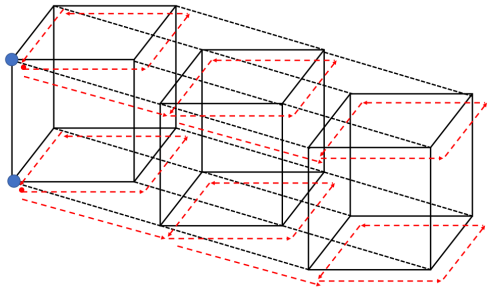
Figure 4.10: The idea of dimensionality reduction

After we transform the problem, then we can use the BVD-qG to solve the PBVD-qG. Let us now have a brief review of the BVD-qG strategy: there are two agents exploring the graph: one is called exploring agent (EA) and one is called leader exploring agent (LEA). They follow the "four step cautious walk" strategy to explore the graph: before exploring a node $v = (x_1, \ldots, x_q)(1 \leq x_i \leq d_i)$ from a node u, the shadowing agents(SA) move to the already explored neighbours of v (whose coordinated can be precisely computed). When EA visits the BV node (and is destroyed there), the LEA and the SAs are aware of the location of the new BV nodes (its coordinates can be computed precisely). So, once the node v containing the BV is identified, $2q$ agents surround each node $u \in N_{un}(v)$ and an additional agent enters it destroying the BV resident there and the instances that it generates.

Assume that we employ a "p dimensional" agent group, which consists of $d_1 \times \ldots d_q$ agents with coordinates: $(x_1, \ldots, x_p, \ldots, x_q)$, $(1 \leq x_1 \leq d_1, \ldots, 1 \leq x_p \leq d_p, x_{p+1} = 0, \ldots, x_q = 0)$. Then the coordinates of the "big agent" in the $q - p$ dimensional grid are $(x_{p+1}, \ldots, x_q)$. The first p dimensional coordinates of the agents would not change in the whole exploring phase, but from that $(p+1)^{th}$ dimensional coordinates, they change as the "big agent" changes. More specifically, the coordinates of the $(p+1)^{th}$ dimension of the agents make the same change as the first dimensional coordinate of the "big agent"; the coordinates of the $(p+2)^{th}$ dimension of the agents make the same change as the second dimensional coordinate of the "big agent", and so on. Assume now that we employ four agents (two agents are viewed as the LEA in the BVD-qG; two agents are viewed as the EA in the BVD-qG) to explore the 4-dimensional grid (they traverse the grid with the same routes following "four step cautious walk"), then the routes of the agents are as below (see Fig.4.11):



(a) The route of the "big agent"(LEA in the BVD-qG)



(b) The routes of the 2 agents viewed as the "big agent"

Figure 4.11: The route of agent when exploring a $1 \times 1 \times 1 \times 2$ grid

Analysis and Comparisons We now compare the time cost, the number of movement and the casualties with different number of agents we employ. (Assuming that we are in a $d_1 \times \ldots \times d_q$ q-dimensional grid) The theorems in [**?**] show us that the protocol BVD-qG performs a BV decontamination of a q-dimensional Grid using $3q+1$ agents with at most $q+1$ casualties and at most $O(qn)$ movements and $\Theta(n)$ time. Based on these theorems, we now discuss the number of agents, the casualties, the time cost and the number of movement in PBVD-qG.

Theorem 3. *PBVD-qG performs a decontamination of a q-dimensional grid(size of* $n = d_1 \times \ldots d_p (d_i > 2, 1 \leq i \leq p)$ *using* $2 \times d_1 \times \ldots d_p (1 \leq p \leq q)$ *agents and at most* $q - p + 1$ *casualties, with at most* $O(qn)$ *and* $\Theta(\frac{n}{d_1 \times \ldots \times d_p})$ *time.*

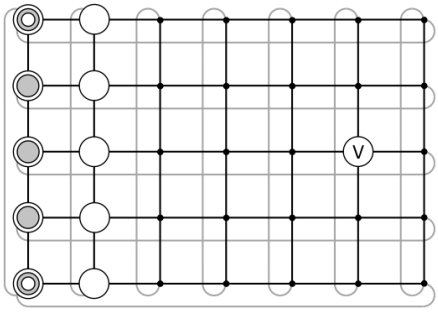
Proof. In PBVD-qG, we can choose different number of agents to start the exploration, and that number results in different casualties and so on. Assuming that we use $2 \times d_1 \times \ldots d_p (1 \leq p \leq q)$ agents to explore the graph, then actually we view $d_1 \times \ldots d_p$ agents as a "big agent", As we mentioned before, totally we need $d_1 \times \ldots d_p \times (3q+1)$ agents (every $d_1 \times \ldots d_p$ agents play the role of one agent in the BVD-qG). Now our problem changes into solving the BVD-qG problem in a $q-p$ dimensional grid with $d_{p+1} \times \ldots d_q$ agents, so the casualties and the times cost follow the same as the situation when we use BVD-qG in a $q-p$ grid. More specifically, the casualties are $q-p+1$ and the time is $\Theta(\frac{n}{d_1 \times \ldots \times d_p})$. In BVD-qG, the number of movements by LEA, EA and each SA is $O(n)$ and since there are at most q shadowing agents, the total number of movements until the BV is found is $O(qn)$ in the worst case. In PBVD-qG, the new "n" should be $\frac{q \times d_1 \times \ldots d_p \times n}{d_1 \times \ldots \times d_p}$ the number of movement should be $O(qn)$. □

4.2.3 Tori

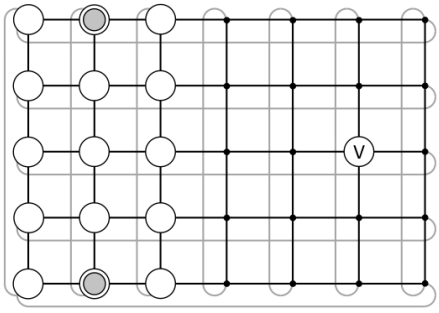
Informally, the torus is a mesh with "wrap-round" links that transform it into a regular graph. A torus of dimensions $d_1 \times d_2$ has $n = d_1 \times d_2$ nodes $v_{i,j} (1 \leq i \leq d_1, 1 \leq j \leq d_2)$ and each node has four neighbours which are $v_{i,j+1}, v_{i,j-1}, v_{i+1,j}, v_{i-1,j}$. The algorithm to parallelly decontaminate the BV in a torus, called PBVD-T, follows a strategy very similar to the one used for the 2-dimensional grid described in section 4.2.1. There is one difference between the two strategies: In 2-dimensional grid, all the agents move EAST in the exploration phase, while in the torus, because of the lack of borders, the spread of the BV might happen even if it reside in $v_{d_2,j}$ $(1 \leq j \leq d_2)$; therefore, we place another group of agents at $v_{i,0}$ $(i = 0, \ldots d_1 - 1)$ and these agents will stay here until the end of the exploration phase.

Initially, $2d_1$ agents are placed at $v_{0,i}$, $v_{1,i}$ $(i = 0, \ldots, d_1 - 1)$ (first round). If no agents are destroyed, then we place another d_1 agents in the first column: two agents at the top

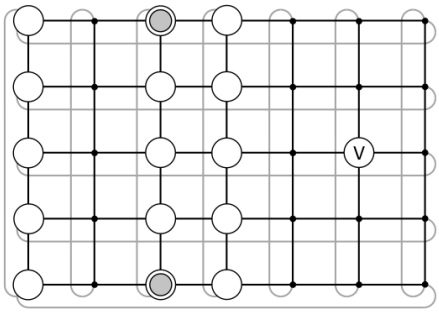
and the bottom as we do in the PBVD-2G. We then start the safe-exploration phase. If one of the agents is destroyed, assuming that the original BV resides in node $v_{0,j}$, $(1 \leq j \leq d_1)$, then all the clones of the BV are destroyed. If the BV resides in node $v_{1,j}$, $(1 \leq j \leq d_1)$, then the elimination phase begin. The movements of agents are the same as the agents in PBVD-2G. Note that in the exploration phase, only $2d_1 + 2$ agents actually move and d_1 agent simply stay in the first column to guard them ensuring the monotonicity. In the surrounding and elimination phase, the movements of the agents are also the same as those in PBVD-2G. Fig4.12 shows the whole process of the PBVD-T when the BV resides in node (5,3) in a 5×6 torus. ("one circle" indicates that there is one agent residing here; "two circle" indicates that there are two agents residing here; "three circle" indicates that there are three agents residing here.)



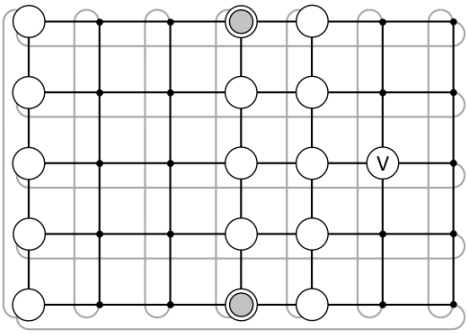
(a) t_0



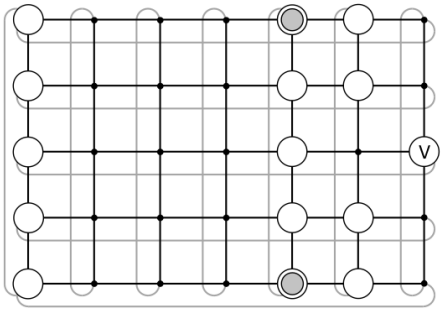
(b) t_1



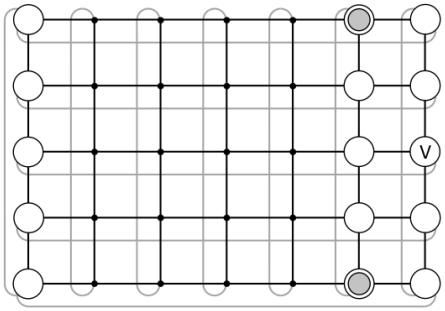
(c) t_2



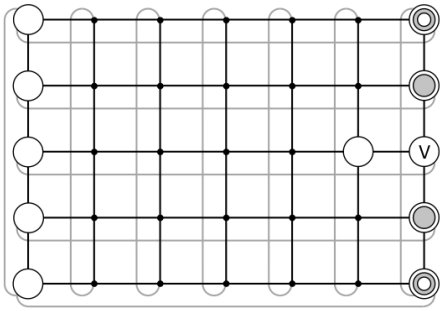
(d) t_3



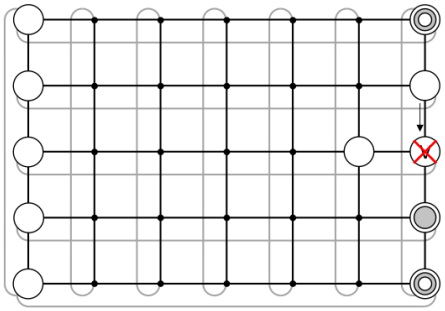
(e) t_4



(f) t_5



(g) t_6



(h) t_7

Figure 4.12: Arrangement of agents PBVD-T

Theorem 4. *The PBVD-T performs a decontamination of a torus (size of* $n = d_1 \times d_2 (d_1 > 2, d_2 > 2)$ *using* $3d_1 + 2 ((d_1 = min(d_1, d_2)$*, in the worst case,* $d_1 = \sqrt{n})$ *agents and at most 1 casualties with at most* $2n - \sqrt{n} + O(1)$ *movements and at most* $\sqrt{n} + 11$.

