# Rocky Project

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The goal of this project is design a control system that maintains the balance of Rocky, an inverted-pendulum on a horizontally-translating cart. We did this using a 32U4 Balancing Robot Kit from Pololu Robotics and Electronics to implement our system and collect the data. As shown in Figure 1, the initial close-loop control diagram of the system includes the PI controller K(s), the transfer function of Rocky H(s), and the motor model M(s).

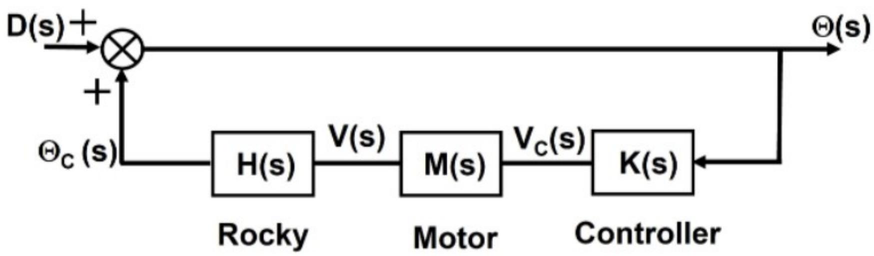



Figure 1: Initial close-loop control system diagram

## 1 Motor Model

To determine the step response of the motor on the cart, we recorded the speed change of the left and right motors within 3 seconds after setting the step input to a magnitude of 300. The data is collected with the wheels of Rocky spinning freely without having any contact to the ground, which means the speed of the wheels is insusceptible to friction and other external forces.

With the motor model assumed to be a first-order system represented as

$$M(s) = \frac{V(s)}{V_c(s)} = \frac{\frac{K}{\tau}}{s + \frac{1}{\tau}} \tag{1}$$

where $V(s)$ is the velocity of the cart (m/s), $V_c(s)$ is the input motor velocity control signal, $K$ is the constant factor, and $\tau$ is the time constant. We can calculate measured speed as a function of M(s)

$$V(s) = V_c M(s) = \frac{\frac{K}{\tau}}{s + \frac{1}{\tau}} \times V_c(s) \tag{2}$$

$$= \frac{\frac{K}{\tau}}{s + \frac{1}{\tau}} \times \frac{300}{s} \tag{3}$$

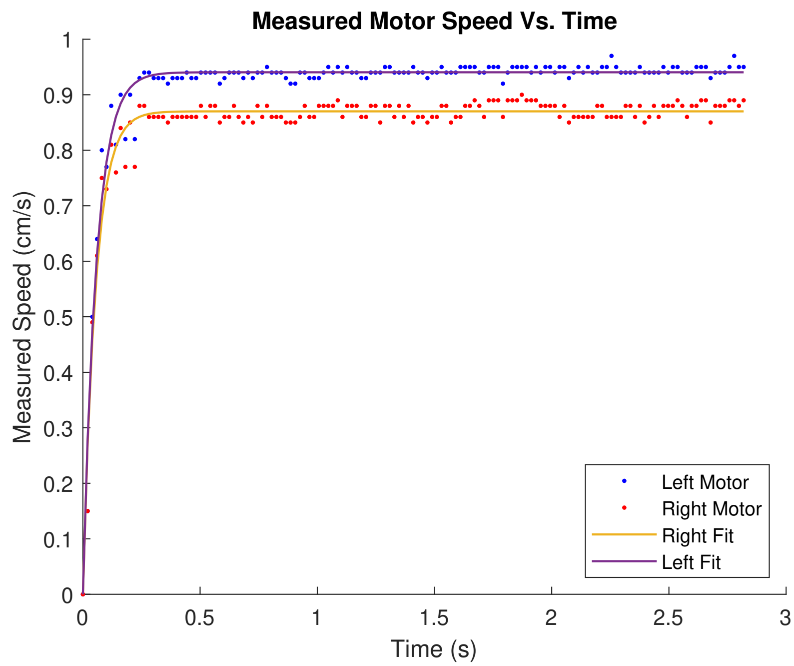



Figure 2: Left and right motor speeds within 3 seconds after setting the speed input at a magnitude of 300. Using MATLAB's curve fitting tool, we found that the theoretical fit lines are $V(t) = 300k(1 - e^{-\tau t})$. Where $\tau = 18.56s, k = 0.0029$ for the right motor and $\tau = 17.48s, k = 0.0031$ for the left.

# 2 Natural Frequency and Effective Length

To find the natural frequency of the inverted-pendulum system structure, we held Rocky cart end up and let it swing freely after briefly applied a force to the lower end. The angular displacement of Rocky measured by the built-in gyroscope is recorded until the measurement reaches a constant. Shown in Figure 3, the peak angle displacements during the swinging motion are marked as red circles. The average time interval between them is approximately the period of the swing motion, which is the reciprocal of the natural frequency, $\omega$. In this case, the natural frequency is calculated to be around 0.828Hz. Rewriting the function calculated in CW10

$$\omega_n = \sqrt{g/l} \tag{4}$$

can calculated l, the effective length of the inverted pendulum

$$l = \frac{g}{\omega_n^2} = \frac{9.8}{0.828^2} = 0.362m \tag{5}$$

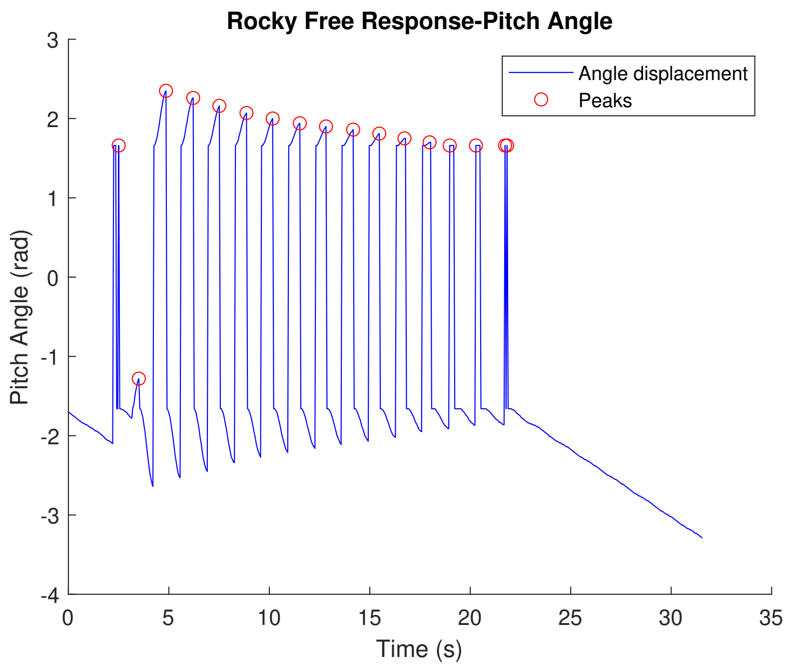



Figure 3: Angular displacement of Rocky when swinging freely in around 30 seconds. The red markers are the peaks of the swinging motion, which are used to calculate the average time interval between swings.

# 3 Initial System

There are three modules in the initial system:

- Controller:

$$K(s) = K_p + \frac{Ki}{s} \tag{6}$$

This is a proportional-integral (PI) controller with the proportional gain being $K_p$ and integral gain being $K_i$. The values of these two parameters are calculated later by tuning the poles of the close-loop system. The input to the K(s) module is the angle $\Theta$ from upright which is converted to the output of K(s) a velocity $V_c$.

- Motor:

$$M(s) = \frac{\frac{K}{\tau}}{s + \frac{1}{\tau}} \tag{7}$$

This is a first-order motor model with constants $\tau = 18.56s, k = 0.0029$, which are calculated in Part 1. The input to the M(s) module is the velocity $V_c$, and the output is the velocity $V$.

