Experiments, Elementary Outcomes, and Events

Probability Properties

- One fundamental property of probability is that the sum of the probabilities of all possible outcomes must

Example:

For the die example, we have:

- Six outcomes: 1, 2, 3, 4, 5, 6
- Each with probability 1/6
- Sum: 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 6/6 = 1

Probability Formula:

The probability of an event E occurring is given by:

$$
P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

Sample Space Probabilities in Python

In Python, simple probability calculations based on sample spaces such as these are easily done using the

- `==` will find the rows that satisfy your event

- `len()` will return the number of rows that satisfy your event

The Law of Large Numbers

The law of large numbers states that the more times an experiment is repeated, the closer the relative freq

Python Exercise:

Below is a Python exercise demonstrating the law of large numbers by estimating the probability of selectin

```python
import random

# Set the seed for reproducibility; the parameter 42 ensures consistency in results
random.seed(42)

def calculate_p(N):
    """
    Function to calculate the probability of selecting a number between 21 and 40
    from a uniform random draw of integers between 1 and 100.
    """
    cnt_21_40 = 0

    for i in range(N):
        random_integer = random.randint(1, 100)

        if 21 <= random_integer <= 40:
```

```
            cnt_21_40 += 1

    return cnt_21_40 / N

p_20 = calculate_p(20)
p_400 = calculate_p(400)
p_10000 = calculate_p(10000)

print(f'{p_20} {p_400} {p_10000}')
```

Venn Diagrams

Venn diagrams illustrate the relationships between different events.

- **Mutually Exclusive Events:** These events cannot happen simultaneously, meaning their probability of i
- **Independent and Dependent Events:** Events are independent if the occurrence of one does not affect

Example:

Consider two independent events:

- A = The first python has 3 spots
- B = The second python has 5 spots

Using the independence formula:

\[

P(A \cap B) = P(A) \times P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}

\]

Python Exercise: Spot Matrix Probability Calculation

```python
import numpy as np

# Define a 6x6 matrix of spot values
spot_matrix = np.array(
    [
        [10, 11, 12, 13, 14, 15],
        [11, 12, 13, 14, 15, 16],
        [12, 13, 14, 15, 16, 17],
        [13, 14, 15, 16, 17, 18],
        [14, 15, 16, 17, 18, 19],
        [15, 16, 17, 18, 19, 20],
    ]
)

spot_counts = {}

for i in range(5, 11):
    for j in range(5, 11):
        total = i + j
        if total in spot_counts:
```

```
            spot_counts[total] += 1
        else:
            spot_counts[total] = 1

spot_probs = {sum_value: freq / 36 for sum_value, freq in spot_counts.items()}

for i in range(10, 21):
    print(i, spot_probs[i])

sum_probs_one = int(sum(spot_probs.values()))
print(sum_probs_one)
```