Decontamination from Black Virus Using Parallel Strategy

by

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Abstract

In this thesis, the problem of decontaminating networks from *black virus* (BVs) using parallel strategy with a team of system mobile agents (the BVD problem) is studied. The BV is a harmful process whose initial location is unknown a priori. It destroys any agent arriving at the network site where it resides, and once triggered, it spreads to all the neighboring sites, i.e, its clones, thus increasing its presence in the network. In order to permanently remove any presence of the BV with as less execution time as possible and minimum number of site infections (and thus casualties), we propose parallel strategy to decontaminate the BVs: instead of exploring the network step by step we employ a group of agents who follow the same protocol to explore the network at the same time, thus dramatically reducing the time needed in the exploration phase and minimizing the casualties. Different protocols are proposed in meshes, tori, and chordal rings following the monotonicity principle. Then we analyze the cost of all our solutions and compare to the asynchronous BV decontamination. Finally conclusion marks are presented and future researches are proposed.

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Chapter 1

Introduction

A distributed system is a group of computational entities cooperating with each to achieve one or more tasks. This thesis deals with distributed computing by mobile agents in network. More specifically, we deal with the problem of deploying a group of mobile agents who follow the same protocol to explore the network and decontaminate the dangerous virus (called Black Virus) present on the network nodes. In this chapter, the motivations of the problem are provided, following is a brief summary of the contributions.Finally, an overview of the organization of the thesis is presented.

1.1 Problem and Motivation

Mobile agents are widely used in distributed and network systems while the applications of them can cause some security issues, thus threatening to the network: A contaminated or infected host can destroy working agents for various malicious purposes; A malicious agent can contaminate or infect other computer nodes so they become malfunctional or crash. The harmful hosts, often called *Black Holes* trigger the problem called *Black Hole Search* (BHS), the focus of which is to locate their positions since it is statics. This problem has been studied in many variants. For example, different topologies and different settings

(synchronous and asynchronous). The harmful agents trigger the problem called *Intruder Capture* (IC). Its main focus is to deploy a group of mobile agents to capture a extraneous mobile agent (the intruder) who moves arbitrarily fast through the network and infects the visiting sites. Also it has been investigated in a variety of topologies. More detailed literature review will be provided in Chapter 2. Note that BH is static and only damage the agents reaching it without leaving any detectable trace. Intruder is mobile and harmful to the network nodes but does not cost any harm to other system agents. A new harmful presence called *black virus* BV has been initially introduced by Cai et al. in[**?**]. It is a dangerous process resides at an unknown site in a network and destroys any upcoming agents, but unlike the BH, the node where the original BV resides thus become clean. At the same time, the original BV multiplies (called clones) and spread to all neighbouring nodes, thus increasing its number,and damage in the network. A BV is destroyed when it moves to a site where there is already an agent. Based on this harmful presence, a new problem called Black Virus Decontamination(BVD) is presented by Cai et al., the main focus of which is to use a group of system agents to permanently remove any presence of the BV from the network. A protocol defining the actions of the agents solves the BVD problem if at least one agent survives and the network is free of BVs. Also, a desirable property of a decontamination protocol is that the nodes which have been explored or cleaned by mobile agents are not be recontaminated by the BV spreading. A solution protocol with such a property will be called *monotone*. see[**?**]. Some important cost measure is the number of node infections by the BVs (casualties); size of the team, i.e, the number of agents employed by the solution, the time needed by the solution. Solutions in which the agents explore the network's node in sequence have been proposed in [**?**], [**?**] and [**?**]. The size of the team is minimum in [**?** **?**] and the number of site infections is also minimum in such case, i.e., exploring the network nodes in sequence but the units of time that these protocol (including the protocol in chordal rings) cost is usually several times as much as the total number of the network's nodes (assuming in synchronous setting). Now

we are interested in the solution using parallel strategy where we deploy a larger number of mobile agents following the same protocol to decontaminate the network in the exploring phase with the goal to minimize the *total working time* (TWT) which is calculated by multiplying the number of agents and the total execution time and also the casualties.

1.2 Our Contribution

1. In this thesis, we propose parallel strategy to solve the BVD problem. It is the first attempt to deal with this issue in a parallel way. Agents are not allowed to communicate with each other unless they are in the same network node so the protocol should enable the agents in different nodes to move properly, i.e, the route of every agent is different but they are served to explore the network; when a BV is triggered, other agents should bypass the new-formed BVs... We give simple but efficient solution to deal with this problem with acceptable cost. Also we give the size of the exploring team which is minimum to guarantee both the TWT and the casualties we reach.

2. The BVD problem is investigated for three important topologies: *meshes*, *tori*, *chordal rings*. All the protocol are optimal both in term of TWT and casualties. We compare our solution with [**?**] and [**?**] in which the exploring route is in sequence and the result is that our solution is better than that of them in both TWT and casualties. One should be point out that in chordal ring especially, the more complicated the chordal ring becomes, the more TWT that we save comparing to [**?**].

1.3 Thesis Organization

The thesis is organized as followed:

Chapter 2 contains a literature review on related problems. We begin by reviewing the Black Hole Search and Intruder Capture problem, and then focus on the solution of BVD problem where the mobile agents explore the network in sequence, the issue has been studies in different topologies: two-dimensional grids, three-dimensional grids, tori, chordal rings, hypercubes and arbitrary network. Also, the variant of this problem, which is decontamination of an arbitrary network from multiple black virus is also reviewed. We also present our assumption and the topologies (meshes, tori, chordal rings).

Chapter 3 introduces terminology, definitions and model for the BVD problem used in the rest of the thesis. Also we describe the high level ideas that serve as the basic of all our solutions. Since monontone is the necessary condition for spread optimality, we go through the principle and finally make a conclusion.

Chapter 4 focus on the BVD problem for two simple classes of network topologies: *meshes* and *tori*. For each kind of network, an optimal algorithm in terms of casualties and TWT is developed. Complexity analysis in terms of casualty and TWT are performed and obtained. Some comparison and analysis are also made between our solution and [**?**] and the result shows that our solution is better.

Chapter 5 presents the BVD problem for the chordal ring topology. In this chapter, we introduce the *Three Jump Notifying Technique* (TJNT) to manipulate each mobile agent efficiently go through their route in the exploring phase and avoid any new-formed BV after the original BV is triggered. Based on this technique, we develop the parallel strategy for the mobile agents to decontaminate the choral ring from BV. Complexity analysis in terms of casualty and TWT are performed. Finally some comparison and analysis are made between our solution and [**?**] and the result shows that our solution performs better.

Chapter 6 summaries the main conclusion of our work and present some open problems and future work.

Chapter 2

Literature Review

Mobile agents have been widely used in the field of distributed computing due to their features especially the mobility which allow them to migrate between computers at any time during their execution. A group of agents can be used to perform a various tasks, for example, network exploration, maintenance, and etc. However, the introduction of mobile agent tend to cause security problem, thus threatening the network. Various security issues and solution algorithms have been proposed by Flocchini and Santoro in [**?**]. Generally, the threaten that the mobile agents cause are divided into two categories: in first case,the malicious agents can cause network nodes malfunction or crash by contaminating or infecting them (the harmful agent); in second case, the contaminated or infected hosts can destroy working agent for various malicious purposes (the harmful hosts). These two threaten trigger two problems: Black Hole Search (BHS) and Intruder Capture (IC) which will be introduced in the following sections. Then we review the BVD problem which deal with the decontamination of a harmful presence which cause the network node malfunction but leaves the network node clean when it is triggered and spreads to all its neighbouring nodes, thus increases it presences. In the section introducing BVD problem, we present the the abilities of mobile agents that has been proposed and different decontamination strategies based on different strategies.

2.1 Black Hole Search, BHS

The BHS problem assumes there is a BH or multiple static BHs residing at certain network nodes and will destroy any upcoming agents without leaving any detectable trace.The task is to use a team of agent to locate the black hole(s) and is completed when at least one agent survives and reports the location(s) of the black hole(s). Note that the solution is based on graph exploration and the goal can be reached totally depending on the sacrifice of some agents. In [**?**], Das et al. considered a model for unknown environment with dispersed agents under the weakest possible setting, many exploration models and works were included in this article. The BHS problem has been widely studies in various topologies and settings: the timing is synchronous or asynchronous; the number of black hole(s) is known or unknown. For example: by Chalopin [**?** **?**] in asynchronous rings and tori, Dobrev et al. [**?**] in arbitrary graph, [**?**] in anonymous ring and [**?**] in common interconnection networks...What is worth pointing out is that the number of BHs remains the same as it of the beginning, thus not causing harm to other sites of the network.

2.2 Intruder Capture

The IC problem assumes that there is an intruder moves with an arbitrary speed from node to node in the network and contaminate the sites it visits, the goal of which is to deploy a group of mobile agents to capture the intruder;the intruder is captured when it comes in contact with an agent. Note that the intruder does not cause any harm to the upcoming agents. It is equivalent to the problem of decontaminating a network contaminated by a virus while avoiding any recontamination. This problem is first introduced in [**?**] and has been widely investigated in a variety network topologies: trees [**?** **?** **?**], hypercubes[**?**], multi-dimensional grids[**?**], chordal rings[**?**] etc. The studies of arbitrary graph has been started in [**?** **?**]. Note that monotone is a critical principle in the solutions of IC

problems.

2.3 Agent Capabilities

Different capacities granted to the mobile agents have an impart on solving the BHS problem,IC problem and also the BVD problem.now we discuss these capabilities in the following section.

Communication Mechanisms Mobile agents can communicate with each other only when they are in the same node in a network.Some essential communication methods have been studied in literature: whiteboard,tokens and time-out. In [**?** **?** **?** **?**], the whiteboard model is used, which is a storage space located at each node and agents arriving there are able to read and write. In the token model, (see [**?** **?**]), tokens are like memos of the agents which can be dropped off and picked by agents at nodes or edges. While the time-out mechanism can only be used in synchronous setting where each agent has a pre-determined amount of time. (see [**?** **?** **?**]).

Knowledge of the topology Different assumption of mobile agents' knowledge of the topology has an impact on solutions of some of the problems mentioned above, for example, the BHS problem. In [**?**], Dobrev et al. present three types of topological knowledge in an asynchronous arbitrary network and show the results of the BHS problem based on different setting of the topology knowledge.

Other capabilities In some studies, agents are endowed with the visibility, which mean that they can see whether or not their neighbouring nodes are clean or contaminated (see [**?** **?**]).They observe that the visibility assumption allow them to drastically decrease the time and move complexities in torus, chordal ring and hypercubes when dealing with IC

problem. For example, in chordal ring $C_n\{d_1 = 1, d_2, ..., d_k\}$, the number of agents, the time and the moves required in local model are $(2d_k + 1)$, $3n - 4d_k - 1$, $4n - 6d_k - 1$ respectively, while in visibility model, they are $2d_k$, $\left\lceil \frac{n-2d_k}{2(d_k - d_{k-1})} \right\rceil$, $n - 2d_k$. In torus , the number of agent, the time and the moves required are $2h + 1$, $hk - 2h$, $2hk - 4h - 1$ and in visibility model are $2h$, $\lceil \frac{k-2}{2} \rceil$, $hk - 2h$ respectively. They also compare the complexity of both models in hypercubes, a algorithm requiring Θ $(\frac{n}{\sqrt{logn}})$ number of agents and $O(n \log n)$ moves while the algorithm they propose in the visibility model requires $\frac{n}{2}$ agents and $O(n \log n)$ moves.

In [**?**], the concept of *k-hop visibility* is presented. The agents have the *full topologies* if each of them have a map in their memory of the entire network including the identities of the node and the labels of the edges. If a agent has *k-hop visibility*, then at a node v a agent can see the k-neighbourhood $N^{(v)}$ of v, including the node identities and the edge labels. Note that *Diam-hop* visibility is equivalent to full topological knowledge.

Another interesting capability of agents is cloning which is introduced in [**?**]. Cloning is the capacity for an agent to create copies of itself. In this paper, they also discuss how the combination of different capacities reaches different optimal strategy in IC problem in hypercube. For example, the strategy is both time and move-optimal when visibility and cloning are assumed or when cloning, local and synchronicity are assumed. But the time and move-optimal strategy can be obtained at the expense of increasing the number of agents. The last capability of agents be discussed is immunity which means that a node is immune from recontamination after an agent departs. Two kinds of immunity have been proposed:local and temporal. In local immunity, (see [**? ?**], the immunity of a node depends on the state of its neighbouring nodes. More specifically, a node remains clean after the departure of an agent until more than half of its neighbours are contaminated. In the temporal immunity, a node is immune for a specific amount of time (t). The node remains clean until time expires and becomes recontaminated if at least one of its neighbours are contaminated. In models without immunity assumption, a node becomes recontaminated if it has at least one contaminated neighbours.

2.4 Black Virus Decontamination

2.4.1 Overview

The BVD problem is first introduced by Cai at al. in citeCai: A black virus is a extraneous harmful process endowed with capabilities for destruction and spreading. The location of the initial BV(s) is known a priori. Like a BH, a BV destroy any agents arriving at the network where it resides. When that happen, the clones of the original BV spread to all it neighbouring nodes and remain inactive until an agent arrives. The BVD problem is to permanently remove any BVs in the network using a team of mobile agents. They proposed that the only way to decontaminate a BV is to surround all its neighbouring nodes and send an agent to the BV node. In this case, the node where the original BV resides is clean and all its clones are destroyed by the guarding agents in its neighbouring nodes.They have presented different protocols in various topologies: q-grid, q-torus, hypercudes in [**?**] and arbitrary graph [**?**]. A basic idea of implementing the decontamination has also been propose by them assuming that the timing is asynchronous which divides the whole decontamination process into two part: '*shadowed exploration*' and '*surround and eliminate*'. In order to minimize the spread of the virus, they use a '*safe-exploration*' technique which is executed by at least two agents: the *Explorer Agent* and the *Leader Explorer Agent* who both residing at a safe node u at the beginning, for example,the homebase. The *Explorer Agent* moves to a node v to explore it and it needs to return to node u to report the node v is safe. The *Leader Explorer Agent* determines if the node v is safe or not by *Explorer Agent*'s arriving or a BV's arriving. If node v is safe, both of them move to node v. For the purpose of insuring monotonicity, at any point in time the already explored nodes must be protected so they are not be recontaminated again. After the BV is detected, the '*surround and eliminate*' begins. In this phase, some agents are employed to surround the new-formed BVs (the clones of the original BV) then some agents are sent to the clones to permanently destroy them. This is called the '*Four-step*

Cautious Walk' and is widely used in BVD problem with synchronous setting. Also, BVD problem in chordal ring has been discussed in [**?**].

2.4.2 BVD in different topologies

Protocols regarding to BVD problems in grid are BVD-2G and BVD-qG which deal with BVD problems in 2-dimensional grid (meshes) and q-dimensional grid respectively. BVD-2G performs a BV decontamination of a 2-dimensional grid of size n using $k = 7$ agents and 3 casualties, within at most $9n + O(1)$ moves and at most $3n$ time. While protocol BVD-qG performs a decontamination of a q-dimensional grid of size $d_1 \times d_2 ... \times d_q$ using $3q + 1$agents and at most $q + 1$ casualties, within at most $O(qn)$ moves and at most $\Theta(n)$ time. Algorithm to decontaminate the BV in a q-dimensional torus,called BVD-qT uses $4q$ agents with 2 casualties with at most $O(qn)$ moves and $\Theta(n)$. Protocol BVD-qH is to perform a BV decontamination of a q-hypercube using $2q$ agents and q casualties with at most $O(n \log n)$ moves and $\Theta(n)$. In arbitrary graph (see [**?**]), two protocols are presented: GREEDY EXPLORATION and THRESHOLD EXPLORATION. In these two protocols, $\Delta + 1$ agents are needed and both of the protocols are worst-case optimal with respect to the team size. Though the protocols are described for a synchronous setting, they easily adapt to asynchronous ones with an additional $O(n)$ moves for the coordinating activities. An advantage of these protocols is that the agents can use only local information to execute the protocol. Another interesting fact based on these two protocols is that both GREEDY ROOTED ORIENTATION and THRESHOLD ROOTED ORIENTATION produce an optimal acyclic orientation rooted in the homebase.

In [**?**], solution for BVD in chordal ring is discussed. In Alotaibi's thesis, she discuss solutions based on different kinds of chordal ring: double loops, triple loops, consecutive-chords rings and finally general chordal ring. In double loops, she proposed three strategies in elimination phase and the upper bound of moves is $4n - 7$ in the whole protocol and

a maximum of 12 agents are employed.In triple loops, she discusses two classes of chordal ring: $C_n(1, p, k)$ and $C_n(1, k-1, k)$. In any triple loop $C_n(1, p, k)$, a maximum of $5n - 6k + 22$ moves and 24 agents are needed for the decontamination while in any triple loop $C_n(1, k-1, k)$, a maximum of $5n - 7k + 22$ moves and 19 agents are needed. Finally in the consecutive-chords ring, a maximum of $(k+2)n - 2k - 3$ moves and $4k + 1$ agents are needed. She described the decontamination strategies in synchronous setting but only with a cost of $O(n)$ moves can the strategies be used in asynchronous setting.

Chapter 3

Definitions and Terminology

3.1 Model

3.1.1 Network, Agent, Black Virus

Network The environment in which mobile agents operate is a network modelled as simple undirected connected graph with $n = |V|$ nodes (or sites) and $m = |E|$ edges (or links). We denote by $E(v) \subseteq E$ the set of edges incident on $v \in V$, by $d(v) = |E(v)|$ its degree, and by$\Delta(G)$ (or simply Δ) the maximum degree of G. Each node v in the graph has a distinct $id(v)$. The links incident to a node are labelled with distinct port numbers. The labelling mechanism could be totally arbitrary among different nodes; without loss of generality, we assume the link labelling for node v is represented by set $l_v = 1, 2, 3, ..., d(v)$.

Agent A group of mobile agents are employed to decontaminate the network. The agent is modelled as a computational entity moving from a node to neighbouring node. More than one agents can be at the same node at the same time. Communication among agents occurs at this time; there are no a priori restrictions on the amount of exchanged information. In our thesis, we employ two groups of agents (the exploring group and the shadowing group) operating the same protocol and we assume that all the agent?s moves follow the same

clock. Agents in the exploring group Also, agents are endowed with 1-hop visibility which means at a node v, it can see the labels of the edges incident to it and the identities of all its 1-hop neighbours. We do not guarantee other capability introduced in Chapter2 to the agents in our protocols.

Black Virus In G there is a node infected by a black virus (BV) which is a harmful process endowed with reactive capabilities for destruction and spreading. The location of the BV is not known at the beginning. It is not only harmful to the node where it resides but also to any agent arriving at that node. In fact, a BV destroy any agent arriving at the network site where it resides, just like the black hole. Instead of remaining static as the black hole, the BV will spread to all the neighbouring sites leaving the current node clean. The clones can have the same harmful capabilities of the original BVs (fertile) or unable to produce further clones(sterile). In this thesis, we only consider the situation of fertile clones but actually the protocol can be simply modified to adapt to the sterile situation. A BV will be destroyed if and only if the BV arrive at a node where there is already an agent. Thus, the only way to eliminate the BV is to surround it completely and let an agent attack it. In such situation, the attacking agent will be destroyed while the clones of the original BV will be permanently eliminated by the agents residing the neighbouring nodes of the original node. We assume that at the same node, multiple BVs (clone or original) are merged. More precisely, at any time, there is at most one BV at each node. Another important assumption is that when a BV and an agent arrive at an empty at the same time, the BV dies and the agent survive remaining unharmed.

