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CIS 581 HW 1

1.2 0,0 is the top left

$I \odot f: g(0,0) = 0\cdot(.5+1+0+0+.5) + .5\cdot 1 + 2\cdot .5 + .5\cdot 0 + 1\cdot .5 = 2$

$f \odot I: g(0,0) = 0\cdot(.5+2+1.5+.5+2) + .5\cdot 1 + 1\cdot 0 + 0\cdot .5 + 1\cdot 1 = 1.5$

so $I \odot f \neq f \odot I$

2.1

$g' = I \otimes f_x$

$g'(0,0) = 0\cdot1 + 0\cdot0 + 1\cdot-1 = -1$

$g'(1,0) = 0\cdot1 + 0\cdot0 + -1\cdot-1 = 1$

$g'(2,0) = 1\cdot1 + -1\cdot0 + 0\cdot-1 = 1$

$g'(0,1) = 0\cdot1 + 2\cdot0 + 1\cdot-1 = -1$

$g'(1,1) = 2\cdot1 + 1\cdot0 + 0\cdot-1 = 2$

$g'(2,1) = 1\cdot1 + 0\cdot0 + 0\cdot-1 = 1$

$g'(0,2) = 0\cdot1 + 0\cdot0 + 3\cdot-1 = -3$

$g'(1,2) = 0\cdot1 + 3\cdot0 + -1\cdot-1 = 1$

$g'(2,2) = 3\cdot1 + -1\cdot0 + 0\cdot-1 = 3$

$$g' = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$g_1 = g' \otimes f_y$

$g_1(0,0) = 0\cdot1 + -1\cdot1 + -1\cdot1 = -2$

$g_1(1,0) = 0\cdot1 + 1\cdot1 + 2\cdot1 = 3$

$g_1(2,0) = 0\cdot1 + 1\cdot1 + 1\cdot1 = 2$

$g_1(0,1) = -1\cdot1 + -1\cdot1 + -3\cdot1 = -5$

$g_1(1,1) = 1\cdot1 + 2\cdot1 + 1\cdot1 = 4$

$g_1(2,1) = 1\cdot1 + 1\cdot1 + 3\cdot1 = 5$

$g_1(0,2) = -1\cdot1 + -3\cdot1 + 0\cdot1 = -4$

$g_1(1,2) = 2\cdot1 + 1\cdot1 + 0\cdot1 = 3$

$g_1(2,2) = 1\cdot1 + 3\cdot1 + 0\cdot1 = 4$

$$g_1 = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

f_{xy}:

$f_{xy}(0,0) = -1$

$f_{xy}(1,0) = 0$

$f_{xy}(2,0) = 1$

$$f_{xy} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$g_2(0,0) = 0\cdot(1+0+-1+1+1) + 0\cdot0 + -1\cdot1 + 2\cdot0 + 1\cdot-1 = -2$

$g_2(1,0) = 0\cdot(0+1+-1) + 0\cdot1 + 1\cdot0 + -1\cdot-1 + 2\cdot1 + 1\cdot0 + 0\cdot-1 = 3$

$g_2(2,0) = 0\cdot(1+0+-1+-1+-1) + 1\cdot1 + -1\cdot0 + 1\cdot1 + 0\cdot-1 = 2$

$g_2(0,1) = 0\cdot(1+1+1) + 0\cdot0 + 1\cdot-1 + 2\cdot0 + 1\cdot-1 + 0\cdot1 + 3\cdot-1 = -5$

$g_2(1,1) = 0\cdot1 + 1\cdot0 + -1\cdot-1 + 2\cdot1 + 1\cdot0 + 0\cdot-1 + 0\cdot1 + 3\cdot0 + -1\cdot-1 = 4$

$g_2(2,1) = 0\cdot(-1+-1+-1) + 1\cdot1 + -1\cdot0 + 1\cdot1 + 0\cdot0 + 3\cdot1 + -1\cdot0 = 5$

