Dynamics

Kinematics - study of motions of objects independent of cause of motion.
Study is based on the relationship between displacement, velocity, acceleration and time.

Kinetics - relates to action of forces and the resulting motion.

Trajectory - Path followed by body during its motion.
Straight line path is observed in rectilinear motion, whereas curved path is observed in curvilinear motion.

(x) Displacement (m) The change of position of a particle or a body with respect to a certain fixed reference point is termed as displacement.

Distance (m) Total path covered.

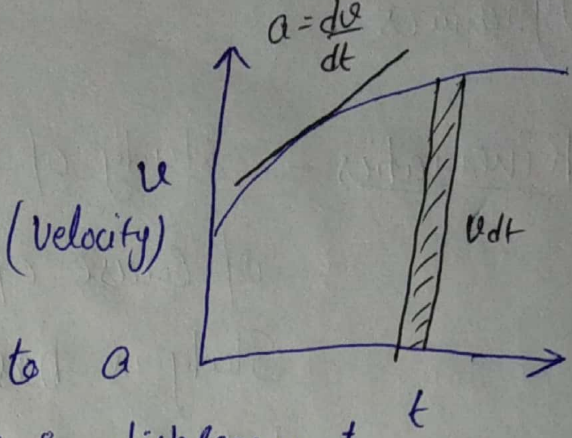
(v) Velocity (m/s) = $\frac{dx}{dt}$

(a) Acceleration (m/s^2) = $\frac{dv}{dt}$

#

$$\int dx = \int v\,dt$$

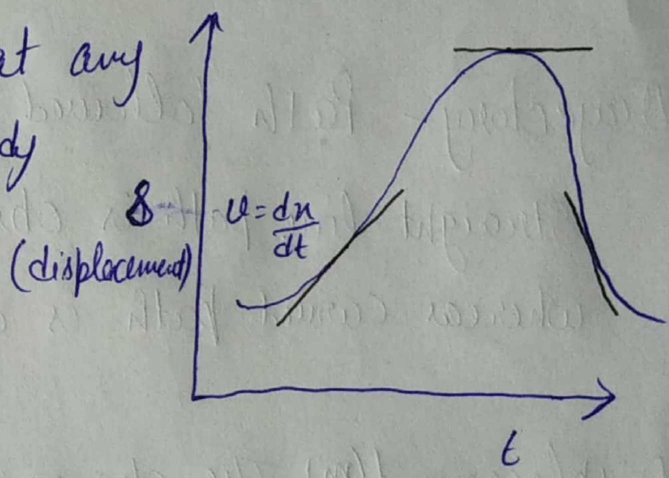
$$(x_2 - x_1) = \int v\,dt$$



⇒ Area under v-t curve corresponding to a given time interval gives the change in displacement

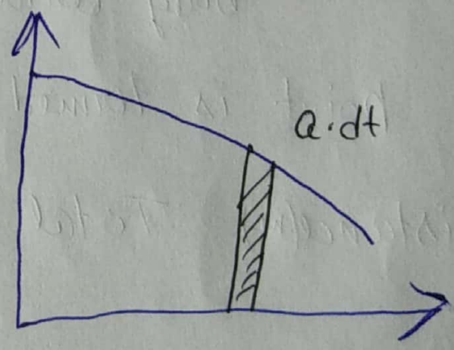
⇒ The slope of v-t curve at any instant gives the acceleration of the body at that instant.

⇒ The slope of x-t curve at any instant gives the velocity of body at that instant.



⇒ $\int dv = \int a\,dt$

Area under a-t curve corresponding to a given time interval gives the change in velocity of body.



Equations of rectilinear motion

$$v = u \pm at$$

$$s = ut \pm \frac{1}{2} at^2$$

$$v^2 = u^2 \pm 2as$$

v = final velocity , u = initial velocity

s = displacement , t = time interval

a = acceleration ,

Distance travelled in nth second

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Q On turning a corner, a motorist driving at 54 Km/hr finds a child on the road 50 m ahead. He instantaneously stops the engine, applies the brakes and vehicle is brought to halt within 5 m of the child. Make calculations for the uniform rate of deceleration and the time for which the brakes were applied.

Sol. $u = 54$ Km/hr $= 15$ m/s

$s = 50 - 5 = 45$ m

$v = 0$

$v^2 - u^2 = 2as$

$\boxed{a = -2.5 \ m/s^2}$ (Retardation)

$v = u + at$

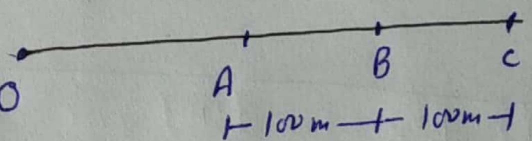
$\boxed{t = 6s}$

Q Three marks A, B and C spaced at a distance 100 m are made along a straight road. A car starting from rest and accelerating uniformly passes the mark A and takes 10 seconds to reach the mark B and further 8 sec to reach mark C. Make calculations for

a) the magnitude of the acc. of the car

b) the velocity of the car at A.

c) the velocity of the car at B and



d) the distance of the mark A from the starting point.