Summarizing, there are five possible situations when an agent arrive at a node v:

- agents arrive at a node which is empty or contains other agents, they can communicate with each other and the node v is clean.
- agents arrive at a node which contains a BV, the agent dies and the clones of the BV (BVC) spread to all the neighbours of v leaving node v clean.

- A BVC arrives at a node which is empty or there is already a BV: the node becomes/stays contaminated; it merges with other BVs.
- BVCs arrive at a node v which contains one or more agents, the BVCs are destroyed but the agents are unharmed.
- A BVC and an agent arrive at an empty node at the same time, the BVC dies while the agent remains unharmed.

3.1.2 Problem, Cost

The BLACK VIRUS DECONTAMINATION (BVD) problem is to permanently remove the BV, and its clones from the network using a team of mobile agents starting from a given node, called home base(HB). The solutions where the agents explore the network sequentially have been proposed in some classes of topologies. chordal rings, hypercubes and arbitrary graph. In this thesis, we are interested in parallel strategies in BVD problem: instead of exploring the network in sequence, we explore it in parallel; in chordal ring, we also propose a parallel solution to surround the clones of the original BV. In this thesis, the efficiency measurements we have are: *spread* of BV (also measures the number of agents *casualties*; the *size* of the team, i.e, the number of agents employed by the solution; total working time(TWT) (calculated by multiplying the *size* of the team and the time cost by the solution. Note that TWT does not contains any practical meaning but exist only as a measurement. We propose TWT to compare more fairly the time of two protocols when the number of agents is different.

3.1.3 Monotone, Synchrony

A desirable property of a decontamination protocol is to prevent the nodes which been explored or cleaned by mobile agent from being recontaminated which will occur if the

clones of the BV are able to move to a explored node in absence of agent. A BVD protocol with such property is called monotone. Monotone property is the necessary condition for spread optimality.

Asynchrony refers to the execution timing of agent movement and computations. The timing can be *Synchronous* or*Asynchronous*. When the timing is synchronous, there is a global clock indicating discrete time unit; it takes one unit for each movement (by agent or BV); computing and processing is negligible. When we have asynchronous agents, there is no global clock, and the duration of any activity (e.g., processing, communication, moving) by the agents, the BV, and its clones is finite but unpredictable. In this thesis, all our protocols work in synchronous setting.

3.2 General strategy

Following the solution in sequential case, we decomposed the BVD process into two separate phase: *Shadowed Exploration* and *Surrounding and Elimination*. The task of the first phase is to locate the BV and the second phase is to decontaminate the BV and its clones. Apart from these two basic phase, we have initialization which is to deploy the agents properly at the beginning of executing the protocol because we explore the network in parallel, and the arrangement of the agents is crucial to successfully execute the protocol.

Phase1: Shadowed Exploring Agents employed are divided into two group: shadowing group and exploring group, and the number of agents in two group is the same. For convenience, we call the agent in the shadowing group the shadow agent (SA and those in the exploring group the exploring agent EA (one EA is accompanied by one SA. As the name indicated, agents in exploring group explore the network and the agents in shadowing group follow the agents protecting the node which have been explored. More precisely, at T_1, EA moves to node v; at T_2, SA moves to node v and EA moves to node u supposing that node v is clean.

Phase2: Surrounding and Elimination In this phase, since we already know the position of the BV, we employ agents to surround all the neighbours of the BVCs. Once all the agent arrive the proper positions(all the neighbours of the BVCs are guarded), we employ another group of agents (the number of them is equal to the number of BVCs) to move to the BVCs, thus permanently destroy them. Usually, some of the agents moving to the neighbours of the BVCs are from the shadowing group and exploring group because in this way we can save the number of agents used in the whole protocol. Note that not all the agents are informed when the BV is detected. More specifically, only the agents who receives the clones know the existence of the BV, and other agents keep moving in the network. In some simple topologies, such as meshes and torus, the second phase begins when the BV is detected since the number of agents are enough to proceed the second phase. In some more complicated topologies, for example, the chordal ring which we discuss later, we take some other measures to call back enough number of agents to finish the second phase.

3.3 Conclusion

In this Chapter, we presented the model of our problem and also some important terminologies. Also we described a general strategy for our problem depending on particular setting: synchronous timing, parallel strategy... In the next chapter, we discuss the parallel strategies in BVD problem in two simple topologies: meshes and torus.

Chapter 4

Black Virus Disinfection in Double Loops

In this chapter we consider the process of using agents to locate and clear the *black virus* in the *double loop* chordal ring $C_n(1, k)$ in a *synchronous* environment. The *Double Loop* ring is an interconnected ring of n nodes $v_0, \ldots v_{n-1}$, where each node is connected to two additional neighbours at distance of k. The neighbourhood of node v_i is thus given by: $N(v_i) = \{v_{i+1}, v_{i+k}, v_{i-1}, v_{i-k}\}$.

As with all the other topologies, the leader and the exploring agent locate the BV and then surrounding agents are sent to surround the new triggered viruses and eliminate them so that the entire topology is disinfected. We propose several variants of the main strategy depending on the routing procedure used to surround the new triggered viruses. We evaluate the complexity of our strategies by considering the overall number of agents employed, the number of agent casualties and the number of moves required. The table below summarizes the worst case complexities of the three main strategies.

Double Loop	Spread W.C	Agents W.C	Moves
Move-Optimal	4	12	$3k + 31$
Simple Greedy	4	12	$6k + 12$
Smart Greedy	4	12	$5k + 17$

4.1 Exploring and Shadowing

This phase is common to all of the strategies. The two agents, LEA and EA, explore each node on the outer ring in a clockwise direction starting from the homebase using the *Safe Exploration* technique: LEA and EA are at node v_j. EA moves to the next node v_{j+1} while LEA waits at v_j. If EA returns to its leader they both move to v_{j+1}, otherwise, the BV is detected and EA is destroyed. In order to diminish the effect of the BV, a shadow agent SH is deployed to guard the explored neighbours of the next node to be visited. This cannot happen unless LEA and EA have passed through at least k nodes. In other words, if the safe area is $\geq k$.

Lemma 1. *The Exploring and Shadowing phase is a monotone protocol that locates the black virus correctly.*

Proof. The exploring team follows the outer ring and EA always precedes LEA, so that the *black virus* is eventually detected. The monotonicity of this phase is apparent in the presence of shadow agents that were all created in the beginning at the homebase and then synchronize their movements with the exploring team. □

EXPLORING AND SHADOWING

let $HB = v_0$

Agents EA and LEA at safe node v_j.

if $(k \leq j+1 < n-k)$

$N_{ex}(v_{j+1}) = \{v_j, v_{j+1-k}\}$

1 SH is deployed to protect v_{j+1-k}

if $(n-k \leq j+1 < n-1)$

$N_{ex}(v_{j+1}) = \{v_j, v_{j+1-k}, v_{j+1+k}\}$

2 SHs are deployed to protect v_{j+1-k},v_{j+1+k}

if $(j+1 = n-1)$

$N_{ex}(v_{j+1}) = \{v_j, v_{j+1-k}, v_{j+1+k}, v_0\}$

3 SHs are deployed to protect v_{j+1-k},v_{j+1+k},v_0

EA moves to v_{j+1}.

Some observations can be made following the implementation of this strategy:

Theorem 2. *In the worst case scenario, the black virus is detected in* $4n-7$ *moves.*

Proof. The worst case for the number of moves is when the *black virus* is found at node (v_{n-1}) after exploring $n-1$ nodes. The BV triggers no new *black viruses* since all the neighbours are already explored in the safe area and all are protected by SHs.

The complexity of this case is $3(n-1)-2$ for the movement of LEA and EA, $n-1-k$ for one SH and $(k-1)$ for the other SH. □

Theorem 3. *In any double loop chordal ring* C*, the worst case scenario in term of the number of agents required occurs when three new black viruses are created after triggering the original virus.*

Proof. The worst case for the number of *black viruses* created upon activation of the original is when that one is found at node v_i where $1 \leq i < k$. In this case, the *black virus* (x_0) triggers three new *black viruses*: x_1, x_k and x_{-k} because no SH has been deployed at this time. Note that x_{-1} is always occupied by LEA. □

4.2 Surrounding and Eliminating

In the double loop chordal ring, when the BV is triggered it may affect up to three neighbouring nodes. If x_0 represents the original BV, node x_{-1} is clearly protected by LEA. x_1, x_k and x_{-k} are protected only if they belong to the safe area and are occupied by shadow agents SH. In summary, the best case scenario in terms of the number of *black viruses* occurs if no more BVs are created after EA triggers the original black virus. The worst case scenario involves the creation of three new BVs.
To handle the spread of the BVs, LEA has to send agents to surround and clear those faults. The only way to destroy BVs is when they arrive at guarded nodes.

To do that, LEA creates agents that are sent to specific targets, the neighbours of the new created BVS. Once all the agents are in position, LEA sends cleaning agents (casualties) to trigger all the BVs at once. This means that we require the same number of SAs as the number of neighbours of BVs and as many CAs as BVs.

Lets take a look at the necessary notation. x_0 represents the discovered BV. V represents the set of all vertices. The set of black viruses in the system is denoted by $\mathcal{BV}$. $\mathcal{S} = V - \mathcal{BV}$ represents the set of clean nodes (not containing any BV). $\mathcal{T}$ represents the set of targets (the nodes to be occupied). S_{area} represents the *safe area* and $|S_{area}|$ is the number of nodes in that area. D_{area} represents the *danger area* where recontamination needs to be avoided.

SURROUNDING AND ELIMINATING

LEA and SHs covering all $N_{ex}(v)$

BV comes back from $v = x_0$.

if ($|S_{area}| < k$) (* LEA is covering x_{-1} *)

$N_{un}(x_0) = \{x_1, x_k, x_{-k}\}$

if ($k \leq |S_{area}| < n - k$) (* LEA and SH covering x_{-1}, x_{-k} *)

$N_{un}(x_0) = \{x_1, x_k\}$

if ($n - k \leq |S_{area}| < n - 1$) (* LEA and SHs covering x_{-1}, x_{-k}, x_k *)

$N_{un}(x_0) = \{x_1\}$

Else (* LEA and SHs covering x_{-1}, x_{-k}, x_k, x_1 *)

$N_{un}(x_0) = \emptyset$

All SH make one move in clockwise direction.

For each $u \in N_{un}(x_0)$:

DEPLOY an agent to each $z \in \{N(u) \setminus N_{un}(x_0)\}$

When $N(u)$ is covered:

DEPLOY one agent to u

From the previous algorithm we can see that once a SH receives a *black virus* it moves to the neighbouring node through chord +1 in order to be part of the surrounding team. Thus, we can see that agents sometimes change roles.

Synchronicity is one of the assumptions of this model. All agents are synchronized and it takes one time unit to traverse a link connecting two neighbouring nodes.

It is convenient for our purposes to divide the chordal ring into three segments starting from the homebase as seen in figure 4.1.

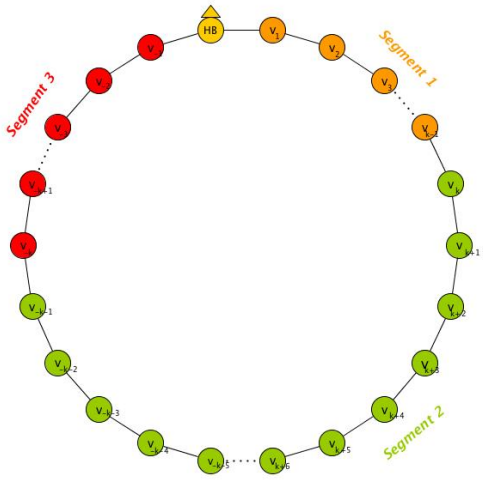


Figure 4.1: Dividing the double loop into three segments.

- *Segment 1* contains nodes $v_1, \ldots v_{k-1}$. If the *black virus* is in this segment, the size of the safe area is $1 \leq |S_{area}| \leq k-1$.
- *Segment 2* contains nodes $v_k, \ldots v_{n-k-1}$. If the *black virus* is in this segment, the size of the safe area is $k \leq |S_{area}| \leq n-k-1$.
- *Segment 3* contains nodes $v_{n-k}, \ldots v_{n-1}$. If the *black virus* is in this segment, the size of the safe area is $n-k \leq |S_{area}| \leq n-1$.

We will see that complexity changes depending on the location of the black virus relative to the segment.

The number of agents required to disinfect the double loop chordal ring C is the same regardless of the strategy employed, whereas the number of moves varies depending on the deployment method.

Theorem 4. *Regardless of deployment strategy and chord length, a maximum of 12 agents are employed in any double loop $C_n(1,k)$ for the black virus disinfection protocol.*

Proof. The number of agents required is determined by the location of the original black virus, regardless of the deployment strategy.

- When the BV is located at any node in *Segment 1*, we would obtain the worst complexity in terms of CA and SA since activating the *black virus* will create three more *black viruses* at x_1, x_k, and x_{-k}. LEA then moves to x_0 and deploys seven SAs to occupy $x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-k+1}, x_{-k-1}$ and x_{2k}. The total number of agents employed is then 12: 4 CAs, 7 SAs and one LEA. Thus, in this case, $Spread(C) = 4$ and $Size(C) = 12$.

- If BV is found in *Segment 2* , activating the original *black virus* would create two more *black viruses* at x_1 and x_k because x_{-k} is guarded by a SH. SH then moves to x_{-k+1} and LEA moves to x_0. LEA then deploys 4 SAs to occupy x_2, x_{k-1}, x_{k+1} and x_{2k}. The total number of agents employed is then 9: 3 CAs, 4 SAs, 1 SH and LEA. Thus, in this case, $Spread(C) = 3$ and $Size(C) = 9$.

- If BV is found in *Segment 3*, there are two possible scenarios:

  - When $n - k \leq i < n - 1$:

    This segment is part of what is called the *Danger area* (D_{area}). If the BV is located at any node in this segment, activating the original *black virus* would create only one *black virus* at x_1, while x_{-k} and x_k are guarded by SHs. Subsequently, in addition to the SH at x_{-k+1}, the SH at x_{k+1} and LEA at x_0, the LEA deploys 1 SA to occupy x_2. The total number of agents employed is then 6: 2 CAs, 1 SA, 2 SH and one LEA. Thus, in this case, $Spread(C) = 2$ and $Size(C) = 6$.

  - When $i = n - 1$, no more *black viruses* are created since all neighbouring nodes of the *black virus* are guarded by SHs and LEA. There are then no SAs to be deployed and all the moves are done in the first phase. The total number of agents employed is then 5: 1 CA, 3 SHs and one LEA. Thus, in this case, $Spread(C) = .1$ and $Size(C) = 5$.

□

For the deployment phase we propose two types of strategies: non-local and local. In non-local strategies, LEA calculates the shortest path to reach the various targets and gives each agent the information about the corresponding path. In local strategies, each SA decides the next node locally, based on the indication of the destination target.

4.2.1 Move-Optimal Deployment

In this section we describe a non-local approach where SAs follow paths set up by their leader LEA who has full topological knowledge. Once the *black virus* is triggered, LEA, knowing the topology and the location of the black viruses, computes the best route to each target and gives the information to the corresponding SAs. Because of the simple structure of the chordal ring, we can calculate the exact length of the shortest paths to reach the target and verify experimentally that they are indeed optimal.

In this section we describe the path to be followed by each agent for the simpler case of shortly-chorded double loops, and then for the case of general double loops. Before we describe these two situations we will identify some special constant size paths that can be used to reach all targets in both situations. The special path to reach node x_i from x_0 is denoted by σ_i. Depending on the chord structure, it may be possible to devise shorter paths in some cases. A detailed analysis of all possible situations is carried out in the next two sections.

Target x_i	Special Path σ_i
x_{k-1}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1}$
x_2	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k} \xrightarrow{+1} x_{2k+1} \xrightarrow{+1} x_{2k+2} \xrightarrow{-k} x_{k+2} \xrightarrow{-k} x_2$
x_{2k}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k}$
x_{k+1}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k} \xrightarrow{+1} x_{2k+1} \xrightarrow{-k} x_{k+1}$
x_{-k-1}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1}$
x_{-k+1}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-k} x_{-2k-1} \xrightarrow{+1} x_{-2k} \xrightarrow{+1} x_{-2k+1} \xrightarrow{+k} x_{-k+1}$
x_{-2k}	$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-k} x_{-2k-1} \xrightarrow{+1} x_{-2k}$

4.2.1.1 Shortly-chorded Double Loops

A shortly-chorded double loop is a double loop where the size is significantly bigger than the length of the largest chord, that is: $k << n$. All results of this section hold for $n > 4k$.

After triggering the original *BV* (x_0) the neighbouring nodes are in one of two states: guarded or contaminated. Node x_{-1} is guarded by *LEA*. Depending on the size of the *safe area*, nodes x_1, x_k and x_{-k} could be in either state. As mentioned earlier, the *black virus* could be located in one of three segments.

For some of the targets it is easy to identify the shortest path from x_0. For example, x_{k-1} cannot be reached directly but it can be reached in two moves from x_0 through node x_{-1} (i.e., following the special path σ_{k-1}). For other targets it is harder to determine whether σ_i represents the best alternative.

We now consider the different routes to each target in four different cases depending on the location of the *black virus* where $\pi[x_0, x_i]$ denotes a path to reach target x_i from x_0.

- **Case 1**: Consider the case where the *black virus* is in the second segment of the chordal ring, that is, when $k \leq |S_{area}| < n - k$. In this case, triggering the original *black virus* creates two more *black viruses*: x_1 and x_k, and thus $\mathcal{T}$=$\{x_2, x_{k-1}, x_{k+1}, x_{2k}\}$.

- x_{k-1}: Node x_{k-1} is reached through σ_{k-1}, which is clearly the shortest path.
- x_2: Since for $k = 3$ we know that $x_2 = x_{k-1}$, we consider $k > 3$.

  Depending on the length of k, x_2 could be reached optimally in different ways (note that all of them are actually shorter than the special path σ_2 identified earlier):

  * If $k = 4$ and $x_{k-1} = x_3$, the shortest path is the following:

$$\pi[x_0, x_2] = \pi_1 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} = x_3 \xrightarrow{-1} x_2$$

  * If $k > 4$, taking advantage of the fact that x_{-k} is known to be safe:

$$\pi[x_0, x_2] = \pi_2 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+1} x_{1-k} \xrightarrow{+1} x_{2-k} \xrightarrow{+k} x_2$$

- x_{2k}: Node x_{2k} is reached through σ_{2k}.
- x_{k+1}: If $k > 4$, x_{k+1} is reached through σ_{k+1}. Otherwise, it is reached faster through:

$$\pi[x_0, x_{k+1}] = \pi_3 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} = x_3 \xrightarrow{-1} x_2 \xrightarrow{+k} x_{k+2} \xrightarrow{-1} x_{k+1}$$

- **Case 2**: When the *black virus* is in the third segment excluding the last node before the homebase (i.e., $n-k \leq |S_{area}| < n-1$), only one *black virus* is generated (x_1) since the rest of the neighbours have been explored and guarded. Thus, $\mathcal{T}=\{x_2, x_{k+1}\}$.
  - x_2: If $k \leq 4$, x_2 is reached using π_1, otherwise it is reached using π_2
  - x_{k+1} is reached in one move from node x_k because x_k contains a SH agent, which received a copy of the original *black virus*.

