A Glossary of Hyperbolic Geometry

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1   Definitions

Fuchsian group A discrete group of orientation preserving isometries of $\mathbb{H}$. Called *of the first kind* if its limit set is $\partial\mathbb{H}$, and *of the second kind* otherwise.

Limit set The limit set of a fuchsian group G is the closure (under the Euclidean topology on $\partial\mathbb{H}$) of the set of non-elliptic fixed points of elements of G.

2   Theorems

2.1   Limit sets

- If G is fuchsian with limit set Λ, then

  - Λ is closed and G-invariant.
  - $|\Lambda| \leq 2$ iff G is elementary. Otherwise $|\Lambda| = \infty$.
  - Every G-orbit is dense in Λ (i.e. Λ is minimal in the sense that it has no proper closed G-invariant subsets).
  - If G is non-elementary, then Λ is uncountable, closed and has no isolated points.
  - If G is of the second kind, Λ is nowhere dense in $\partial\mathbb{D}$.
  - For any $x \in \overline{\mathbb{D}}$, Λ is the set of accumulation points of $G \cdot x$.

- Corollary: if G is of the second kind, its limit set is homeomorphic to the Cantor set.