Sol: Motion from A to B

$s = u_a t + \frac{1}{2} a t^2$

$100 = u_a \cdot 10 + \frac{1}{2} \cdot a (10)^2$

$100 = 10 u_a + 50 a \quad -(i)$

Considering motion from A to C

$200 = u_a \cdot 18 + \frac{1}{2} a (18)^2$ — (ii)

from (i) & (ii)

$\boxed{u_a = 8.61 \text{ m/s}}$, $\boxed{a = 0.278 \text{ m/s}^2}$

Considering motion from B to C

$s = ut + \frac{1}{2} at^2$

$100 = u_b \cdot 8 + \frac{1}{2}(0.278)(8)^2$

$\boxed{u_b = 11.388 \text{ m/s}}$

Considering motion from O to A

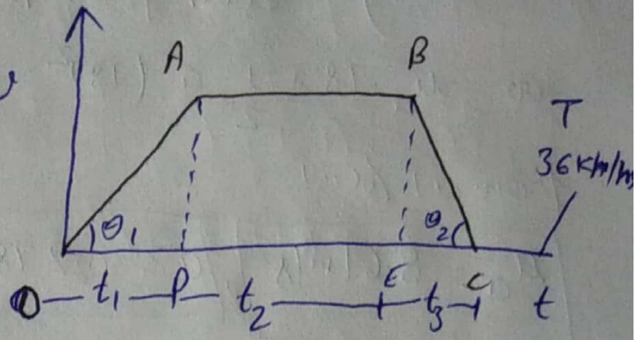
$v^2 - u^2 = 2as$

$(8.61)^2 = 2(0.278) \cdot s$

$\boxed{s = 133.3 \text{ m}}$

Q. An electric train runs between two stations which are 2.5m apart and takes 6 minute from start to stop. The train has a constant running speed of 36 Km/hr before the the end of acc. and beginning of retardation. If the acc. and retar. are uniform and numerically equal to each other, make calculations for the values

Sol.



Since acc. and retar. are equal

$\theta_1 = \theta_2$

as $\Delta OAD \cong \Delta BCE$

$OD = EC$

$t_1 = t_3$

$t_1 + t_2 + t_3 = 0.1$

$2t_1 + t_3 = 0.1$ (i)

Area of v-t graph give displacement

$s = s_1 + s_2 + s_3$

$= \frac{1}{2} \cdot 36 \cdot t_1 + t_2 \cdot 36 + \frac{1}{2} \cdot 36 \cdot t_1$

$t_1 + t_2 = 0.0694$ — (ii)

from (i) & (ii)

$t_1 = 0.0306$ hr

$a = \frac{V_a - V_o}{t} = \frac{36}{0.0306} = 1176.47 \text{ km/hr}^2$

(4)

Q A particle moves along a straight line and its motion is represented by equation

$$s = 16t + 4t^2 - 3t^3 \text{ ; where } s \text{ is in meter \& } t \text{ is in seconds}$$

Determine

a) displacement, velocity and acceleration 2 sec. after start

b) s and a when velocity is zero.

c) s and v when a is zero.

Sol:

$$s = 16t + 4t^2 - 3t^3$$

$$v = \frac{ds}{dt} = 16 + 8t - 6t^2$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 8 - 18t$$

a) when $t = 2$

$s = 24\ m$

$v = -4\ m/s$

$a = -28\ m/s^2$

b) when $v = 0$

$16 + 8t - 6t^2 = 0$; $t = 1.85\,s$

$s_{(1.85)} = 24.3\ m$

$a_{(1.85)} = -25.3\ m/s^2$

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c) when $a = 0$

$8 - 18t = 0$

$t = 0.444 s$

$s_{(0.444)} = 7.63 m$

$v_{(0.444)} = 17.775 m/s$

Q A particle move along a straight line with an acc. $(a = 4t^2 - 3t + 2)$ where a is in m/s^2, t is in sec.

The particle has a velocity of 10 m/s at $t = 3$ seconds and it is located 12 m to the right of origin at $t = 2$ secon. Determine the position and velocity of the particle after 5 seconds

Sol. $a = \frac{dv}{dt}$

$dv = a\,dt$

$v = \int (4t^2 - 3t + 2)dt$

$v = \frac{4t^3}{3} - \frac{3t^2}{2} + 2t + C_1$

By using giving condition $v = 10$ m/s at $t = 3$ sec

$C_1 = -18.5$

at $t = 5$

$\boxed{v = 120.67 \text{ m/s}}$

b) $dx = v \cdot dt$

$$x = \frac{t^4}{3} - \frac{t^3}{2} + t^2 - 18.5t + C_2$$

But ~~at~~ $x = 12\,m$ at $t = 2$ sec

$$C_2 = 43.67$$

at $t = 5$

$$\boxed{x = 122\,m}$$

Motion under gravity

$$v = u \pm gt$$

$$h = ut \pm \frac{1}{2}gt^2$$

$$v^2 = u^2 \pm 2gh$$

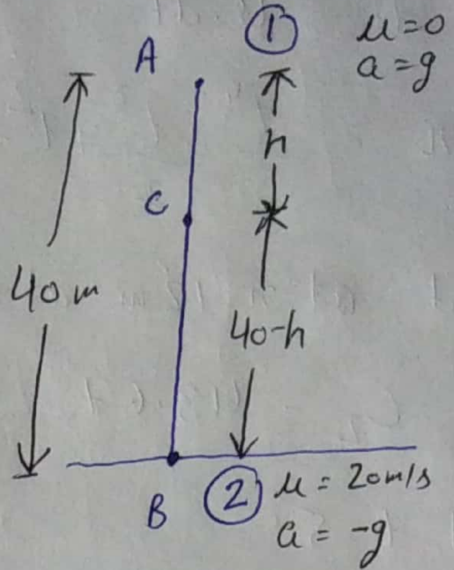
Distance travel in nth second

$$S_n = u \pm \frac{g}{2}(2n-1)$$

Q A stone is dropped from the top of a tower 40 m heigh. At the same instant, another stone is thrown upward from foot of tower with an initial velocity of 20 m/s. At what distance from the top and after how much time the two stones cross each other?

Further proceed to calculate the relative velocity with which the stones cross.