- **Case 3**: We have a special case in which the *black virus* is located on the last node before the homebase. In this case, all neighbours are guarded and no more *black viruses* are created. The surrounding phase is no longer needed.

- **Case 4**: When the *black virus* is in the first segment (i.e., $|S_{area}| < k$) we have to consider the nodes that might be contaminated if they do not belong to the area already protected by shadows. In fact, when $|S_{area}| < k$, LEA has to deploy agents to the neighbours of the three new black viruses $\mathcal{BV}$=$\{x_1, x_k, x_{-k}\}$ while x_{-1} is being guarded by LEA and no SH has been deployed.

  Node x_{-k} has three unguarded neighbours, $\mathrm{N}_{un}(x_{-k})$= $\{x_{-k-1}$,x_{-k+1},$x_{-2k}\}$. Thus, $\mathcal{T} = \{x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-k-1}, x_{-k+1}, x_{-2k}, \}$.

  We have:

  - Nodes x_{k-1} and x_{2k} are reached through paths σ_{k-1} and σ_{2k}.
  - x_{k+1} is reached through π_3 or σ_{k+1} as explained in **Case 1**.
  - x_{-k-1} is reached through path σ_{-k-1}.
  - x_{-2k} is reached through path σ_{-2k}.
  - x_{-k+1} is reached through path σ_{-k-1}.
  - x_2: In this case, node x_{-k} is a *black virus* and any path that passes through it to reach x_2 should be avoided.
    - if $k < 8$, the path to reach x_2 is:

      $$\pi[x_0, x_2] = \pi_4 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{-1} x_{k-2} \xrightarrow{-1}, ... \xrightarrow{-1} x_2$$

    - otherwise, (i.e. $k \geq 8$), x_2 is reached through σ_2.

The following theorems summarize the number of moves:

Theorem 5. *In any shortly-chorded double loop $C_n(1,k)$, if $|S_{area}| \geq k$, the number of moves to complete the* Surrounding and Eliminating *phase is a maximum of* 20.

Proof. The first move is made by LEA to move to x_0. Then, by construction, we know that:

- $k \leq |S_{area}| < n - k$
  - Node x_{k-1} is reached in two moves through path σ_{k-1}
  - In the case of node x_2 we have three possibilities: if $k > 4$, the target is reached in four moves; if $k = 4$, the target is reached in three moves; if $k = 3$, x_2 coincides with x_{k-1} and the target is reached in two moves. The maximum amount of moves is thus four.
  - x_{2k} is reached in four moves through path σ_{2k}
  - x_{k+1} follows the same route as x_{2k} in the clockwise direction with the addition of two moves, or from x_2 with the addition of two more moves. The algorithm selects the minimum. The maximum number of moves in this case is then six.
- $n - k \leq |S_{area}| < n - 1$

  In this case, one *black virus* is generated (x_1) since the neighbouring nodes are guarded. Thus, $\mathcal{T}$=$\{x_2, x_{k+1}\}$.
  - x_2 is reached in maximum four moves, corresponding to path π_2
  - x_{k+1} is reached in one move since the SH at node x_k received a copy of the original *black virus*, and SHs always make one move in the clockwise direction when they receive a *black virus* in order to be part of the surrounding team.

Since all of the agents are sent at the same time they will arrive at their destinations within six time units. After waiting six time units, the LEA will send two agents to clear the two *black viruses* , thus performing two more moves. □

Theorem 6. *In any shortly-chorded double loop $C_n(1,k)$, if $|S_{area}| < k$, the number of moves to complete the* Surrounding and Eliminating *phase is a maximum of* 36.

Proof. One move is made by LEA to move to x_0. Then, by construction, if $|S_{area}| < k$, we know that: node x_{k+1} is reached within six moves through σ_{k+1} or π_3; node x_{k-1} is reached within two moves through σ_{k-1}; node x_{2k} is reached within four moves through σ_{2k}; node x_{-k-1} is reached within two moves through σ_{-k-1}; node x_{-2k} is reached within four moves through σ_{-2k}; x_{-k+1} is reached through x_{-2k} with the addition of two more moves through path σ_{-k+1}, which means a maximum of six moves; node x_2. In this case x_2 is reached through σ_2 or π_4 and the number of moves would be a maximum of eight. After all the agents arrive at their destinations, three more moves are needed to clear the black viruses.

□

4.2.1.2 General Double Loop

In this section, we handle double loop chordal rings in general $C_n(1,k)$, where $2 < k < \lfloor \frac{n}{2} \rfloor$. Finding the exact optimal number of moves in this case is much more complicated and difficulties arise because, depending on the length of the chords and the number of nodes, it may be more convenient to wrap around the ring in order to reach certain targets.

In order to describe the *Surrounding and Eliminating* process we use the same notations and cases. In other words, we have two cases: when $|S_{area}| \geq k$ and when $|S_{area}| < k$. Therefore, the set of targets $\mathcal{T}$ is $= \{x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-2k}, x_{-k+1}, x_{-k-1}\}$, $= \{x_2, x_{k-1}, x_{k+1}, x_{2k}\}$, $= \{x_2, x_{k+1}\}$ or $\emptyset$. The only difference between this section and the previous one is in the *deployment* part of the algorithm since, as mentioned above, we have more possibilities in the general structure of the double loop.

Case $|S_{area}| \geq k$. Let us first consider the case in which $|S_{area}| \geq k$. For some targets, the path to be followed is the same as the one employed for shortly chorded rings. This is

the case for x_{k-1} and x_2. Node x_{k-1} is reached through σ_{k-1}. If $k = 4$, node x_2 is reached through π_1; otherwise, it is reached through π_2.

In the following discussion we will refer to the special paths σ_i identified in the previous section and compare them to other possible routes in order to find the best one.

In order to find the best routes to reach nodes x_{k+1} and x_{2k}, we divide the ring into "windows" of size $(k+1)$ starting from x_0 in both directions, and consider different cases depending on the number of windows covering the ring.

1. $\lceil \frac{n}{k} \rceil > 4$.

   Intuitively, n is big enough for us to consider this a Shortly-chorded double loop and we follow the same paths indicated in the previous section to reach the target.

2. $\lceil \frac{n}{k} \rceil = 4$.

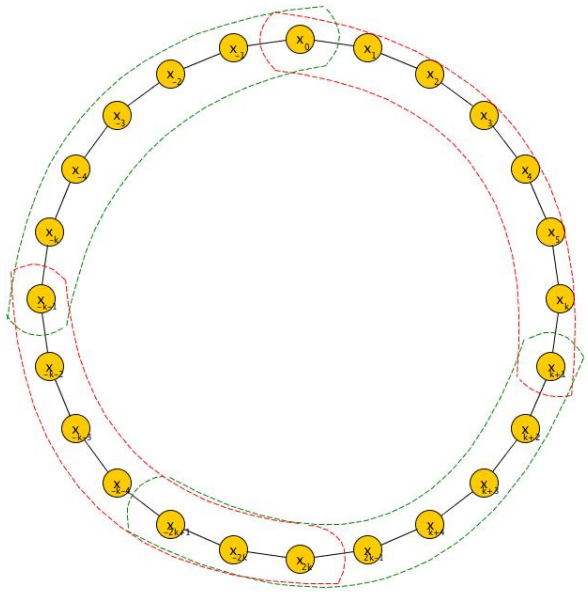


Figure 4.2: Dividing the outer ring into 4 windows of size $k+1$.

   In this case it might be more efficient to "wrap-around" the chordal ring to reach both x_{k+1} and x_{2k}.

- x_{2k}.

  To reach target x_{2k} we distinguish between different situations depending on the relationship between n and k:

  — If $n = 4k$

  $$\pi[x_0, x_{2k}] = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-k} x_{-2k} \xrightarrow{-1} x_{2k}$$

  — If $k < \frac{2}{7}n - 1$

  $$\pi[x_0, x_{2k}] = \min\{\pi_5, \sigma_{2k}\}$$

  where

  $$\pi_5 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-k} x_{-2k} \xrightarrow{+1} x_{-2k+1} \xrightarrow{+1} ... \xrightarrow{+1} x_{2k}$$

  and σ_{2k} is the special path we referred to in shortly-chorded loops.

  $$\sigma_{2k} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k}$$

  — If $k \leq \frac{2}{7}n - 1$

  $$\pi[x_0, x_{2k}] = \min\{\pi_6, \sigma_{2k}\}$$

  where

  $$\pi_6 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-1} x_{-k-1} \xrightarrow{-1} x_{-k-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{2k}$$

- x_{k+1}

  — If $n = 4k$

  $$\pi[x_0, x_{k+1}] = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-k} x_{-2k} \xrightarrow{-k} x_{k+1}$$

—Else

$$\pi[x_0, x_{k+1}] = \min\{\pi_7, \sigma_{k+1}\}$$

where

$$\pi_7 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-k} x_{-2k} \xrightarrow{-1} x_{-2k-1} \xrightarrow{-1} ... \xrightarrow{-1} x_{k+1}$$

and σ_{k+1} is the path to reach x_{k+1} in shortly-chorded loops.

$$\sigma_{k+1} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k} \xrightarrow{+1} x_{2k+1} \xrightarrow{-k} x_{k+1}$$

3. $\lceil \frac{n}{k} \rceil < 4$

- x_{k+1}. In this case, in addition to σ_{k+1}, there are three other possibilities and we have:

$$\pi[x_0, x_{k+1}] = \min\{\sigma_{k+1}, \pi_8, \pi_9, \pi_{10}\}$$

  – (* when the $+k^{th}$ neighbour of x_{k+1} is between x_{-1} and x_{-k}, it might be advantageous to move to x_{-1} first *)

$$\pi_8 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{2k+1} \xrightarrow{-k} x_{k+1}$$

  – (* if instead x_{-k} is closer to x_{2k+1} , then it might be better to move to first x_{-k} *)

$$\pi_9 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+1} x_{-k+1} \xrightarrow{+1} ... \xrightarrow{+1} x_{2k+1} \xrightarrow{-k} x_{k+1}$$

  – (* finally, if x_{-k} is closer to x_{k+1} *)

$$\pi_{10} = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{-1} x_{-k-1} \xrightarrow{-1} ... \xrightarrow{-1} x_{k+1}$$

- x_{2k}. Target x_{2k} resides between x_{-1} and x_{-k}, so we first have to determine which

of them is closer. We have:

$$\pi[x_{11}, x_{k+1}] = \min\{\sigma_{2k}, \pi_{11}, \pi_{12}\}$$

where

$$\pi_{11} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{2k}$$

$$\pi_{12} = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+1} x_{-k+1} \xrightarrow{+1} ... \xrightarrow{+1} x_{2k}$$

Case $|S_{area}| < k$. Consider now the case in which $|S_{area}| < k$. In this case: $\mathcal{T}$= {x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-2k}, x_{-k+1}, x_{-k-1}}. Notice here that in all cases, any route that passes through x_{-k} should be avoided because x_{-k} is now a *black virus*.

- Nodes x_{k-1}, x_{-2k}, x_{-k+1}, and x_{-k-1} are reached through the paths σ_{k-1}, σ_{-2k}, σ_{-k+1} and σ_{-k-1} respectively.
- To find the best routes to reach nodes x_2, x_{k+1} and x_{2k}, we also use the concept of "windows" mentioned earlier.
  1. $\lceil \frac{n}{k} \rceil > 4$

     This case is the same as the shortly-chorded chordal ring structure and uses the methodology described in the previous section.
  2. $\lceil \frac{n}{k} \rceil = 4$
     - x_{2k}. (* The only option besides the route σ_{2k} is π_{13} described below, which should be taken only if $n = 4k + 1$. *)

$$\pi_{13} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-k} x_{2k}$$

– x_{k+1}

$$\pi[x_0, x_{k+1}] = \min\{\sigma_{k+1}, \pi_{14}\}$$

where

$$\pi_{14} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-k} x_{-2k-1} \xrightarrow{-1} x_{-2k-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{k+1}$$

– x_2:

$$\pi[x_0, x_2] = \min\{\sigma_2, \pi_4, \pi_{15}\}$$

where

$$\pi_{15} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-k} x_{-2k-1} \xrightarrow{-k} x_{-3k-1} \xrightarrow{-1} ... \xrightarrow{-1} x_2$$

Notice here that if $n = 4k + 1$, the agent cannot take route π_{15} because node $x_{-3k-1} = x_k$.

3. $\lceil \frac{n}{k} \rceil < 4$

– x_{k+1} In this case, in addition to the path σ_{k+1}, there are the following possibilities leading to:

$$\pi[x_0, x_{k+1}] = \min\{\sigma_{k+1}, \pi_{16}, \pi_{17}\}$$

* (* The $+k^{th}$ neighbour of x_{k+1} is between x_{-1} and x_{-k}, so if x_{-1} is closer then: *)

$$\pi_{16} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{2k+1} \xrightarrow{-k} x_{k+1}$$

$*$ (* x_{-k} is closer to x_{k+1} *)

$$\pi_{17} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-1} ... \xrightarrow{-1} x_{k+1}$$

– x_{2k}

$$\pi[x_0, x_{2k}] = \min\{\sigma_{2k}, \pi_{18}\}$$

$$\pi_{18} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2} \xrightarrow{-1} ... \xrightarrow{-1} x_{2k}$$

– x_2

$$\pi[x_0, x_2] = \min\{\sigma_2, \pi_4, \pi_{15}\}$$

We have seen that the case of the shortly-chorded ring is simpler to study, however, the number of moves employed to reach the targets is the same in both cases and leads to the same complexity.

Theorem 7. *In any double loop $C_n(1, k)$, if $|S_{area}| \geq k$, the number of moves to complete the* Surrounding and Eliminating *phase is a maximum of* 20*.*

Proof. In any double loop chordal ring, if $|S_{area}| \geq k$, the maximum number of moves required to reach all targets is 20. One move is done by LEA to reach x_0. By construction, whether $\lceil \frac{n}{k} \rceil > 4$ or $\lceil \frac{n}{k} \rceil \leq 4$, the nodes are reached as follows: node x_{2k} is reached within four moves; node x_{k+1} is reached within six moves; node x_{k-1} is reached within two moves and node x_2 is reached within four moves.

Since we always compare the resulting routes π_z, where $z \in \mathbb{Z}$, to the special ones in the shortly-chorded loops σ_i, none of π_z would be greater than the corresponding σ_i.

After all the agents have arrived at their destinations, one move is done by the SH and two moves are needed to send CAs. □

Theorem 8. *In any double loop $C_n(1,k)$, if $|S_{area}| < k$, the number of moves to complete the Surrounding and Eliminating phase is a maximum of 36.*

Proof. At the beginning of the second phase, one move is made by the LEA to move to x_0. Then, whether $\lceil \frac{n}{k} \rceil > 4$ or $\lceil \frac{n}{k} \rceil \leq 4$, the nodes are reached as follows: node x_{2k} is reached within four moves; node x_{k+1} is reached within six moves and node x_{k-1} is reached within two moves. Node x_2 is reached by taking any route except the one that passes through x_{-k}, and the number of moves would be a maximum of eight. Node x_{-k-1} is reached within four moves; node x_{-k+1} is reached within six moves and node x_{-2k} is reached within four moves.

Since we always compare the resulting routes π_z, where $z \in \mathbb{Z}$, to the ones in the shortly-chorded loops σ_i, none of π_z would be greater than the corresponding σ_i.

After all agents arrive at their destinations, only three moves are needed to disinfect the topology. □

The following table summarizes the number of moves required in the *Surrounding and Eliminating* phase in any double loop chordal ring.

Destination	Number of moves	
	$\lvert S_{area}\rvert < k$	$\lvert S_{area}\rvert \geq k$
x_0	1	1
x_2	≤ 8	≤ 4
x_{k-1}	2	2
x_{k+1}	≤ 6	≤ 6
x_{2k}	≤ 4	≤ 4
x_{-k-1}	2	-
x_{-2k}	4	-
x_{-k+1}	6	1
x_1	1	1
x_k	1	1
x_{-k}	1	-
Total	≤ 36	≤ 20

Table 4.1: Move complexity in double loops.

Theorem 9. *The algorithm following the move-optimal deployment strategy successfully disinfects a double loop chordal ring from black viruses in a monotone synchronous way.*

Proof. Destroying a *black virus* is done only if it moves to a guarded node. Therefore, in this phase, agents surround all the neighbouring nodes and then LEA activates them so they move to neighbouring nodes. As previously mentioned, the crucial part is the routing, and in this strategy (Move-optimal), LEA is responsible for directing agents to their destinations. Since LEA has the topological knowledge, it correctly calculates the targets and sets up the paths for SAs. □

4.2.1.3 On Optimality and Other Observations

We ran a simulation to construct a partial *Breadth-First Search* tree rooted in x_0 in double loops with "missing nodes" corresponding to the black viruses triggered in the various scenarios. The spanning tree was constructed until all the targets appeared as leaves. In this type of spanning tree, any path from x_0 to a target leaf is the shortest path from x_0 to that destination.

We exhaustively tested all the different scenarios depending on the location of the black virus. Note that the size of the chordal ring influences the length of the shortest path for each scenario: 1) Shortly-chorded double loops when $|S_{area}| < k$, 2) Shortly-chorded double loops when $|S_{area}| \geq k$, 3) General double loops where $n \leq 4k$ when $|S_{area}| < k$ and 4) General double loops where $n \leq 4k$ when $|S_{area}| \geq k$. In all the scenarios above we verified that the paths indicated are indeed the shortest paths.

As a final note, if all agents have full knowledge of the topology and of the targets, the aforementioned approach can be transformed into a *local strategy* where agents decide their next step without referring back to the LEA. In each step, an agent could locally construct a breadth first search tree in order to find the shortest path to their target. They could then pick the neighbour that satisfies the following conditions: not a BV, not a predecessor and on the shortest path. After all the agents reach their destinations, LEA triggers the BVs by sending the cleaning agents. This strategy is optimal and local, yet it is compute-intensive. It requires the same number of agents and moves as the aforementioned move-optimal algorithm.

4.2.2 Simple Greedy Deployment

In the previous section, the proposed algorithm is controlled by the LEA who has a map for the entire topology. Once he finds the original BV, he decides the best routes and

sends agents through them. The only job the agents have to do is following the paths set by the leader and stored in their memories. In this section we introduce some strategies that rely completely on local decisions. In other words, instead of carrying the whole path, the agent calculates its next move for each step until it reaches its target.

Consider the following simple local Greedy strategy: an agent at some node *source*, having to reach a node *dest*, chooses as a next link the one that takes it closest to *dest*. The distance is calculated on the outside ring. Note that, in a double loop *without viruses*, this simple greedy strategy would correspond to optimal routing between any pair of nodes.

Let $link(source, dest)$ be the next link to be taken from node *source* to eventually reach node *dest*. It is easy to see that the greedy strategy described above corresponds to the following local choice:

$$link(x_i, x_j) = \begin{cases} \text{if } (i-j) \leq (n-j+1) & go-clockwise: \\ \quad \text{if } (j-i) < k & \text{take-link } +1 \\ \quad \text{otherwise} & \text{take-link } +k \\ & \\ \text{if } (i-j) > (n-j+1) & go-counter-clockwise: \\ \quad \text{if } (n-j+i) < k & \text{take-link -1} \\ \quad \text{otherwise} & \text{take-link } -k \end{cases}$$

In order to occupy the target nodes, we have the following optimal paths where the BV has been underlined:

Target	Greedy path
x_2	$[x_0, \underline{x_1}, x_2]$
x_{k-1}	$[x_0, \underline{x_k}, x_{k-1}]$
x_{k+1}	$[x_0, \underline{x_k}, x_{k+1}]$
x_{2k}	$[x_0, \underline{x_k}, x_{2k}]$

Table 4.2: Greedy path without *black viruses.*

In the presence of BVs to be avoided, the greedy strategy will give correct but non-optimal routing. The advantage of this approach is that the surrounding agents do not carry a pre-determined path and movement decisions are made locally.