Proof. To complete the exploration and the elimination, we need $2(d_1 + 1)$ agents which has been proven in Theorem 2, plus d_1 agents to guard the first column, we need $3d_1 + 2$ agents (which is $3\sqrt{n} + 2$ in the worst case). The computation of casualties, number of movements, and time follow from Theorem 2. □

Chapter 5

Parallel Black Virus Decontamination in Chordal Ring

5.1 Introduction

In this chapter, we discuss a parallel strategy for BVD problem in chordal rings. A chordal ring is a circulant graph with $d_1 = 1$, i.e., it is an augmented ring, and will be denoted by $C_n(1, d_2, ..., d_k)$. More specifically, in chordal ring each node is directly connected to the nodes at distance d_i by additional links called chords. The link connecting two nodes is labeled by the distance that separates these two nodes on the ring. (see Fig.5.1)

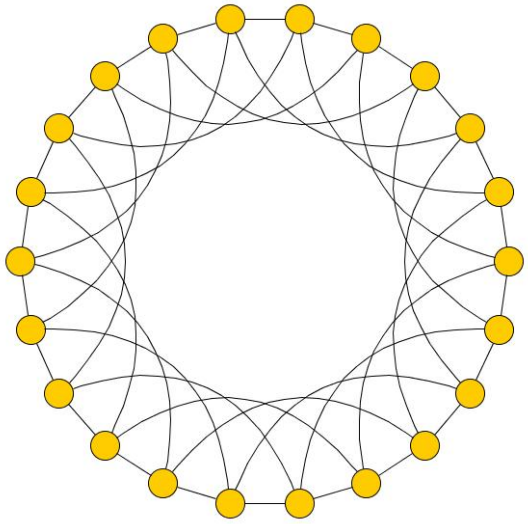


Figure 5.1: An example of chordal ring $C_n(1, 4)$

Agents are anonymous, but for the purpose of easier description, we refer to the nodes as x_0, x_1 .... Agents are not allowed to communicate with each other unless they are in the same node so the protocol should enable agents in different nodes to move properly. That is, the route of every agent is different but they are served to explore the network; when a BV is triggered, other agents should bypass the new-formed BVs. We give simple but efficient solution to deal with the problem with acceptable cost. Our goal is to minimize the time to complete the whole decontamination process and at the same time the casualties. In order to do that, we propose a parallel strategy for decontaminating the chordal ring and this is the first attempt to deal with this issue in a parallel way.

5.2 Shadowed Exploration

Initialization

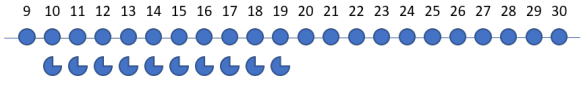
The chordal ring is a complete symmetrical structure, so we can randomly choose a node x_0 as the start node. The initial setup consists of deploying three groups of agents. Initially, we place one agent in each of the first $2d$ nodes $x_0, x_1, \ldots, x_{2d-1}$. The agents residing in

nodes from x_0 to x_{d-1} form the *shadowing group*, while the ones from x_d to x_{2d-1} form the *exploring group*. If the BV is within this window of nodes, then it is easily detected. We then assume that the first $2d$ nodes do not contain the BV, and we place d additional agents at nodes $x_0, x_1, \ldots, x_{d-1}$ (*guarding group*). Only the shadowing and exploring groups move to explore the graph. The ones in the guarding group remain dormant for now, guarding the nodes to guarantee monotonicity.

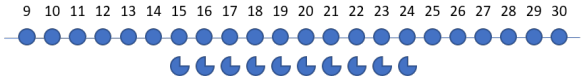
Route of the agent in exploring phase

The exploration proceeds in synchronized rounds composed each by one movement step, when selected groups of agents move to proceed with the exploration, and three steps for synchronization purposes. We call these different steps *move step* and *notification steps*. A round T_i is composed by four time units, one for the move step, and 3 for the notification steps:$T_1 = T_{move\_1}, T_{noti\_1^{(1)}}, T_{noti\_1^{(2)}}, T_{noti\_1^{(3)}}, T_2 = T_{move\_2}, T_{noti\_2^{(1)}}, T_{noti\_2^{(2)}}, T_{noti\_2^{(3)}}, \ldots$

The agents that move during a move step T_{move_i}, always do so along their longest chord d_k. That is, agents move along d_k in steps $T = 1 + 4t$ ($t \in \mathbb{N}$). An example of how agents move in chordal ring $C_n(1, 2, 4, 5)$ at T_{move_i} in the exploring phase is shown in Fig.5.2.



(a) Arrangement of agents at T_{move_i}



(b) Arrangement of agents at T_{move_i+1}

Figure 5.2: Arrangement of agents when moving

Synchronization: Three Jump Notifying Technique In the sequential strategy [**?**], two agents explore the graph (exploring agent and leader agent) using "cautious walk". That is, the Exploring Agent moves to the next node in its route, and if the node is safe, it moves back to the Leader Explorer Agent, and then move forward to that safe node

together. If instead the node contains a BV, the Leader Explorer Agent becomes aware of that because a BV arrives through that link instead of Explorer Agent. However, in our strategy, we employ $2d$ agents in the exploring phase but we do not use the "casual walk". We then have to guarantee that they all find out whether or not the BV has been found in the current round. In fact, if they are not properly informed, when one agent is destroyed by the BV, in their next step some of them (the risky agents RAs) may be destroyed by the new formed BVs. In order to avoid these potential casualties, we propose the $Three\,Jump\,Notifying\,Technique$ to properly notify the agents who otherwise would move to the new formed BVs in the next round.

The idea is the following: when/if a node receives a clone and becomes aware of the presence of the BV, it becomes a $Notification\,Agent$ (NA). The NA's role is to make the risky agents aware of the presence of the BV in an efficient way. They will do so in parallel, each following a special route of length 3. More precisely, let the BV be at node x_0 (refer to Figure 5.3), the $Notification\,Agent$ located at node x_{n-d_i} will follow the following route:

$$x_{n-d_i} \xrightarrow{move\,along\,chord\,d_i} x_0 \xrightarrow{move\,along\,chord\,d_k} x_{n-d_k} \xrightarrow{move\,along\,chord\,d_i} x_{n-d_k+d_i}.$$

In this case, the notifying route of the NA whose coordinate is x_{n-d_k} is $x_{n-d_k} \rightarrow x_0 \rightarrow x_{-d_k} \rightarrow x_0$.

Note that with the $Three\,Jump\,Notifying$ technique, we only inform the RAs (agents who will be destroyed by the BV next step)(i.e., agents residing in nodes $x_{n+d_i-d_k}$, $(i = 1, \ldots, d_{k-1})$) but still, agents (if existing) residing in node $x_{n+d_i-2d_k}$, $(i = 1, \ldots, d_{k-1})$ will be destroyed by the BVs in their next two moves. So they should be properly informed as well.

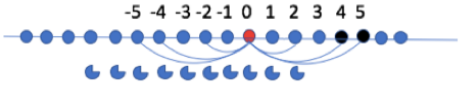
Actually, this could be easily performed: when one agent $A1$ moves to a node where there is an agent $A2$ knowing the position of the original BV, $A1$ would be informed and directly moves along the longest chord to its own position.

In this way, all the agents that might be destroyed by the BV would be properly

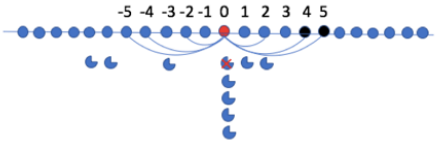
informed.

confusion between node, chord, etc. Let us always use x_i for the nodes (not i), d_i for the chords, etc..

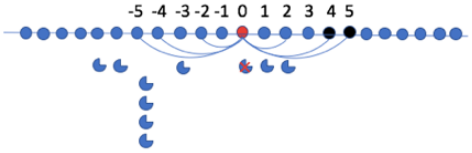
We would make some modification of this agents route in the *Surrounding and Elimination*, but now let us assume it still follows the route above. The whole process of *Three Jump Notifying* technique in chordal ring $C_n(1, 2, 4, 5)$ is shown in Fig.5.3.



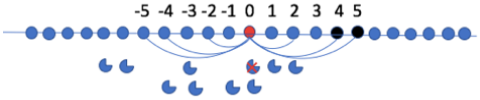
(a) Arrangement of agents at T_{move_i}



(b) Arrangement of agents at $T_{noti\_i^{(1)}}$



(c) Arrangement of agents at $T_{noti\_i^{(2)}}$



(d) Arrangement of agents at $T_{noti\_i^{(3)}}$

Figure 5.3: The whole process of the *Three Jump Notifying* technique in chordal ring $C_n(1, 2, 4, 5)$

If the original BV is residing in the red node, then once an agent moves to it, the agent and the BV are destroyed but the clones of the BV spread to all its neighbours. According to our technique, agents residing in nodes x_{-4}, x_{-3}, x_{-1}, x_0 are the ones to be notified; agents residing in nodes x_{-5}, x_{-4}, x_{-2}, x_{-1} are the NAs. The routes for agents residing in nodes x_{-5}, x_{-4}, x_{-2}, x_{-1} are $x_{-5}\rightarrow x_0\rightarrow x_{-5}\rightarrow x_0$; $x_{-4}\rightarrow x_0\rightarrow x_{-5}\rightarrow x_{-1}$; $x_{-2}\rightarrow x_0\rightarrow x_{-5}\rightarrow x_{-3}$; $x_{-1}\rightarrow x_0\rightarrow x_{-5}\rightarrow x_{-4}$ respectively.

Safe Exploring with Three Jump Notifying technique

After the initialization, the exploring and shadowing agents move following the longest

chord. The subsequent three time units are either used for the notification phase (if one of the agents is destroyed at time T_{move_i}), or they are spent simply by waiting before the next move. If executing the $Three\,Jump\,Notifying$ technique, the NAs move back to where they are before the notification. For example, in the example in $Three\,Jump\,Notifying$ technique, NA residing in node -1 moves back to node x_{-4} following the reverse route in the notifying phase which is $x_{-1}\rightarrow x_{-5}\rightarrow x_0\rightarrow x_{-4}$.

5.3 Surrounding and Elimination

When a BV is found, the Three Jump Notifying technique guarantees that the risky agents are now aware of the presence of the BV. Other agents, however, might not have received the notification and might proceed to the next round without such knowledge; we call these agents $KeepMoving$ agents.

In this section, we introduce the process of eliminating the BVs after the original BV is triggered. For the purpose of saving the number of agents, we prefer to chase the $Keep\,Moving$ agents, but it is not necessary to complete the process especially when you care most about the execution time; In that case, we may instead carry enough agents and proceed to the $Surrounding\,and\,Elimination$ phase immediately. The number of agents that should be carried in order to successfully proceed the $Surrounding\,and\,Elimination$ will be discussed later. We now describe how to chase the $KeepMoving$ agents.

5.3.1 Notifying Moving Agents

Overview of the Notifying Moving Agents

When the $Shadowed\,Exploring$ ends, it is possible that some of the agents in the array are not informed and do not realize the existence of the BV, so they keep moving following the routes in $Shadowed\,Exploring$ phase but it is obvious that they would not encounter

any BV. In order to reduce waste, we employ the agent who receives the clone from chord d_k (*Coordinator Agent*) to notify the other *Keep Moving* agents to move back to their position they occupied before the BV was triggered.

The Process of the Notifying Phase of the Coordination Agent

To do that we employ one of the agents as a *Coordinator Agent*(CA). The CA follows a specific path that will guarantee to meet all the *Keep Moving* within a certain amount of rounds.

