A Glossary of Rings and Modules

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1 Notation

2 Definitions

Algebra If R is a commutative ring, an R-algebra is a ring A together with a structure map, a homomorphism $f : R \to A$ such that $f(R) \subset \mathrm{Z}(A)$.

Cokernel If $f : M \to N$ is an R-module homomorphism, $\operatorname{coker}(f) = N / \operatorname{im}(f)$.

Direct sum If I is an index set and $M_i : i \in I$ are R-modules, then $\bigoplus_{i \in I} M_i = \{(x_i) \in \prod M_i : \text{ all but finitely many } x_i = 0\}$ is an R-module.

Group ring R a ring and G a group. $R[G]$ has elements $rg : r \in R, g \in G$ and multiplication $(rg)(sh) = (rs)(gh)$.

Ideal An ideal I of R is a subring which also has $ai, ia \in I$ for every $a \in R, i \in I$. R/I is another ring.

Module A (left) R-module is an abelian group M with an operator $R \times M \to M$ which distributes over R- and M-addition and is associative with R-multiplication, and $1 \cdot x = x \forall X$. We can quotient a module by another module.

Product If I is an index set and $M_i : i \in I$ are R-modules, $\prod_{i \in I} M_i$ is an R-module.

Representation K a field, G a group; a K-representation of G is a K-vector space V with multiplication $v \mapsto gv$ linear and $g(hv) = (gh)v$. Equivalent to a $K[G]$-module.

Ring We assume all rings have a 1. A **division ring** has all elements except 0 units (and the zero ring doesn't count); a **field** is a commutative division ring.

Tensor product Given R-modules M, N, their tensor product is an R-module $M \otimes_R N$ together with an R-bilinear map $b : M \times N \to M \otimes_R N$

Unit A unit of a ring is an element with both left and right inverses.