Sol -



For downward motion

$h = 0 \cdot t + \frac{1}{2} \cdot gt^2$

$h = 4.905t^2$ — (i)

for upward motion

$40 - h = 20t - \frac{1}{2} \cdot gt^2$

$40 - h = 20t - 4.905t^2$ — (ii)

from (i) & (ii)

$$\boxed{\begin{array}{l} t = 2 \text{ sec} \\ h = 19.62 \text{ m} \end{array}}$$

At Crossing

$v_1 = 0 + 9.81 \times 2 = 19.62 \text{ m/s} \downarrow$

$v_2 = 20 - 9.81 \cdot 2 = 0.38 \text{ m/s} \uparrow$

$\boxed{\text{Relative velocity} = 20 \text{ m/s}}$

Q A stone is thrown vertically upwards with a velocity 20 m/s from the top of a tower 25 m high. Make Calculations for the following parameter

i) The max. height to which the stone will rise.

ii) Velocity of stone during its downward travel at a point in the same level as the point of projection.

iii) time required for the stone to reach the ground.

Take $g = 10 \text{ m/s}^2$

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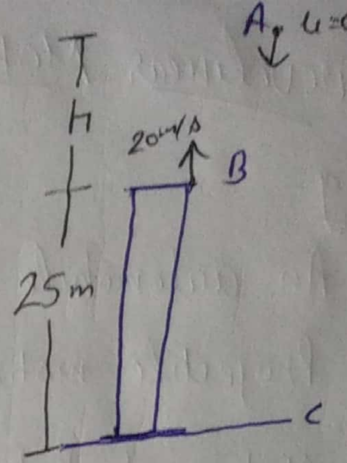


i) $v^2 - u^2 = 2gh$

$0 - (20)^2 = -2 \cdot 10 \cdot h$

$\boxed{h = \frac{400}{20} = 20\,m}$

From ground $25 + 20 = \boxed{45\,m}$

ii) $v = u + gt$

$= 0 + 10 \cdot 2$

$\boxed{v = 20\,m/s}$

iii) Time of flight

$s = ut + \frac{1}{2} g t^2$

$-25 = 20t - \frac{1}{2} \cdot 10 t^2$

$\boxed{t = 5\ sec}$

Curvilinear Motion:

e.g

i) An automobile vehicle negotiating a turn on the road.

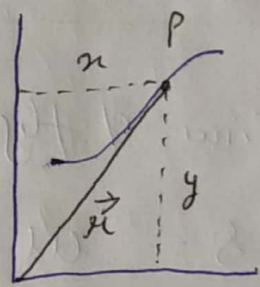
ii) Projectile motion of a bullet

iii) Motion of bob of pendulum

Rectangular cartesian x, y coordinates

$\vec{r} = x i + y j$; where i and j are unit vectors

$$\boxed{\text{Magnitude } (r) = \sqrt{x^2 + y^2}}$$



$$\vec{V} = \frac{dx}{dt} i + \frac{dy}{dt} j$$

$$\vec{V} = V_x i + V_y j$$

$$\boxed{\text{Resultant velocity} = \sqrt{V_x^2 + V_y^2} \text{ ; } \tan\alpha = \frac{V_y}{V_x}}$$

Acceleration (a)

$$\vec{a} = \frac{dV_x}{dt} i + \frac{dV_y}{dt} j$$

$$\vec{a} = a_x i + a_y j$$

Resultant acceleration $\boxed{a = \sqrt{a_x^2 + a_y^2}}$

$$\boxed{\tan\beta = \frac{a_y}{a_x}}$$

Q The motion of a particle is defined by the relations $x = t^2 + 3t$ and $y = t^3 - 8t^2 + 3$, where x, y is in meters. t is in second.

a) write the equations defining the motion of particle in vectorial form

b) Calculate the velocity and acceleration of particle at $t = 2$ sec.

Sol:

$x = t^2 + 3t$

$v_x = \frac{dx}{dt} = 2x + 3$

$a_x = \frac{dv_x}{dt} = 2$

$y = t^3 - 8t^2 + 3$

$V_y = \frac{dy}{dt} = 3t^2 - 16t$

$a_y = \frac{dV_y}{dt} = 6t - 16$

a)

Position vector $\vec{r} = x\hat{i} + y\hat{j} = (t^2 + 3t)i + (t^3 - 8t^2 + 3)j$

Velocity vector $\vec{v} = V_x\hat{i} + V_y\hat{j} = (2t + 3)i + (3t^2 - 16t)j$

Acc. $\vec{a} = a_x\hat{i} + a_y\hat{j} = 2i + (6t - 16)j$

b) at $t = 2$ seconds

$V_x = 7$ m/s

$V_y = -20$ m/s

$V = 21.19$ m/s

$\alpha = 70.71^\circ$

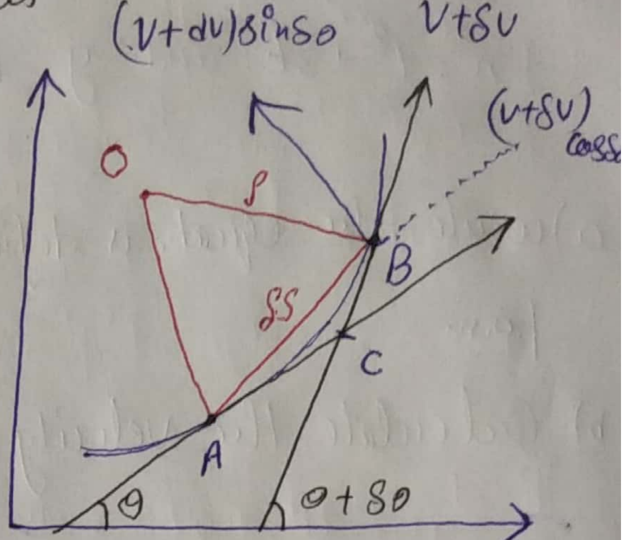
$a_x = 2$ m/s^2

$a_y = -4$ m/s^2

$\beta = 63.43^\circ$

Tangential and normal co-ordinates

Consider a particle that moves along a curved path from point A to point B and transverse an infinitely small distance δs in a small interval of time δt.