The general outline of the technique is the following:

1. the CA is chosen to be the agent who receives a BV clone from its longest chord.
2. following three moving rules (the rules of moving are described later) the CA moves to occupy an node as the starting point for chasing and set a notification window $[x_y, x_z]$ (the range computation are described later) based on its own coordinate.
3. the CA waits an appropriate amount of time to allow the agents that are still moving to reach this window of nodes
4. the CA now moves in synchronization with the movement of the agents and follows a specific paths. More precisely, while the moving agents proceed as usual with one move and 3 waiting steps, the CA will take its longest chord in correspondence of an agents move and 3 consecutive nodes in correspondence of the waiting steps of the moving agents. In doing so, with $O(d_k)$ moves, the CA is guaranteed to have encountered all of them.
5. when a moving agent encounters the CA, it stops and waits for a second agent that will arrive at the next round. When both agents are there, they go back to their original positions to start the surrounding phase.

For convenience, we consider the chordal ring as arranged in rows of size d_k where the last node of a row is connected to the first node of the following row and the last node is

connected to the first. Depending on the size of the chordal ring, the last row could be incomplete. So in this matrix, moving down a column corresponding to using the longest chord d_k.(see Fig.5.4).

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40	41	42	43
44	45	46	47	48	49	50	51	52	53	54

Figure 5.4: Viewing a chordal ring whose longest chord is 11 as a matrix

In the following step, we would use a concept called "Notification Window". In a chordal ring $C_n(1, d_2, \ldots, d_k)$, given a coordinate x, the $Notification\,Window$ $[x_y, x_z]$ contains a set of consecutive nodes where x_y is the $Beginning\,Flag$ and x_z is the $End\,Flag$. When we mention "marking a flag", it does not mean that the agent has to move to the node to do that but only needs to remember the positions of the two flags in its memory. The relations between x, x_y, x_z is defined as follows:

- x_y is the biggest number smaller than or equal to x_i and such that x_y mod d_k =0;
- x_z is the smallest number s bigger than x_i and that x_z mod $d_k = d_{k-1}$.

For example, in the matrix of Figure 5.4), given a node, the $Beginning\,Flag$ of its "Notification Window" is the first node of its row while the $End\,Flag$ is the last node of its row.

The selection of the CA is simple: when an agent receives a BV clone from its longest chord, then it realizes that it is chosen as the CA. More specifically, if the coordinate of the original BV is x_i, then the coordinate of the CA would be x_{i-d_k}. After being selected as the CA, the CA should move to a node as the starting point for chasing. In the matrix, if the original BV reside in row i, then the destination of the CA (the starting point for chasing) should be in any node in row $i + 2$ to avoid being destroyed by the clones when

chasing the "Keep Moving" agents. In this case, the $Notification\,Window$ set by the CA would be from $x_{d_k\times(i+2)}$ to $x_{d_k\times(i+2)+d_k-1}$.

Supposing the coordinate of the original BV is x_i ,the coordinates of the positions where the clones spread are: x_{i-d_k} (which is the original coordinate of the CA), $x_{i-d_{k-1}}$, $x_{i-d_{k-2}}$, $\ldots$, x_{i-1}, x_{i+1}, x_{i+d_2}, $\ldots$, $x_{i+d_{k-1}}$, x_{i+d_k}. There are three different situations to consider to describe the behavior of the CS. Now we describe the three scenarios:

- Scenario 1: The last agent of the $Exploring\,Group$ is destroyed by the BV and the positions of the clones satisfy: $x_{i-d_{k-1}+d_k} = x_{i+1}$, $x_{i-d_{k-2}+d_k} = x_{i+d_2}$, $\ldots$, $x_{i-1+d_k} = x_{i+d_{k-1}}$.

- Scenario 2: The last agent of the $Exploring\,Group$ is destroyed by the BV and the at least pair of the positions of the clones does not satisfy: $x_{i-d_{k-1}+d_k} = x_{i+1}$, $x_{i-d_{k-2}+d_k} = x_{i+d_2}$, $\ldots$, $x_{i-1+d_k} = x_{i+d_{k-1}}$.

- Scenario 3: One of the agents in the $Exploring\,Group$ except the last agent is destroyed by the BV.

In scenario 1, the CA needs to move for 5 steps to reach its destination while in the other two scenarios, it only needs to move for 4 steps to arrive the destination. Now we describe the route for each scenario.

- For CA in scenario 1: Let us denote by x_i the coordinate of the node in the $Notification\,Window$ set by the coordinate of the original BV which does not receive any clone and his left neighbour receives a clone (the coordinate of which is x_{i-1}). The CA first moves to the original BV, then to node x_{i-1}, finally to x_i. At this point, it only needs to move along chord d_k twice to reach its destination.

- For CA in scenario 2: There is at least one pair of positions of the clones that does not satisfy the equations, so there should be one node (say, x_i) who receives a clone

from the original BV but node x_{i+d_k} is empty. The route now for the CA is first to move to the original BV, then to node x_i, and then along the chord d_k twice to reach its destination.

- For CA in scenario 3: The CA here simply needs to move one step to its right neighbour and move along the chord d_k three times to reach its destination.

The $Keep\,Moving$ agents reach the CA's $NotificationWindow$ 8 unit of time after the BV is triggered. Because it takes at most 6 units of time for the CA to reach its destination, the CA need to wait for the $Keep\,Moving$ agents before beginning the chasing. The CA simply counts the time it costs to reach its destination (say. t), and compute the time it need to wait which is $8-t$. The chasing phase starts from $T_{move\_i+2^{(1)}}$. The "Keep Moving" agents proceed with one move and three waiting steps while the CA takes its longest chord in correspondence of an agent's move and three consecutive nodes in correspondence of the waiting steps of the "Keep Moving" agents. Every time when the CA moves along a chord d_k, it resets the "Notification Window" as below:

$New\,Beginning\,Flag = Old\,Beginning\,Flag + d_k$

$New\,End\,Flag = Old\,End\,Flag + d_k$

When the CA moves along three consecutive nodes, it notifies the agents it encounters (if any) to go back. To ensure that the CA and the "Keep Moving" agents are in the same "Notification Window", during the movement of the CA, when it realizes that it just passes the $End\,Flag$, the CA should move along the longest chord counter-clockwise to the node marked $Beginning\,Flag$ and should continue to move along three consecutive nodes to notify agents it encounters . The chasing phase terminates when the CA arrives at a node x_y which satisfies $x_y = x_z + t \times d_k$ ($t \in \mathbb{N}$) (The relative starting point).

Example: The route of the CA in the chasing phase is shown for an example in Fig.5.5 for a chordal ring $C_n(1, 2, 7, 11)$.

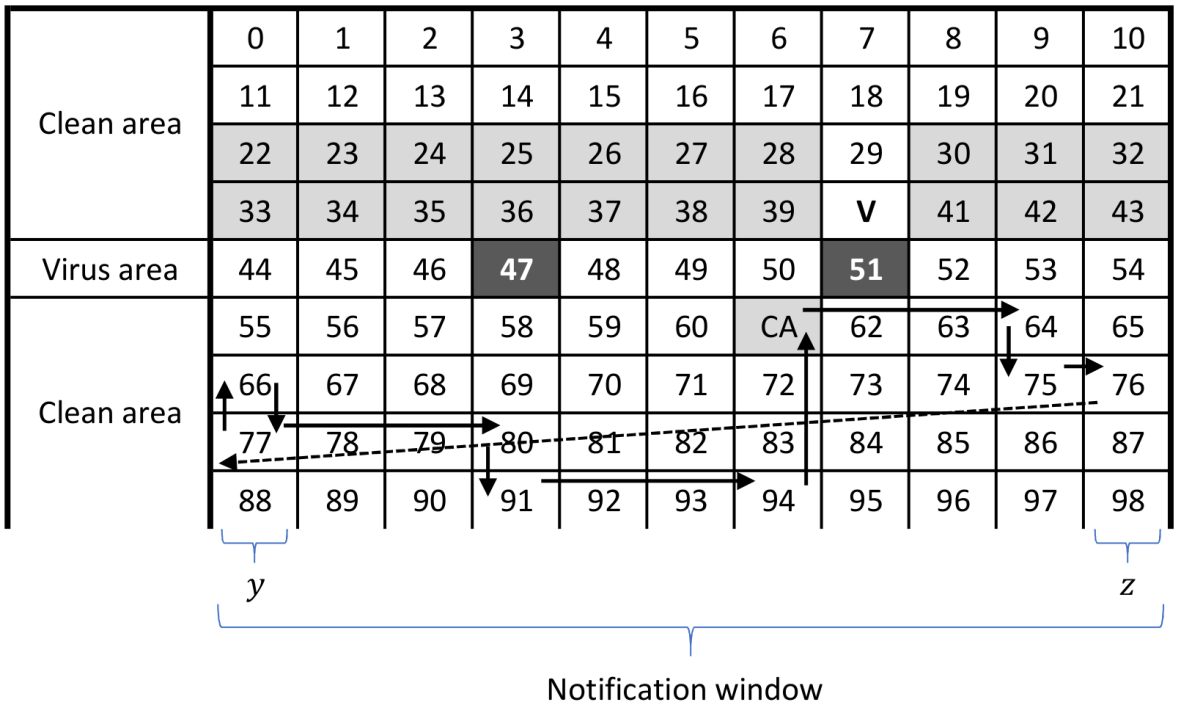


Figure 5.5: The route of the CA in the chasing phase in a chordal ring C(1, 7, 11)

In this example, node x_{61} is the starting point of the chasing phase and at this time, the "Notification Window" is $[x_{55}, x_{65}]$.The CA moves along three consecutive nodes x_{62}, x_{63} and x_{64} while the "Keep Moving" agents are waiting and then move along the longest chord to node x_{75}. At this time, the CA updates its "Notification Window" to $[x_{66}, x_{76}]$. After that, it continues to move along three consecutive nodes, but when it arrives x_{77}, it realizes that it just passes the "End Flag" which is x_{76}, so it moves along the longest chord counter-clockwise to the node marked *Beginning Flag* which is x_{66}. After that, it moves along the longest chord with the "Keep Moving" agents to x_{77} and again updates its "Notification Window", then moves along the nodes to notify the agents...

Let us assume that the time when the original BV is triggered is T_{move_i} , then the CA should remember the T_{move_i} and informs the agents he encounters of it. The agent $A1$ who encounters the CA should remember the time when they encounter ($T_{noti\_y^{(a)}}$ $a \in (1, 2, 3)$) and stop moving until next T_{move} (T_{move_y+1}) when it will meet another agent $A2$. Then

$A1$ moves along d_k anticlockwise for $y+1-i$ times while $A2$ moves for $y+2-i$ times. When arriving its relative starting point at T_{move_a}, the CA knows that it has finished the chasing task and moves along d_k to its position when the original BV is triggered.

We now describe how the agents and the CA move in coordination in $C_n(1,2,7,11)$ (see Figure 5.6).

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65

Figure 5.6: Arrangement of agent at T_{move_2} when the BV is triggered

Yellow nodes are connected to the original BV but guarded by agents while grey nodes are the new formed BVs. The node marked V is the original BV is now clean. The agent residing in node x_{29} receives a clone from chord d_k so it knows it is the CA. During the notifying time, agents residing in nodes x_{33}, x_{38}, x_{39} notify agents residing in nodes x_{36}, x_{31}, x_{30} respectively following the $Three\,Jump\,Notifying\,Technique$, while the CA moves to x_{28}, x_{39}, x_{50} and finally x_{61} following the route in scenario 3.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76

Figure 5.7: Agents' roles after $Three\,Jump\,Notifying\,Technique$. (for convenience, we denote the CA by a red spot, more specifically, node x_{50} is where CA resides)

Agents in purple nodes would be notified at T_{move_3} and move back. Agents in light green nodes are the $Keep\,Moving$ agents, while agents in dark green nodes are informed to

stop in $Three\,Jump\,Notifying\,Technique$. In the meantime, the CA moves to node x_{28}, x_{39}, x_{50}, and finally x_{61}. It is obvious that the CA can reach its destination before T_{move_4}, so it waits until $T_{noti\_4^{(1)}}$ to start its notifying phase.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76

Figure 5.8: Arrangement of agent at T_{move_3}. The CA has arrived its destination

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76

Figure 5.9: Arrangement of agents at T_{move_4}. The CA starts its chasing phase

In the chasing phase, CA starts to notify other $Keep\,Moving$ agents. First, it computes the $Notification\,Window$ which is from node x_{55} to node x_{65}. It moves to node x_{62} at $T_{noti\_4^{(1)}}$, node x_{63} at $T_{noti\_4^{(2)}}$, node x_{64} at $T_{noti\_4^{(3)}}$ and to node x_{75} at T_{move_5}.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87

Figure 5.10: Arrangement of agents at T_{move_5}.