The greedy strategy can be described as follows. $dist(x, y)$ represents the shortest distance between nodes x and y along the ring.

SIMPLE GREEDY

Deploying agent A arriving at v from y with destination z

Let $FD = \{N(v) - \mathcal{BV} - y\}$ be the set of possible destinations.

Agent A moves to $w \in FD$ that minimizes $dist(w, z)$

Note that the first step in any surrounding strategy is always counter-clockwise, regardless of the destination, because both clockwise neighbours of x_0 are BVs. For example, an agent wants to reach x_2 from x_0 when $k > 5$. The agent would move to x_{-1} and would continue to move counter-clockwise until reaching node x_{-i} such that $i + 3 > -i + k - 2$

$(2i > k-5,\ i = \lceil \frac{k-5}{2} \rceil)$. At this point the agent would take the clockwise chord $+k$ which gets it closer to x_2, and then would proceed counter-clockwise towards it by using chords -1. Notice that when $k \leq 5$, i=1.

All the resulting greedy paths when $|S_{area}| \geq k$ are outlined in the table below. For simplicity, we assume that $5 < k < \lfloor \frac{n}{2} \rfloor$ and $k << n$.

Target	Greedy path with BVs	Number of moves
x_2	$[x_0, x_{-1}, x_{-2}, \ldots, x_{-i}, x_{-i+k}, x_{-i+k-1}, \ldots....x_2]$ $i = \lceil \frac{k-5}{2} \rceil$	$k-1$
x_{k-1}	$[x_0, x_{-1}, x_{k-1}]$	2
x_{k+1}	$[x_0, x_{-1}, x_{k-1}, x_{k-2}, x_{k-3}, ...x_{k-i}, x_{2k-i}, x_{2k-i-1}, \ldots, x_{k+1}]$ $i = \lceil \frac{k-3}{2} \rceil$	$k+1$
x_{2k}	$[x_0, x_{-1}, x_{k-1}, x_{2k-1}, x_{2k}]$	4

Table 4.3: Greedy paths with *black viruses* when $|S_{area}| \geq k$

Theorem 10. *In any double loop $C_n(1,k)$, if $|S_{area}| \geq k$ and $k > 5$, to surround and eliminate the black viruses, the total number of moves performed by the Simple Greedy strategy is $\leq 2k + 10$.*

Proof. The first move in this phase is performed by LEA in order to move from x_{-1} to x_0, then the routing begins. When $|S_{area}| \geq k$, after activating the original *black virus*, we might have two *black viruses*, one *black virus* or none depending on where the original *black virus* resides. If we have two *black viruses*, $\mathcal{T}$= $\{x_2, x_{k-1}, x_{k+1}, x_{2k}\}$, and the agents reach them as in table 4.3 and the number of moves is obtained as follows: node x_2 is reached in $(k-1)$ moves; node x_{k-1} is reached in two moves; node x_{2k} is reached in four moves; node x_{k+1} is reached in $(k+1)$ moves. The SH at x_{-k} makes one move to occupy x_{-k+1}, and LEA sends two CAs in two moves to disinfect the whole topology.

□

All the greedy paths when $|S_{area}| < k$ are outlined in the following table.

Target	Greedy path with BVs	Number of moves
x_2	$[x_0, x_{-1}, x_{-2}, \ldots, x_{-i}, x_{-i+k}, x_{-i+k-1}, \ldots....x_2]$ $i = \lceil \frac{k-5}{2} \rceil$	$k-1$
x_{k-1}	$[x_0, x_{-1}, x_{k-1}]$	2
x_{k+1}	$[x_0, x_{-1}, x_{k-1}, x_{k-2}, x_{k-3}, ...x_{k-i}, x_{2k-i}, x_{2k-i-1}, \ldots, x_{k+1}]$ $i = \lceil \frac{k-3}{2} \rceil$	$k+1$
x_{2k}	$[x_0, x_{-1}, x_{k-1}, x_{2k-1}, x_{2k}]$	4
x_{-k-1}	$[x_0, x_{-1}, x_{-k-1}]$	2
x_{-k+1}	$[x_0, x_{-1}, x_{-k-1}, x_{-k-2}, ...x_{-k-i}, x_{-i}, x_{-i-1}, x_{-i-2}, \ldots, x_{-k+1}]$ $i = \lceil \frac{k-3}{2} \rceil$	$k+1$
x_{-2k}	$[x_0, x_{-1}, x_{-k-1}, x_{-2k-1}, x_{-2k}]$	4

Table 4.4: Greedy paths with *black viruses* when $|S_{area}| < k$

As mentioned above, the greedy algorithm does not produce optimal paths. For example, consider the case $C_n\{1, 6\}$ with $\lceil \frac{n}{k} \rceil > 4$. To reach x_2 using the strategy described in the previous section requires four moves while in the greedy strategy the agent requires five moves.

Theorem 11. *In any double loop $C_n(1, k)$, if $|S_{area}| < k$ and $k > 5$, to surround and eliminate the black viruses, the total number of moves performed by the Simple Greedy strategy is $3k + 17$.*

Proof. The first move in this phase is performed by LEA to move from x_{-1} to x_0, then the routing begins. When $|S_{area}| < k$, after activating the original *black virus*, we have three *black viruses*. Thus $\mathcal{T}$= $\{x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-k-1}, x_{-k+1}, x_{-2k}\}$ and the agents reach them as in table 4.4 as follows: node x_2 is reached in $(k-1)$ moves; node x_{k-1} is reached in

two moves; node x_{2k} is reached in four moves; node x_{k+1} is reached in $(k+1)$ moves; node x_{-k-1} is reached in two moves; node x_{-2k} is reached in four moves; node x_{-k+1} is reached in $(k+1)$ moves. The LEA then sends three CAs in three moves to disinfect the whole topology. □

4.2.3 Smart Greedy Deployment

This algorithm is similar to the greedy algorithm except that this one takes the k^{th} chords into consideration.

SMART GREEDY

Deploying agent A arriving at v from y with destination z

Agent A calculates *next*.

Let $FD = \{N(v) - \mathcal{BV} - y\}$ be the set of possible destinations.

For each $v_i \in N(v)$

Let $d_i = dist(v_i, z)$

If $d_i \geq k$

$d_i = \lfloor \frac{d_i}{k} \rfloor + (d_i \mod k)$

next $\in FD$ that has v_i with the minimum d_i

If there is a tie between two or more of $N(v)$

TIE-BREAK.

Agent A moves to *next*

TIE-BREAK

$v_i, v_j \in N(v)$ where $v_i, v_j \notin BV$ and $dist(v_i, z) = dist(v_j, z)$

Agent A calculates $next$

Check $N(v_i)$ and $N(v_j)$

For each $u \in N(v_i)$ and $w \in N(v_j)$ where $u, w \notin N(v)$ — are not predecessors

Let $u_i = dist(v_i, z)$ and $w_j = dist(v_j, z)$

for each u_i or w_j that is $\geq k$

change the distance to the value $\lfloor \frac{dist}{k} \rfloor + (dist \mod k)$

$next \in N(v)$ the has the neighbour with the minimum distance.

The smart greedy algorithm is local, yet it requires agents to do more calculations than the simple greedy algorithm. The complexity of this algorithm is not optimal. For example, to reach x_{k+1} in a shortly-chorded double loop, the agent needs $> six$ moves.

In the smart greedy algorithm, we consider the longest chord which plays an important role in decreasing the distance between the two nodes. Therefore, if the $dist(x, y) \geq k$, there is one or more long chords between x, y which minimizes the distance significantly. For example, if $dist(x, y) = 10$ in $C_n(1, 7)$, the actual distance is not 10 but 4 as we can see in the following equation: $dist(x, y) = \lfloor \frac{10}{7} \rfloor + (10 \mod 7) = 4$. Also, in this strategy, we attempted to resolve the situation in which two neighbours have the same distance to a target. If there is a tie, before the SA chooses, it checks the neighbours and finds the minimum among them. If the tie persists, the SA checks the neighbours of the neighbours until it breaks the tie. However, if the SA finds two neighbours have a common neighbour that gives the minimum distance, it can pick any of them as in the case of reaching x_{k+1} as we will see.

All the resulting smart greedy paths when $|S_{area}| \geq k$ are outlined in the table below. For simplicity, we assume that $5 < k < \lfloor \frac{n}{2} \rfloor$ and $k << n$.

Target	Smart Greedy path with BVs	Number of moves
x_2	$[x_0, x_{-k}, x_{-k}, x_{-k+1}, x_{-k+2}, x_2]$	4
x_{k-1}	$[x_0, x_{-1}, x_{k-1}]$	2
x_{k+1}	$[x_0, x_{-1}, x_{k-1}, x_{k-2}, ...x_{k-i}, x_{2k-i}, x_{2k-i-1}, \ldots, x_{k+1}]$ $i = \lceil \frac{k-3}{2} \rceil$	$k+1$
x_{2k}	$[x_0, x_{-1}, x_{k-1}, x_{2k-1}, x_{2k}]$	4

Table 4.5: Smart greedy paths with *black viruses* when $|S_{area}| \geq k$

Theorem 12. *In any double loop $C_n(1,k)$ where $|S_{area}| \geq k$ and $k > 5$, to surround and eliminate the black viruses, the total number of moves performed by the Smart Greedy strategy is $\leq k + 15$.*

Proof. The first move in this phase is performed by LEA to move from x_{-1} to x_0, then the routing begins. When $|S_{area}| \geq k$, after activating the original *black virus*, we might have two *black viruses*, one *black virus* or none depending on where the original *black virus* resides. If we have two *black viruses*, $\mathcal{T}$= $\{x_2, x_{k-1}, x_{k+1}, x_{2k}\}$ and the agents reach them as in table 4.5 as follows: node x_2 is reached in four moves; node x_{k-1} is reached in two moves; node x_{2k} is reached in four moves; node x_{k+1} is reached in $(k+1)$ moves. The SH at x_{-k} makes one move to occupy x_{-k+1}, and LEA sends two CAs in two moves to disinfect the whole topology. □

When $|S_{area}| < k$, all the resulting greedy paths are outlined in the table below.

Target	Smart Greedy path with BVs	Number of moves
x_2	$[x_0, x_{-1}, x_{-2}, \ldots, x_{-i}, x_{-i+k}, x_{-i+k-1}, \ldots....x_2]$ $i = \lceil \frac{k-5}{2} \rceil$	$k-1$
x_{k-1}	$[x_0, x_{-1}, x_{k-1}]$	2
x_{k+1}	$[x_0, x_{-1}, x_{k-1}, x_{k-2}, x_{k-3}, ...x_{k-i}, x_{2k-i}, x_{2k-i-1}, \ldots, x_{k+1}]$ $i = \lceil \frac{k-3}{2} \rceil$	$k+1$
x_{2k}	$[x_0, x_{-1}, x_{k-1}, x_{2k-1}, x_{2k}]$	4
x_{-k-1}	$[x_0, x_{-1}, x_{-k-1}]$	2
x_{-k+1}	$[x_0, x_{-1}, x_{-k-1}, x_{-2k-1}, x_{-2k}, x_{-2k+1}, x_{-k+1}]$	6
x_{-2k}	$[x_0, x_{-1}, x_{-k-1}, x_{-2k-1}, x_{-2k}]$	4

Table 4.6: Smart greedy paths with *black viruses* when $|S_{area}| < k$

Theorem 13. *In any double loop $C_n(1,k)$, if $|S_{area}| < k$ and $k > 5$, to surround and eliminate the black viruses, the total number of moves performed by the Smart Greedy strategy is $2k + 22$.*

Proof. The first move in this phase is performed by LEA to move from x_{-1} to x_0, then the routing begins. When $|S_{area}| < k$, after activating the original *black virus*, we have three *black viruses*. Therefore, $\mathcal{T} = \{x_2, x_{k-1}, x_{k+1}, x_{2k}, x_{-k-1}, x_{-k+1}, x_{-2k}\}$ and the agents reach them as in 4.6 as follows: node x_2 is reached in $(k-1)$ moves; node x_{k-1} is reached in two moves; node x_{2k} is reached in four moves; node x_{k+1} is reached in $(k+1)$ moves; node x_{-k-1} is reached in two moves; node x_{-2k} is reached in four moves; node x_{-k+1} is reached in six moves. The LEA then sends three CAs in three moves to disinfect the whole topology. □

4.3 Conclusion

In this chapter we investigated the two phases required to disinfect a double loop chordal ring from *black viruses*. The first phase is common to all deployment strategies. In fact, as we will see throughout this thesis, this phase uses the same technique for all chordal rings. On the other hand, the second phase of double loop chordal rings has different possible routing strategies: move-optimal, simple greedy and smart greedy. The whole algorithm correctly disinfects any double loop chordal ring, regardless of the routing strategy.

In the following tables we combine the complexities of the two phases for the three routing strategies proposed for disinfecting any double loop chordal ring.

BV at node v_i	Phase 1	Move-Optimal		Simple Greedy		Smart Greedy	
		Agents	Moves	Agents	Moves	Agents	Moves
$1 \leq i < k$	$\leq 3k-5$	12	≤ 36	12	$\leq 3k+17$	12	$\leq 2k+22$
$k \leq i < n-k$	$\leq 4n-5k-6$	9	≤ 20	9	$\leq 2k+10$	9	$\leq k+15$
$n-k \leq i < n-1$	$\leq 4n-12$	6	≤ 8	6	$k+3$	6	≤ 8
$i = n-1$	$\leq 4n-7$	5	0	5	0	5	0

Table 4.7: Comparison between the complexities of the three strategies.

BV at node v_i	Phase1+ Move-Optimal	Phase1+ Simple Greedy	Phase1+Smart Greedy
$1 \leq i < k$	$\leq 3k+31$	$\leq 6k+12$	$\leq 5k+17$
$k \leq i < n-k$	$\leq 4n-5k+14$	$\leq 4n-3k+4$	$\leq 4n-4k+9$
$n-k \leq i < n-1$	$\leq 4n-4$	$4n+k-9$	$\leq 4n-4$
$i = n-1$	$4n-7$	$4n-7$	$4n-7$

Table 4.8: The overall move complexity of the two phases in the three strategies.

By examining the tables above we can see that the complexity of the first phase remains the same regardless of the strategy chosen in the second phase. Moreover, the number of

agents is the same for all of the strategies, while the number of moves varies. We can also see that when the *black virus* is found at node v_{n-1}, no moves are necessary in the *Surrounding and Eliminating* phase.

Even though we have only considered the problem in a synchronous setting, the algorithm would work in an asynchronous environment with some modifications. In order to make it work we would need an extra process for the termination phase which can be conducted by the LEA. After the LEA sends agents to their destinations, it will then move around and visit each site to confirm the successful arrival of each agent. This process would only add a constant number of moves.

Chapter 5

Black Virus Disinfection in Triple Loops

5.1 Introduction

In this chapter, we discuss parallel strategy on BVD problem in chordal ring. A chordal ring is a circulant graph with $d_1 = 1$, i.e., it is an augmented ring, and will be denoted by $C_n(1, d_2, ..., d_k)$. More specifically, in chordal ring each node is directly connected to the nodes at distance d_i and $n - d_i$ by additional links called chords. The link connecting two nodes is labeled by the distance that separate these two nodes on the ring. For convenience, if we say the agents or the clones move along chord i, then they actually move along d_i. If we say the agents or the clones move i, then they actually move along chord d_x with its length equal to i. Let us donated by d the half degree of the chordal ring (for example, for the chordal ring structure $C_n(1, 2, 4, 5)$, $d = 4$), by l the length of the longest chord of the chordal ring (for example, for the chordal ring structure $C_n(1, 2, 4, 5)$, $l = 5$). In order to simplify the process, we assume that all the nodes in the network are marked with a number: the starting point is marked 0, then the second node is marked 1... Our goal is to minimize the time to complete the whole decontamination process and at the same time

the casualties (number of agents destroyed by the BV) In order to do that, we propose parallel strategy for decontaminating the chordal ring. In the elimination phase, we propose two strategies to surround the neighbours of the clones sequentially and parallelly. In the parallel case, an tricky situation might happen: supposing that the sites of the two clones are connected, in this case, after these two clones are triggered, one of their clones spread to another sites and since the agents sent to destroy them die, these two sites are empty when the second round clones arrive, which make our decontamination invalid. In order to solve this problem, we make an assumption when we talk about parallel strategy in elimination phase in chordal rings: when a BV is trigger at T_i, it take negligible time for its clones to move to all its neighbours, for example, at T_i.

5.2 Shadowed Exploration

Initialization The chordal ring is a complete symmetrical structure, so we can randomly choose a node x_0 as the start node. Initially, we place $2d$ agents at node 0, 1, ..., $2d-1$ (first round). Agents residing in nodes from 0 to $d-1$ are in shadowing group, while from d to $2d-1$ are in exploring group. If none of the agent is destroyed, we then place d agents at nodes 0,1, ..., $d-1$ (second round). If not, we can easily know the location of the BV and employ agents to surround the new formed BVs, then destroy them permanently. Only the agents employed in the first round move in the exploring phase. The agents employed in the second round remain dormant, guarding the nodes to guarantee the monotone.

Route of the agent in exploring phase We separate the time of exploring phase into two part: moving time and notice time. In the whole phase of exploring phase, they are arranged as below: T_{move_1}, T_{notice_1}, $T_{notice\_1'}$, $T_{notice\_1''}$, T_{move_2}, T_{notice_2}, $T_{notice\_2'}$, $T_{notice\_2''}$ ...More specifically, every cycle contains one unit of time for moving and three units of time for notice. We discuss why we arrange the time as above and what exactly the agents do in the notice time later.

In chord ring $C_n(1, d_2, \ldots, d_k)$, all the agents in the array move along their longest chord d_k in T_{move_i}. That is, agents move along d_k in $T = 1 + 4t$ ($t \in \mathbb{N}$). An example of how agents move in chordal ring $C_n(1, 2, 4, 5)$ at T_{move_i} in exploring phase is shown in figure **??**. For our convenience, in some case, we consider the chordal ring as arranged in rows of d_k where the last node of a row is connected to the first node of the following row and the last node is connected to the first. Depending on the size of the chordal, the last row could be incomplete. So in this matrix, moving down a column corresponding to using the longest chord d_k. In the matrix, we also mark the number of nodes.