Chord AB = δS = arc AB

as $\angle BAC = \delta\theta$

The velocity of the particle at point A in the tangential direction is

$$V_t = \frac{\text{Distance moved along tangential direction}}{\text{time interval}} = \underset{\delta t \to 0}{\delta t} \frac{\delta S \cos\delta\theta}{\delta t}$$

as $\delta\theta$ is small, $\cos\delta\theta \to 1$

$$\boxed{V_t = \frac{dS}{dt}}$$

The velocity of the particle in normal direction

$$V_n = \frac{\text{Distance moved along normal direction}}{\text{time interval}} = \underset{\delta t \to 0}{\delta t} \frac{\delta S \sin\delta\theta}{\delta t}$$

as $\delta\theta$ is small; $\sin\delta\theta \to \delta\theta$

$$\boxed{V_n = 0}$$

Tangential acc. (a_t) = $\dfrac{\text{Change in tangential velocity}}{\text{time interval}}$

$$= \lim_{\delta t \to 0} \frac{(V+\delta V)\cos\delta\theta - V}{\delta t}$$

$$\boxed{a_t = \frac{dV}{dt}}$$

Normal acceleration $(a_n) = \lim_{\delta t \to 0} \dfrac{(V+\delta V)\sin\delta\theta - 0}{\delta t}$

$$a_n = \lim_{\delta t \to 0} V \frac{\delta\theta}{\delta t} = V\frac{d\theta}{dt} = V\frac{d\theta}{ds} \times \frac{ds}{dt} = \frac{V \cdot V}{\frac{ds}{d\theta}} = \frac{V^2}{\rho}$$

$$\boxed{a_n = \frac{V^2}{\rho}}$$

[for circular orbit of radius r)

$$\boxed{a_n = \frac{V^2}{\mu}}$$

$$\boxed{\text{Resultant acc.} = \sqrt{a_n^2 + a_t^2}}$$

Q A motorist is driving at 80 km/hr on the curved position of a highway of 400 m radius. He suddenly applies brakes and that causes the speed to decrease to 45 km/hr at a constant rate in 8 sec. Determine the tangential and normal components of acc. immediately after the application of brakes and 4 seconds later.

Sol: $u = 22.22$ m/s

$v = 12.5$ m/s

Tangential acc. $(a_t) = \frac{12.5 - 22.22}{8} = -1.215$ m/s^2

$a_n = \frac{v^2}{r} = \frac{22.22^2}{r} = 1.234$ m/s^2 [at instant of brakes]

Resultant(a) = 1.732 m/s^2

- after 4 second

Since the tangential component of acc. is uniform,

$V = u + at$

$V = 22.22 - (1.215 \times 4) = 17.36$ m/s

$a_n = \frac{(17.36)^2}{400} = 0.753$ m/s^2 ; $a_t = -1.215$ m/s^2

Resultant $a = 1.429$ m/s^2

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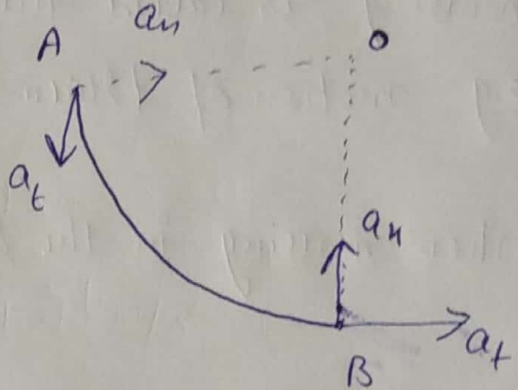
Q An automobile enters a curved road at 30 Km/hr and then leaves at 48 Km/hr. The curved road is in the form of quarter of a circle and has a length of 400m. If the car travels at constant acc. along the curve, Calculate the resultant acc. at both ends of curve.

Sol.



$V_a = 30\,Km/hr = 8.33\,m/s$

$V_b = 48\,Km/hr = 13.33\,m/s$

$\because \; v^2 - u^2 = 2as$

$a_t = 0.135\,m/s^2$

$\frac{2\pi r}{4} = 400$

$r = 254.77\,m$

At A $\quad a_n = \frac{V_a^2}{r} = \frac{8.33^2}{254.77} = 0.272\,m/s^2 \;;\; a_t = 0.135\,m/s^2$

$\boxed{\text{Resultant } (a) = 0.304\,m/s^2 \;;\; \alpha = 63.60'}$

At B

$a_n = \frac{V_b^2}{r} = \frac{13.33^2}{254.77} = 0.697\,m/s^2 \;;\; a_t = 0.135\,m/s^2$

$\boxed{\text{Resultant } a = 0.71\,m/s^2 \;;\; \beta = 79.04'}$

$\tan\beta = \frac{a_n}{a_t}$

Relative Velocity

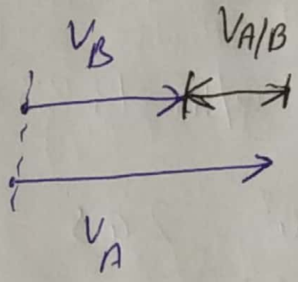
The motion of a particle with respect to a fixed frame of reference is called absolute motion of the particle.

e.g motion of train w.r.t. platform.

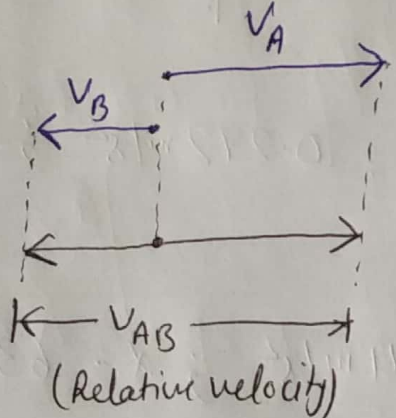
The motion relative to a set of axes which are moving, is called the relative motion.

e.g. motion of train A w.r.t. to another moving train B.