After that, CA moves to node x_{76} at $T_{noti\_5^{(1)}}$. We can see that it encounters the agent residing in node x_{76}, so CA informs node x_{76} about the $T_{trigger}$, which is T_{move_2}. *The agent residing in node* x_{76} *should remember* T_{noti_now} *which is* $T_{noti\_5^{(1)}}$ *and wait additional time* T_{move} *to inform agent (Following Agent) who resides in node* x_{65} *now but would move to node* x_{76} *next* T_{move}. After encountering its *Following Agent*, the CA informs it to move back along chord d_k for $T_{move\_now} - T_{trigger} + 1$ times which is $T_{move\_5} - T_{move\_2} + 1$ times, while it moves for $T_{move\_5} - T_{move\_2}$ times.

At $T_{noti\_5^{(2)}}$, when the CA arrives at node x_{77}, it knows that it just pass its *Ending Flag* so it moves along the longest chord anticlockwise to its *Beginning Flag* (node x_{66} at $T_{noti\_5^{(3)}}$.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87

Figure 5.11: Arrangement of agents at $T_{move\_5^{(3)}}$.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87
88	89	90	91	92	93	94	95	96	97	98
99	100	101	102	103	104	105	106	107	108	109

Figure 5.12: Arrangement of agents at $T_{move\_7^{(3)}}$.

We could know that the CA moves back to its relative starting point (node x_{94}) at

$T_{noti\_7^{(3)}}$ and it knows that it has finished the chasing phase and moves back to its original starting point which is node x_{29}.

5.3.2 Overview of the Elimination

After all the agents move back to where they are when the BV is triggered, they start the *Surrounding and Elimination* phase.

The BVs can be destroyed sequentially, which is simple, but might be time consuming in some cases. In this way, because all the agents are aware of the positions of the new-formed BVs, for each BV, at most $d-1$ agents are sent to surround it and one agent is sent to destroy. This elimination strategy has been used in [**?**].

An alternative strategy, which allows a faster decontamination at the expense of a larger number of agents, is to the BVs concurrently. First we need to guard all the neighbouring nodes of all the newly formed BVs. In order to avoid collision and efficiently leverage the agents, we allocate different Destination Tables to all the agents in the array to inform them where should they move in different situations (e.g., when the first agent in the exploring team is destroyed, then every agent except the first agent have a distinct destination, when the second agent is destroyed, then every agent except that agent destroyed have again a distinct destination). More specifically, for a Chordal Ring with degree $2k$, every agent in the array carries a $Destination Table$ with $k-1$ destinations. If we need more agents, then we give their $Destination Table$ to the last agent in the shadowing group,. When the elimination begins, the agent clones a sufficient number of agents and give the $Destination Table$ to them. Before moving to its destination, the agent computes the shortest route from its own position to its destination using Dijkstra Algorithm. There are two kinds of agent in the Elimination phase: *surrounding agents* that are responsible for guarding the neighbouring nodes of the BVs and *destroying agents* who move to the BVs after all the neighbouring nodes are guarded. We want the BVs to be destroyed at one

time, so it is important that the destroying agents move to the BVs at the same time and only after all the neighboring nodes are guarded by agents. In fact, if the destroying agents know the longest time $t_{longest}$ to move to the destination taken by all the agents (including the destroying agents and the surrounding agents), then they can move to the last node prior to the destination and wait until time $t_{longest}$ to move to the BVs simultaneously. So, in the *Destination Tables* for the destroying agents, we also add the time $t_{longest}$. We now describe how to compute the shortest routes and how to design the *Destination Tables*. Note that we design *Destination Tables* for all the agents and allocate them to the agents before the exploring phase begins.

5.3.3 Destination Table and Elimination

Suppose that there are some BVs and agents in the chordal ring, it is obvious that the BV nodes are in the clockwise side of the agents. In order to use Dijkstra, first we need to map the chordal ring with BVs into a graph where we would run the Dijkstra algorithm to find out the shortest route from every node to its destination.**what do you mean by "map into a graph" ? The chordal ring is already a graph.**We include nodes from the node containing the first agent to the node which is d_k away from the last BV node, then delete the chords from the BV nodes to build the graph where we run Dijkstra Algorithm. Here is an example how we built the graph for running Dijkstra Algorithm. Below we show the situation when the third agent in the exploring group is destroyed by the BV (see Figure 5.13). Only the chords of the original BV node are shown.

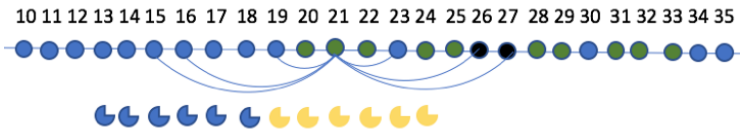


Figure 5.13: Situation when the third agent in the exploring group is destroyed

The black node is the BV node while the green nodes need to be guarded. In this case,

we need 12 agents (10 surrounding agents and 2 destroying agents). We add nodes from 13 to 33 with their chords within this area and delete chords connected with the BV nodes to get the graph where we use Dijkstra Algorithm. Below is the graph we build. (see Figure 5.14). For convenience, we show all the nodes we included and the chords we delete.

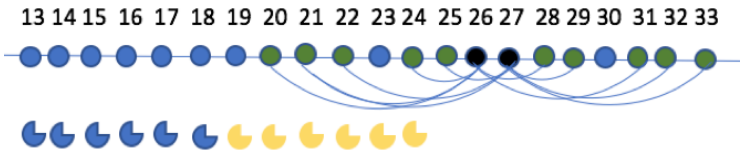


Figure 5.14: The graph we build for Dijkstra Algorithm

Using the graph and Dijkstra Algorithm, we compute the routes from every agent to every node. Then we use enumeration to choose an allocation of every agents's destination considering that:

1) the maximum length of the route should be minimum.

2) after the allocation, in every needed position there should be exactly one agent.

Eery agent in the exploring group should hold a $Destination\,Table$ of $d-1$ parts (Part i records its moving information when $No.i$(anticlockwise) agent in the exploring group is destroyed) while every agent in the shadowing group should hold a $Destination\,Table$ of d parts. Note that an agent in the exploring group have only $d-1$ parts because tit does not need to record moving information when itself is destroyed. Besides, every part contains 1 items (for surrounding agents) or 2 items (for destroying agents).

After we get the optimal allocation, we can know the destination of every agent when $No.i$ agent in the exploring group is destroyed and we can also know whether it is a surrounding agent or a destroying agent in the situation when $No.i$ agent is destroyed.

For each agent in the chordal ring, each item in Part i of its $Destination\,Table$ is arranged as below.

1) The first item records the agent's destination

2) The second item records the longest time $t_{longest}$ among all agents.

After one of the agent is destroyed, the agent can check their $Destination\,Table$ to get the information of their destination. Then using Dijkstra Algorithm they can compute the shortest route separately and starts to move. The destroying agents should move to the last node prior to the destination and wait until $t_{longest}$ to move to the BVs together.

5.4 Analysis

Paola: I added the new proof. This part is still to be checked.

Theorem 5. *If the structure of a chordal ring is fixed, regardless of the position of the BV, the number of $Keep\,Moving$ agents is constant.*

Proof. Define: Going column, the column in which the agents keep going when an explorer encounters a virus.

Define: Stop column, the column in which the agents stop moving when an explorer encounters a virus. We assume that the structure of the ring is $C_n(d_1, \ldots, d_{k-1}, d_k)$, and there are k columns in the matrix which are $M = c_0, \ldots, c_{k-1}$

We assume the number of node where the original BV is $node_V$, then it should be in the column $c_{V \bmod k}$.

A column c_S is a stop column if and only if $\exists i \in [1, k]$, so that $(V + d_i) \bmod k = S$ or $(V - d_i) \bmod k = S$.

If the explorer encounters the virus at the position $node_{V+a}$ instead of $node_V$, since $(V + d_i + a) \bmod k = (S + a) \bmod k$ or $(V - r_i + a) \bmod k = (S + a) \bmod k$, the column $c_{(S+a) \bmod d}$ should be a stop column.

In another word, if c_S is a stop column when virus position is V, then $c_{(S+a) \bmod k}$ is a stop

column when the virus position is $V + a$.

We assume we have two different stop columns c_x and c_y ($0 \leq x \leq d-1$, $0 \leq x \leq d-1$) when the black virus's position is V. So when the black virus's position is $V + a$, the stop columns are $c_{(x+a)\,mod\,k}$ and $c_{(y+a)\,mod\,k}$. It is obvious that $\forall a$, if $x \neq y$, then $(x+a)\,mod\,d \neq (y+a)\,mod\,d$. That means we still have two different stop columns. So that the number of stop column does not decrease, which means that given a fixed chordal ring C and the number of agent keeping moving when the BV is in V (V can be any position), then the number of agent keeping moving when the BV is in other position does not decrease.

Let us assume that the number of agents keeping moving in different case when the longest chord remains the same is different and denotes the minimum number of going columns among them by $N_{minimum}$ while the maximum number of going columns among them by $N_{maximum}$, then according to our conclusion $N_{minimum} \geq N_{maximum}$, which means that $N_{minimum} = N_{maximum}$. In another word, the number of agents keeping moving is fixed when the structure of the chordal ring is fixed. □

Time cost analysis and Comparisons

We only consider the situation when n (the number of nodes of the chordal ring) is much larger than d_k, and since the time cost in the elimination phase is $O(1)$, we only compute the time cost in the exploring phase. Finally, we compute the TWT(calculated by multiplying the size of the team and the time cost by the solution) **Add reminder of TWT?** of both protocols to present a more fair comparison.

Let us assume that the total number of moves is M, then the worst case costing the most time is when the BV is located at any nodes within the range from x_{n-d_k+1} to x_{n-1}. In this case, it cost $M = \lfloor 2n - 2d_k \rfloor$ moves and $4\frac{M}{2d_k}$ units of time which is $4 \times \left\lfloor \frac{n}{d_k} - 1 \right\rfloor$ to finish the exploring phase. In [**?**], she give the number of move in three case.

- 1) In double loops the upper bound of moves is $4n - 7$.

- 2) In the triple loops, she discusses two classes of chordal ring: $C_n(1, p, k)$ and $C_n(1, d_k - 1, d_k)$.In the first case, the number of moves needed is $5n - 6d_k + 22$ while in the second case, a maximum of $5n - 7d_k + 22$ moves are needed.

- 3) In the consecutive-chordal rings, a maximum of $(d_k + 2)n - 2d_k - 3$ moves are needed.

Since in the sequential strategy, agents do not need to wait so the time cost is equal to the number of moves. And it is obvious that our protocol is much faster than the sequential strategy. But since we use much more agents, so in order to gain a fair comparison, now we compute TWT of both protocol.

In the exploring phase, we use $2d_k$ agents, so the TWT of our protocol is $8n - 8d_k$. In the exploring phase of the sequential strategy, it need at least 2 agent to explore and some other shadow agents to guard the explored nodes but the number of shadow depends on the structure of the chordal ring so now we ignore them. Now we compute TWT of the sequential strategy.