(a) Arrangement of agents at T_{move_i}

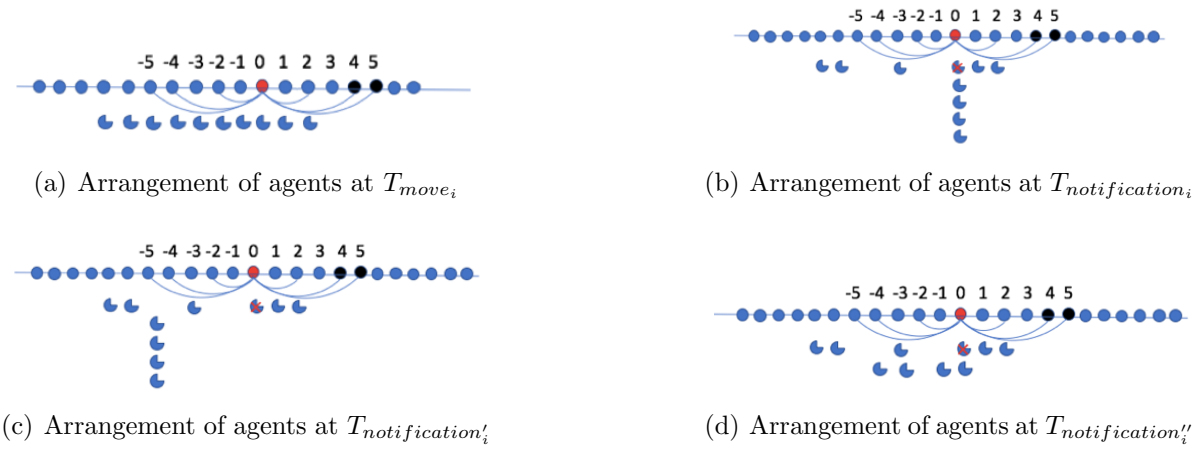
(b) Arrangement of agents at T_{move_i+1}

Figure 5.1: Arrangement of agents when moving

Three Jump NotificationTechnique In the model of BV decontamination exploring the network sequentially, explore agent moves forward for one step. If the node is safe, it move back to the leader agent, and then move forward to the safe node together. In this case, if the leader agent does not meet the leader in next T after it moves forward, the leader agent learns that the original BV resides in the next node so it stop moving. But in our model, we employ $2d$ agents in the exploring phase, if they are not properly informed when one agent is destroyed by BV, in their next step of moving, some of them may be destroyed by the new formed BVs. In order to avoid the casualties, we propose $Three\,Jump\,Notification\,Technique$ to properly notice the agents who will move to the new formed BVs. For convenience, let us take the node where the original BV resides as the original of the one-dimensional coordinate system. In a chordal ring $C_n(1, d_2, \ldots, d_k)$, if the original BV is triggered, the clones of it spread to nodes whose coordinates are $-d_k$,

$-d_{k-1}$, ..., $-d_2$, -1, 1, d_2, ..., d_{k-1}, d_k. Obviously the nodes whose coordinates are 1, d_2, ..., d_{k-1}, d_k may become the BV (Possible BV Nodes) now, our goal is to notify the agents which will move to these nodes to stop (Risky Agent). The coordinates of them are $1-d_k$, d_2-d_k, ..., $d_{k-1}-d_k$, d_k-d_k (which is exactly the coordinate of the original BV) respectively. It is obvious that not all of the nodes from 1 to d_k become BV nodes after the triggering because there might be some agents already there, but since notifying all the *Risky Agents*(RAs) does not add more cost comparing to notifying some of them, in our strategy, we notify all of the RAs. Let us donate one of the Possible BV Nodes by d_i, and in our $Three\,Jump\,Notification\,Technique$, agent residing in node $-d_i$ (Notification Agent) is employed to notify the agent residing in node $-d_k+|d_i|$ (RA) who will move to the BV node.

We show that $Notification\,Agent$ are able to meet *Risky Agent* in three steps: $-d_i \xrightarrow{move|d_i|} 0 \xrightarrow{move\ along\ chord\ k} -d_k \xrightarrow{move\ |d_i|} -d_k+|d_i|$. In this case, the notifying route of the $Notification$ *Agent* whose coordinate is $-d_k$ is $-d_k \to 0 \to -d_k \to 0$. We would make some modification in the $Surrounding\,and\,Elimination$ phase, but now let us assume it still follow the route above. The whole process of $Three\,Jump\,Notification\,Technique$ in chordal ring $C_n(1,2,4,5)$ is shown in figure **??**.



(a) Arrangement of agents at T_{move_i}

(b) Arrangement of agents at $T_{notification_i}$

(c) Arrangement of agents at $T_{notification'_i}$

(d) Arrangement of agents at $T_{notification''_i}$

Figure 5.2: The whole process of the Three Notifying Technique in chordal ring $C_n(1,2,4,5)$

If the original BV residing in the red node, then once an agent moves to it, the agent and the BV are destroyed but the clones of the BV spread to all its neighbours. According to our technique, nodes whose coordinates are 1, 2, 4, 5 become *Possible BV Nodes*; agents residing in nodes -4, -3, -1, 0 are the *Risky Agents*; agents residing in nodes -5, -4, -2, -1 are the *Notice Agents*. The routes for agents residing in nodes -5, -4, -2, -1 are $-5\rightarrow 0\rightarrow -5\rightarrow 0$; $-4\rightarrow 0\rightarrow -5\rightarrow -1$; $-2\rightarrow 0\rightarrow -5\rightarrow -3$; $-1\rightarrow 0\rightarrow -5\rightarrow -4$ respectively.

Safe Exploring with Three Jump Notifying Technique

After the initialization, agents employed in the first round move as the route we introduced in "Initialization" . When one of the agents is destroyed at $T - move\_i$, then $Three\ Jump\ Notification\ Technique$ begins. In "Route of the agent in exploring phase", we say that in the exploring phase every cycle contains one unit of time for moving and three units of time for notifying. Actually, the three units of time for notice are reserved for $Three\ Jump\ Notice\ Technique$, even though it only executes when one of the agents is destroyed. In another word, before encountering a BV, all the agents just stay where they are in the notice time. After executing the $Three\ Jump\ Notice\ Technique$, the *Notice Agents* move back to where they are before the notification. For example, in the example in "Three Jump Notification Technique", Notification Agent residing in node -1 moves back to node -4 following the reverse route in the notice phase which is $-1\rightarrow -5\rightarrow 0\rightarrow -4$.

5.3 Surrounding and Elimination

In this section, we introduce the process of eliminating the BVs after the original BV is triggered. For the purpose of saving the number of agents, we prefer to chase the *Keep Moving* agents, but it is not necessary to complete the process, for example, you can carry enough number of agents so you can proceed the *Surrounding and Elimination* immediately. As we mentioned before, we first chase the *Keep Moving* agents then begin the *Surrounding and Elimination*. We propose two methods: the first one is to surround

the new formed BVs sequentially, and the second one is to surround the new formed BVs parallelly. The former cost more time but save the number of agents in some cases; while the latter save time but uses more agents comparing to former method. Here we are confronted with two tradeoffs: whether to chase the *Keep Moving* agents and whether to surround the new formed BVs parallelly. In the following section we first introduce these two techniques and in next chapter we discuss the tradeoffs between them.

5.3.1 notification Moving Agents Technique

Overview of the notification Moving Agents Technique

When the *Shadowed Exploring* ends, it is possible that some of the agents in the array are not informed and do not realize the existence of the BV, so they keep moving following the routes in *Shadowed Exploring* phase but it is obvious that they would not encounter any BV. In order to reduce waste, we employ the agent who receives the clone from chord d_k (*Coordinate Agent*) to notice the other *Keep Moving* agents to move back to their position when the BV is triggered. Now we introduce how we choose the *Coordinate Agent* and how the *Coordinate Agent* notify the other agents.

The Process of the notification Phase of the Coordination Agent

We can see that the relative position of agents does not change when they keep moving along the longest chord. For example, agents residing in nodes 1, 5, 10 (noted as $A1$, $A5$, $A10$) move along the longest chord and then $A5$ moves forward for one step. The relative position of them would be exactly the same as the situation where all of them remain dormant except $A5$ moves forward for one step. So now we discuss how to notify all the *Keep Moving* agents assuming they are dormant and then make some modification to fit the scenario where the *Coordinate Agent* notifies the *Keep Moving* agents. The problem we need to solve is that given a range which the agents are in and a *Coordinate Agent* located in this range, let the *Coordinate Agent* to notify all the agents in this range. In a

chord ring $C_n(1, d_2, \ldots, d_k)$, the *Coordinate Agent* sets a $Notification\,Window$ using the modular arithmetic. Let us assume the number of the node where the $Coordinate_Agent$ resides in is x, the number of the nodes where should be marked the *Beginning Flag* and the $End\,Flag$ are y and z.

The relations between x, y, z should be as follow:

- y is the biggest number which satisfies that it is smaller than x and that y mod d_k =0;
- z is the smallest number which satisfies that it is bigger than x and that z mod d_k = d_{k-1}.

When the agent is chosen to be the *Coordinate Agent*, it computes its $Notification\,Window$. When we mention marking a flag, it does not mean that the agent has to move to the node to do that but only needs to remember the positions of the two flags in its memory. Also, after the position of the notice window is set, it remains stable. More specifically, when the *Coordinate Agent* moves, he does not set another notice window using its new position. The *Coordinate Agent* moves step by step to every possible position and notifies the agents to go back if there is one until it realizes that it just passes the $End\,Flag$. After that, he moves along the longest chord anticlockwise to the node marked a *Beginning Flag* and continues moving again step by step to notify agents until it arrives its relative departing node. For example, if the *Coordinate Agent* starts to notify other from node x, then its relative departing node is $x' = x + t \times d_k$ ($t \in \mathbb{N}$).

We make some modifications to let the solution fit the real scenario where the agents move along the longest chord:

- The Beginning Flag and the End Flag move along the longest chord also to keep the relative position the same.

- When one agent $A1$ moves to a node where there is an agent $A2$ knowing the position of the original BV, $A1$ would be informed and directly moves along the longest chord to its own position.

- Let us assume that the time when the original BV is triggered is T_{move_i} ($T\_trigger$), then the *Coordinate Agent* should remember the $T_{trigger}$ and informs the agents he encounters of it. The agent $A1$ who encounters the *Coordinate Agent* should remember the time when they encounter (T_{notice_now}) and stop moving until next T_{move} when it will meet another agent $A2$. Then $A1$ moves along d_k anticlockwise for $T_{move\_now} - T_{trigger}$ times while $A2$ moves for $T_{move\_now} - T_{trigger} + 1$ times.

- When arriving its relative departing position at T_{move_a}, the *Coordinate Agent* knows that it has finished the task and moves along d_k for $T_{move\_a} - T_{trigger} + 1$ times to its position when the original BV is triggered.

- Let us assume that the time when the BV is triggered is T_{move_i}, the *Coordinate Agent* starts to move step by step to notify other agents after T_{move_i+2}, because we want to ensure the security of the *Coordinate Agent*. If it starts to notify other agents at T_{move_i+1}, it might encounter a BV. Also, it should be ensured that the *Keep Moving* agents in the exploring group are in the *Notice Window* of the *Coordinate Agent*, or the *Coordinate Agent* would never encounter them. We would talk about how to ensure this in next section.

Election of the Coordination Agent

We choose the agent who receives a BV clone from its chord d_k to be the *Coordinate Agent*, which means when an agent receives a BV clone from its longest chord, then it realizes that it is chosen as the *Coordinate Agent*. We know that, the notice phase starts from T_{move_i+2} assuming the time when the BV is triggered is T_{move_i}, which means the notification Phase begins only after all the *Keep Moving* agents move twice. At T_{move_i+2}, all the

Keep Moving agents are in a notice window from nodes $d_k \times (move\_i+1)$ *to* $d_k \times (move\_i+1) + d_k - 1$, and let us donate it by *Initial Notice Window*. The *Coordinate Agent* would directly move to any node in *Initial Notice Window*. Now we propose three kinds of routes for the *Coordinate Agent* to move to his destination:

The coordinates of the positions where the clones spread are: $x - d_k$ (which is the coordinate of the *Coordinate Agent*), $x - d_{k-1}$, $x - d_{k-2}$, ..., $x - 1$, $x + 1$, $x + d_2$, ..., $x + d_{k-1}$, $x + d_k$ supposing the coordinate of the original BV is x and we are in a chordal ring $C_n(1, d_2, \ldots, d_k)$. Now we describe three scenarios:

- Scenario 1: The last agent of the *Exploring Group* is destroyed by the BV and the positions of the clones satisfy: $x-d_{k-1} = x+1$, $x-d_{k-2} = x+d_2$, ..., $x-1 = x+d_{k-1}$.
- Scenario 2: The last agent of the *Exploring Group* is destroyed by the BV and the at least one pair of the positions of the clones does not satisfy: $x - d_{k-1} = x + 1$, $x - d_{k-2} = x + d_2$, ..., $x - 1 = x + d_{k-1}$.
- Scenario 3: One of the agents in the *Exploring Group* except the last agent is destroyed by the BV.

In scenario 1, the *Coordinate Agent* needs to move for 5 steps to reach its destination while in the other two scenarios, it only needs to move for 4 steps to arrive the destination. Now we propose the route for each scenario.

- For CA in scenario 1: Let us donate by y the coordinate of the node in the *Notice Window* set by the original BV which does not receive any clone and his left neighbour receives a clone (the coordinate of it is $y - 1$). The CA first moves to the original BV, then to node $y - 1$, finally to y. After that it only need to move along the chord d_k for twice to reach its destination.
- For CA in scenario 2: There is at least one pair of the positions of the clones does not satisfy the equations so there should be one node (assuming its coordinate is z)

who receives a clone from the original BV but node $z + d_k$ is empty. The route now for the CA is first move to the BV, then to node z, and then moves along the chord d_k for twice to reach its destination.

- For CA in scenario 3: The CA here simply need to move for one step to its right neighbour and move along the chord d_k for three times to reach its destination.

In any case, the CA can reach its destination within 6 unit of time which is required for the $Keep\,Moving$ agents to move to the $Initial\,Notification\,Window$, so the CA start to chase the $Keep\,Moving$ agents as we introduce from T_{notify_i+2}.

An example of how the agents and the CA move in chordal ring $C_n(1, 2, 7, 11)$is shown in ??.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65

Figure 5.3: Arrangement of agent at T_{move_2} when the BV is triggered

Yellow nodes are connected to the original BV but guarded by agents while the grey nodes are the new formed BVs. The node marked V is the original BV but now is clean. Agent residing in node 29 receives clone from chord d_k so it knows it is the CA. During the notification time, agents residing in nodes 33, 38, 39 notify agents residing in nodes 36, 31, 30 respectively following the ?Three Jump Notice Technique? while the CA moves to 28, 39, 50 and finally 61 following the route in scenario 3.

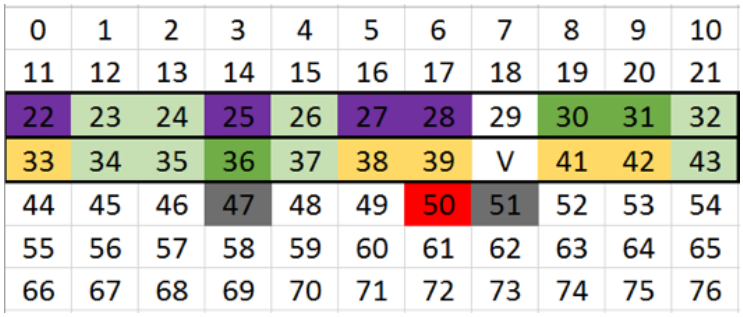


Figure 5.4: Agents roles after $Three\,Jump\,Notification$ and choosing CA. (for convenience, we donate the CA by a red spot, more specifically, node 50 is where CA resides)

Agents in purple nodes would be noticed at T_{move_3} and move back. Agents in light green nodes are the $Keep\,Moving$ agents while agent in dark green nodes are informed to stop in $Three\,Jump\,Notification$. In the meantime, the CA moves to node 28, 39, 50, and finally 61. It is obvious that the CA can reach its destination before T_{move_4}, so it waits until T_{notice_4} to start its notification phase.

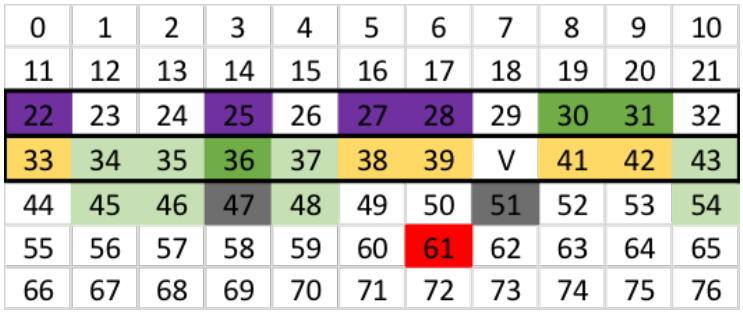


Figure 5.5: Arrangement of agent at T_{move_3}. The CA has arrived its destination)

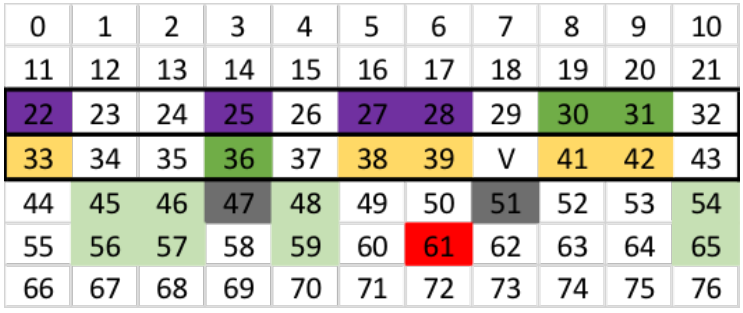


Figure 5.6: Arrangement of agents at T_{move_4}. The CA starts its notice phase

In notice phase, CA starts to notify other $Keep\,Moving$ agents. First, it computes the $Notice\,Window$ which is from node 55 to node 65. Note that the $Notice\,Window$ would move along chord d_k at every T_{move}. It moves to node 62 at T_{notice_4}, node 63 at $T_{notice\_4?}$, node 64 at $T_{notice\_4??}$ and to node 75 at T_{move_5}.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87

Figure 5.7: Arrangement of agents at T_{move_5}.

Again, in the notification phase, CA moves to node 76 at T_{notice_5}. We can see that it encounters agent residing in node 76, so CA informed it the $T_{trigger}$ which is T_{move_2}. Agent residing in node 76 should remember T_{notice_now} which is T_{notice_5} and wait until next T_{move} to inform agent ($Following\,Agent$) who resides in node 65 now but would move to node 76 next T_{move}. After encountering its $Following\,Agent$, it informs it to move back along chord d_k for $T_{move\_now} - T_{trigger} + 1$ times which is $T_{move\_5} - T_{move\_2} + 1$ times while itself moves for T_{move_5}- T_{move_2} times. At $T_{notice\_5?}$ when the CA arrives at node 77, it knows that it just pass its $Ending\,Flag$ so it moves along the longest chord anticlockwise to its $Beginning\,Flag$ at $T_{notice\_5??}$.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87

Figure 5.8: Arrangement of agents at $T_{move''_5}$.

0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21
22	23	24	25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	V	41	42	43
44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76
77	78	79	80	81	82	83	84	85	86	87
88	89	90	91	92	93	94	95	96	97	98
99	100	101	102	103	104	105	106	107	108	109

Figure 5.9: Arrangement of agents at $T_{move''_7}$.

We could know that the CA moves back to its relative original position at $T_{notice\_7??}$. It would wait until next T_{move} to inform its *Following Agent* of $T_{trigger}$ and moves back with it.