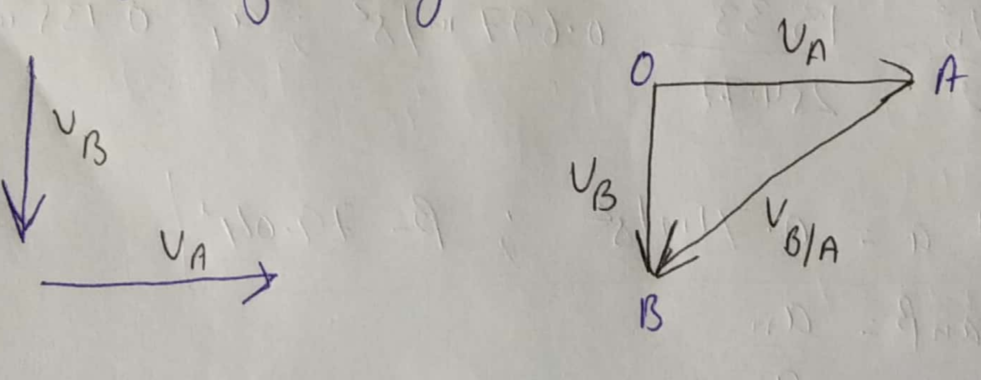
when moving in the same direction



when moving in opposite direction



Velocities at right angles

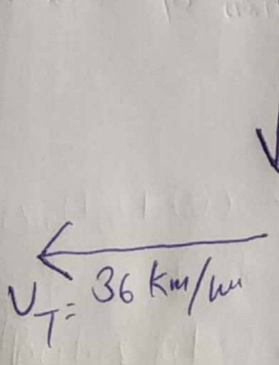


To find relative velocity, place the two velocity vectors with their tails joining at a common point and representing the two sides of a triangle then, the closing side of the triangle represents the relative velocity.

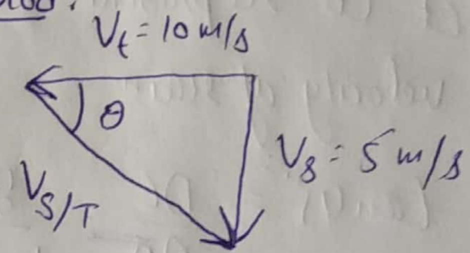
Determination of the relative velocity of A w.r.t B is essentially the determination of the vector difference of their velocities V_A and V_B.

Q A train moving at 36 km/hr is hit by a stone thrown at right angles to it with a velocity of 18 km/hour. Find the velocity and direction with which the stone appears to hit a person travelling in the train.

Sol



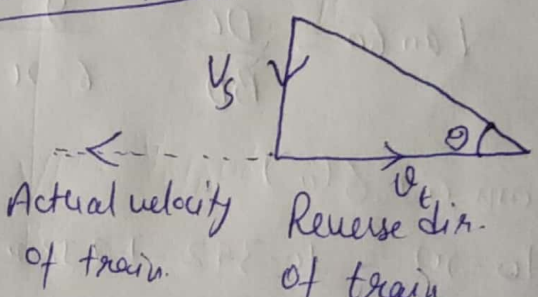
I method:



$V_{St} = 11.2 \, m/s$

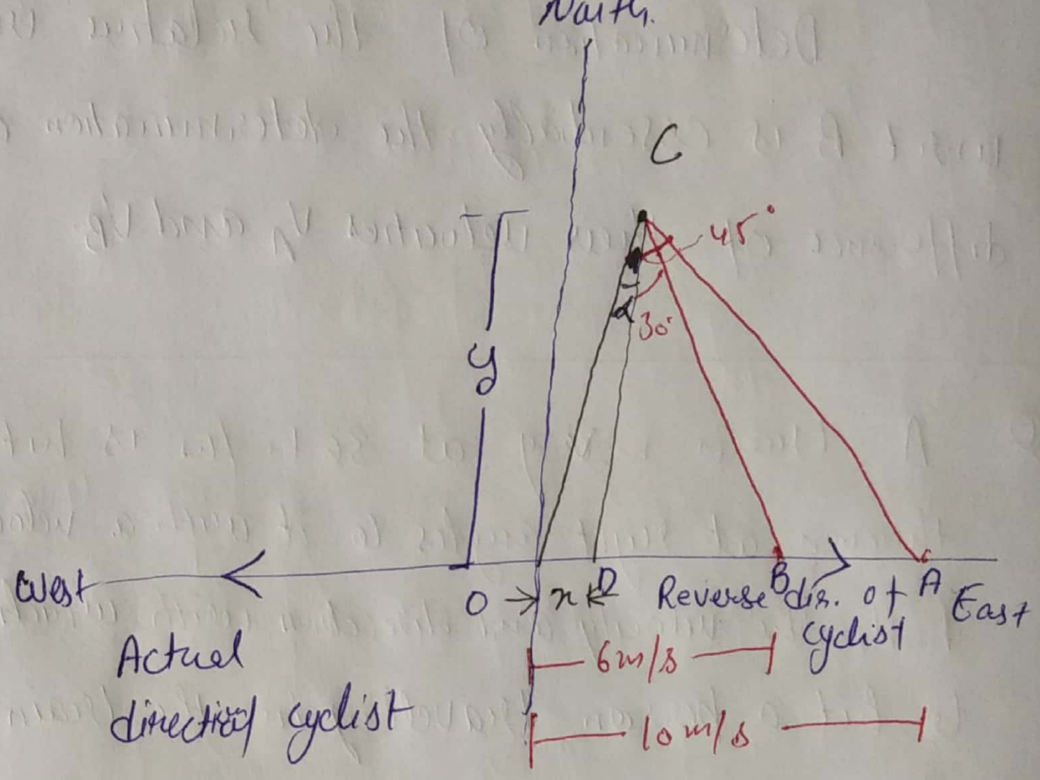
$\theta = 26.5°$ Ans

II method:



Q A motor cyclist traveling west with speed of 10 m/s feels the rain coming down at 45° to the vertical. On reducing speed to 6 m/s, he feels the rain to be at 30° with the vertical. Make calculations for the magnitude and direction of the actual velocity of rain.