- 1)In double loops the upper TWT is $8n - 14$

- 2)The TWT in chordal ring $C_n(1, p, d_k)$ is $10n-6d_k+44$ and in chordal ring $C_n(1, d_k-1, d_k)$ is $10n - 14d_k + 44$.

- 3)The TWT in consecutive-chordal rings is $2(d_k + 2)n - 4d_k - 6$.

It is obvious that when $d_k \geq 2$, our protocol is faster in first case; when $d_k \leq \frac{1}{3}n + 7$,our protocol is faster in the second case (both $C_n(1, p, k)$ and $C_n(1, d_k - 1, d_k)$); when $d > 2 - \frac{1}{n+2}$, our protocol is faster in the third case.

Casualy Analysis

Casualty is the number of agents destroyed by the BV. In chordal ring $C_n(1, d_2, \ldots, d_k)$, the worst case is that the first agent in the exploring group is destroyed by a BV and the

clones of it spread to all its neighbouring nodes. The casualties in this case are $d_k + 1$ because another d_k nodes are guarded by agents while in sequential case, the casualties are $2d_k$. So in terms of casualty, our protocol is better than the sequential strategy.

Chapter 6

Parallel Black Virus Decontamination in Arbitrary Graph

6.1 Introduction

In [**?**], Cai proposes two exploration strategies: Greedy Exploration and Threshold Exploration, both spread optimal and total number of agents asymptotical optimal. Since these strategies are sequential, they are time consuming($O(\Delta n^2)$). In order to explore the graph parallelly, we propose two different strategies:

(1) Flood Strategy

(2) Castle First Strategy

The general idea of the Flood Strategy is simple, supposing that an agent resides in node v, and it has i neighbours excepted to be explored, then it simply clones v agents and send them to its neighbours. In Castle First Strategy, we build castles which is a node or the combination of several nodes(rules are introduced later), the exploration phase can be viewed as the combination of many smaller scale exploration in the graph which begins with the location of one of the exploring group and ends with one of the unexplored castles. After all the castles are explored, all the nodes in the graph are explored. The

general exploring strategy for these two strategies is based on the one described in Chapter 3 which consists of performing a *Shadowed Exploration* phase to locate the BV, followed by a *Surrounding and Elimination* phase to eliminate the cloned BVs.

Strategy *Flood* is time optimal with the cost of a great number of agents while strategy *Castle First* comes to a compromise between the strategy *Flood* and the sequential strategies: it employ much less agents than the strategy *Flood* while cost much less time than the sequential strategies.

In the arbitrary graph, we assume that any node does not disconnect the graph.

6.2 Parallel Strategies for BV Decontamination in Arbitrary Graph

6.2.1 Flood Strategy

Initialization In this strategy, all the agents are endowed with 3-hop visibility. Also, all the agents do not need to remember the routes they pass. Finally, in this strategy, we use an important ability of agents which is clone. As we introduced in Chapter 2, clone means that an agent is endowed with the capacity to generate one or more agent.

We use the Dijkstra Algorithm to compute the shortest route from the homebase to every node and write the route step on the white board on each node, so for each node, there should be a number (Shortest Route Number) recording the number of steps of the shortest route from the homebase to it. Let us denote by v_{SRN} the Shortest Route Number of node v, and nodes $v_1, \ldots, v_i$ are neighbours of node v assuming that v has i neighbours. Then edges connecting node v and its neighbours are $e(v, v_1), \ldots, e(v, v_i)$. We write the v_{SRN} to the end of these edges (the end connecting to its neighbours), so when an agent resides in any node from v_1 to v_i, it can see the Shortest Route Number of node v because

of the local visibility. (For example, see Fig6.1)

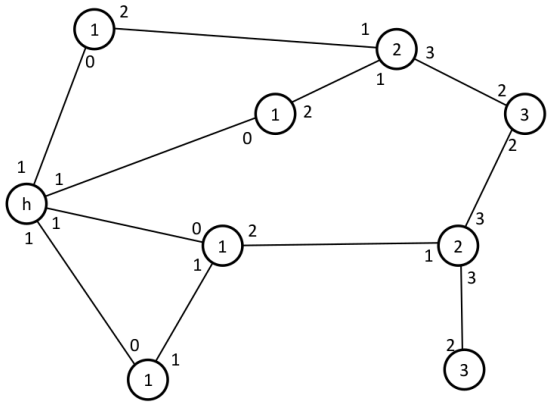


Figure 6.1: Initialization of the graph

Exploration Phase All the agents in this strategy follow the same rules in the exploration phase.

Rules for agents in the exploration phase: Let us denote by SNR the Shortest Route Number.

1. Agents can only move from node with lower SNR to node with higher SNR.

2. Assuming that there are x agents residing in node v, and the next destination(s) are $\{v_1, \ldots, v_i\}$.

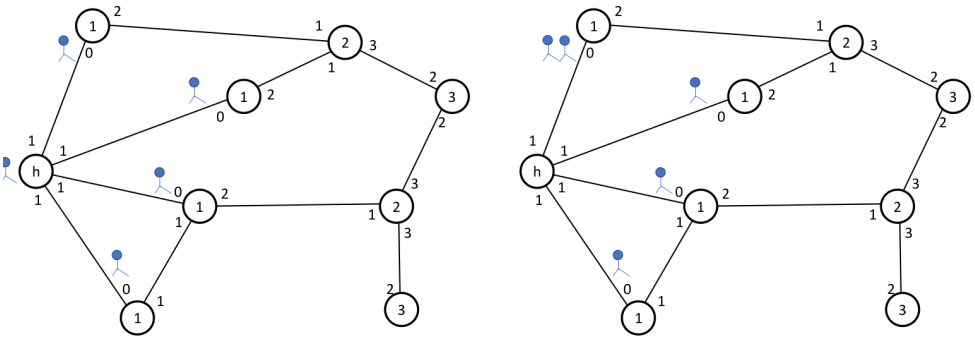
   if $x \geq i+1$, then i agents move to the destinations respectively while the left agents stay in node v to guard v at T_i. If one of the agents is destroyed, the Elimination phase begins; if none of the agents is destroyed, the left $x-(i+1)+1$ (the one who guards the node v) agents evenly move to the destinations at T_{i+1}.

   if $x < i+1$, then one of the agents residing in node v clones $i+1-x$ agents, and these i agents move to all the neighbours at T_i. If one of the agents is destroyed, the Elimination phase begins; if none of the agents is destroyed, the agent guarding the node v randomly moves to one neighbour with SNR equal to $a+1$ at T_{i+1}.

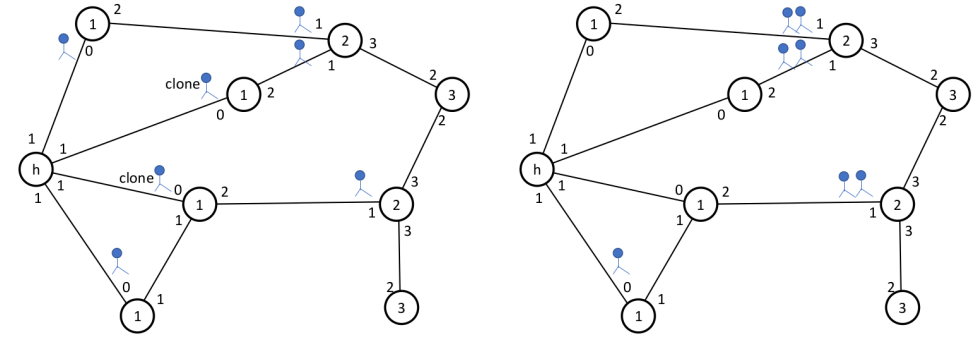
3. If an agent resides in a node (assuming its SNR is a) without any neighbour whose SNR equal to $a + 1$, then it simply stays there.

   Since all the action of agents happen after they meet each other at the same node, they can communicate with each other and make sure that all their routes do not conflict.

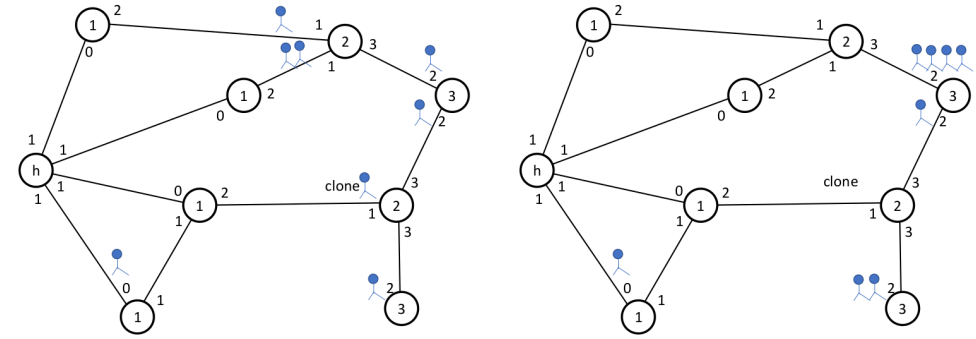
An example of how the agents move is showed in Fig6.2



(a) T_1 (b) T_2



(c) T_3 (d) T_4



(e) T_5 (f) T_6

Figure 6.2: An example of how the agents move in the Flood Strategy

Elimination Phase Let us assume that the node where the original BV resides is v with SNR equal to a and it is triggered at T_i. Then at this time, the clones spread to all neighbours of node v with SNR equal to $a+1$ and survive while leaving node v clean (no agent and no BV) and let us denote by v_{BV}s all these BV nodes.

Now we introduce how agents residing in different positions move in the Elimination

phase.

1. Rule 1: For convenience, we call agents residing in the these v_{BV}s' neighbours with SNR equal to a the Witness Agents, and these Witness Agents can easily realize whether or not node v is the place where the original BV resides. For example, since they have 3-hop visibility, if they see that one of their "2-distance" neighbours (say node v') does not contains an agent but some neighbours of node v' contain agents, they can know that the original BV resides in node v. If the Witness Agents realize the existence of BV at T_i, then they simply stop cloning and moving to the new formed BV nodes. Note that if the Witness Agents (say it resides in node u) have some higher SNR neighbours except the new formed BVs, they should still explore these nodes following the rules in the exploration phase but leave an agent in node $u$to guard it.

2. Rule 2: For all of the agents residing in node v's neighbours with SNR equal to $a-1$ (say the number of them is y), they receive clones and realize the location of the BV and would move to v at T_{i+2}. Let us denote by z the number of v's neighbours with SNR equal to $a+1$, then if $y < z+1$, one of the agents residing in node v should clone another $z+1-y$ agents and then z agents move to v's neighbours with SNR equal to $a+1$ at T_{i+4}.

Note that in our assumption, any node does not disconnect the graph, so there should be other routes from the homebase to the new BV nodes except through the original BV node. So when z agents are sent to v's neighbours with SNR equal to $a+1$ at T_{i+4}, some other agents move to these nodes's higher SNR neighbours at the same time, which ensure that all the neighbours of the new formed BVs are guarded when they are triggered.

See Fig6.3. For convenience of description, we give every node in the graph a distinct ID from 1 to 13. As shown in the picture, when the BV is triggered, the clones spread to

all neighbours of node 3, and node 4 and 5 become new BV nodes while nodes 1, 2 and 12 destroy a clone respectively and realize that the original BV resides in node 3.