5.3.2 Overview of the Elimination

After all the agents move back to where they are when the BV is triggered, we start the *Surrounding and Elimination*. We are interested in destroying the BVs at one time. So we need to first guard all the neighbouring nodes of the new formed BVs. In order to avoid collision and efficiently leverage the agents, we allocate different Destination Tables to all the agents in the array to inform them where should they should move in different

situations (e.g., when the first agent in the exploring team is destroyed, then every agent except the first agent have a distinct destination, when the second agent is destroyed, then every agent except that agent destroyed have a distinct destination. More specifically, for a Chord Ring with half degree d, every agent in the array carries a $Destination\,Table$ with $d-1$ destinations. If we need more agents, then we will give their $Destination\,Table$ to the last agent in the shadowing group, when the elimination begins, it clones enough number of agents and give the $Destination\,Table$ to them. Before moving to its destination, the agent computes the shortest route from its own position to its destination using Dijkstra Algorithm. There are two kinds of agent in the Elimination phase: surrounding agents who are responsible for guarding the neighbouring nodes of the BVs and destroying agents who move to the BVs after all the neighbouring nodes are guarded. We want the BVs to be destroyed at one time, so it is important that the destroying agents move to the BVs at the same time and only after all the neighboring nodes are guarded by agents. In fact, if the destroying agents know the longest time $t_{longest}$ to move to the destination taken by all the agents (including the destroying agents and the surrounding agents), then they move to the last node prior to the destination and wait until $t_{longest}$ to move to the BVs together. So in the $Destination\,Tables$ for the destroying agents, we also add an item which is the $t_{longest}$. Now we introduce how to compute the shortest routes and how we design the $Destination\,Tables$. Note that we design $Destination\,Tables$ for all the agents and allocate them to the agents before the exploring phase begins.

5.3.3 Destination Table and Elimination

Supposing there are some BVs and agents in the chordal ring, it is obvious that the BV nodes are in the clockwise side of the agents. In order to use Dijkstra, first we need to map the chordal ring with BVs into a graph. We include nodes from the node containing the first agent to the node which is d_k away from the last BV node, then delete the chords from the BV nodes to build the graph where we run Dijkstra Algorithm. Here is an example

how we built the graph for running Dijkstra Algorithm. Below we show the situation when the third agent in the exploring group is destroyed by the BV (see **??**). Only the chords of the original BV node are shown.

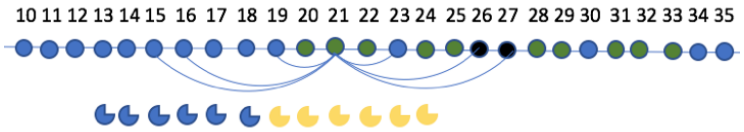


Figure 5.10: Situation when the third agent in the exploring group is destroyed

The black node is the BV node while the green nodes need to be guarded. So in this case, we need 12 agents (10 surrounding agents and 2 destroying agents). We add nodes from 13 to 33 with their chords within this area and delete chords connected with the BV nodes to get the graph where we use Dijkstra Algorithm. Below is the graph we build. (see **??**) For convenience, we show the all the nodes we included and the chords we delete.

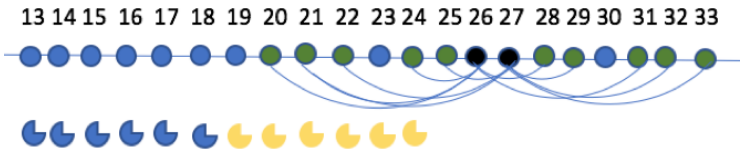


Figure 5.11: The graph we build for Dijkstra Algorithm

Using the graph and Dijkstra Algorithm, we compute the routes from every agent to every node. Then we use enumeration to choose an allocation of every agents's destination satisfying:

- 1) the maximum length of the route should be minimum.
- 2) after the allocation, in every needed position there should be exactly one agent.

After we get the optimal allocation, we record every agent?s distinct destination and the position of the third agent in the exploring group in their *Destination Table*. Also, we add

the length of the longest chord to every destroying agent. More specifically, here we talk about the situation when the third agent in the exploring group is destroyed. For every agent, the information we compute would be in the third part of its $Destination\,Table$ recording the position of the third exploring agent (for example, it connects this agent through chord x), the destination it should move if the third exploring is destroyed. For a destroying agent, there should be another item recording the length of the longest route in this part. Above we introduce how to design one part of $Destination\,Table$ of an agent, for every agent in chordal ring, it should hold a $Destination\,Table$ of $d-1$ parts, and every part contains 2 items (for surrounding agent) or 3 items (for destroying agent). After the one of the agent is destroyed, the agent can check their $Destination\,Table$ to get the information of their destination. Then using Dijkstra Algorithm they can compute the shortest route separately and starts to move.

Chapter 6

Black Virus Disinfection in Consecutive-Chords Rings

In this chapter, the black virus disinfection problem is considered in the *consecutive-chords* ring $C_n(1, 2, 3, .., k-1, k)$, with $k < \lfloor \frac{n}{2} \rfloor$, in a *synchronous* environment.

For the deployment phase, unlike the other classes of chordal rings, the local strategy that we propose, the *One-Direction Greedy* strategy, is also a move-optimal solution. We thus only concentrate on local deployment. As usual, we evaluate the complexity of the solution by considering the overall number of agents employed, the number of agent casualties and the number of moves required. The following table summarizes some of the results.

Consecutive-Chords(C)	$\|S_{area} \geq k$	$\|S_{area} < k$
Spread(C)	$\leq k+1$	$\leq 2k$
Size(C)	$\leq 3k+1$	$\leq 4k+2$
Move(C)	$\leq ((k+3)n - \frac{3}{2}(k^2+k) - 3$	$\leq n+6k-1$

6.1 Exploring and Shadowing

The exploring phase is the same as in all the other chordal rings. We instantiate it here for the particular case of the consecutive-chords chordal ring.

EXPLORING AND SHADOWING

Let $HB = v_0$.

Agents EA and LEA at safe node v_j.

if $j+1 < k$ (* $N_{ex}(v_{j+1}) = \{v_j, v_{j-1}, ..., v_0\}$ *)

$(j-1)SH$s are deployed to protect $N_{ex}(v_{j+1})$

Else if $k \leq j+1 < n-k$ (* $N_{ex}(v_{j+1}) = \{v_j, v_{j-1}, v_{j-2}, ..., v_{j+1-k}\}$ *)

$k-1$ SHs are deployed to protect $N_{ex}(v_{j+1})$

Else if $n-k \leq j+1 \leq n-1$ (* $N_{ex}(v_{j+1}) = \{v_j, v_{j-1}, \ldots, v_{j+1-k}, v_{j+1+k}, v_{j+k}, \ldots, v_0\}$ *)

let $j+1 = n-i$, where $i \in \mathbb{Z}_{\geq 0}$

$(2k-i)$ SHs are deployed to protect $N_{ex}(v_{j+1})$

EA moves to v_{j+1}.

The observations below were obtained following the application of this strategy:

Theorem 14. *In the worst case scenario, the black virus is detected in* $(k+2)n - 2k - 3$ *moves.*

Proof. The worst case scenario for the number of moves required occurs when the *black virus* is found at node (v_{n-1}) after exploring $n-1$ nodes. In this case, the BV triggers no new *black viruses* since all of the neighbouring nodes in the safe area have been explored and are protected by SHs. The complexity of this case is $3(n-1) - 2$ for the movement

of LEA and EA, $(\sum_{i=2}^{k} n-1-i)$ for the movement of $k-1$ SHs in the counter clockwise direction and $(\sum_{i=1}^{k-1} i)$ for the movement of SHs in the other direction. □

Theorem 15. *In any consecutive-chord ring, the worst case scenario in term of the number of agents required occurs when* $(2k-1)$ *new black viruses are created after triggering the original virus.*

Proof. The worst case scenario for the number of *black viruses* created upon activating the original *black virus* is when the original is found at node v_i where $\leq i = 1$. In this case, the *black virus* (x_0) triggers $(2k-1)$ new *black viruses*: x_1, x_2, x_3, ..., x_k, and x_{-2}, x_{-3}, ..., x_{-k} because no SH have been deployed. Note that x_{-1} is always occupied by LEA.

□

6.2 Surrounding and Eliminating

The deployment phase of the consecutive-chords chordal ring is different from all the other structures considered in this thesis because when the *black virus* is triggered, the chordal ring gets disconnected in the clockwise direction.

SURROUNDING AND ELIMINATING

LEA and SHs covering all $N_{ex}(v)$

BV comes back from $v = x_0$.

if ($|S_{area}| = 1$) (* LEA is covering x_{-1} *)

$N_{un}(x_0) = \{x_1, x_2, ..., x_k, x_{-2}, x_{-3}, ..., x_{-k+1}, x_{-k}\}$

if ($1 < |S_{area}| < k$)

let $|S_{area}| = k - i$, where $i \in \mathbb{Z}_{\geq 0}$

(* LEA is covering x_{-1} and SHs covering $N_{ex}(x_0)$*)

$N_{un}(x_0) = \{x_1, x_2, ..., x_k, x_{-k}, x_{-k+1}, x_{-k+2}, ..., x_{-k+i-1}\}$

if ($k \leq |S_{area}| < n - k$) (* LEA and SH covering $x_{-1}, x_{-2}, ..., x_{-k}$ *)

$N_{un}(x_0) = \{x_1, x_2, ..., x_k\}$

if ($n - k \leq |S_{area}| < n - 1$)

let $|S_{area}| = n - i$, where $i \in \mathbb{Z}_{\geq 0}$

(* LEA and SHs covering $x_{-1}, x_{-2}, ..., x_{-k}, x_i, x_{i+1}, ..., x_k$ *)

$N_{un}(x_0) = \{x_1, x_2, ..., x_{i-1}\}$

Else (* LEA and SHs covering $x_{-1}, x_{-2}..., x_{-k}, x_1, x_2, ..., x_k$ *)

$N_{un}(x_0) = \emptyset$

All SH make one move in clockwise direction.

For each $u \in N_{un}(x_0)$:

DEPLOY an agent to each $z \in \{N(u) \setminus N_{un}(x_0)\}$

When $N(u)$ is covered:

DEPLOY one agent to u

As with the previous chapters, we will consider different cases depending on the location of the *black virus* . More precisely, the number of agents required to disinfect the consecutive-

chords ring C_n depends entirely on the chords structure and the location of black viruses in respect to the safe area. We first partition the chordal ring into four distinct segments using v_0 as the homebase as seen in figure 6.1.

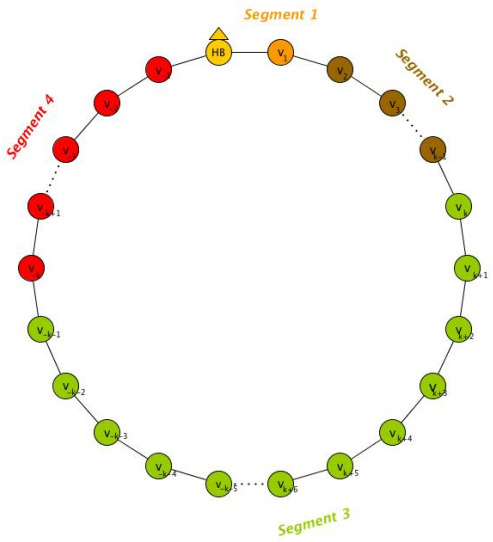


Figure 6.1: Dividing the consecutive-chords into four segments.

- *Segment 1* contains node v_1. If the *black virus* is in this segment, the size of the safe area is $|S_{area}| = 1$.
- *Segment 2* contains nodes $v_2, \ldots v_{k-1}$. If the *black virus* is in this segment, the size of the safe area is $2 \leq |S_{area}| \leq k-1$.
- *Segment 3* contains nodes $v_k, \ldots v_{n-k-1}$. If the *black virus* is in this segment, the size of the safe area is $k \leq |S_{area}| \leq n-k-1$.
- *Segment 4* contains nodes $v_{n-k}, \ldots v_{n-1}$. If the *black virus* is in this segment, the size of the safe area is $n-k \leq |S_{area}| \leq n-1$.

Theorem 16. *For black virus disinfection in consecutive-chords ring $C_n(1, 2, ..., k)$, a maximum of $(4k+1)$ agents are employed.*

Proof. The number of agents is determined according to the location of the original BV and the chord structure.

- When the BV is located at any node in *Segment 1*, we have the worst complexity in terms of CA and SA since activating the *black virus* will create $(2k-1)$ *black viruses* at $x_1, x_2, ..., x_k, x_{-2}, x_{-3}, ..., x_{-k}$. Subsequently, the LEA moves to x_0 and then deploys $2k$ SAs while one SH is created in the x_{-1}, which is the homebase. The SAs are then to occupy $x_{k+1}, x_{k+2}, x_{k+3}, ..., x_{2k}, x_{-k-1}, x_{-k-2}, ..., x_{-2k}$. The total number of agents employed is then $(4k+2)$: $(2k)$ CAs, $(2k)$ SAs, one SH and one LEA. Therefore, $Spread(C) = 2k$ and $Size(C) = 4k+2$.

- If the BV is located at any node in *Segment 2* , activating the original *black virus* would create $k+i$ *black viruses* at x_1, x_2,..., x_k, x_{-k}, x_{-k+1} ,..., x_{-k+i}, where $i = k-|S_{area}|$. The same number of SHs as nodes in the safe area are then deployed and each makes one move through $+1$ chord when it receives a BV. LEA also moves to x_0, and then deploys $2k$ SAs to occupy x_{k+1}, x_{k+2}, x_{k+3}, ..., x_{2k}, x_{-k-1}, x_{-k-2}, ..., x_{-2k}. Therefore, the team of agents consists of $(4k+2)$ agents: $(k+i+1)$ CAs, $(2k)$ SAs, $(|S_{area}|)$ SHs and LEA. In this case, $Spread(C) = k+i+1 = 2k-|S_{area}|+1$ and $Size(C) = 4k+2$.

- If the BV is located at any node in *Segment 3*, activating the original *black virus* would create k *black viruses* at x_1, x_2,..., x_k, while the other neighbouring nodes are already guarded. Once the LEA and SHs receive BVs, they make one move through their $+1$ chords. Then LEA deploys k SAs to occupy $x_{k+1}, x_{k+2}, x_{k+3}, ..., x_{2k}$. Therefore, the team of agents consists of $(3k+1)$ agents: $(k+1)$ CAs, (k) SAs, $(k-1)$ SHs and LEA. In this case, $Spread(C) = k+1$ and $Size(C) = 3k+1$.

- If the BV is located at any node in *Segment 4* , also called the *Danger area* (D_{area}), we have two possible cases:

- When $n - k \leq |S_{area}| < n - 1$: Let us assume that $|S_{area}| = n - i$. If the BV is located at any node in this segment, activating the original *black virus* would create $i - 1$ *black viruses* at x_1, x_2,..., x_{i-1}, while the other neighbouring nodes are guarded by SHs. In addition to the SHs at x_{-1}, x_{-2},..., x_{-k}, x_i, x_{i+1}, x_{i+2},..., x_k, and the LEA at x_0, the LEA deploys $i - 2$ SA to guard the unoccupied neighbours by SHs . Therefore, the team of agents consists of $(2k + i - 1)$ agents: (i) CAs, $(i - 2)$ SA, $(2k - i)$ SH and LEA. In this case, $Spread(C) = i = n - |S_{area}|$ and $Size(C) = 2k + i - 1 = 3k - |S_{area}| - 1$.
- When $|S_{area}| = n - 1$: No more *black viruses* are created since all of the neighbouring nodes of the *black virus* are guarded by SHs and the LEA. No SAs to be deployed and all of the moves are made in the first phase. In this case, the team consists of $2k+1$ agents: 1 CA, $2k-1$ SHs and LEA. Therefore,$Spread(C) = 1$ and $Size(C) = 2k + 1$.

□

As previously mentioned, triggering the *black virus* disconnects the chordal ring in one direction and in order to reach the targets to surround the new black viruses, the agents must move in a counter-clockwise direction.

The idea is for the surrounding agents to take the longest chord $-k$ to reach the area to be occupied as quickly as possible, and then reach their own target in one additional step through a shorter chord. Each agent can locally decide the following step at each intermediate node, simply on the basis of its target.

The one-direction greedy consists of two stages: the long jumps and the simple greedy. The long jumps portion forces SAs to move from x_0 to their targets through the edge$(-k)$ in a counter-clockwise direction making $(\lceil \frac{n-k}{k} \rceil - 2)$ hops. At that point an extra hop will cause them to reach their destination. Figure 6.2 shows an example of this strategy.

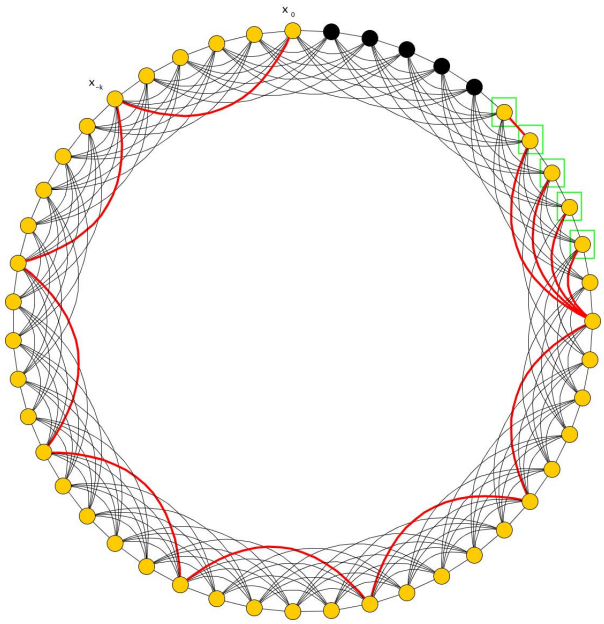


Figure 6.2: An example of One-direction strategy where red links show the paths to the targets.

ONE-DIRECTION GREEDY

Agent A at node x_{-i} $(i \geq 0)$ with target x_j

If $i = 0$

If $|BV| \leq k$

Agent A moves through $\lambda = -k$

If $|BV| > k$

Agent A moves through $\lambda = -1$, then moves through $\lambda = -k$

If $i < k(\lceil \frac{n-k}{k} \rceil - 2)$

move through $\lambda = -k$

Else

Let $FD = \{N(v) - \mathcal{BV} - y\}$ be the set of feasible destinations.

Agent A moves to $w \in FD$ that minimizes $dist(w, x_j)$

Theorem 17. *In any consecutive-chords ring* $C_n(1, 2, 3, ..., k)$*, with* $k < \lfloor \frac{n}{2} \rfloor$*, using the one-direction greedy approach in the* Surrounding and Eliminating *phase disinfects the whole*

topology from black viruses.

Proof. After finding the original *black virus*, and depending on the size of the safe area, we get BVs in the clockwise direction or in both directions. In the first scenario we have $|S_{area}| \geq k$ and all BVs reside in one direction (clockwise). From the location of the LEA, the other direction is completely safe and clean, and all agents can traverse that part of the chordal ring until they reach their targets. In order to do so, the agents must be aware of the locations of BVs according to our algorithm. In the second scenario we have $|S_{area}| < k$, kBVs reside in the clockwise direction, and $\leq k$ BVs reside in the counter clockwise direction. From the location of the LEA the counter clockwise direction is not quite as safe as it was in the first scenario. Agents can still traverse that direction safely due to the fact that the node x_{-1} is always safe no matter the size of the safe area. Through node x_{-1}, the LEA will send agents to their targets, and the agents will avoid all the faulty nodes in that direction since the longest chord connected to x_{-1} surpasses the furthest possible BV (i.e., $x_{-1-k} < x_{-k}$).