- Rocky:

$$H(s) = \frac{-s}{ls^2 - g} \tag{8}$$

The effective length is a constant calculated in Part 2, where $l = 0.362m$. The input to the H(s) module is the velocity $V$, and the output is converted back to the angle from upright, $\Theta_c$. This model is based on the force analysis of the physical setup:

$$gsin(\Theta) - l\dot{\Theta} = \ddot{x}cos(\Theta) \tag{9}$$

With the assumption that $\Theta \approx 0$, there is $cos(\Theta) \approx 1, sin(\Theta) \approx \Theta$, so the relation between velocity and angle can be rewritten as the transfer function

$$\frac{\theta(s)}{V(s)} = \frac{-s/l}{s^2 - g/l} = \frac{-s}{ls^2 - g} = H(s) \tag{10}$$

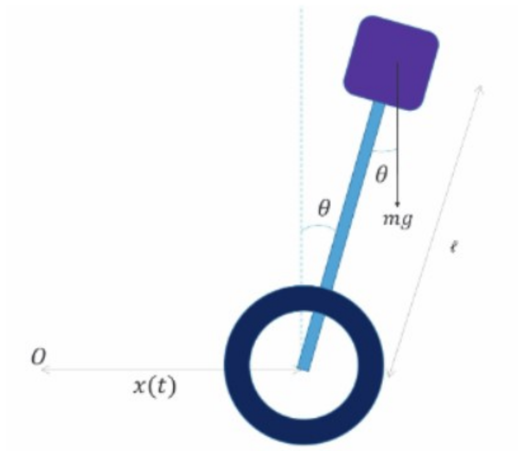



Figure 4: Physical setup and force diagram of Rocky.

To determine our poles, we used arbitrary values such that our poles were closer to the imaginary axis to provide a smaller overshoot, rise time, and settling time for our angular response. Our poles were

$$p_1 = -1 + 1i$$
$$p_2 = -1 - 1i$$
$$p_3 = -2$$
$$p_4 = -5$$
$$p_5 = -5$$

We used MATLAB to calculate the controller parameters from the poles. We did this using MATLAB's syms function to create a simulation of the system and create a target characteristic polynomial to the fifth degree to find the constants.

$$Kp = 2.1455 * 10^5$$
$$Ki = 1.1167 * 10^6$$
$$Ji = -3.9534 * 10^4$$
$$Jp = 7.8619 * 10^4$$
$$Ci = -2.0807 * 10^4$$

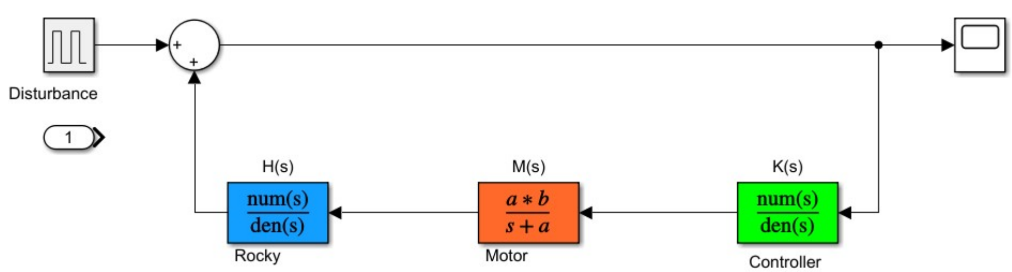



Figure 5: Simulink model of the initial system.

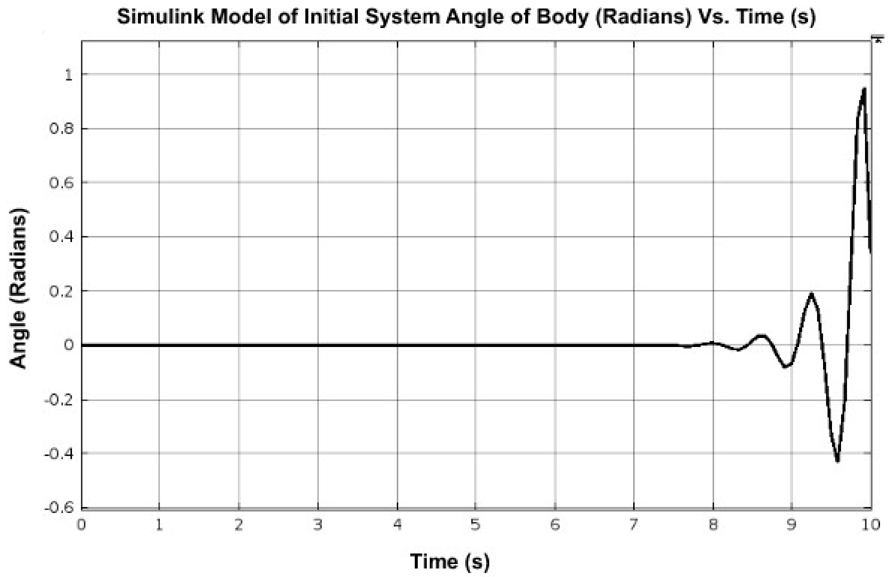



Figure 6: Simulink response plot of the angle (radians) of the initial balancing system versus time (s).

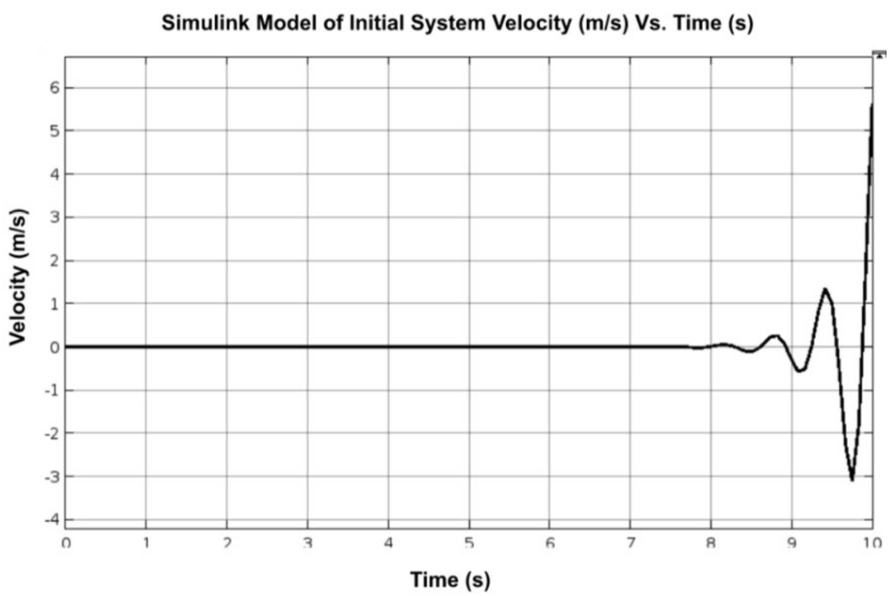



Figure 7: Simulink response plot of the velocity (m/s) of the initial balancing system versus time (s).

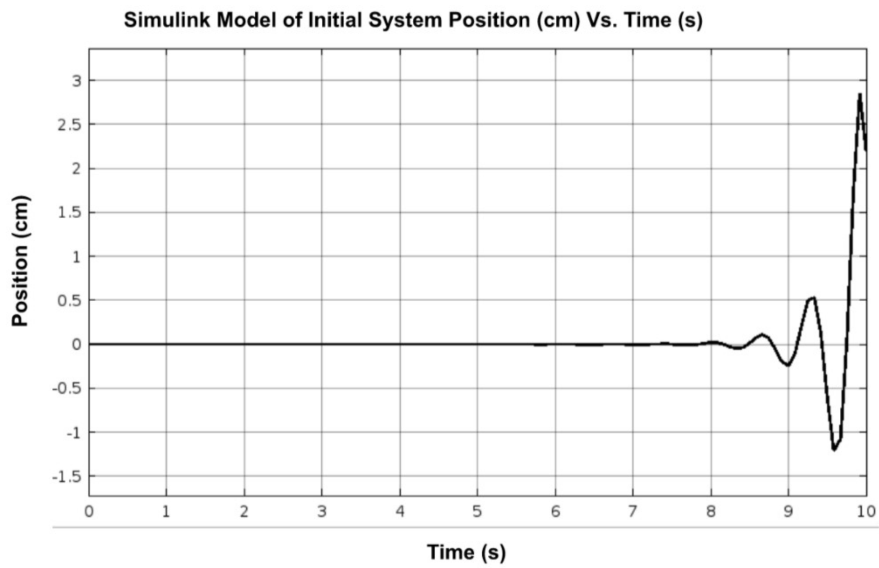



Figure 8: Simulink response plot of the position (cm) of the initial balancing system versus time (s).