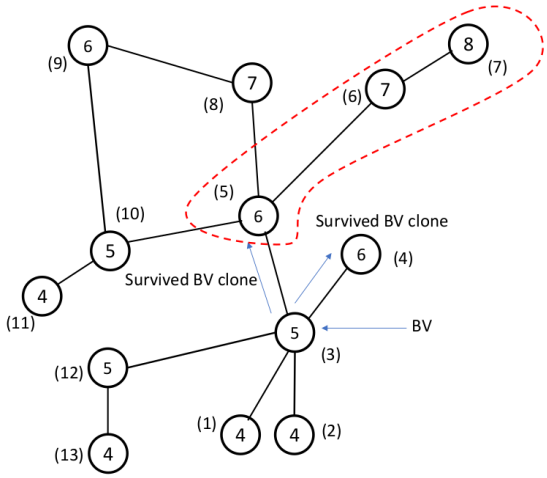


Figure 6.3: An possible situation when the BV is detected

In Fig.6.3, agent residing in node 10 is a Witness Agent and because of the 3-hop visibility, it can "see" that there is no agent residing in node 3 but there are agents residing in node 3's neighbours which are node 1 and node 2. By this way, it knows that the original BV resides in node 3 and it stops moving and cloning. Also, according to the Rule1, it still has a higer SNR neighbour (node 9) which does not contain a BV, so for this node, it continue to explore it following the rule in the exploration phase. For example, it sends an agent to node 9, then all the agent except one move to node 9. After that, one agent is sent to node 8.

6.2.2 Castle First Strategy

Introduction In the Castle First Strategy, all the agents have only the local visibility. But the leader agent in each exploration group have the map of the graph in its memory. Also, the leader agents are endowed with the ability of clone.

In the Castle First Strategy, we built some castles based on the graph with SNR. More

than one group of agents are sent to explore the graph respectively. Their exploration of the graph are separated into many "sub-exploration"s and each "sub-exploration" begins with where the agents are and ends with a new unexplored castle. The map of the graph with all the castles being pointing out is recorded on the whiteboard on nodes with more than two neighbours (the intersection) so when agents move across these nodes, they can update the information on the whiteboard (for example, changing the state of some castles into explored) or update its own memory. When agents cannot find an unexplored castle in the graph, then they terminate.

Initialization Based on the graph marked SNR for every node, we built another graph called "Castle Graph". We now give the definition of "Castle":

1. Case 1: Node who has one neighbour is a "Castle".
2. Case 2: If a node has at least one neighbour with the same SNR as it, then the combination of them is a castle. If different castles have at least one common node, then we merge them into a bigger castle;
3. Case 3: If a node has more than two neighbours with lower SNR than itself, than this node is a castle.

Some examples of castles are shown in Fig6.4

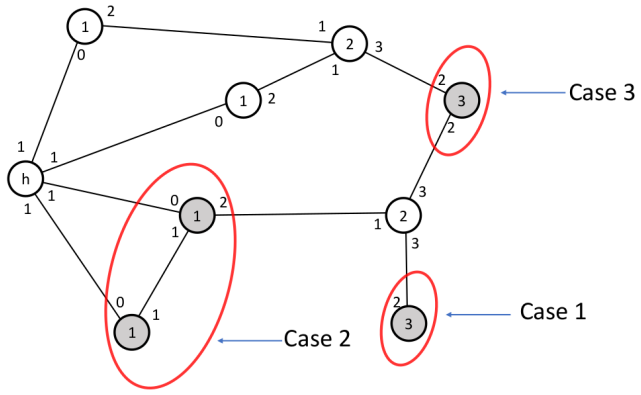


Figure 6.4: Examples of Castles

After the processing, we have a new graph called "Castle Graph" and this graph is presented on the whiteboard on the nodes which have more than two neighbours, so when the agent reaches this node, it can read the graph to update its own information or using its own information to update the "Castle Graph".

Exploration Phase

The Exploration Phase of the Castle First Strategy is shown in Fig6.4 and is introduced in detail in the following.

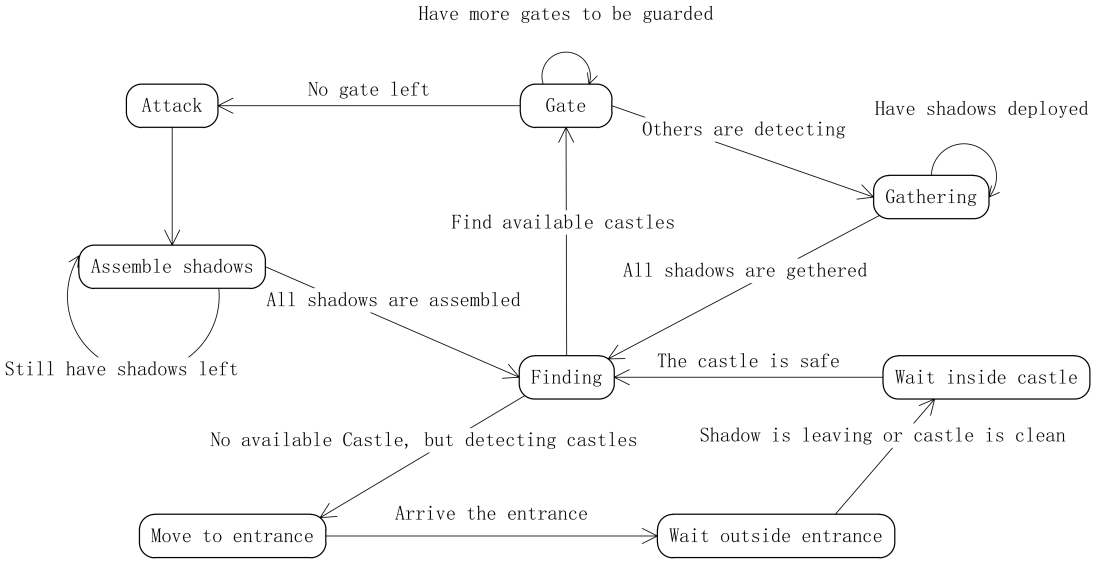


Figure 6.5: Exploration phase of the Exploration Phase

There are two principle in the exploration phase for the agents:

1. when agents move in an unexplored area, they should strictly move from node with smaller SNR to node with bigger SNR and should follow the "casual walk" while when move in an explored area, they can move in opposite (from bigger SNR to smaller SNR) and when the agents move in this area, they are allowed not to follow the "casual walk" (for example, when they plan to explore node v from node u, the

LA can directly move to node v when node v is in the explored area) which is called "normal walk".

2. The castle selected to be the destination of the "sub exploration" should meet the follow the following requirement: assuming that the SNR of the destination castle is a and the node from where the agents start the "sub exploration" is v , then for every node with SNR equal to $a-1$ connected to the castle, there should be a route from v to these nodes and no castle(s) which are not explored (the castles can be in the state of "under exploring" or "explored") existing on these routes. In another word, when the agents move from v to these node, except the destination castle, they do not need to "attack" other castles.

There are two kind of agents in this strategy: Leader Agent (LA) and Shadow Agent(SA) and the LA has the map of the whole graph in its mind. When more agents are needed, the LA would clone the agents. At the beginning of the exploration phase, the LAs are in the homebase, so they respectively pick a castle to be the destination of the their first "sub exploration" and since they have the map of the graph, they can compute a shortest route for the "sub exploration" and then are sent randomly from the homebase.

The agents are divided into three status: Finding Castle, Attacking Castle and Waiting in Line.

Initially, the status of all agent group are "Finding Castle".In a agent group with the status of "Finding Castle", if the LA in that agent group can find an available castle, then the status of this group changes into "Attacking castle". Unless the agent group realizing the existence of BV, the status of the agent group changes into "Finding Castle" at the end; if the LA in that agent group cannot find an available castle but there are still castle unexplored in the graph, then the status of this group changes into "Waiting in line". Unless the agent group realizing the existence of BV, the status of agent group changes into "Finding Castle" at the end.

(1) From "Finding Castle" to "Attacking Castle"

On the way to their destination (the castle), the agents follow the "casual walk" in the unexplored area and "normal walk" in the explored area: when exploring the node u from the node v, one of the SAs moves to node u, when the node u is safe, it returns to node v and moves to the node u with the LA and the other SAs; when the node u contains a BV, then the LA knows the existence of the BV by receiving the clone of the BV.

When one of the agent is destroyed by the BV, then the elimination phase begins.

Along the route to the group's destination, the LA updates the information (changing the state of the destination castle to be "under explored") of the intersections and also read information from them, if it finds that the destination castle of its "sub exploration" has been explored or under exploring, then the status of its group changes into "Finding Castle" again. If not, then this group reach one of the destination castle's lower SNR neighbour and the LA starts to arrange the SAs to guard all the lower SNR neighbour(s) of the castle.

After the arrangement of the SAs, it should be ensured that all the lower SNR neighbours of the castle are guarded and assuming that there are x nodes in the castle $\{castle\_0, \ldots, castle\_x\}$, another x SAs (Attacking Agent) should move to these x nodes at the same time after the arrangement. In order to do that, we propose one possible strategy for the LA to place the SAs: assuming that the sequence of the nodes which should be guarded is $\{node\_0, \ldots, node\_y\}$, and $node\_i$ is the last node in the sequence connected to $castle\_k$ where $0 \leq i \leq y$, $0 \leq k \leq x$, then the LA should place two agents in $node\_i$ while place one agent in the other nodes when it moves in sequence to place the SAs. For example, in Fig6.6, assuming that the sequence of nodes needing guarded is $\{node_1, node_2, \ldots, node_{10}\}$, then after the arrangement, there should be two agents in $node_2$, $node_4$, $node_6$, $node_8$, $node_{10}$ and one agent in other node.

Also, the LA knows the time when it finishes the arrangement of the last agent(s), then

it should inform the agents the exact time to move to the castle nodes and permanently destroy the BVs.

When one agent group starts to surround the castle, by which we mean that the LA has placed SAs in at least one lower SNR of the castle, it is possible that another agent group has started to surround the castle (even the LA updates the information when it moves, still the consistency of all the information cannot be guaranteed). The LA of an agent group realizes this by meeting an SA not from its group when it arranges the SAs to guard the lower SNR neighbours of the castle. We prefer to avoid conflict when two agent group explore one castle (SAs from two different agent group move to the castle nodes). The LA can compute the time when the arrangement is finished and record it on a timestamp, so when the LA places the SAs, it leaves timestamp with these SAs. By doing this, every SA from one agent group guarding the castle hold a timestamp recording when does the arrangement end and if a LA meet a SA from other agent group with a earlier timestamp, it moves back to collect the SAs in its group that have been placed and the status of this agent group turn into "Finding Castle".

When the arrangement is done, the LA resides in the last node needing guarded and in the next unit of time, all the Attacking Agents move to the castle nodes. Note that at this time, if there is a BV in the castle, not all the guarding agents can receive a BV clone. As shown in Fig6.6, when the BV is triggered, except the castle nodes, only the guarding agents 1 and 2 receive the clones.

