CIS580: Machine Perception
Homework 1
Due: To Be Announced

Instructions

- This is an individual homework and is worth a 100 points.
- You must submit your solutions online on Gradescope, the entry code for which is MKYGP8. We recommend that you use LaTeX, but we will accept scanned solutions as well.
- There will be an auto-grader for items 3 and 4. The instructions for their submission will be posted soon on Piazza. We provide a few tests; use `run test/run_tests` to run them. Pleas make sure that the given tests are passing before submitting your work.
- Start early! If you get stuck, please post your questions on Piazza or come to office hours!

Homework

1. We define convolution as

$$(f * g)(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du.$$

Prove the following properties, where f, g, and h are functions and α is a scalar.

(a) (5 pts) **linearity**: $f * (g + h) = (f * g) + (f * h)$,

(b) (5 pts) **linearity**: $f * (\alpha g) = \alpha(f * g)$,

(c) (10 pts) **commutativity:** $f * g = g * f$,

(d) (10 pts) **shift equivariance:** $\tau_z(f * g) = (\tau_z f) * g = f * (\tau_z g)$ [1].

2. (15 pts) We define correlation as

$$(f \star g)(x) = \int_{-\infty}^{\infty} f(u)g(x+u)du.$$

Is it commutative? Prove your answer.

3. (20 pts) Discrete 1D convolution, being a linear operator, can be written as matrix multiplication. Write a MATLAB program to compute the matrix M such that $x * y = Mx$, $\forall x$, given y and the length of x. Follow the template given in `conv_matrix_1d.m`.

4. (10 pts) Write a MATLAB program to generate a sine wave, given its amplitude, frequency, initial phase, sampling frequency, and duration. Follow the template given in `make_sine.m`.

5. (25 pts) Use your `make_sine` function to generate 2 seconds of sinusoids of 220.5 Hz and 441 Hz, sampled at $Fs = 44.1$ kHz, with same amplitude. You can listen to the sound waves using MATLAB's `sound(y, Fs)` command; the 441 Hz tone should sound familiar. Let y_1, y_2 be the two signals, and $f(t) = y_1(t) + y_2(t)$.

   Now generate an amplified Gaussian of form $g(t) = A\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(t-\mu)^2}{2\sigma^2}}$, with $A = 2, \mu = 1, \sigma = 8.5 \cdot 10^{-4}$, with same duration and sampling rate.

   Compute the convolution [2] $f * g$.

   Show, in the same plot, $y_1, y_2, f, f * g$. Listen to each sound; what do you notice?

[1] τ_z is the shift-by-z operator: $\tau_z f(x) = f(x - z)$.

[2] we are looking for an approximation of the continuous convolution here.