# 4 Stationary Balancing System

The stationary system is initial system with feedback looks adds onto the motor module shown in Figure 10:

- Motor Controller:
$$J(s) = J_p + \frac{Ji}{s} \tag{11}$$
This is a proportional-integral (PI) controller with the proportional gain being $J_p$ and integral gain being $J_i$. In Simulink, the proportional and integral parts separated into two branches of the loop shown as orange. The values of these two parameters are calculated later by tuning the poles of the close-loop system.

- Stationary Motor:
$$C(s) = \frac{C_i}{s^2} \tag{12}$$
Shown as yellow in the Simulink model. This feedback loop enables the subtraction of the integral of position from the motor, which ensures the overall zero position change during the balancing motion.

  For finding the poles of the stationary balancing system, we took a different approach from the initial system. As seen in section 2, the natural frequency is 0.828Hz. Converting that to rad/s, we know that $\omega_n = 5.50756$ rads/sec. With this information, we calculated the poles with the following equations.

$$\begin{aligned}
p_1 &= w_n(-\cos\theta_1 - \sin\theta_1)\\
p_2 &= w_n(-\cos\theta_1 + \sin\theta_1)\\
p_3 &= -w_n\\
p_4 &= w_n(-\cos\theta_2 - \sin\theta_2)\\
p_5 &= w_n(-\cos\theta_2 + \sin\theta_2)
\end{aligned}$$

  We wanted our system's $\zeta$ to be as close to 1 as possible for it to be overdamped. We chose the value of 0.94 and found the angles using $\theta = \cos^{-1}(\zeta)$. We found both $\theta_1$ and $\theta_2$ to be equal to

20°. Inputting the natural frequency $\omega_n = 5.50756$, $\theta_1 = 20°$, and $\theta_2 = 20°$, the resulting poles are

$$p_1 = -7.0591$$
$$p_2 = -3.2917$$
$$p_3 = -5.5076$$
$$p_4 = -7.0591$$
$$p_5 = -7.0591$$

Since we used the same angle for both the $\theta_1$ values, we created a three-pole system resulting in our system controlling the velocity and angular position of our rocky.

Using the same method as we did for the initial system to calculate the constants, we found that

$$Kp = 1.2151 * 10^6$$
$$Ki = 6.3253 * 10^6$$
$$Ji = -1.2080 * 10^6$$
$$Jp = 1.6871 * 10^5$$
$$Ci = -1.3269 * 10^6$$

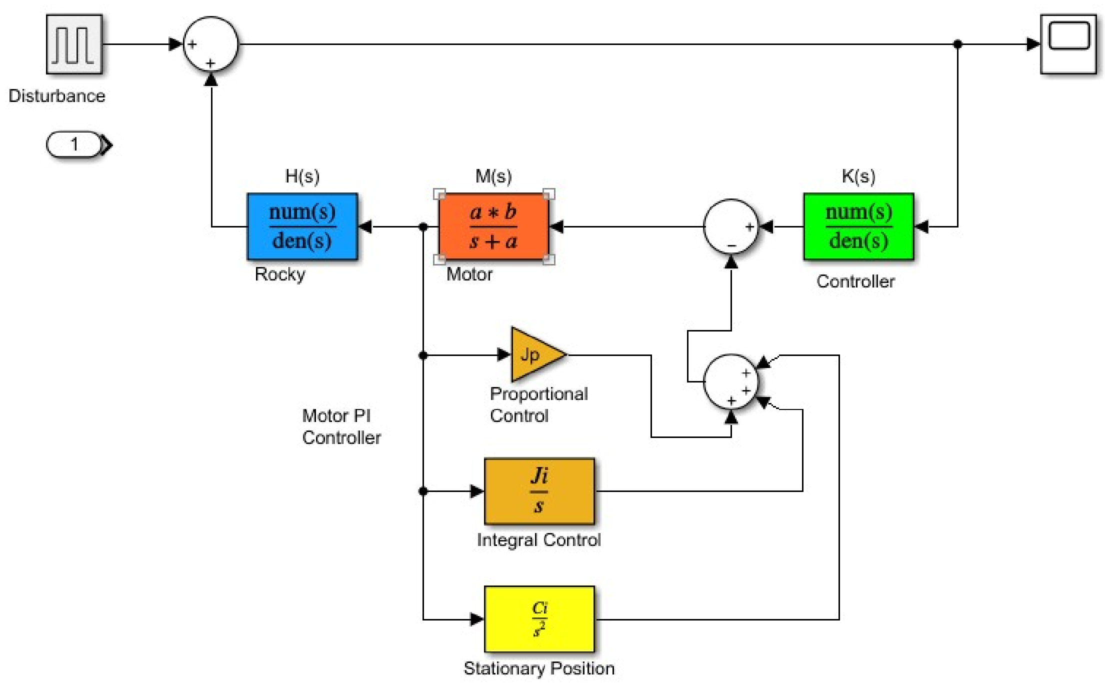



Figure 9: Simulink model of the Stationary balancing system.

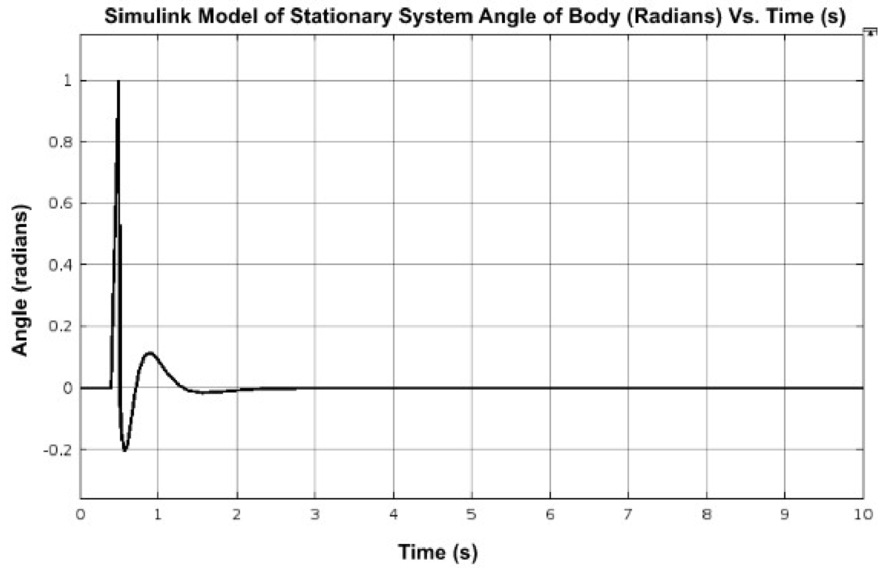



Figure 10: Simulink response plot of the angle of body (radians) of the stationary balancing system.

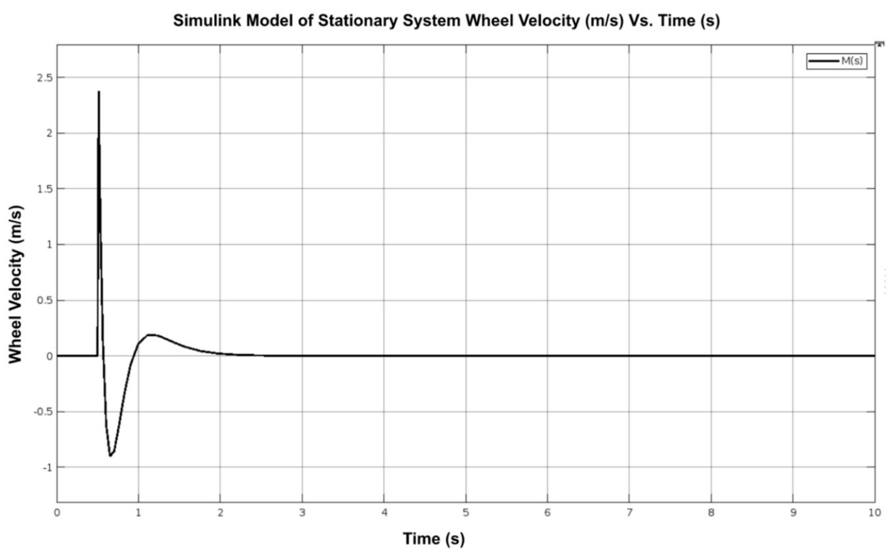



Figure 11: Simulink response plot of the velocity (m/s) of the stationary balancing system.