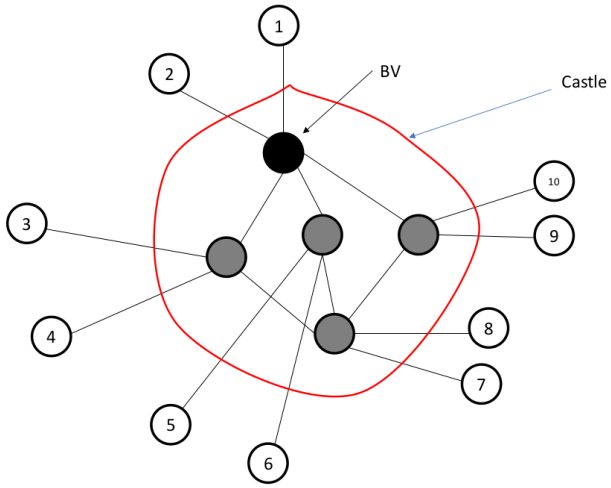
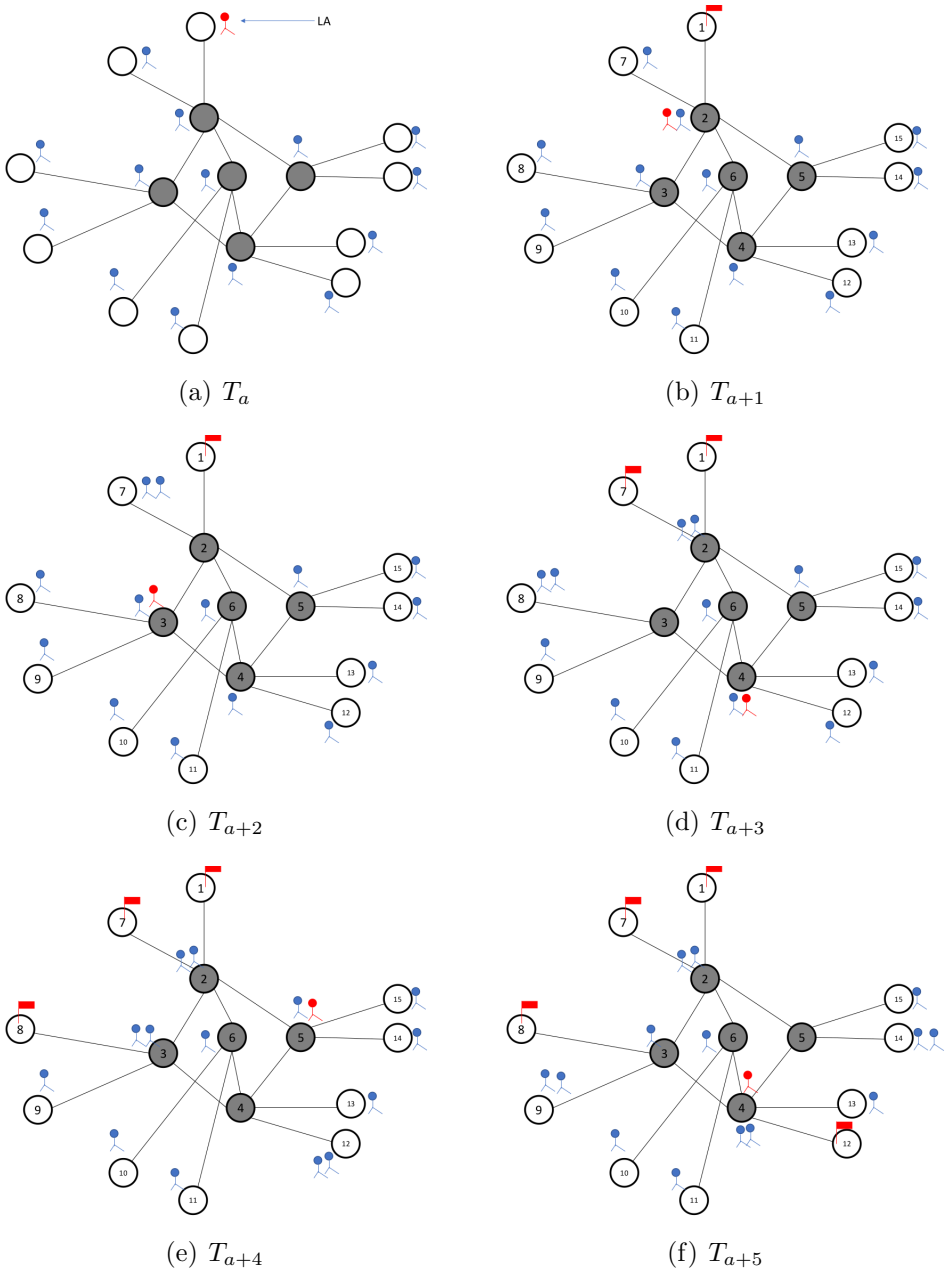


Figure 6.6: An example shows that if there is a BV in the castle, in some cases not all the guarding agents can receive a BV clone

So after the "attacking" by the "Attacking agent", the LA should execute the "Double Patrol". Now we introduce what is the "Double Patrol".

1. The LA first computes a route to traverse all the castle nodes, (say the route is $castle\_0 \rightarrow castle\_1 \rightarrow \ldots, castle\_x$ assuming that there are x nodes in the castle). In this first "patrol" (see Fig.6.7), the LA first leaves a flag saying "First Patrol" and then moves to one castle node, it tells the SA residing in that castle node to collect the SAs residing in all its lower SNR neighbours and finally move to that castle node with all these collecting SAs. Also, when the SA collects the ones residing in the lower SNR neighbours, it places a flag on that neighbour node saying "First Patrol". After the LA traverses all the castle nodes, it can easily know whether there is a BV in the castle: if there is no BV in the castle, then there should be one agent residing in every castle node and the LA would start the "second patrol". if not, then there is a BV and the clones of it have move to all higher SNR neighbours of this node. If the LA realizes that there is a BV, it stops traversing the left castle nodes.

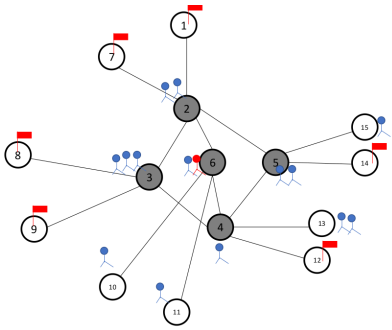


(a) T_a (b) T_{a+1}

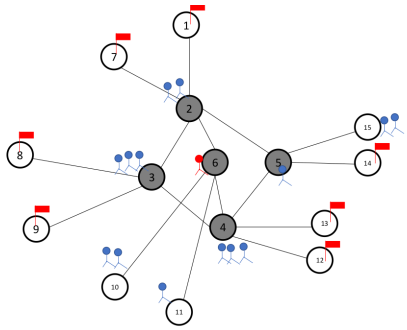
(c) T_{a+2} (d) T_{a+3}

(e) T_{a+4} (f) T_{a+5}

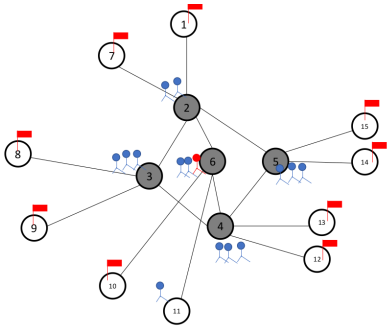
Figure 6.6: An example of "FirstPatrol"



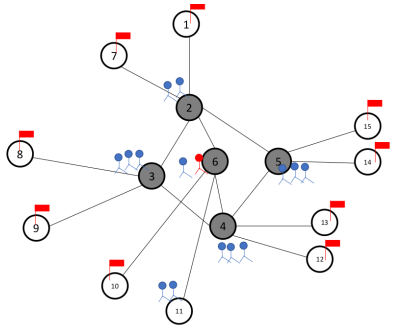
(g) T_{a+6}



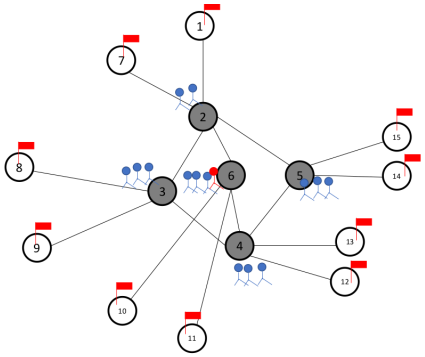
(h) T_{a+7}



(i) T_{a+8}



(j) T_{a+9}



(k) T_{a+10}

Figure 6.7: An example of "FirstPatrol"

2. After the first patrol, the LA knows that if there is a BV. If there is, the Elimination phase starts. If there is not, then the "Second Patrol" starts. In the "Second Patrol" the LA moves along the reverse route in the "First Patrol" which is $castle\_x \rightarrow castle\_x - 1 \rightarrow \ldots, castle\_0$. Also it collects all the SAs in the castle node it passes by and updates the information on the whiteboard on the castle node saying that the castle is explored.

After the "Second Patrol", the status of this agent group changes into "Finding Castle".

(2) From "Finding Castle" to "Waiting in Line"

When the status of the agent group changes into "Waiting in Line", then it means that the LA in that agent group cannot find an available castle but there are still castle unexplored in the graph. At this time, the LA randomly chooses an castle marked "under exploring" and move there with its agent group. When the agent group arrives one of the lower SNR neighbours of that castle we call them the "Guarding Node", three situations may happen:

1. Case 1: The agent group arrives there finding that no SA(s) and no flag in that Guarding Node. In this situation, the agent group who is excepted to attack this castle may not yet arrive this castle or does not finish the arrangement of the SAs. Then the agent group should wait at that guarding node until there is an SA and follows the instruction in case 2.

2. Case 2: The agent group arrives there finding at least one SA. In this situation, it means that the agent group who is excepted to attack the castle has finished arranging the SAs but has not finished the "First Patrol". So this agent group stays with the SA until the SA is called by another SA and moves into the castle. After moving into the castle node, the agent group should wait the LA of the agent group who is attacking the castle to move to this castle node again because at this time the LA would update the situation in the castle: whether there is a BV or not. If there is a BV in the castle, then agent group who is waiting in the castle node ends its exploration phase and start the elimination phase; If not, then the status of this agent group changes into "Finding Castle".

3. Case 3:The agent group arrives there and finds that there is a flag on that guarding node. It means that the LA of the agent group who is excepted to attack the castle is doing the "First Patrol" or has finished the first "First Patrol", or is doing the second

"Second Patrol". In this situation, the agent group moves into the castle node which is connected to the guarding node it arrives and wait the LA of the agent group who is attacking the castle to move to this castle node again. As in the case 2, this agent group can know the if there is a BV in the castle and follows the same instruction as in the case 2: if there is a BV, then the agent group starts the elimination phase, if not, the status of it changes into "Finding Castle".

Eliminaiotn Phase

When the BV is triggered, then the location of the BV can be in a castle or outside the castle. In both situation, the lower SNR neighbours of the BV node or the castle containing the BV have been guarded by the agents. In another word, when the BV is triggered, only the clones spreading to the higher SNR neighbours (say that the number of them is y) survive and their locations are exposed. Also, the locations of all the neighbours of these new formed BVs are exposed.

So the LA computes the routes from where it resides to all the nodes needing guarded including the lower SNR neighbours and the higher SNR neighbours(Surrounding Nodes). When the location of the original BV is outside the castle, then all the SAs stays with the LA, so the LA simply sends the SAs to these Surrounding Nodes. When the original BV is in the castle, then after the "Double Patrol", the LA knows the location of the original BV, so it computes the routes from where it resides to all the Surrounding Nodes and sent the SAs to them (note that all the SAs are with the LA after the "Double Patrol"). Note that in our assumption that any node in the graph does not disconnect the graph, there is always another route to reach the surrounding node except bypassing the new formed BV nodes. Assuming that there are x higher SNR neighbours of the original BV, then finally, the LA sends another x to the new formed BV nodes and permanently destroy the BVs.

6.3 Experimental Study on Black Virus Decontamination on Arbitrary graph

In this section, we experimentally investigate the problem of black virus decontamination by studying the Castle First Strategy described in Chapter 6, and comparing it with the Greedy Exploration Algorithm [**?**]. The simulator we built is called ArbiBV. We introduce its modelling methods and execution. The behaviour, properities and performance have been investigated through an extensive number of computer simulation runs. The simulation results confirm the excepted behaviour/properties of the solution protocol.