□

In the following section we will consider the number of moves required in the two main cases: $|S_{area}| \geq k$ and $|S_{area}| < k$.

Theorem 18. *In any consecutive-chords ring $C_n(1, 2, 3, ..., k)$, with $k < \lfloor \frac{n}{2} \rfloor$, when $|S_{area}| \geq k$, the number of moves required to surround and eliminate the black viruses is $\leq k(\lceil \frac{n-k}{k} \rceil) + 2k$, regardless of the number of nodes.*

Proof. The number of moves required to decontaminate kBVs can be calculated as follows. One move is made by LEA to occupy x_0. The movements of SAs are calculated as follows:

1. Calculating the number of jumps. Agents need to traverse the ring starting from x_0

and using the longest chord as they move greedily:

$$\lfloor \frac{n-k}{k} \rfloor \tag{6.1}$$

2. Finding the number of $leftover$ nodes, which are not reached from the last jump, and require an extra jump:

$$leftover = ((n-k) \mod k) - 1 \tag{6.2}$$

This equation excludes BVs and x_0.

3. Based on the aforementioned equations, we have the following information:

- If 6.2$\geq$ 0, then $((n-k) \mod k) - 1$ neighbours are reached within $\lfloor \frac{n-k}{k} \rfloor + 1$, and $k - (((n-k) \mod k) - 1)$ neighbours are reached within $\lfloor \frac{n-k}{k} \rfloor$. In total we have:
  $(k - leftover)\lfloor \frac{n-k}{k} \rfloor + leftover(\lfloor \frac{n-k}{k} \rfloor + 1)$
- If 6.2$<$ 0, all neighbours except one are reached within $\lfloor \frac{n-k}{k} \rfloor$. The other neighbour is reached in $\lfloor \frac{n-k}{k} \rfloor - 1$. In total, we have:
  $(k-1)\lfloor \frac{n-k}{k} \rfloor + (\lfloor \frac{n-k}{k} \rfloor - 1)$

In the theorem we substitute the floor function with the ceiling function in order to give a general upper bound for all of the possible values.

The SHs movements equal $k-1$ at most since all SHs make one move in the clockwise direction when the original *black virus* is activated. LEA sends k CAs in k moves to trigger the BVs.

□

Theorem 19. *In any consecutive-chords ring* $C_n(1, 2, 3, ..., k)$*, with* $k < \lfloor \frac{n}{2} \rfloor$*, when* $|S_{area}| <$

k, the number of moves required to surround and eliminate the black viruses is $\leq k(\lceil \frac{n-k-1}{k} \rceil)+ 6k-1$*, regardless of the number of nodes.*

Proof. The number of moves required to decontaminate the worst case scenario in terms of *spread* and *size* (i.e., when $|S_{area}| = 1$) can be calculated as follows. One move is made by LEA to occupy x_0. In the worst case scenario we have $2k-1$ BVs, so in addition to $\{x_{k+1}, x_{k+2}, x_{k+3}, ..., x_{2k}\}$, there are more nodes to be guarded: $\{x_{-k-1}, x_{-k-2}, x_{-k-3}, ..., x_{-2k}\}$ The only way to reach those targets is through x_{-1}. k agents reach the consecutive neighbours $\{x_{k+1}, x_{k+2}, x_{k+3}, ..., x_{2k}\}$ by wrapping around in the counter clockwise direction after moving from x_0 to x_{-1} and using the one-direction greedy strategy. The movements of this group of SAs are calculated in the same way as the previous case when $|S_{area}| \geq k$ except that the long jumps start from x_{-1}.

1. Calculating the number of jumps. Agents need to traverse the ring starting from x_{-1} and using the longest chord as they move greedily:

$$\lfloor \frac{n-k-1}{k} \rfloor \tag{6.3}$$

2. Finding the number of $leftover$ nodes, which are not reached from the last jump, and require an extra jump:

$$leftover = ((n-k-1) \mod k) - 1 \tag{6.4}$$

   This equation excludes BVs and x_0.

3. According to the aforementioned equations, we know that:

   - If 6.4≥ 0, then $leftover$ neighbours are reached within $\lfloor \frac{n-k-1}{k} \rfloor + 1$, and $(k - leftover)$ neighbours are reached within $\lfloor \frac{n-k-1}{k} \rfloor$. In total, we have:

$$(k - leftover)\lfloor\frac{n-k-1}{k}\rfloor + leftover(\lfloor\frac{n-k-1}{k}\rfloor + 1) + k$$

- If 6.2$<$ 0, all neighbours except one are reached within $\lfloor\frac{n-k-1}{k}\rfloor$. The other neighbour is reached in $\lfloor\frac{n-k-1}{k}\rfloor - 1$. In total, we have:
  $(k-1)\lfloor\frac{n-k-1}{k}\rfloor + (\lfloor\frac{n-k-1}{k}\rfloor - 1) + k$

Notice that k agents make an extra move from x_0 to x_{-1}.

In the theorem we substitute the floor function with the ceiling function in order to give a general upper bound for all of the possible values.

The consecutive nodes $\{x_{-k-1}, x_{-k-2}, x_{-k-3}, ..., x_{-2k}\}$ are reached in three moves each while node x_{-k-1} which is reached in two moves:

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-k} x_{-k-1} \xrightarrow{-i} ...$$

where $i = \{1, 2, 3, ..., k-1\}$ for a total of $3k-1$. LEA sends $2k-1$ CAs in $2k-1$ moves to trigger the BVs. □

6.3 Conclusion

In this chapter, we have addressed the problem of disinfecting *consecutive-chords* rings from the black virus. We have demonstrated that when we have an undirected consecutive-chords ring and a synchronous execution, the *Exploring and Shadowing* phase is the same as the one employed in other topologies, having an exploring team that uses the safe exploration technique starting from the homebase and moving in a counter-clockwise direction followed by shadows until the *black virus* is located and triggered. We divided the outer ring into segments to demonstrate the monotone nature of our strategy. The *Surrounding and Eliminating* phase begins once the original *black virus* has been located. As previously

mentioned, this class of chordal rings is unlike the other classes in terms of routing and directing the surrounding agents. The local strategy proposed, the *one-Direction Greedy*, is also a move-optimal solution. In this approach, agents must be aware of their targets. Surrounding agents traverse in a counter-clockwise direction using the edge labels to calculate their next move until they reach their targets. We will now summarize and combine the results obtained in the previous two sections for the number of required moves.

BV **at node** v_i	**Phase 1**	**Phase 2**	**Total**
$i = 1$	1	$\leq k(\lceil \frac{n-k-1}{k} \rceil) + 6k - 1$	$\leq n + 6k - 1$
$1 < i < k$	$\leq \frac{k^2+k-2}{2} - 1$	$\leq k(\lceil \frac{n-k-1}{k} \rceil) + 5k - 1$	$\leq n + \frac{k^2+11k}{2} - 4$
$k \leq i < n - k$	$\leq (k+2)n - (\frac{3k^2+7k}{2}) - 3$	$\leq k(\lceil \frac{n-k}{k} \rceil) + 2k$	$\leq ((k+3)n - \frac{3}{2}(k^2+k) - 3$
$n - k \leq i < n - 1$	$\leq (k+2)n - 4k - 4)$	$\leq (j-2)(\lceil \frac{n-k}{k} \rceil) + 2k$ where $j = n - \lvert S_{area} \rvert$	$\leq (\frac{k^2+2k+j-2}{k})n - 2k - 4)$
$i = n - 1$	$(k+2)n - 2k - 3$	1	$\leq (k+2)n - 2k - 2$

Table 6.1: Move complexity of disinfecting a consecutive-chords according to location of BV.

The above table shows the move complexity when we have synchronized mobile agents. The same approach can be applied using an asynchronous execution but includes an extra termination phase that can be applied by a coordinator (e.g. LEA). This extra phase ensures that that all SAs reach their targets before triggering the BVs.

Chapter 7

General Chordal Rings

Now that we have investigated the problem of disinfecting a network from a *black virus* in special classes of chordal rings, we will discuss the problem in general chordal rings, regardless of chords structure.

The solution we propose is based on the idea described for the general chordal ring in Chapter 3, and has already been explained throughout this thesis. It involves an exploring phase, followed by a surrounding phase. Our protocol is based on a general surrounding method that can be applied to any chordal ring structure, regardless of the number or distance of chords. It must be noted that the move-cost for this method is not optimal.

7.1 Exploring and Shadowing

The phase is described in detail in Chapter 3. The following are our observations in general chordal ring structures:

Theorem 20. *In any chordal ring* $C_n = \{1, d_2, d_3, ..., d_m\}$*, in the worst case scenario, the black virus is detected in* $((2+m)n - 2m - 3)$ *moves.*

Proof. The worst case scenario regarding the number of moves required occurs when the

black virus is located at node (v_{n-1}) after exploring $n-1$ nodes. The complexity of this case would be $3n-5$ for the movement of LEA and EA, ($\sum_{i=2}^{m} n-1-d_i$) for the movement of SHs to counter-clockwise neighbours and ($\sum_{i=2}^{m} d_i - 1$) for SHs to clockwise neighbours.

In this case, the *black virus* triggers no new *black viruses* since all of the neighbouring nodes are occupied by SHs, $(2m-1)$ SHs. This case is considered the worst case scenario in terms of calculating the number of moves, but the best case in terms of cleaning and surrounding agents. □

Theorem 21. *In any chordal ring* $C_n = \{1, d_2, d_3, ..., d_m\}$*, the worst case scenario regarding the number of agents required for disinfection would occur when* $2m-1$ *new black viruses are created after triggering the original virus.*

Proof. If the *black virus* is found at node (v_i) where $1 \leq v_i < d_2$, the spread of *black viruses* would be maximized since no SHs have been deployed and the explored neighbours $|N_{ex}(v_i)| = 1$, which is v_{i-1} and is occupied by the LEA. Therefore , the number of unexplored neighbours, o *black viruses* , is $2m-1$. This number of *black viruses* requires a high number of surrounding agents. □

As usual, this phase comes to an end when the *black virus* is detected and triggered. At this point, new *black viruses* have been created and moved to the unexplored neighbouring nodes. It is at this point that the second phase begins.

7.2 Surrounding and Eliminating

As described in Chapter 3, once the *black virus* node is detected, the LEA moves to its location and the *Surrounding and Eliminating* phase begins.

Throughout our study of different classes of chordal rings, we have proposed two routing variations: local and non-local strategies. We have seen in previous chapters that these

strategies work for some topologies and not others. For example, the local greedy approach causes correct routing in double loop chordal rings and infinite loops in triple loop and consecutive-chords rings. We can thus deduce that the simple greedy would not work in general chordal rings with arbitrary chord structures (see 5.3.2.1). The non-local move-optimal strategy would always work for any chordal rings since this approach requires precise information about the chord structure which is available to the LEA and complex to calculate. We have observed the following from both strategies:

- If node x_0 represents the location of the original *black virus* , node x_{-1} is always safe because it is occupied by an agent. This means that it can be used as a starting point to reach any target.
- After triggering the original *black virus*, the new *black viruses* divide the outer ring into enclosed and open segments. Enclosed segments are areas that include the following set of nodes: $\{x_{\pm d_i-1}, x_{\pm d_i-2}, x_{\pm d_i-3}, ..., x_{\pm d_{i\pm1}+1}\}$. Open segments are areas that include the following set of nodes: $\{x_{\pm d_m\pm1}, x_{\pm d_m\pm2}..., x_{\pm d_m}\}$.
- Once the agents reach node $x_{\pm d_i-1}$, they are able to reach close targets greedily. In order to avoid getting stuck in infinite loops, agents can reach their close targets by using the one-direction greedy approach.

Making use of the observations above, we propose an efficient general strategy that is capable of disinfecting any chordal ring and gives upper bounds for the optimal paths. In this approach, all targets are reached through node x_{-1}. The set of targets is calculated based on the location of the original BV and the agents are scattered between BVs. The topology is thus divided into *enclosed areas* and *open areas*.

Note that with respect to node x_{-1}, some targets are in the clockwise direction while others are in the counter-clockwise direction.

In order to reach the targets, the LEA deploys agents through x_{-1}. The agents then move to the closest neighbour that is greater than or equal to its destination. Once the agent reaches $x_{\pm d_i - 1}$, it arrives to the area in which its target resides. The agent then moves greedily in one direction until it reaches its destination. Since this is a one-direction greedy approach, the agent only moves to neighbouring nodes that are greater or equal to its target. Only in one case an agent moves to a neighbour that is smaller than its target, when $t = x_{2d_m}$. Figure 7.1 shows an example of this strategy.

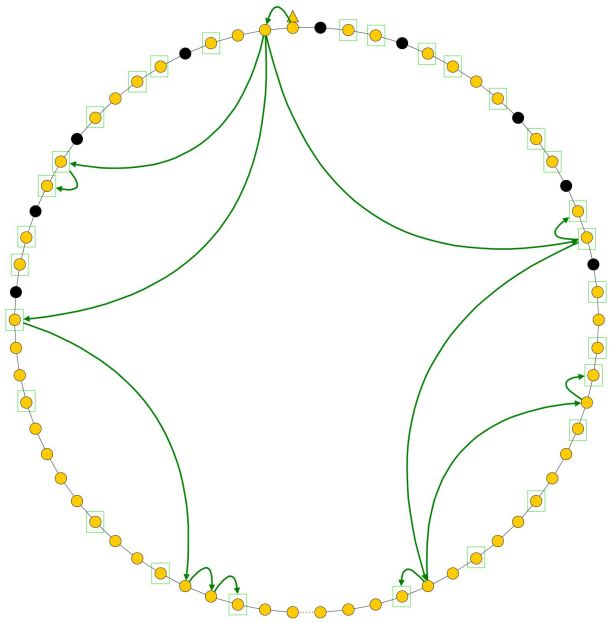


Figure 7.1: The paths to reach some targets using the general strategy.

General Deployment Strategy

Deploying agent A arriving at x_j from y with destination t

If $t = x_z$ where $z \in \mathbb{Z}^+$

if $j = 0$

move to x_{-1}

if $j = -1$

move to x_{d_i-1} such that $x_{d_i-1} \geq t$, and x_{d_i-1} minimizes $dist(x_{-1}, t)$

Else

if $t > x_{d_m}$

move to x_{d_m-1} then to x_{2d_m-1} (i.e., move to the open area)

If $t = x_{2d_m}$

move to x_{2d_m}

Else

compute $next$

let $OA = \{x_{2d_m-1}, x_{2d_m-2}, ..., x_{d_m+1}\}$the open area set

Let $FD = \{N(x_j) - y - BV\}$ be the set of feasible destinations.

let $C = FD \cap OA$

Agent A moves to $next \in C$ that minimizes $dist(x_j, t)$ and $next \geq t$

Else

compute $next$

let $EA = \{x_{d_i-2}, x_{d_i-2}, x_{d_i-3}, x_{d_i-4}, ..., x_{d_{i-1}+1}\}$the enclosed area set

Let $FD = \{N(x_j) - y - BV\}$

let $C = FD \cap EA$

Agent A moves to $next \in C$ that minimizes $dist(x_j, t)$ and $next \geq t$

General Deployment Strategy

Else (i.e., $t = x_z$ where $z \in \mathbb{Z}^-$)

if $j = 0$

move to x_{-1}

if $j = -1$

move to x_{-d_i-1} such that $x_{-d_i-1} \geq t$, and x_{-d_i-1} minimizes $dist(x_{-1}, t)$

Else

if $t < x_{-d_m}$

move to x_{-d_m-1} (i.e., move to the open area)

compute *next*

let $OA = \{x_{-d_m-1}, x_{-d_m-2}, x_{-d_m-3}, ..., x_{-2d_m}\}$the open area set

Let $FD = \{N(x_j) - y - BV\}$

let $C = FD \cap OA$

Agent A moves to $next \in C$ that minimizes $dist(x_j, t)$ and $next \geq t$

Else

compute *next*

let $EA = \{x_{-d_i-1}, x_{-d_i-2}, x_{-d_i-3}, x_{-d_i-4}, ..., x_{-d_{i+1}+1}\}$the enclosed area set

Let $FD = \{N(x_j) - y - BV\}$

let $C = FD \cap EA$

Agent A moves to $next \in C$ that minimizes $dist(x_j, t)$ and $next \geq t$

The following observations were made after using the general strategy:

Theorem 22. *In any chordal ring $C_(1, d_2, d_3, ...d_m)$ with $d_m << n$, the worst case scenario in terms of number of agents required occurs when triggering the original black virus creates $2m-1$ more black viruses. In this case, $size(C) \leq \lfloor \frac{4m^2-8m+3}{2} \rfloor + 6m - 1$ and $Spread(C) = 2m$.*

Proof. In any chordal ring C, the worst case scenario occurs when the original *black virus* is found before deploying any SH. In orther words, when $|S_{area}| < d_2$. In this case, the maximum number of *black viruses* would be created: $2m-1$ and $\mathcal{BV} = \{x_1,\ x_{\pm d_2},\ x_{\pm d_3},\ ...,\ x_{\pm d_m}\}$. If we assume that the chords are well separated (i.e., $d_i - d_{i-1} \geq 1$), we have $2(m-1)$ enclosed areas and 2 open areas. Each $bv \in BV$ has at most $2m-1$ neighbours which represent our targets. Some of these are common neighbours or other *black viruses* , depending on the structure of the chords. Because we are interested in calculating the $size(C)$, we must first find the number of targets. Each $bv \in BV$ has common neighbours and non-common neighbours. $N_{nc}(bv)$ and $N_c(bv)$ denote the set of non-common neighbours and the set of common neighbours of any $bv \in BV$. Note that for any i and j, $x_{d_i} + x_{-d_j}$ (a neighbour of node x_{d_i}) is the same as $x_{-d_j} + x_{d_i}$ (a neighbour of node x_{-d_j}). We have thus found that $|N_{nc}(bv)| \leq 2$ and $|N_c(bv)| \leq 2m-3$ for any $bv \in BV$. In other words, each *black virus* has a maximum of two non-common neighbours. Therefore, we can calculate the maximum possible number of targets as: $|\mathcal{T}| \leq \lfloor \frac{4m^2-8m+3}{2} \rfloor + 4m - 2$. $|\mathcal{T}|$ represents the number of SAs we need to surround the *black viruses* in the system.

The team of agents consists of $\leq \lfloor \frac{4m^2-8m+3}{2} \rfloor + 4m - 2$ SAs, one LEA and $(2m)$ CAs. Therefore $size(C) \leq \lfloor \frac{4m^2-8m+3}{2} \rfloor + 6m - 1$ and $Spread(C) = 2m$.

□

Theorem 23. *In any chordal ring $C_(1, d_2, d_3, ...d_m)$ where $|S_{area}| < x_{d_2}$, the number of moves required to surround and eliminate BVs is $O(m^2)$.*

Proof. Calculating the number of moves is not trivial in the general case. This approach is not optimal but it is efficient and gets the routing done correctly. We have an upper bound for the total number of moves required and it is constant. In the analysis of our strategy we find:

- $2m-1$ targets are reached in 2 moves, which are $N(x_{-1})$:

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{\pm d_i} x_{\pm d_i - 1}$$

- Two targets are reached in 4 moves each which are x_{2d_m} and x_{-2d_m}.