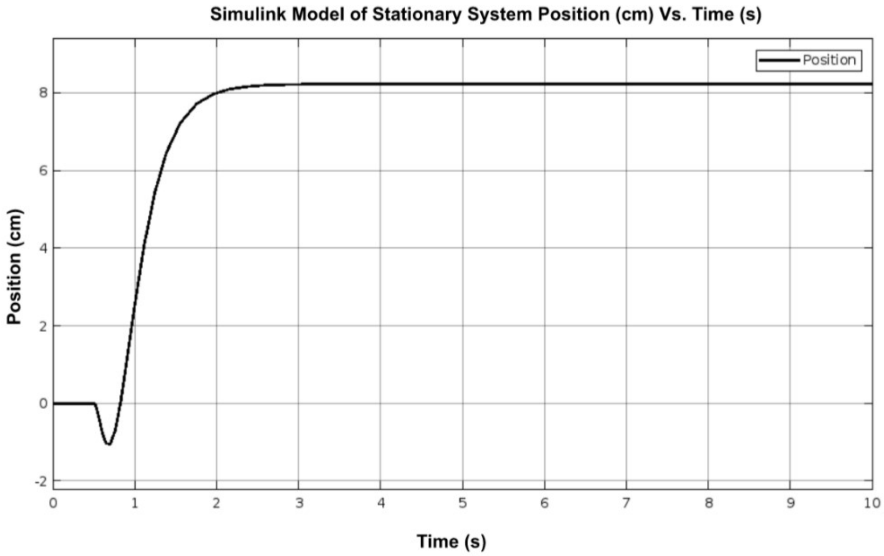



Figure 12: Simulink response plot of the position (cm) of the stationary balancing system.

The results of implementing the control constants from these poles on our physical rocky resulted in a stabilized system, outputting the following graphs to show the wheel speeds, angle, and position of the rocky over a 65 second period. This demonstration can also be seen in our video attached.

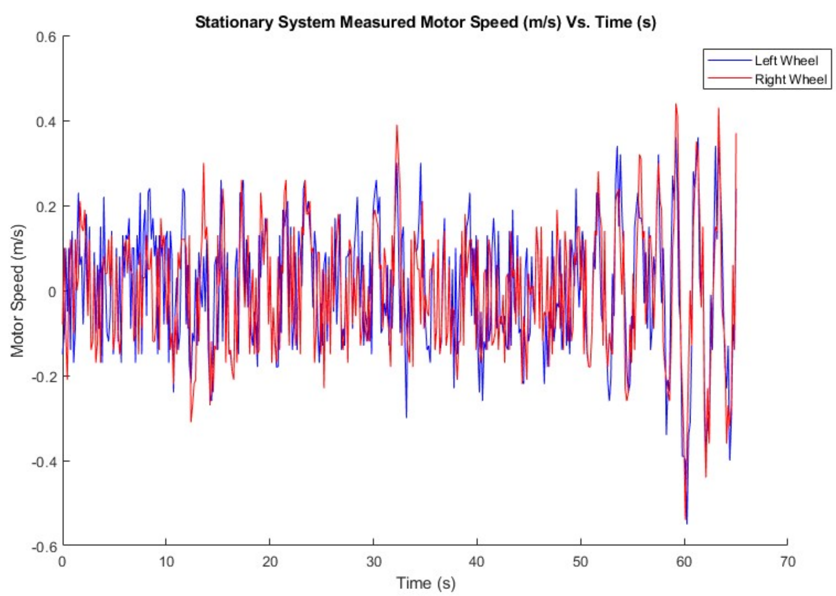



Figure 13: Left and right wheel velocities of Rocky over 65 seconds during the implementation of the stationary system's response-to-disturbance. We added a physical disturbance to Rocky by attempting to push it over at the 60 second mark. You can see in the graph that at this point, it increases the amplitude of the differences in motor speed, but remains stable. It appears to be different from the Simulink model of the velocity, because the physical Rocky has constant disturbances on it, such as friction, wind, etc. However, the simulink model includes just one instance of disturbance at the 0.5 second mark.

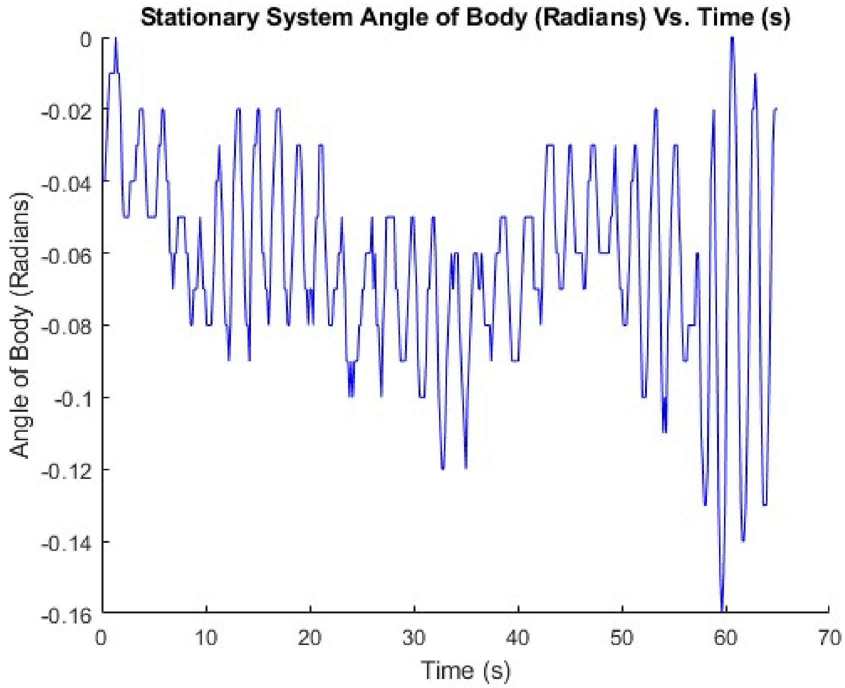



Figure 14: Angle in radians of Rocky in relation to balancing straight up (angle = 0 radians) over 65 seconds during the implementation of the stationary system's response-to-disturbance. The graph shows that instead of oscillating around 0 as expected, it oscillates around roughly -0.07 radians. This is likely due to errors during the calibration of Rocky, such as not holding it directly upright that caused this slight difference. The simulink angular position corresponds to that of the velocity as you can see the mass of change at the 60 second mark, which indicates our disturbance, increasing the angle and the rocky balancing to stabilize to it's original position.

