SIMPLIFICATIONS OF BOOLEAN ALGEBRA

(a) $AB + A(B+C) + B(B+C)$

Applying distributive law, we have

$$AB + AB + AC + BB + BC$$

But $BB = B$ and $AB + AB = AB$, therefore

$$= AB + AC + B + BC$$

$$= AB + AC + B(\cancelto{1}{1+C})$$

$$= AB + AC + B$$

$$= AB + B + AC$$

$$= B(\cancelto{1}{A+1}) + AC$$

$$= B + AC$$

(b) $[A\bar{B}(C+BD) + \bar{A}\bar{B}]C$

Applying distributive law, we have

$$[A\bar{B}C + \cancelto{0}{A\bar{B}BD} + \bar{A}\bar{B}]C$$

$$[A\bar{B}C + \bar{A}\bar{B}]C$$

apply distributive law, we have

$$A\bar{B}CC + \bar{A}\bar{B}C$$

$$= A\bar{B}C + \bar{A}\bar{B}C$$

$$\bar{B}C(\cancelto{1}{A+\bar{A}})$$

$$= \bar{B}C$$

(c) $[AB(C + \overline{BD}) + \overline{AB}]\, CD$

Applying de morgan's theorem, we have

$[AB(C + \bar{B} + \bar{D}) + \bar{A} + \bar{B}]\, CD$

Applying distributive law

$= [ABC + \overset{0}{\cancel{AB\bar{B}}} + AB\bar{D} + \bar{A} + \bar{B}]\, CD$

$= [ABC + AB\bar{D} + \bar{A} + \bar{B}]\, CD$

$= [\bar{A} + ABC + \bar{B} + AB\bar{D}]\, CD$

$= [\bar{A} + BC + \bar{B} + A\bar{D}]\, CD$

$= [\bar{A} + A\bar{D} + \bar{B} + BC]\, CD$

$= [A + \bar{D} + \bar{B} + C]\, CD$

Applying Distributive law, we have

$= ACD + \overset{0}{\cancel{CD\bar{D}}} + \bar{B}CD + CCD$

$= ACD + \bar{B}CD + CD$

$= CD\,(\overset{1}{\cancel{1 + A + \bar{B}}})$

$= CD.$

(d) $\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$

$= \bar{A}BC + ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

$= BC(\cancel{\bar{A}+A})^{1} + \bar{B}\bar{C}(A+\bar{A}) + \bar{A}BC$

$= BC + \bar{B}\bar{C} + \bar{A}BC$

$= BC + \bar{B}(\bar{C} + AC)$

$= BC + \bar{B}(\bar{C} + A)$

$= BC + \bar{B}\bar{C} + A\bar{B}$

(e) $AB\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C}$

$= AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}(\cancel{C+\bar{C}})^{1}$

$= AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}$

$= AB\bar{C} + \bar{A}(\bar{B} + BC)$

$= AB\bar{C} + \bar{A}(\bar{B} + C)$

$= AB\bar{C} + \bar{A}\bar{B} + \bar{A}C$

(f) $\overline{AB + AC} + \bar{A}\bar{B}C$

$= \overline{AB} \cdot \overline{AC} + \bar{A}\bar{B}C$

$= (\bar{A}+\bar{B})(\bar{A}+\bar{C}) + \bar{A}\bar{B}C$

$= \bar{A}\bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A}(\cancel{1+C})^{1} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A}(\cancel{1+\bar{B}})^{1} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{B}\bar{C} + \bar{A}\bar{B}C$

$= \bar{A} + \bar{B}(\bar{C} + \bar{A}C)$

$= \bar{A} + \bar{B}(\bar{C} + \bar{A})$

$= \bar{A} + \bar{B}\bar{C} + \bar{A}\bar{B}$

$= \bar{A} + \bar{A}B + \bar{B}\bar{C}$

$= \bar{A}\cancel{(1+B)}^{1} + \bar{B}\bar{C}$

$= \bar{A} + \bar{B}\bar{C}$

(g) $\overline{AB} + \overline{AC} + \bar{A}\bar{B}\bar{C}$

$= \bar{A} + \bar{B} + \bar{A} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A} + \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A} + \bar{B} + \bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A} + \bar{A}\bar{B}\bar{C} + \bar{B} + \bar{C}$

$= \bar{A}\cancel{(1+\bar{B}\bar{C})}^{1} + \bar{B} + \bar{C}$

$= \bar{A} + \bar{B} + \bar{C}$

(h) $A + AB + A\bar{B}C$

$= A\cancel{(1+B)}^{1} + A\bar{B}C$

$= A + A\bar{B}C$

$= A\cancel{(1+\bar{B}C)}^{1}$

$= A$

(i) $(\bar{A}+B)C + ABC$

$= \bar{A}C + BC + ABC$

$= \bar{A}C + BC(\cancelto{1}{1+A})$

$= \bar{A}C + BC$

$= C(\bar{A}+B)$

(ii) $A\bar{B}C(BD + CDE) + A\bar{C}$

$= \cancelto{0}{A\bar{B}BCD} + A\bar{B}CCDE + A\bar{C}$

$= A\bar{B}CDE + A\bar{C}$

$= A(\bar{C} + \bar{B}CDE)$

$= A(\bar{C} + \bar{B}DE)$