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{\pm d_m} x_{\pm d_m - 1} \xrightarrow{\pm d_m} x_{\pm 2d_m - 1} \xrightarrow{\pm 1} x_{\pm 2d_m}$$

- The rest of targets are nested in the enclosed and open areas and their locations are solely dependent on the chords structure. In order to reach them, the agents first move to node x_{-1} and then to one of the neighbours that satisfies the strategy's conditions. Once an agent is located at the beginning of an interval in which its destination resides, the one-direction greedy strategy begins.

  The destination between $x_{\pm d_i}$ and t can be minimized if there is a chord $d_i > 1$. In our thesis we will consider the worst case scenario in which an agent moves to its target through ± 1 chords. Therefore, each of the other targets, $\leq \lfloor \frac{4m^2-8m+3}{2} \rfloor + 2m - 3$, are reached in $O(m^2)$ moves.

□

7.3 Conclusion

In this chapter we addressed the *black virus disinfection* problem in any general chordal rings, regardless of the chords structure. In this chapter we have shown that the first phase remains the same in all chordal rings and that the second phase can be done non-locally. In this case, the LEA finds the shortest paths to the targets and sends SAs through them. In order to calculate the complexity of the optimal paths, we require information about the chords structure that is not available. As a result, we propose an efficient protocol

that gives us upper bounds to the optimal length. In this protocol we considered some observations from all of the strategies discussed in previous chapters. This protocol is based on the fact that the presence of *black viruses* in the system divides the topology into *enclosed* and *open* areas and that all of those areas are reached through node x_{-1}. Once a SA reaches the area in which its target resides, it moves greedily in one direction until it reaches its destination. We have found that the total complexity of our general algorithm in term of moves is $O(mn)$ for the first phase, where m is the total number of chords in one direction, and $O(m^2)$ for the second phase.

Chapter 8

Conclusion

In this thesis, we investigated the black virus disinfection problem in undirected chordal rings, presenting a solution that is based on the use of the mobile agents model. We addressed the problem with the existence of a single black virus at unknown location of the chordal ring that, when triggered, generates and sends new viruses to unprotected neighbours. Although we assumed synchronous execution by the agents, this strategy can be easily extended to work in asynchronous settings.

The proposed solution is a distributed algorithm that consists of two phases: *Exploring and Shadowing* and *Surrounding and Eliminating*. Regarding cost, the efficiency measures considered include: the total number of black viruses originated in the system, the total number of mobile agents employed for disinfection and the total number of moves required by the agents. We found that the number of black viruses originated and agents required for disinfection is influenced by the location of the original black virus and that these numbers remain constant, regardless of the deployment method used in the surrounding phase. The number of moves varies according to the deployment strategy.

In our study we demonstrated that the first phase remains the same for all chordal rings and consists of the exploration of the graph using the safe exploration technique until the black virus is found. In order to achieve monotonicity, shadow agents are deployed

during the search in order to guard the nodes that have been explored. The only difference between the types of chordal rings in the first phase is the number of shadow agents created at the beginning of the protocol. On the other hand, the second phase varies depending on the chords structure. Since black viruses are only destroyed when they arrive at a node that is being guarded, the agents must occupy all of the neighbouring nodes of the black viruses in the system. Routing is thus a critical part of this phase. Assuming that the leading agent has full topological knowledge, the routing can be done globally by the leading agent. The leading agent always finds the shortest path to all targets and sends the surrounding agents through them. This routing method is optimal, yet it requires that the surrounding agents have larger memories. We also proposed other local methodologies that do not require that the entire path be calculated by the leading agent. We analyze the complexity of all the deployment strategies, providing upper bounds on the path lengths and optimality when possible.

In chapter 4, we addressed the black virus disinfection problem in double loop chordal rings. In order to disinfect the entire topology we require a maximum of 12 agents. The maximum number of black viruses is four. For the surrounding phase we presented three strategies: move-optimal, simple greedy and smart greedy. The move-optimal strategy is a global optimal deployment method that is mainly done by the leading agent. In this method, we demonstrated the optimal path to each target in all possible double loop cases. For the simple greedy and smart greedy strategies, we described two local approaches that allow the surrounding agents to decide their next step according to local information. These two strategies are not optimal.

In chapter 5, we described our solution in triple loop chordal rings. In order to disinfect the entire topology we require a maximum of 24 agents. The maximum number of black viruses originated is six. For the deployment phase we only described the move-optimal strategy since the greedy approach does not work for triple loops. We were unable to calculate the optimal paths for triple loops, however, we provided the upper bounds for

the optimal loops. In order to get a better understanding of the problem, we studied two extreme triple loop cases and were able to find the optimal complexity for both cases. For simplicity, we only considered the shortly-chorded triple loop for the analysis of the surrounding phase.

In chapter 6, we addressed the black virus disinfection problem in consecutive-chords loops. In order to disinfect the entire topology we require a maximum of $4k+2$ agents. The maximum number of black viruses originated is $2k$, where k represents the longest chord. In the second phase we described a local strategy, the one-direction greedy strategy, which gives us the shortest paths to the targets.

In chapter 7, we discussed the black virus disinfection problem in general chordal rings. In order to disinfect a chordal ring $C(d_1 = 1, \ldots, d_m)$ we require $O(m^2)$ agents. The maximum number of black viruses originated is $2m$, where m represents the number of chords in one direction. For the deployment phase we described a general protocol that is not optimal but that works correctly for any chord structure.

The following table summarizes the worst case complexities for the various chord structures considered in this thesis. The indicated complexity of the second phase corresponds with the best technique in terms of the number of moves required.

	$C(1,k)$	$C(1,p,k)$	$C(1,2,\ldots,k)$	$C(d_1,d_2,\ldots,d_m)$
Agents	12	24	$4k+2$	$O(m^2)$
Spread	4	6	$2k$	$2m$
Moves Phase1	$O(n)$	$O(n)$	$O(nk)$	$O(nm)$
Moves Phase 2	≤ 36	≤ 78	$O(k)$	$O(m^2)$

Table 8.1: A summary of the worst case complexities for various chordal ring types.

As previously mentioned, the problem of disinfecting a topology from black viruses is fairly new, and therefore, there remain many problems to be solved:

- In the case of directed chordal rings, how can the agents safely explore in an asynchronous environment? In addition, can the routing always be performed correctly avoiding the black viruses?

- In the case of sequential activation, where the triggering of black viruses is done sequentially, what would be the effect on the number of agents and the number of moves?

- In our study we only considered situations in which there is initially a single black virus. What would the topological impact be if we started with an unknown number of black viruses? In consecutive-chord loops, having two black viruses disconnects the graph. What about other types of chordal rings? What would the minimum number of black viruses that would cause network disconnection?

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APPENDICES

Appendix A

Detailed Paths Analysis in Triple Loops

Now let us consider the different routes to each target in 6 cases depending on the location of the *black virus*. Let $\pi[x_0, x_i]$ denote a path to reach target x_i, and let $dif = k - p$, we have the following situations:

- **Case1**: Let us study the case of finding the *black virus* in the third segment of the chordal ring: $k \leq |S_{area}| < n - k$. In this case, triggering the original *black virus* creates three more *black viruses* : x_1, x_p and x_k, and thus $\mathcal{T}$=$\{x_2, x_{p-1}, x_{p-k}, x_{k-1}, x_{p+1}, x_{k+1}, x_{k-p}, x_{k+p}, x_{2p}, x_{2k}\}$.

  - x_{k-1}: Node x_{k-1} is reached through σ_{k-1}

$$\sigma_{k-1} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1}$$

  - x_{p-1}: Node x_{k-1} is reached through σ_{p-1}

$$\sigma_{p-1} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1}$$

– x_2: could be reached in different ways:

$$\pi[x_0, x_2] = \min\{\pi_1, \pi_2, \pi_3, \}$$

  * taking advantage of the fact that x_{-k} is known to be safe:

$$\pi_1 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+1} x_{1-k} \xrightarrow{+1} x_{2-k} \xrightarrow{+k} x_2$$

  * taking advantage of the fact that x_{-p} is known to be safe:

$$\pi_2 = x_0 \xrightarrow{-p} x_{-p} \xrightarrow{+1} x_{1-p} \xrightarrow{+1} x_{2-p} \xrightarrow{+p} x_2$$

  * If $p = 4$

$$\pi_3 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{-1} x_2$$

  * If $p = 3$

$$\pi_3 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_2$$

– x $_{p+1}$:

  * Taking advantage of the fact that x_{-k} is known to be safe:

$$\pi_4 = x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+p} x_{-k+p} \xrightarrow{+1} x_{-k+p+1} \xrightarrow{+k} x_{p+1}$$

  * If $k = 2p$

$$\pi_5 = x_0 \xrightarrow{-p} x_{-p} \xrightarrow{+1} x_{-k+p+1} \xrightarrow{+k} x_{p+1}$$

  * If $k = 2p + 1$

$$\pi_5 = x_0 \xrightarrow{-p} x_{-k+p+1} \xrightarrow{+k} x_{p+1}$$

– x_{k-p}:

$$\pi[x_0, x_{k-p}] = \min\{\pi_6, \sigma_{k-p}\}$$

where

$$\sigma_{k-p} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{-p} x_{k-p-1} \xrightarrow{+1} x_{k-p}$$

$\pi_6 =$

* If $k < 2p$

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{-1} x_{p-2} \ldots \xrightarrow{-1} x_{k-p}$$

* If $p = dif$, there is no x_{k-p}.

* Else, x_{k-p} is between x_p and x_k.

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{-1} x_{k-2} \ldots \xrightarrow{-1} x_{k-p}$$

If applicable(i.e., $k \neq 2p + 1$)

– x_{2p} is reached through σ_{2p}:

$$\sigma_{2p} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{+p} x_{2p-1} \xrightarrow{+1} x_{2p}$$

– x_{2k} is reached through σ_{2k}:

$$\sigma_{2k} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k}$$

– x_{k+1}:

$$\pi[x_0, x_{k+1}] = \min\{\pi_7, \sigma_{k+1}, \pi_8, \pi_9\}$$

where

$$\sigma_{k+1} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k} \xrightarrow{+1} x_{2k+1} \xrightarrow{-k} x_{k+1}$$

$\pi_7 =$

* If $k < 2p$

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{+p} x_{2p-1} \xrightarrow{-1} x_{2p-2} \xrightarrow{-1}, ..., \xrightarrow{-1} x_{k+1}$$

* If $k > 2p$

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{+k} x_{k+p-1} \xrightarrow{-i} x_{k+p-i}, ..., x_{k+1}$$

where $i = 1$ or $i = p$ depending on whether the differnece between x_{k+1} and x_{k+p-1} is greater than p or not.

If $|\pi[x_0, x_{k-p}]| < 4$, then x_{k+1} is reached through π_6 plus two more moves.

$$\pi_8 = \pi_6 + x_{k-p} \xrightarrow{+1} x_{k-p+1} \xrightarrow{+p} x_{k+1}$$

If $|\pi[x_0, x_2]| < 4$, then x_{k+1} is reached through π_3 plus two more moves.

$$\pi_9 = \pi_3 + x_2 \xrightarrow{+k} x_{k+2} \xrightarrow{-1} x_{k+1}$$

– x_{k+p} is reached through σ_{k+p}:

$$\sigma_{k+p} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{+k} x_{k+p-1} \xrightarrow{+1} x_{k+p}$$

– x_{p-k}

$$\pi[x_0, x_{p-k}] = \min\{\pi_{10}, \sigma_{p-k}\}$$

where

$$\sigma_{k+p} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{-k} x_{p-k-1} \xrightarrow{+1} x_{p-k}$$

$\pi_{10} =$

* If $dif \leq 3$

$$x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2}, ..., \xrightarrow{-1} x_{p-k}$$

* If $p \leq 3$

$$x_0 \xrightarrow{-k} x_{-k} \xrightarrow{+1} x_{-k+1} \xrightarrow{+1}, ..., \xrightarrow{+1} x_{p-k}$$

* If $p - dif < 3$

$$x_0 \xrightarrow{-p} x_{-p} \xrightarrow{+1} x_{-p+1} \xrightarrow{+1}, ..., \xrightarrow{+1} x_{p-k}$$

– Nodes x_{-p+1}, and x_{-k+1} are occupied by SHs that were at nodes x_{-p} and x_{-k} respectively when the original *black virus* got triggered..

We have to take into consideration the fact that some of the above paths might be not applicable if they pass thorough a BV. However, the special paths are always applicable.

- **Case2**: In the case of finding the *black virus* in the fourth segment $n-k \leq |S_{area}| < n-p$, two *black viruses* are generated, which are at x_1 and x_p, since the rest neighbours are explored and guarded. Thus, $\mathcal{T}$=$\{x_2, x_{p-1}, x_{p+1}x_{p-k}x_{p+k}x_{2p}\}$

  – x_2 is reached using π_1 or π_2 orπ_3 as we discussed in **Case1**.

  – x_{p-1} is reached using σ_{p-1}

  – x_{p+1} is reached using π_4 or π_5 as we discussed in **Case1**.

  – x_{p-k} is reached using σ_{p-k} or π_{10} as we discussed in **Case1**.

  – x_{p+k} is reached using σ_{p+k}.

  – x_{2p} is reached using σ_{2p}.

  – x_{k+1}, x_{-p+1}, and x_{-k+1} are occupied by SHs that were at nodes x_k, x_{-p} and x_{-k} respectively when the original *black virus* got triggered.

- **Case3**: In the case of finding the *black virus* in the fifth segment $n-p \leq |S_{area}| <$

$n-1$, one *black virus* is generated, which is at x_1 since the rest neighbours are explored and guarded. Thus, $\mathcal{T}$=$\{x_2\}$

- x_2 is reached using π_1 or π_2 orπ_3 as we discussed in **Case1**.
- Nodes x_{p+1}, x_{k+1}, x_{-p+1}, and x_{-k+1} are occupied by SHs that were at nodes x_p, x_k, x_{-p} and x_{-k} respectively when the original *black virus* got triggered.

- **Case4**: We have a special case when the *black virus* is located at node $n-1$. In this case, all neighbours are guarded and no more *black viruses* are created. No more moves are made in the second phase since all moves are done in the first phase as we explained in the previous section.

- **Case5**: In the case of finding the *black virus* in the second segment $p \leq |S_{area}| < k$, four *black viruses* are generated, which are at $\mathcal{BV}$=$\{x_1, x_p, x_k, x_{-k}\}$ since only one SH has been deployed so far at node x_{-p}. Thus, $\mathcal{T}$=$\{x_2,\ x_{p-1},\ x_{k-1},\ x_{p+1},\ x_{k+1},\ x_{k-p},\ x_{k+p},\ x_{2p},\ x_{2k},\ x_{-k+p},\ x_{-k+1},\ x_{-k-1},\ x_{-k-p},\ x_{-2k}\}$. To reach any of these targets, we should avoid any path that has x_{-k}.

  - Node x_2 is reached as the following:

$$\pi[x_0, x_2] = \min\{\pi_2, \pi_3\}$$

  - Node x_{p-1} is reached using σ_{p-1}
  - Node x_{k-1} is reached using σ_{k-1}
  - Node x_{p+1} is reached as the following:

$$\pi[x_0, x_{p+1}] = \min\{\pi_5, \sigma_{p+1}\}$$

– Node x_{k+1} is reached as the following:

$$\pi[x_0, x_{k+1}] = \min\{\pi_7, \pi_8, \pi_9, \sigma_{k+1}\}$$

– Node x_{k-p} is reached as the following:

$$\pi[x_0, x_{k-p}] = \min\{\pi_6, \sigma_{k-p}\}$$

– Node x_{k+p} is reached using σ_{k+p}

– Node x_{2p} is reached using σ_{2p}

– Node x_{2k} is reached using σ_{2k}

– Node x_{-k+p} is reached as the following:

$$\pi[x_0, x_{-k+p}] = \min\{\pi_{10}, \sigma_{-k+p}\}$$

– Node x_{-k+1} is reached as the following:

$$\pi[x_0, x_{-k+1}] = \min\{\pi_{11}, \sigma_{-k+1}\}$$

where

$$\pi_{11} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-p} x_{-p-1} \xrightarrow{-i}, \ldots \xrightarrow{-i}, x_{-k+1}$$

where $i = 1$ or $i = p$ depending on the distance $dist$ between x_{-p-1} and x_{-k+1}. If $dist \geq p$, then $i = p$, otherwise, $i = 1$.

– Node x_{-k-1} is reached using σ_{-k-1}

– Node x_{-k-p} is reached using σ_{-k-p}

– Node x_{-2k} is reached using σ_{-2k}

– Node x_{-p+1} is already guarded by a SH that was at node x_{-p} when the original *black virus* got triggered.

- **Case6**. Finding the *black virus* in the first segment $1 \leq |S_{area}| < p$, five *black viruses* are generated, which are at $\mathcal{BV}$=$\{x_1, x_p, x_k, x_{-p}, x_{-k}\}$ since no SHs have been deployed yet-p. Thus, $\mathcal{T}$=$\{x_2,\ x_{p-1},\ x_{k-1},\ x_{p+1},\ x_{k+1},\ x_{k-p},\ x_{k+p},\ x_{2p},\ x_{2k},\ x_{-p+1},\ x_{-p-1},\ x_{-2p}$,$x_{-k+p},\ x_{-k+1},\ x_{-k-1},\ x_{-k-p},\ x_{-2k}\}$. To reach any of these targets, we should avoid any path that has x_{-p} or x_{-k} as the following:

  – Node x_2. Since x_{-p} and x_{-k} are BVs, we have

$$\pi[x_0, x_2] = \min\{\pi_{12}, \sigma_2\}$$

  where

$$\pi_{12} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+p} x_{p-1} \xrightarrow{-1} x_{p-2} \ldots \xrightarrow{-1} x_2$$

$$\sigma_2 = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{+k} x_{k-1} \xrightarrow{+k} x_{2k-1} \xrightarrow{+1} x_{2k} \xrightarrow{+1} x_{2k+1} \xrightarrow{+1} x_{2k+2} \xrightarrow{-k} x_{k+2} \xrightarrow{-k} x_2$$

  – Node x_{p-1} is reached using σ_{p-1}

  – Node x_{k-1} is reached using σ_{k-1}

  – Node x_{p+1} is reached using σ_{p+1}

  – Node x_{k+1} is reached as the following:

$$\pi[x_0, x_{k+1}] = \min\{\pi_7, \pi_8, \pi_9, \sigma_{k+1}\}$$

– Node x_{k-p} is reached as the following:

$$\pi[x_0, x_{k-p}] = \min\{\pi_6, \sigma_{k-p}\}$$

– Node x_{k+p} is reached using σ_{k+p}

– Node x_{2p} is reached using σ_{2p}

– Node x_{2k} is reached using σ_{2k}

– Node x_{-p+1} is reached using σ_{-p+1}

– Node x_{-p-1} is reached using σ_{-p-1}

– Node x_{-2p} is reached using σ_{-2p}

– Node x_{p-k} is reached as the following:

$$\pi[x_0, x_{p-k}] = \min\{\pi_{10}, \sigma_{p-k}\}$$

where

$$\pi_{10} = x_0 \xrightarrow{-1} x_{-1} \xrightarrow{-1} x_{-2}, ..., \xrightarrow{-1} x_{p-k}$$

– Node x_{-k+1} is reached as the following:

$$\pi[x_0, x_{-k+1}] = \min\{\pi_{11}, \sigma_{-k+1}\}$$

– Node x_{-k-1} is reached using σ_{-k-1}

– Node x_{-k-p} is reached using σ_{-k-p}

– Node x_{-2k} is reached using σ_{-2k}