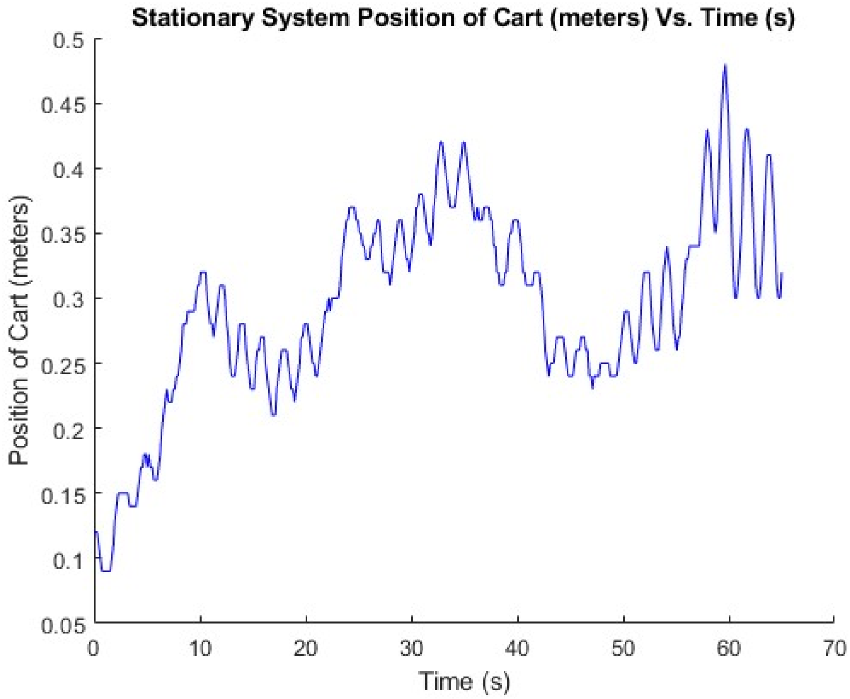



Figure 15: Position of Rocky in relation to the starting point (position = 0 meters) over 65 seconds during the implementation of the stationary system's response-to-disturbance. Since we used a 3-pole system, we only controlled the angular position and velocity of the Rocky, which is why it didn't end up at the same linear position it started at.

```
% Rocky_5_closed_loop_poles.m
%
% 1) Symbolically calculates closed loop transfer function of PI disturbannce
% rejection control system for Rocky.
% No motor model (M =1). With motor model (1st order TF)
%
% 2) Specify location of (target)poles based on desired reponse. The number of
% poles = denominator polynomial of closed loop TF
%
% 3) Extract the closed loop denomiator poly and set = polynomial of target
% poles
%
% 4) Solve for Ki, Kp, Ji, Jp, Ci to match coefficients of polynomials. In general,
% this will be underdefined and will not be able to place poles in exact
% locations. In this case (5th order), the control constants can be found exactly
%
% 5) Plot impulse response to see closed-loop behavior.

clear all;
close all;

syms s a b l g Kp Ki Jp Ji Ci   % define symbolic variables

Hvtheta = -s/l/(s^2-g/l);       % TF from velocity to angle of pendulum

K = Kp + Ki/s                  % TF of the PI angle controller
M = a*b/(s+a)                  % TF of motor (1st order model)
J = Jp + Ji/s + Ci/s^2;         % TF of controller around motor-combined PI of x and v
Mfb = M/(1+M*J);                % Black's formula to get tf for motor with PI feedback control

% closed loop transfer function from disturbance d(t)totheta(t)
% Hcloop = 1/(1-Hvtheta*M*K)    % use this for no motor feedback
% with motor feedback
Hcloop = 1/(1-Hvtheta*Mfb*K)    % use this for motor with feedback

pretty(simplify(Hcloop))       % to display the total transfer function

% Substitute parameters and solve
% system parameters
g = 9.81;
l = 0.362;  %effective length
a = 1/17.48; %nominal motor parameters (tau)
b = 0.0031;  %nominal motor parameters (k) %both left motor

Hcloop_sub = subs(Hcloop) % sub parameter values into Hcloop

% specify locations of the target poles,
% choose # based on order of Htot denominator
% e.g., want some oscillations, want fast decay, etc.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% POLES FOR INITIAL SYSTEM
p1 = -1 + 1*i   % dominant pole pair
p2 = -1 - 1*i    % dominant pole pair
```

```
p3 = -2
p4 = -5   % dominant pole pair
p5 = -5 % dominant pole pair
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% POLES FOR STATIONARY SYSTEM
w_n = 0.877*6.28                            %natural frequency
angle1 = 20;                                %target angle1
angle2 = 20;                                %target angle2
p1 =  w_n*(-cosd(angle1) - sind(angle1))  % dominant pole pair
p2 = w_n*(-cosd(angle1) + sind(angle1))    % dominant pole pair
p3 = -w_n
p4 = w_n*(-cosd(angle2) - sind(angle2))   % dominant pole pair
p5 = w_n*(-cosd(angle2) - sind(angle2)) % dominant pole pair
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% target characteristic polynomial
% if motor model (TF) is added, order of polynomial will increases
% tgt_char_poly = (s-p1)*(s-p2)*(s-p3)

% check polynomial-expand to fifth order
tgt_char_poly = (s-p1)*(s-p2)*(s-p3)*(s-p4)*(s-p5)
exp_tgt_char_poly = expand(tgt_char_poly)

% get the denominator from Hcloop_sub
[n d] = numden(Hcloop_sub)

% find the coefficients of the denominator polynomial TF
coeffs_denom = coeffs(d, s)

% divide though the coefficient of the highest power term
coeffs_denom = coeffs(d, s)/(coeffs_denom(end))
% num_coeff_denom = length(coeffs_denom)

% find coefficients of the target charecteristic polynomial
coeffs_tgt = coeffs(tgt_char_poly, s)
% num_coeff_tgt = length(coeffs_tgt)

% for check. reorder the coefficients to match the denomimator polynomial
for ii = 1:length(coeffs_denom)
    reord_coeffs_tgt(ii) = coeffs_tgt(length(coeffs_tgt) + 1 - ii);
end
% check roots of target polynomial-should be same as selected poles
roots_target = vpa(roots(reord_coeffs_tgt),4)


% solve the system of equations setting the coefficients of the
% polynomial in the target to the actual polynomials
solutions = solve(coeffs_denom(1:5) == coeffs_tgt(1:5), Jp, Ji,  Kp, Ki, Ci);

% display the solutions as double precision numbers
Kp = double(solutions.Kp)
Ki = double(solutions.Ki)
Ji = double(solutions.Ji)
Jp = double(solutions.Jp)
```

```
Ci = double(solutions.Ci)

%write out denominator polynomial
aaa = vpa(subs(coeffs_denom),4)

% reorder coefficients for the check polynomial
for ii = 1:length(coeffs_denom)
    chk_coeffs_denom(ii) = coeffs_denom(length(coeffs_denom) + 1 - ii);
end

% check poles should be same as chosen input poles
check_closed_loop_poles = vpa (roots(subs(chk_coeffs_denom)), 4)

% write out target polynomial
% bbb = vpa( expand( (s-check_closed_loop_poles(1))*(s-check_closed_loop_poles(2)) ...
%      *(s-check_closed_loop_poles(3))*(s-check_closed_loop_poles(4)) ...
%      *(s-check_closed_loop_poles(5)) ) )

% Plot impulse and step responses of closed-loop system
    TFstring = char(subs(Hcloop));
    % Define 's' as transfer function variable
    s = tf('s');
    % Evaluate the expression
    eval(['TFH = ',TFstring]);
    figure (1)
    impulse(TFH);   %plot the impulse reponse
    figure(2)
    step(TFH)       %plot the step response
```

## Initial System Reponse-to-Disturbance Data

```
clear length

initial = [
0.06    0.01    0.36    0.34    0.00;
-0.03    0.04    -0.04    0.31    0.00;
-0.03    0.05    0.26    0.05    0.00;
-0.05    0.06    0.23    0.21    0.00;
-0.04    0.07    0.10    0.09    0.00;
-0.02    0.06    -0.12    0.21    0.00;
-0.01    0.08    0.19    0.04    0.00;
-0.02    0.10    0.32    0.30    0.00;
-0.04    0.14    0.58    0.43    0.00;
-0.07    0.19    0.57    0.61    0.00;
-0.10    0.24    0.48    0.43    0.00;
-0.13    0.28    0.26    0.28    0.00;
-0.15    0.31    0.14    0.13    0.00;
-0.15    0.32    0.23    0.21    0.00;
-0.14    0.32    -0.28    -0.32    0.00;
-0.11    0.30    -0.09    -0.23    0.00;
-0.07    -0.03    -0.57    -0.61    0.00;
0.04    -0.09    -0.79    -0.76    0.00;
0.18    -0.16    -0.77    -0.73    0.00;
```

```
0.38    -0.22    -0.05    -0.22    0.00;
0.50    -0.19    0.63    0.31    0.00;
0.67    0.03    0.63    0.62    0.00;
0.94    0.01    0.66    0.64    0.00;
1.30    0.01    0.66    0.64    0.00;
1.24    0.01    0.66    0.64    0.00;
1.33    0.01    0.66    0.64    0.00;
1.37    0.01    0.66    0.64    0.00;
1.30    0.01    0.66    0.64    0.00;
];

t = linspace(0, 2, 28);
```

