1 a) Kirchoff's voltage law, $i_c = C\frac{dv}{dt}$

$\rightarrow E(t) = IR + \int \frac{I}{c}\,dt + L\frac{dI}{dt}$

Solve for homogeneous solution.

$\rightarrow E(t) = 0$

$\hookrightarrow IR + \int \frac{I}{c}\,dt + L\frac{dI}{dt} = 0$

Differentiate both sides

$\hookrightarrow \frac{dI}{dt}R + \frac{I}{c} + L\frac{d^2I}{dt^2} = 0$

Guess solution, $I_h = e^{\lambda t}$

$\hookrightarrow \lambda e^{\lambda t}R + \frac{e^{\lambda t}}{c} + \lambda^2 e^{\lambda t}L = 0$

Cancel out $e^{\lambda t}$

$\hookrightarrow \lambda^2 L + \lambda R + \frac{1}{c} = 0$

Sub in values, $L = 1$, $R = 5$, $c = 0.1$

$\hookrightarrow \lambda^2 + 5\lambda + 10 = 0$

Solve using quadratic equation

$\hookrightarrow \lambda = \frac{-5 \pm \sqrt{5^2 - 4(1)(10)}}{2}$

$\lambda = \frac{-5 + \sqrt{15}\,i}{2}$, $\lambda = \frac{-5 - \sqrt{15}\,i}{2}$

Homogeneous solution (General)

$\hookrightarrow I_h = C_1 e^{\left(\frac{-5+\sqrt{15}i}{2}\right)t} + C_2 e^{\left(\frac{-5-\sqrt{15}i}{2}\right)t}$

OR (other form)

$I_h = e^{\frac{-5}{2}t}\left(C\cos\left(\frac{\sqrt{15}}{2}t\right) + D\sin\left(\frac{\sqrt{15}}{2}t\right)\right)$