Sol:



Let CO represent the magnitude & direction of actual velocity of rain

$\tan 45° = \frac{DC}{AD} = \frac{y}{10-x}$ — (i)

from Δ BDC

$\tan 60° = \frac{DC}{BD} = \frac{y}{6-x}$ — (ii)

from (i) & (ii)

$(10-x) = 10.392 - 1.732x$

$x = 0.535$ m/s

$y = 9.465$ m/s

$OC = \sqrt{OD^2 + CD^2}$

$= 9.48$ m/s

$\alpha = 3.23°$

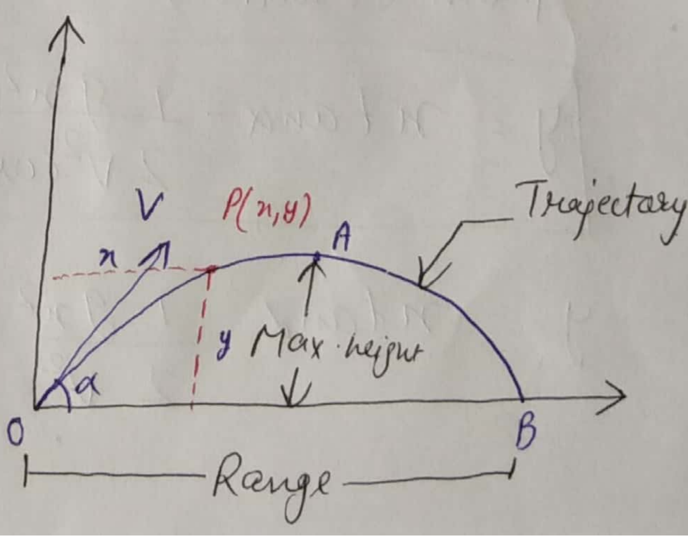
Projectiles:

A projectile is any object thrown into space. A projectile travels both in the horizontal and vertical directions and traces a curvilinear path.

e.g.
i) a bullet fired from a gun.
ii) throwing a ball

Terms related to projectile motion

1) Velocity of projection(α)
2) Range(R) - horizontal distance between the point of projection and the point where the projectile hits the ground.
3) Time of flight - Time interval during which the projectile remains in motion.
4) Max. height - Max. vertical distance of projectile from the point of projection.



Equation of projectile path - Consider the motion of an object which is projected from point O with velocity V inclined at an angle α with the horizontal.

Consider a point P on Trajectory

Velocity consists of two components; velocity u in x direction and velocity v in y direction

$$u = V\cos\alpha$$
$$v = V\sin\alpha$$

The horizontal component does not experience any acc. So it remains constant during the entire flight.

$\therefore$ Horizontal and vertical distances travelled by the object are

$$x = V\cos\alpha \cdot t \quad - (i)$$

$$y = V\sin\alpha t - \frac{1}{2} g t^2 \quad -(ii)$$

$$t = \frac{x}{V\cos\alpha} \quad \{\text{from (i)}\}$$

from (i) and (ii)

$\therefore$

$$\boxed{y = x\tan\alpha - \frac{1}{2}\frac{g x^2}{V^2\cos^2\alpha}}$$

$$y = x\tan\alpha - \frac{1}{2}\frac{g x^2}{V^2}(1+\tan^2\alpha)$$

Max. height attained -

By using relation

$v^2 - u^2 = 2as$

$u = V\sin\alpha$

$v = 0$

$-(V\sin\alpha)^2 = -2gH$

$$\boxed{H = \frac{V^2}{2g}\sin^2\alpha}$$

Time of flight:

$y = V\sin\alpha . t - \frac{1}{2}gt^2$

when the object reaches B, $y = 0$, $t = T$

$0 = V\sin\alpha\, T - \frac{1}{2}gT^2$

$$\boxed{T = \frac{2V}{g}\sin\alpha}$$

Horizontal Range :-

R = horizontal range x Time of flight

$= V\cos\alpha . \frac{2V}{g}\sin\alpha$

$$\boxed{R = \frac{V^2}{g}\sin 2\alpha}$$

Q Two adjacent guns having the muzzle velocity of 400 m/s fire simultaneously at angle α_1 and α_2 for the same target at a range 4800 m. Calculate the time difference between the hits. [$g = 9.80 \, m/s^2$]

Sol: $R = \frac{V^2 \sin 2\alpha}{g}$

$$\frac{V^2}{g} \sin 2\alpha_1 = \frac{V^2}{g} \sin 2\alpha_2 = 4800 \Rightarrow \boxed{\alpha_1 = 8.55^\circ}$$

$$\sin 2\alpha_1 = \sin 2\alpha_2 = \sin(\pi - 2\alpha_2)$$

$$\left[\alpha_1 = \frac{\pi}{2} - \alpha_2\right]$$

$$\boxed{\alpha_2 = 81.45^\circ}$$

Diff. between hits $= t_2 - t_1$

$$= \frac{2V}{g} (\sin\alpha_2 - \sin\alpha_1)$$

$$\boxed{\Delta t = 68.53 \, s}$$

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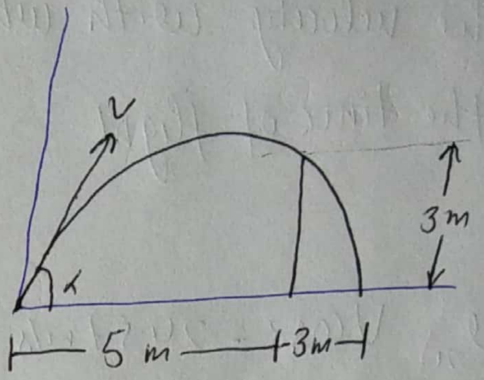
Q There is a 3m high wall 3m in front of enemy target. If the gunman is 5m away from the wall on the opposite side of the target and on the same horizontal plane as the target, find the angle of projection and the least velocity of projection so that the shell strikes the target.