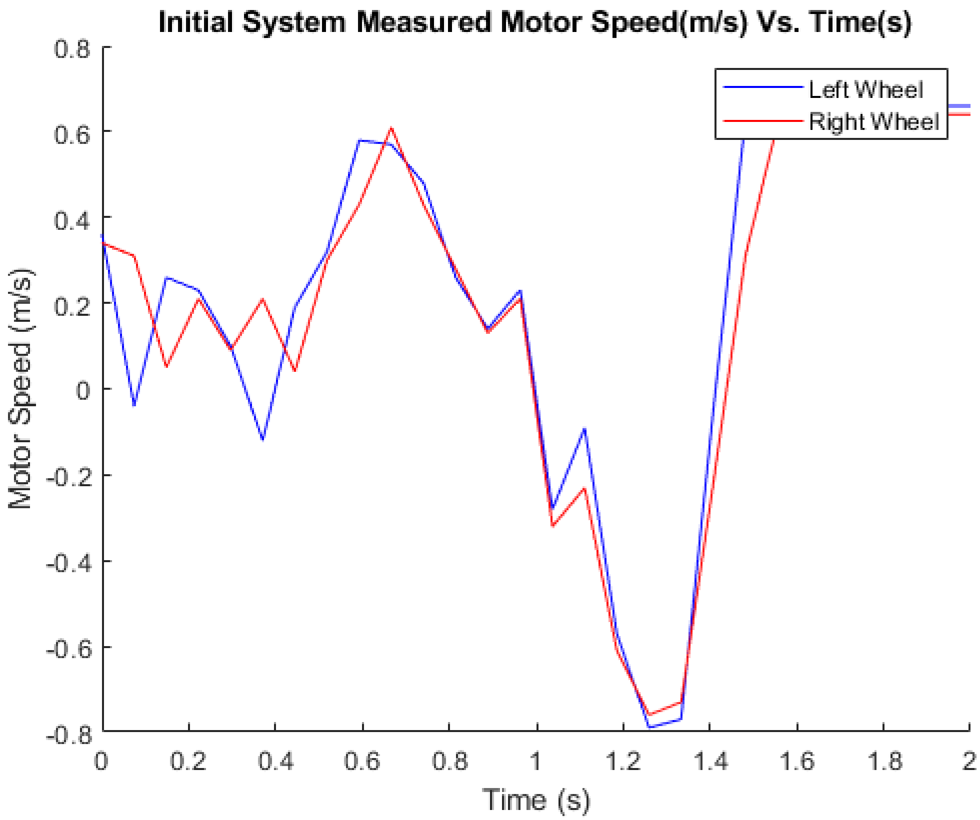
### Initial System: Wheel Velocities

```
% Initial System Wheel Velocities

figure
clf
hold on

left_v = initial(:,3);
right_v = initial(:,4);

plot(t, left_v, 'b')
plot(t, right_v, 'r')
legend('Left Wheel', 'Right Wheel')
ylabel("Motor Speed (m/s)")
xlabel("Time (s)")
title("Initial System Measured Motor Speed(m/s) Vs. Time(s)")
```



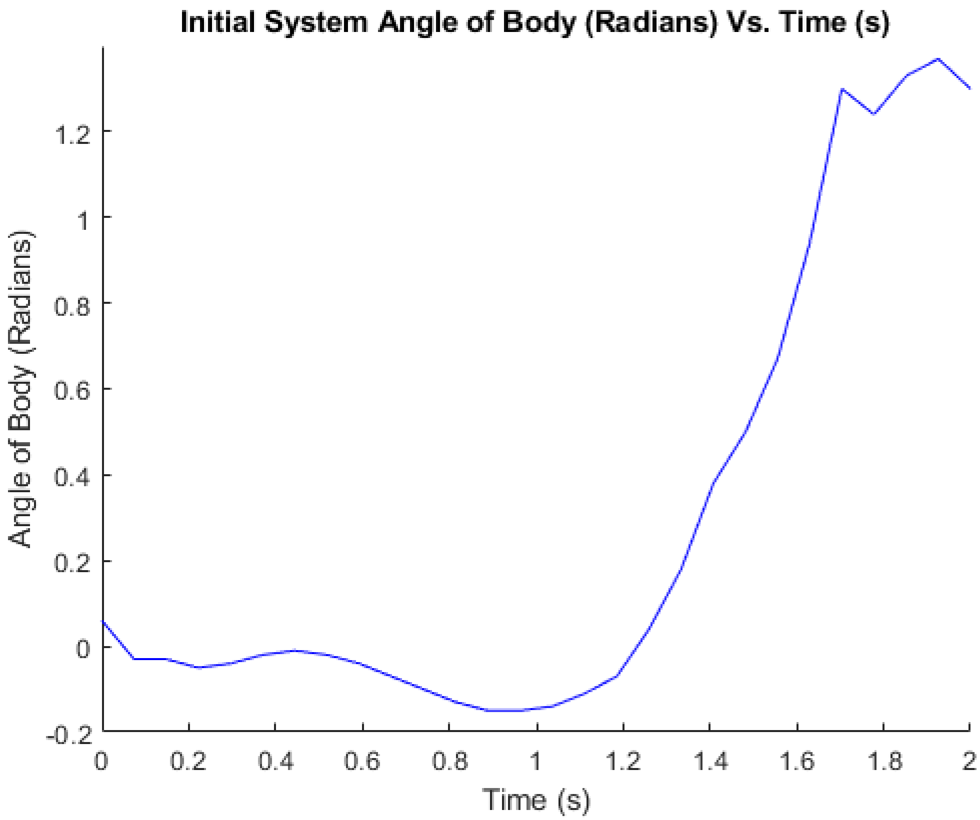

### Initial System: Position of Cart

```
% Initial System Angle of Body

figure
clf
hold on

angle = initial(:,1);

plot(t, angle, 'b')
ylabel("Angle of Body (Radians)")
xlabel("Time (s)")
title("Initial System Angle of Body (Radians) Vs. Time (s)")
```