6.4 Introduction of the simulator

6.4.1 Simulation Model

Our simulator only operates the process in the exploration phase, since the parameters (for example, the number of agent, the execution time . . .) can be easily derived (the time in elimination phase is $O(1)$) after the location of the BV is exposed. In the simulator, we do not set a BV, so the agents have to explore the whole graph. The environment where the agents operate is a network modeled as a simple arbitrary undirected connected graph and each node has an unique ID (without loss of generality, from 1 to n where n is the number of node in the graph) . An agent is modelled as an entity with computational or information processing capability. All the agents follow the same protocol while the role of them and the state can be different (for example, some of the agents are Leader Agent (LA) while some of the agents are Shadowing Agent(SA)) Also, our simulator describes the movement of the agent through language but not through Graphical User Interface(GUI). For example, "Leader Agent in group 1 moves from node x to node y". By doing this, we can monitor the movement of the agents. In each run of the simulator, we provide the

number of node, the average degree of the graph, and the simulator generates a arbitrary graph based on that. After each run, the simulator calculates the number of agent, the units of time.

6.4.2 Simulation Sample graph

In this section, we compare the number of agents and the execution time between the Castle First Strategy and the sequential strategy[**?**], also between Castle First Strategy with different settings (sending different number of group to explore the graph). When we compare our strategy with the sequential strategy, we run our protocol in different graph setting (the size of the graph are 20, 40, 60, 80, 100 nodes and network connectivity densities are 10%, 20%, ..., 100%). The network connectivity density is defined as the ratio of the number of links of a graph to the number of links in complete graph with the same nodes. For each specific setting (size of the graph, the network connectivity densities and number of group of agent), we run the experiment for 50 times: the average execution time and the average number of agent needed would be the result of the execution time and the number of agent of this setting. So overall, we run 9020 experiments to obtain statistic data of our strategy. By doing the comparison, we are interested in observing how faster can our strategy be than the sequential strategy and in the cost (more agents) that we should pay for the faster speed.

When we compare the results of different strategy settings of Castle First Strategy, we are interested in the following aspects:

1. What is the relationship between the execution time and the number of agent group that completing the strategy;

2. Since in our strategy, given a certain group of agent group, we except them to explore as many castles at the same time as possible (the advantage of distributed strategy

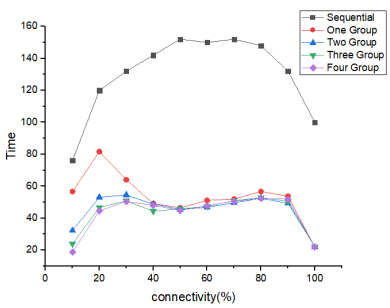
would be more obvious), we also investigate how the efficiency of the the strategy depending on various topologies factors (the size of node, the connectivity, ...) .

6.5 Simulation Results

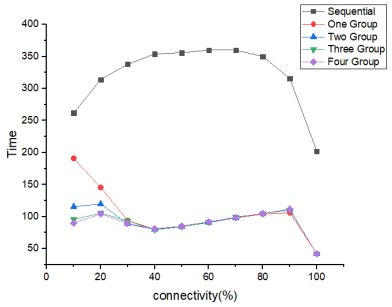
Comparison between the Castle First Strategy and the Sequential Strategy (GREEDY Exploration)

As a worst case scenario, we let the exploration proceed until all nodes without placing any black virus. We do the comparison of the Castle First Strategy and the GREEDY Exploration. For each size category, we create 10 connectivity levels and run our protocol with the number of group ranging from one to four.

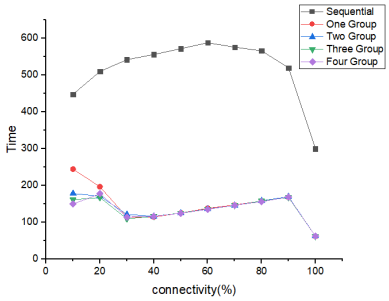
Simulation results demonstrate Castle First Strategy is never slower than the GREEDY Exploration even when only one group of agent explore the graph and the cost (more agents) that we pay is acceptable. The time needed to finish the exploration of the graph in GREEDY Exploration and Castle First Strategy (where number of exploring group ranging from one to four) is as shown in Figure6.8



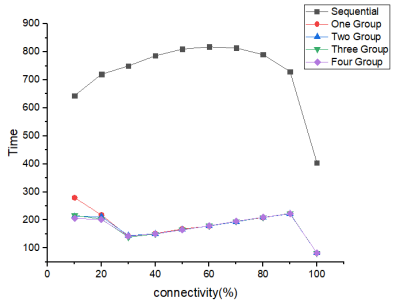
(a) size of graph:20 nodes



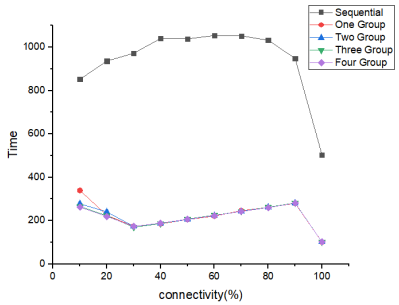
(b) size of graph:40 nodes



(c) size of graph:60 nodes



(d) size of graph:80 nodes



(e) size of graph:100 nodes

Figure 6.8: Comparison between GREEDY Exploration and Castle First Strategy

From Figure6.8, we have the following observations of the time:

1. Sequential Strategy(GREEDY Exploration) vs Castle First Strategy:

   In all size of graph, the time of the GREEDY Exploration increases gradually to a maximum at 40%-60% connectivities, after that, it gradually decreases. It should be pointed out that the time costs are close for different graphs at 100% connectivity level and are comparable or less than those at all 10% connectivity level. While in the

Castle First Strategy, the time cost first increases slightly and then decreases gradually to a minimum at 28%-40% connectivities, after that, they gradually increase to a number comparable to those at all 10%-20% and then decrease sharply to reach a minimum when the connectivity is 100%. Note that though experiencing an increase from connectivity of 10 to 20 when the size of node is 20, the time increases less and less obviously as the size of the graph grows (except the time cost by the "one group" agent in all size of graph which always experiences decreasing until it reaches the local minimum). When the size of graph is 80, the time cost of the Castle First Strategy executed by more than one group stays at almost the same level from connectivity of 10 to 20. When the size of the graph is 100, the time directly decreases to the local minimum. This is reasonable because as the size of the graph grows, even at the same connectivity, graph with more nodes is denser, so when the agents move from one castle to another, they have shorter route to choose which saves time. The behaviour of the "one group" agent are largely influenced by this factor, so except in the graph of 20 nodes, the time cost by "one group" agent does not experience an slight increase but decreases directly to the local minimum.

After reaching the local minimum, the movement starts to increase. The reason is that though there are more shorter routes to choose as the connectivity increases when moving to the next castle which saves more time, this advantage becomes less obvious because the length of the route from one node to another node does not change a lot as the connectivity increases. At the same time, as the graph becomes denser, the castles are becoming larger, so we need to complete the "Double Patrol" in large castles which costs lots of time. The result of the counteracting is that the movement increases after the local minimum movement.

When the connectivity is larger than 90%, the situation is that except the home base, the combination of the other nodes becomes a extreme large castle, so that in the exploration phase, the only thing we need to do is to explore one castle which results

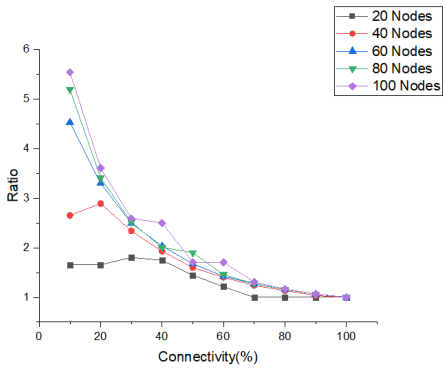
in the decrease of movement.

2. Performance of the Castle First Strategy when the number of agent group executing the strategy is different.

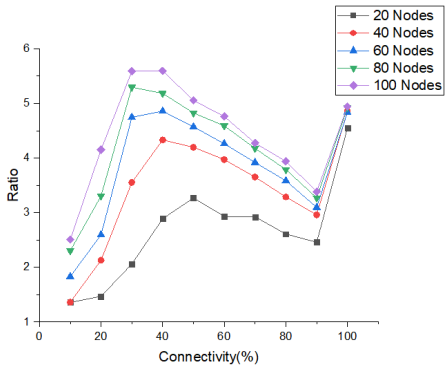
   From the Fig.6.8, we can see that when the connectivity is larger than 30%, then performances of movement are almost the same no matter how many groups we send to explore the graph. This is reasonable because according to our algorithm, the castles in a graph should be explored strictly sequentially following some rules, which means in some case, though there are many castles unexplored and many group of agent not exploring the castle or on the way to explore the castle, these agent group cannot explore these unexplored castles because they have to wait until some castles are explored first. At this time, even we sent more agent group to explore the graph, we cannot decrease the time.

   Also, another important thing is that when the connectivity is larger than 20 (or maybe less), the time cost by different number of group of agent is similar (almost the same). In another word, we prefer to use one group of agent executing the Castle First Strategy when the connectivity is larger than 20%.

Based on the statistics presented in Fig.6.8, we now compare the time and the number of agent used by these two strategies. Since the time cost between the Castle First Strategy executed by different group of agent is similar from connectivity 10%to 100% especially when the connectivity is larger than 20% , so we only compare the statistics between the GREEDY Exploration and the the Castle First Strategy with one group of agent. In figure 6.9, we give the ratio of the number of agent used in Castle First Strategy using one group of agent to the number of agent used in GREEDY Exploration. Also we give the ratio of the time cost by Castle First Strategy using one group of agent to the time cost by the GREEDY Exploration.



(a) Ratio of the number of agent used in Castle First Strategy using one group of agent to the number of agent used in GREEDY Exploration



(b) Ratio of the time cost by Castle First Strategy using one group of agent to the time cost by the GREEDY Exploration

Figure 6.9: Ratio of the time and the agent cost of Castle First Strategy by one group to GREEDY Exploratioin

As we can see on the figure 6.9, in (a), the ratio is equal to number of agent used in Castle First Strategy using one group of agent/ the number of agent used in GREEDY Exploration (for convenience, we call it the ratio 1) while in (b), the ratio is equal to the time cost by Castle First Strategy using one group of agent/ the time cost by the GREEDY Exploration (for convenience, we call it the ratio 2).

From the Fig6.9, we have the following observations:

1. As the connectivity grows, we can see that the ratio 1 is decreasing and when the connectivity is larger than 30%, the ratio does not change a lot. When the connectivity is larger than 70, the ratio is near 1. This tell us that when the connectivity is smaller than 20, we use much more agents (up to 5.5 times) than the GREEDY Exploration while when the connectivity is between 30 to 60, we use 1.2 to 2.5 times as many agents as that in the GREEDY Exploration. When the connectivity exceeds 70, we use almost the same number of agent as that used in GREEDY Exploration.

2. The ratio 2 first increases to the local maximum when the connectivity reaching 30% to 50%, then decreases to the local minimum when the connectivity reaching 90%,

then increases sharply to reach a maximum when the connectivity reaching 100%.

3. Based on the two observations above, we come to an conclusion that when the connectivity of the graph reach 30% to 50%, the Castle First Strategy using one group of agent work most efficiently.

Now we would like to focus on the graph with small connectivity, it is obvious that when the size of the graph is different, even with the same connectivity, the degrees of the nodes are different. The average degree of a graph is an important factor because it influences how the castles of the graph form. Now we would like to introduce the factor "Average Degree" and to see if the results of the Castle First Strategy executed by different number group of agent have large difference. Note that from the conclusion above, we know that when the connectivity is larger than 20% (or less), there are no big difference between the the time finished by different group of agent, so now we would like to start from small degree and increase by 0.5 degree at a time until no big difference appears.