Sol.

Range = 8 m



$$y = x\tan\alpha - \frac{g x^2}{2v^2\cos^2\alpha}$$

$$3 = 5\tan\alpha - \frac{g(5)^2}{2v^2\cos^2\alpha} \quad -(i)$$

$$R = \frac{v^2}{g}\sin 2\alpha$$

$$8 = \frac{v^2}{g}\sin 2\alpha \Rightarrow v^2 = \frac{8g}{\sin 2\alpha} \quad -(ii)$$

from (i) and (ii)

$$\boxed{\alpha = 58^\circ}$$

$$\boxed{V = 9.348\ m/s}$$

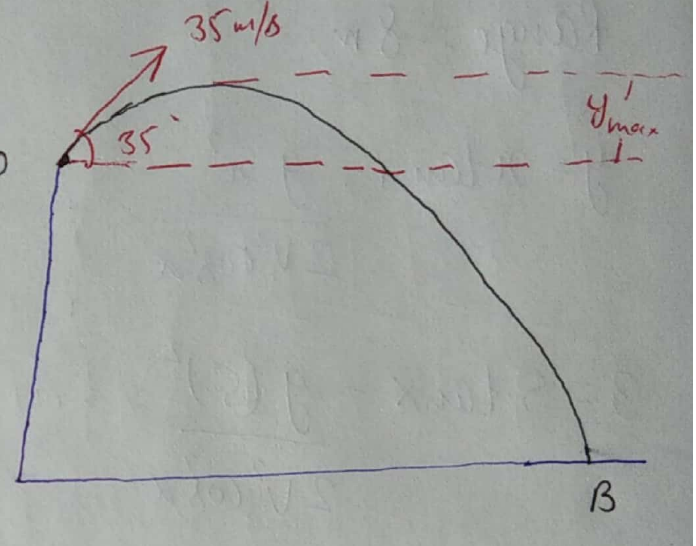
Q. A stone is projected upwards from the top of a 30 m high building with a velocity of 30 m/s at angle of 35° with horizontal. Make calculations for

a) the greatest height reached by the stone above the ground.

b) the horizontal distance from the point of projection to the point where the stone strikes the ground.

c) the velocity with which it strikes the ground

d) the time of flight.



Sol.

$v_x = V\cos\alpha = 24.57\,m/s$

$U_y = V\sin\alpha = 17.21\,m/s$

a) $H = \frac{v^2\sin^2\alpha}{2g} = 15.1\,m$

$\boxed{y_m = 30 + 15.1 = 45.1\,m}$

(b) Let B be the point at which the stone strikes the ground

$y_B = -30\,m$

$-30 = 17.21t - \frac{1}{2}\cdot 9.81t^2$

$\boxed{t = 4.786\,s}$ Time of flight (d)

$\boxed{AB = V_x \cdot t = 24.57 \times 4.786 = 117.59\,m}$

(c) $V_y = V_{y_0} - gt$

$= 17.21 - 9.81 \cdot 4.786$

$= -29.74\,m/s$

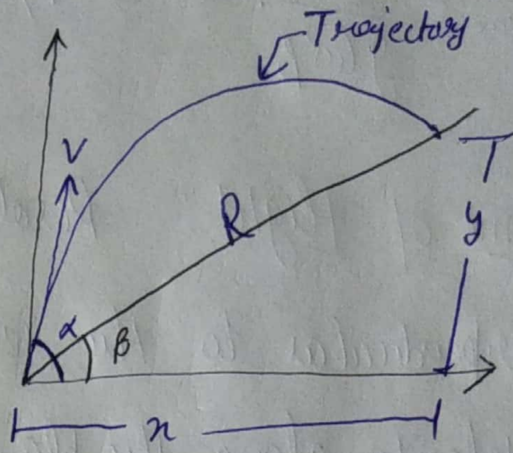
$V_B = \sqrt{V_x^2 + V_y^2}$

$= 38.57\,m/s$

$\theta = 50.42°$

Projectile on an inclined plane

$x = R\cos\beta$
$y = R\sin\beta$



The eq. of projectile

$$y = x\tan\alpha - \frac{g x^2}{2V^2\cos^2\alpha}$$

$$R\sin\beta = R\cos\beta\tan\alpha - \frac{g R^2\cos^2\beta}{2V^2\cos^2\alpha}$$

$$R = \frac{2V^2\cos^2\alpha}{g\cos^2\beta}(-\sin\beta + \cos\beta\tan\alpha)$$

$$= \frac{2V^2\cos^2\alpha}{g\cos^2\beta}\left(\frac{-\sin\beta\cos\alpha + \cos\beta\sin\alpha}{\cos\alpha}\right)$$

$$= \frac{2V^2\cos\alpha}{g\cos^2\beta}\sin(\alpha-\beta)$$

$$= \frac{V^2}{g\cos^2\beta}[2\cos\alpha\cdot\sin(\alpha-\beta)]$$

But $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$$2\cos\alpha\sin(\alpha-\beta) = \sin(2\alpha-\beta) - \sin\beta$$

$$\boxed{R = \frac{V^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]}$$

for max. range

$\sin(2\alpha-\beta) = 1$

$\boxed{2\alpha-\beta = 90^\circ}$

$$R_{max} = \frac{v^2}{g\cos^2\beta}\cdot(1-\sin\beta)$$

Time of flight: Resolving initial velocity V along and perpendicular to the inclined plane

Velocity along the plane $= V\cos(\alpha-\beta)$

Velocity perpendicular to the plane $= V\sin(\alpha-\beta)$

For motion normal to the plane, the net distance moved is zero.

$$s = ut + \frac{1}{2}at^2$$

$$0 = V\sin(\alpha-\beta)\cdot T - \frac{1}{2}g\cos\beta\, T^2$$

$$\boxed{T = \frac{2V\sin(\alpha-\beta)}{g\cos\beta}}$$

when the body is projected down the plane β is replaced by $-\beta$.