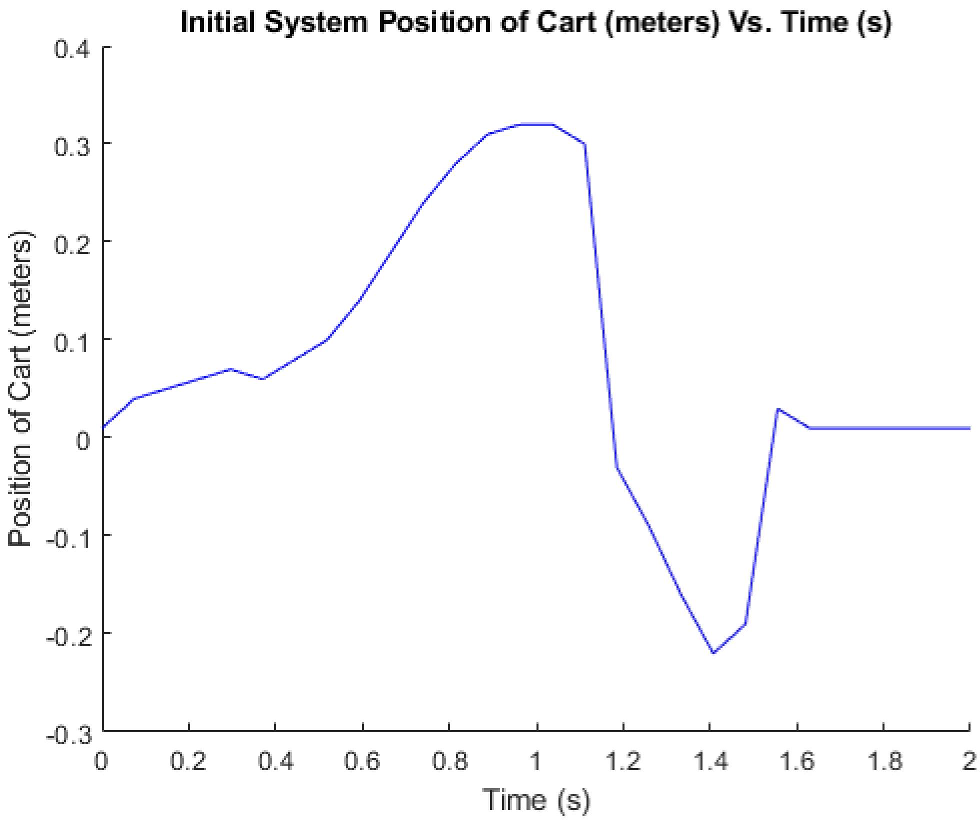

### Initial System: Position of Cart

```
% Initial System Position of Cart

figure
clf
hold on

position = initial(:,2);

plot(t, position, 'b')
ylabel("Position of Cart (meters)")
xlabel("Time (s)")
title("Initial System Position of Cart (meters) Vs. Time (s)")
```




## Stationary Response-to-Disturbance Data

```
clear length

stationary = [
-0.04    0.12    -0.15    -0.08    0.00;
-0.04    0.12    -0.10    0.10    0.00;
-0.04    0.12    0.10    -0.10    0.00;
-0.03    0.11    -0.05    -0.21    0.00;
-0.02    0.10    0.10    0.01    0.00;
-0.01    0.09    -0.14    0.12    0.00;
-0.01    0.09    0.14    0.05    0.00;
-0.01    0.09    -0.17    -0.10    0.00;
-0.01    0.09    -0.09    0.12    0.00;
-0.00    0.09    0.08    0.00    0.00;
-0.01    0.09    0.23    0.03    0.00;
-0.01    0.10    0.06    0.21    0.00;
-0.02    0.11    0.08    0.15    0.00;
-0.04    0.13    -0.08    0.14    0.00;
-0.05    0.14    0.08    0.19    0.00;
-0.05    0.15    0.18    0.04    0.00;
-0.05    0.15    -0.06    -0.06    0.00;
-0.05    0.15    0.15    0.12    0.00;
-0.04    0.15    -0.14    -0.14    0.00;
-0.04    0.15    -0.12    -0.12    0.00;
-0.04    0.15    0.09    0.06    0.00;
-0.04    0.15    -0.10    -0.17    0.00;
```

```
 -0.03    0.14    0.06    -0.12    0.00;
 -0.03    0.14    -0.09    0.05    0.00;
 -0.02    0.14    0.15    -0.17    0.00;
 -0.02    0.14    -0.17    0.05    0.00;
 -0.02    0.14    0.22    0.08    0.00;
 -0.03    0.15    0.03    0.04    0.00;
 -0.04    0.16    -0.10    0.04    0.00;
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-0.05    0.29    0.23    0.17    0.00;
-0.06    0.30    0.17    0.32    0.00;
-0.08    0.32    0.17    0.31    0.00;
-0.09    0.33    0.15    0.12    0.00;
-0.09    0.33    -0.03    0.14    0.00;
-0.09    0.34    0.09    -0.08    0.00;
-0.08    0.34    -0.09    -0.01    0.00;
-0.08    0.34    0.15    0.12    0.00;
-0.08    0.34    -0.14    0.13    0.00;
-0.08    0.34    -0.09    0.05    0.00;
-0.08    0.34    0.08    -0.09    0.00;
-0.06    0.34    0.03    0.03    0.00;
-0.06    0.36    0.19    0.21    0.00;
-0.08    0.38    0.32    0.30    0.00;
-0.11    0.40    0.10    0.21    0.00;
-0.12    0.42    -0.03    0.19    0.00;
-0.13    0.43    0.10    -0.12    0.00;
-0.13    0.42    -0.04    -0.18    0.00;
-0.12    0.41    -0.34    -0.21    0.00;
-0.09    0.38    -0.21    -0.24    0.00;
-0.04    0.36    -0.24    -0.26    0.00;
-0.03    0.35    0.09    -0.12    0.00;
-0.02    0.36    0.27    0.10    0.00;
-0.06    0.39    0.21    0.17    0.00;
-0.09    0.42    0.36    0.44    0.00;
-0.13    0.45    0.23    0.41    0.00;
-0.15    0.47    -0.03    0.15    0.00;
-0.16    0.48    -0.08    -0.12    0.00;
-0.15    0.46    -0.39    -0.23    0.00;
-0.13    0.44    -0.39    -0.34    0.00;
-0.08    0.40    -0.40    -0.54    0.00;
-0.05    0.36    -0.55    -0.37    0.00;
-0.02    0.32    -0.34    -0.18    0.00;
-0.00    0.30    -0.31    0.00    0.00;
0.00    0.30    -0.12    -0.13    0.00;
-0.01    0.31    0.28    0.15    0.00;
-0.03    0.33    0.24    0.24    0.00;
```
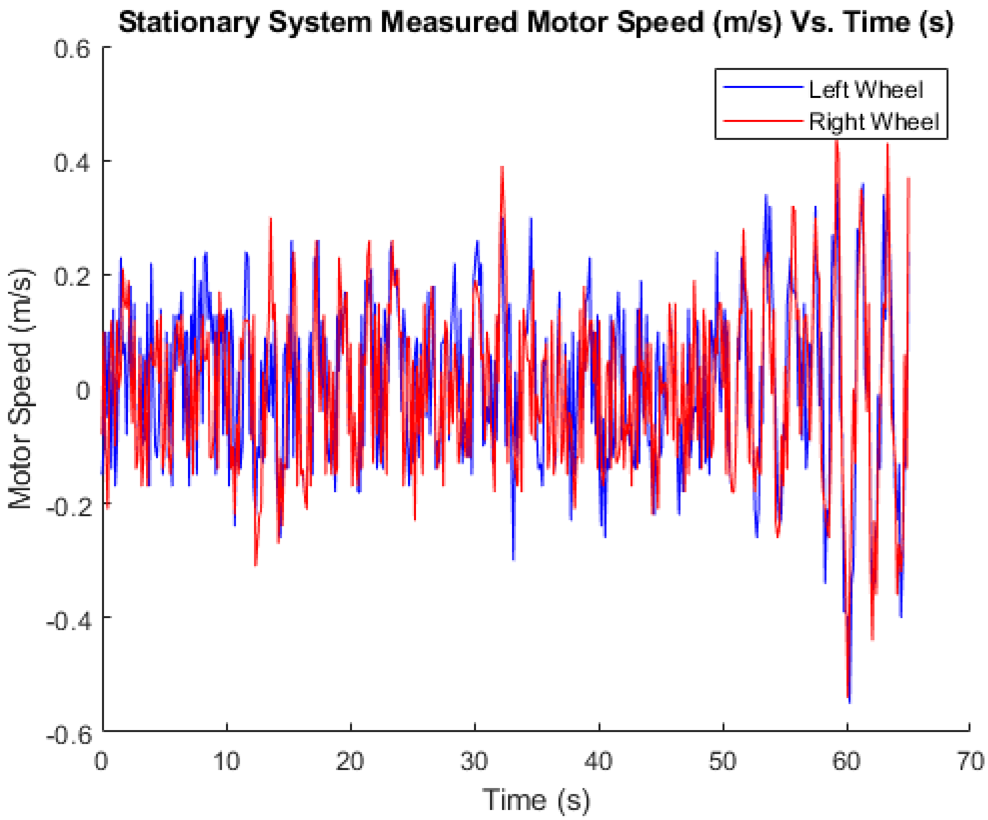

```
-0.06    0.35    0.30    0.35    0.00;
-0.10    0.39    0.36    0.32    0.00;
-0.13    0.42    0.03    0.18    0.00;
-0.14    0.43    -0.03    -0.04    0.00;
-0.14    0.43    0.15    0.15    0.00;
-0.13    0.42    -0.23    -0.30    0.00;
-0.11    0.40    -0.40    -0.44    0.00;
-0.08    0.37    -0.30    -0.23    0.00;
-0.05    0.34    -0.35    -0.36    0.00;
-0.02    0.31    -0.01    -0.14    0.00;
-0.02    0.30    -0.14    -0.04    0.00;
-0.01    0.30    0.12    0.04    0.00;
-0.02    0.31    0.34    0.15    0.00;
-0.04    0.33    0.12    0.14    0.00;
-0.08    0.36    0.35    0.43    0.00;
-0.11    0.39    0.26    0.26    0.00;
-0.13    0.41    0.14    -0.04    0.00;
-0.13    0.41    -0.04    -0.08    0.00;
-0.13    0.41    -0.18    -0.13    0.00;
-0.11    0.39    -0.23    -0.36    0.00;
-0.09    0.37    -0.13    -0.27    0.00;
-0.06    0.34    -0.40    -0.32    0.00;
-0.03    0.31    -0.30    -0.27    0.00;
-0.02    0.30    -0.08    0.06    0.00;
-0.02    0.30    -0.14    -0.14    0.00;
-0.02    0.32    0.24    0.37    0.00;
];

t = linspace(0, 65, 426);
```