$g_2(0,2) = 0\cdot(1+1+1+0+-1) + 2\cdot0 + 1\cdot-1 + 0\cdot0 + 3\cdot-1 = -4$

$g_2(1,2) = 0\cdot(1+0+-1) + 2\cdot1 + 1\cdot0 + 0\cdot-1 + 0\cdot1 + 3\cdot0 + -1\cdot-1 = 3$

$g_3(2,2) = 0\cdot(-1+-1+1+0+-1) + 1\cdot1 + 0\cdot0 + 3\cdot1 + -1\cdot0 = 4$

$$g_2 = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

2.2 each line in $I \otimes f_x$

has 2 addition, 3 multiplication

so $I \otimes f_x$ has 18 + and 27 ×

same for $(I \otimes f_x) \otimes f_y$

so g_1 took 36 additions

54 multiplications

constructing f_{xy} took 18 additions

27 multiplications

each line in $I \otimes f_{xy}$ took 8 +, 9 ×

so in total g_2 took 90 additions

108 multiplications

2.3 This convolution identifies changes in values in the rows. The greater the change in g_1 the great the value change is reflected in g_1 & g_2.

The convolution acts like the horizontal sobel operator

3

$$I \otimes k = g$$

$$k = k_1 + k_2$$

$$I \otimes k_1 + I \otimes k_2 = g$$

$$\underbrace{\begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix}}_{g} - \underbrace{5\begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix}}_{I \otimes k_1} = \underbrace{\begin{bmatrix} 24 & 18 & 0 \\ 27 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{I \otimes k_2}$$

$$k_1 = 5\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad k_2 = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$k = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4.1

$$I_n = \frac{1}{4}\left[\begin{bmatrix}1&1&1&0\\1&1&1&0\\1&1&1&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&1&1&1\\0&1&1&1\\0&1&1&1\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\1&1&1&0\\1&1&1&0\\1&1&1&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&1&1&1\\0&1&1&1\\0&1&1&1\end{bmatrix}\right]$$

$$f_1 = \frac{1}{4}\left[\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix} + \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}\right] = \frac{1}{4}\begin{bmatrix}0&0&0\\0&1&1\\0&1&1\end{bmatrix}$$

$$f_2 = \frac{1}{4}\left[\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix} + \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\right] = \frac{1}{4}\begin{bmatrix}1&1&0\\1&1&0\\0&0&0\end{bmatrix}$$

4.2

$$f = \frac{1}{8}\begin{bmatrix}1&1&0\\1&2&1\\0&1&1\end{bmatrix} \quad I_Q' = \frac{1}{8}\begin{bmatrix}5&6&4&1\\6&8&6&2\\4&6&5&2\\1&2&2&1\end{bmatrix} \quad I_T' = \frac{1}{8}\begin{bmatrix}1&2&2&1\\2&5&6&4\\2&6&8&6\\1&4&6&5\end{bmatrix}$$

$$D(I_Q', I_T') = \frac{1}{64}\sum_{i,j}\left(I_Q'(i,j) - I_T'(i,j)\right)^2$$

$$= \frac{1}{64}\left(4^2+4^2+2^2+0^2+4^2+3^2+0^2+2^2+2^2+0^2+(-3)^2+(-4)^2+0^2+(-2)^2+(-4)^2+(-4)^2\right)$$

$$= \frac{1}{64}(130) = 2.03125$$

$$D(I_Q', I_T') < D(I_Q, I_T)$$

The kernel blurs the edges which makes the difference between the images. Since we do not know the orientation of the barrel, this method allows us to more easily find the object using the template

5.1 My iPhone 6 has a rear-facing sensor of 4.80 x 3.60 mm

http://www.cameradebate.com/2015/sensor-size-comparison-iphone-6-plus-vs-samsung-galaxy-s6-edge-vs-note-4/

The larger the sensor, the larger the fov.