$$R = \frac{V^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)+\sin\beta\right]$$

$$T = \frac{2v}{g}\frac{\sin(\alpha+\beta)}{\cos\beta}$$

5

Q A projectile is fired from a point on an inclined plane with a velocity of 50 m/s. The angle of projection and the angle of plane are 60° and 25° to the horizontal resp. If the motion of particle is up the plane, determine

a) time of flight
b) range up the plane
c) Condition for max. range
d) max. range up the plane

Sol:

$\alpha = 60°$

$\beta = 25°$

a) Time of flight $(T) = \dfrac{2V \sin(\alpha-\beta)}{g \cos\beta} = \boxed{6.45\,s}$

b) Range $(R) = \dfrac{2V^2 \cos\alpha \sin(\alpha-\beta)}{g \cos^2\beta} = \boxed{177.96\,m}$

(c) Condition for max. Range

$2\alpha - \beta = 90°$

$\boxed{\alpha = 57.5°}$

(d) Max. Range $= \dfrac{V^2}{g}(1+\sin\beta)$

$= \boxed{179.14\,m}$

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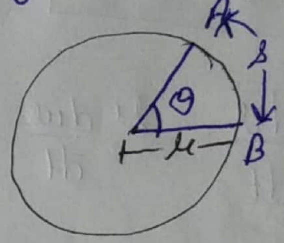
Rotational Motion

In rotary motion, the movement of the particle is along a circular path.

Terms related to circular motion

(i) Angular displacement (θ) - the angle through which body moves.



$\boxed{\text{Unit : radian}}$

ii) Angular Velocity (ω) - The rate of change of angular displacement of a body w.r.t. time is called angular velocity.

$$\boxed{\omega = \frac{d\theta}{dt}}$$

$\boxed{\text{Unit : radian/sec}}$

$$\boxed{\omega = \frac{2\pi N}{60}}$$

iii) Angular acceleration (α): $\frac{d\omega}{dt}$

$\boxed{\text{Unit} = \text{rad/sec}^2}$

Relation b/w circular motion and linear motion.

$$\boxed{s = r\theta}$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\boxed{v = r\omega}$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{a = r\alpha}$$

Equations of circular motion

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

Q The speed of a flywheel changes from 10 rad/s to 30 rad/s in 5 seconds. Determine the angular acceleration of the wheel. How many revolutions the wheel would turn to attain a speed of 600 rev/min?

Sol.

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega - \omega_0}{t} = 4 \text{ rad/s}^2$$

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b) $\omega = \frac{2\pi N}{60} = \frac{2\pi \cdot 600}{60} = 62.8$ rad/s

By using relation

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = 480.48$ radians

1 revolution = 2π radians

Total revolution made by wheel = $\frac{480.48}{2\pi} = 76$

Q A flywheel 0.5m in diameter accelerates uniformly from rest to 360 rpm in 12 seconds. Determine the velocity and acceleration of a point on the rim of the flywheel 0.1 secon after it has started from rest.

Sol $\omega_0 = 0$

$\omega = \frac{2\pi N}{60} = 37.68$ rad/s

By using relation

$\omega = \omega_0 + \alpha t$

$\alpha = \frac{37.68 - 0}{12} = 3.14$ ra/s

Angular velocity of wheel 1 sec after start

$$\omega = \omega_0 + \alpha t$$
$$= 3.14 \text{ rad/s}$$

As we know

$$v = r\omega$$
$$= 3.14 \times \frac{0.5}{2} = 0.785 \text{ m/s}$$

The linear velocity will be tangential to the rim.

Tangential acc $(a_t) = \alpha r$

$$= 3.14 \times \frac{0.5}{2} = 0.785 \text{ m/s}^2$$

Normal acc $(a_n) = \omega^2 r$

$$= 1.232 \text{ m/s}^2$$

Resultant $a = \sqrt{a_n^2 + a_t^2} = 1.461 \text{ m/s}^2$

$$\phi = \tan^{-1} \frac{a_t}{a_n} = 32.5^\circ$$

Q A shaft is accelerated uniformly from 600 rev/min to 900 rev/min in 2 seconds. It continues accelerating at this rate for a further period of 4 seconds and then continues to rotate at the max. speed attained. Make calculations for the time taken to complete the first 180 revolutions.

Sol: During uniform acc. from 600 rpm to 900 rpm

$$\omega_0 = \frac{2\pi \cdot 600}{60} = 62.8 \text{ rad/s}$$

$$\omega_1 = \frac{2\pi \cdot 900}{60} = 94.2 \text{ rad/s}$$

$$\alpha_1 = \frac{\omega_1 - \omega_0}{t} = 15.7 \text{ rad/s}^2$$

The angular displacement θ_1 for the entire accelerating period of 6 seconds.

$$\theta = \omega t + \frac{1}{2}\alpha t^2$$

$$\theta_1 = 62.8 \times 6 + \frac{1}{2} \times 15.7 \times 6^2 = 659.4 \text{ radians}$$

Revolutions turned by the wheel during the acc. period $= \dfrac{659.4}{2\pi} = \boxed{105 \text{ revolutions}}$

Revolutions to be made by shaft at uniform speed attained at the end of accelerating period (180 - 105 = 75 revolutions)

Corresponding angular displacement $= 75 \cdot 2\pi = 471$ radians

Angular velocity at the end of acceleration period,

$$\omega = \omega_0 + \alpha t$$

$$\boxed{= 62.8 + 15.7 \times 6 = 157 \text{ rad/s}}$$

Time taken to turn 471 radians at uniform speed of 157 rad/s

$$\boxed{t = \frac{\theta}{\omega} = \frac{471}{157} = 3 \text{ seconds}}$$

$$\boxed{\text{Total time} = 2 + 4 + 3 = 9 \text{ seconds}}$$