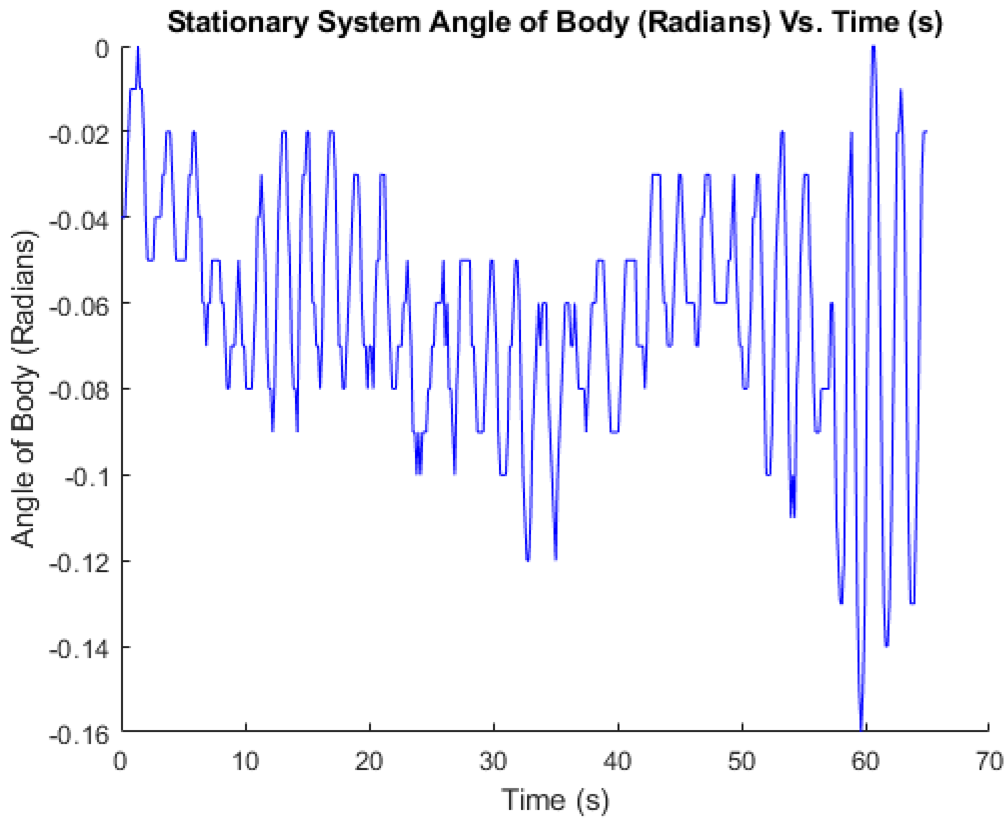
### Stationary System: Wheel Velocities

```
% Stationary System Wheel Velocities
figure
clf
hold on

left_v = stationary(:,3);
right_v = stationary(:,4);

plot(t, left_v, 'b')
plot(t, right_v, 'r')
legend('Left Wheel', 'Right Wheel')
ylabel("Motor Speed (m/s)")
xlabel("Time (s)")
title("Stationary System Measured Motor Speed (m/s) Vs. Time (s)")
```



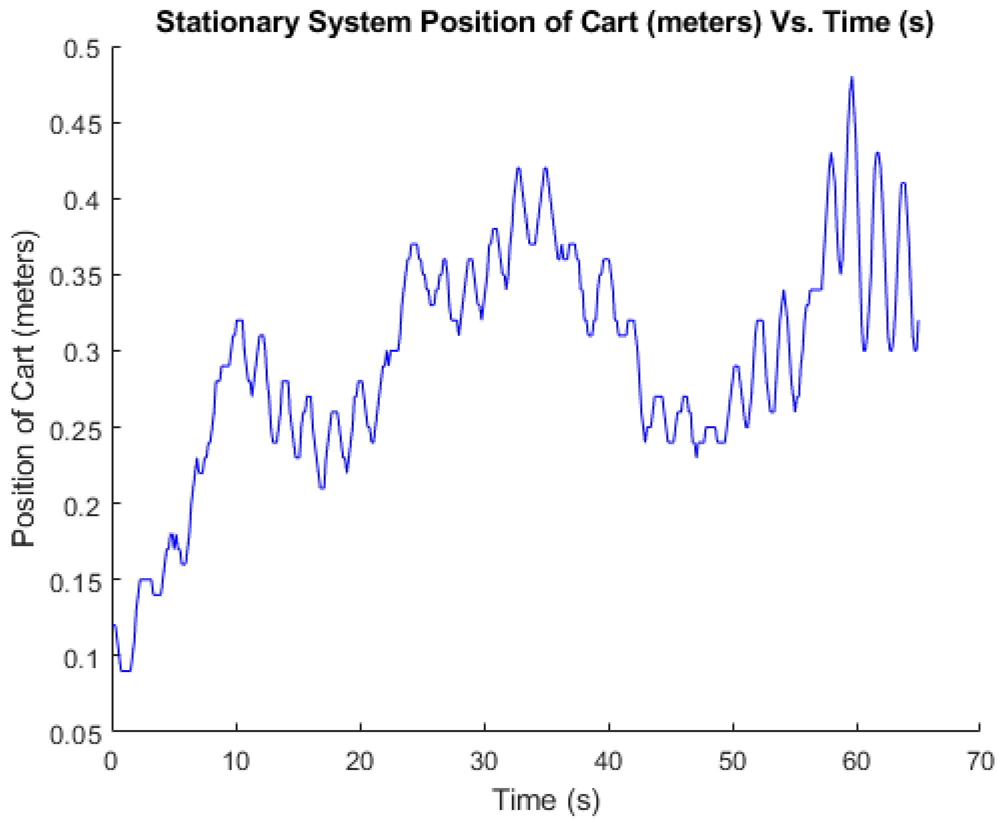

### Stationary System: Angle of Body

```
% Angle of Body

figure
clf
hold on

angle = stationary(:,1);

plot(t, angle, 'b')
ylabel("Angle of Body (Radians)")
xlabel("Time (s)")
title("Stationary System Angle of Body (Radians) Vs. Time (s)")
```




### Stationary System: Position of Cart

```
% Position of Cart

figure
clf
hold on

position = stationary(:,2);

plot(t, position, 'b')
ylabel("Position of Cart (meters)")
xlabel("Time (s)")
title("Stationary System Position of Cart (meters) Vs. Time")
```


Stationary System Position of Cart (meters) Vs. Time (s)
Position of Cart (meters)
Time (s)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
50
60
70