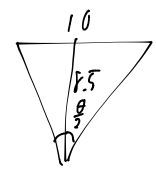
5.2 The image sensor resolution is 3264 x 2448 pixels

1 pixel size is $\frac{.0048}{3264} = 1.47\ \mu m = .00147\ mm$

5.3 The focal length is 4.15 mm

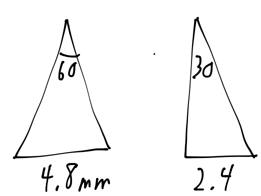
http://www.anandtech.com/show/9686/the-apple-iphone-6s-and-iphone-6s-plus-review/9

I took a picture of a 10 inch object 8.5 inch away
The object took the whole height.



$\frac{\theta}{2} = \tan^{-1}\frac{5}{8.5} \simeq 30$

fov = 60°

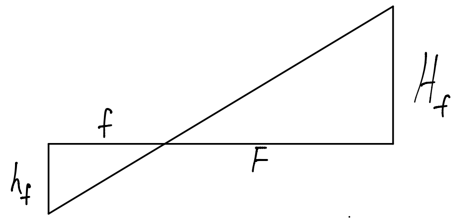


focal length $= 2.4\sqrt{3} = 4.157$ mm

The focal length are basically the same.
The focal length is inversely proportional to the fov.

5.4 Intrinsic camera matrix: $\begin{bmatrix} 4.15 & 0 & 1632 \\ 0 & 4.15 & 1224 \\ 0 & 0 & 1 \end{bmatrix}$

5.5



$h_f = 1213 \times .00147$

$= 1.78311$ mm

5.5.1

$\frac{F}{f} = \frac{H_f}{h_f}$ $\qquad H_f = 1955.8$ mm

$F = f\frac{H_f}{h_f} = 4.15\text{m}\,\frac{1955.8}{1.78311} = 4552\text{mm} = 4.552\text{m}$

5.5.2

$h_B = 1381 \times .00147 = 2.03$ mm

$\frac{B}{f} = \frac{H_B}{h_B} = \frac{5410.2}{2.03}$ $\qquad B = \frac{5410.2}{2.03} \cdot 4.157\text{mm} = 11.06\text{m}$

5.6.1 $f'' = \frac{h_f''}{H_f}(F+\Delta d) \quad h_f'' = h_f' = \frac{f'}{F} H_f$

$f'' = f' + f' \frac{\Delta d}{F}$

$\frac{B+\Delta d}{f''} = \frac{H_B}{h_B''} \rightarrow h_B'' = \frac{f''}{B+\Delta d} H_B = \frac{f' H_B + \frac{\Delta d}{F} H_B f'}{B+\Delta d} = \frac{h_B' B + \frac{\Delta d}{F} h_B' B}{B+\Delta d}$

$f' = \frac{h_B'}{H_B} B$

5.6.2 $h_B'' = 2.474\,mm = 1682$ pixels

This isn't close to the original height

5.6.3 $\frac{h_B' B + \frac{\Delta d}{F} h_B' B}{B+\Delta d} = 3h_B' \quad h_B' B + \frac{\Delta d}{F} h_B' B = 3B h_B' + 3h_B' \Delta d$

$\frac{\Delta d}{F} h_B' B - 3h_B' \Delta d = 3B h_B' - h_B' B$

$\Delta d = \frac{2B h_B'}{\frac{h_B' B}{F} - 3h_B'}$

$f'' = f' + f' \frac{\Delta d}{F} = \frac{F h_f'}{H_f} + \frac{h_f'}{H_f} \Delta d = \frac{F h_f'}{H_f} + \frac{2B h_B' h_f'}{H_f \left(\frac{h_B' B}{F} - 3h_B'\right)}$

$$\Delta d = \frac{2 \cdot 11.06 \cdot .00203}{\frac{.00203 \cdot 11.06}{4.552} - 3 \cdot .00203} = -38.79\,m$$

$f'' = 4.15\,mm + 4.15\,m \frac{-38.79\,m}{4.552\,m} = -31.21\,mm$

This means that the statue needs to be in front of the person to achieve $3h_B'$