Shear force and Bending Moment

"Shear force at a section in a beam is the force that is trying to shear off the section.

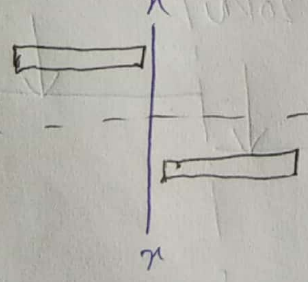
It is equal to the algebraic sum of all the forces acting normal to the axis of beam; either to the left or to the right of the section."

Bending Moment at a section in a beam is the moment that tends to bend the beam.

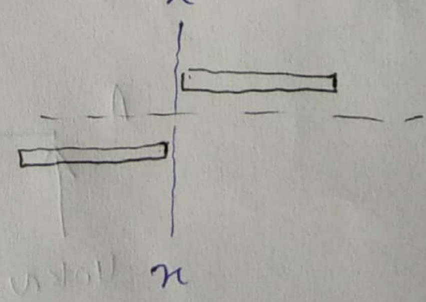
It is obtained as algebraic sum of moment of all the forces about the section, acting either to the left or to the right of the section.

Sign Convention:

(i) Shear force is taken positive if its tends to move the left portion upward with respect to the right portion.

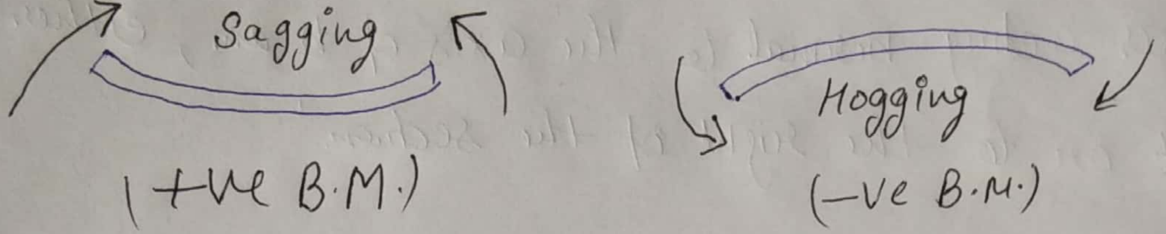


+ve S.F

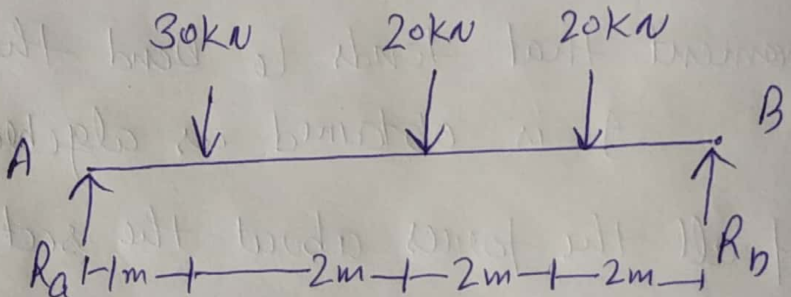


−Ve S.F.

ii) BM is taken positive if it tends to sag (concave upward) the beam and it is taken negative if it tends to hog (concave down) the beam.



e.g

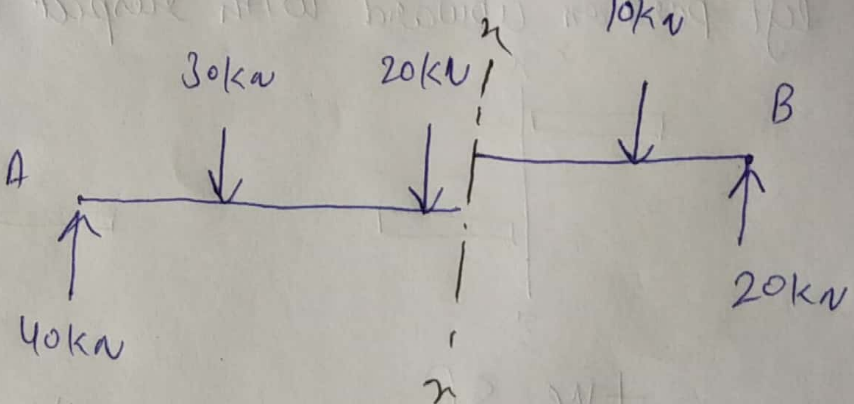


By using eq. condition.

$R_a = 40 kN$

$R_b = 20$ ~~40~~ kN

Consider a section xx at a distance x from point A.



Consider equilibrium condition on each portion of beam, the net resultant vertical forces are

$$F_{left} = 40 - 30 - 20 = -10 \text{ kN}$$

$$F_{right} = -20 + 10 = -10 \text{ kN}$$

For B.M.

$$M_{left} = 40 \times 4 - 30 \times 3 - 20 \times 1 = 50 \text{ kNm} \quad (x = 4)$$

(clockwise)

$$M_{right} = 20 \times 3 - 10 \times 1 = 50 \text{ kNm} \text{ (Anti-clockwise)}$$

Relation Between Load, SF and BM

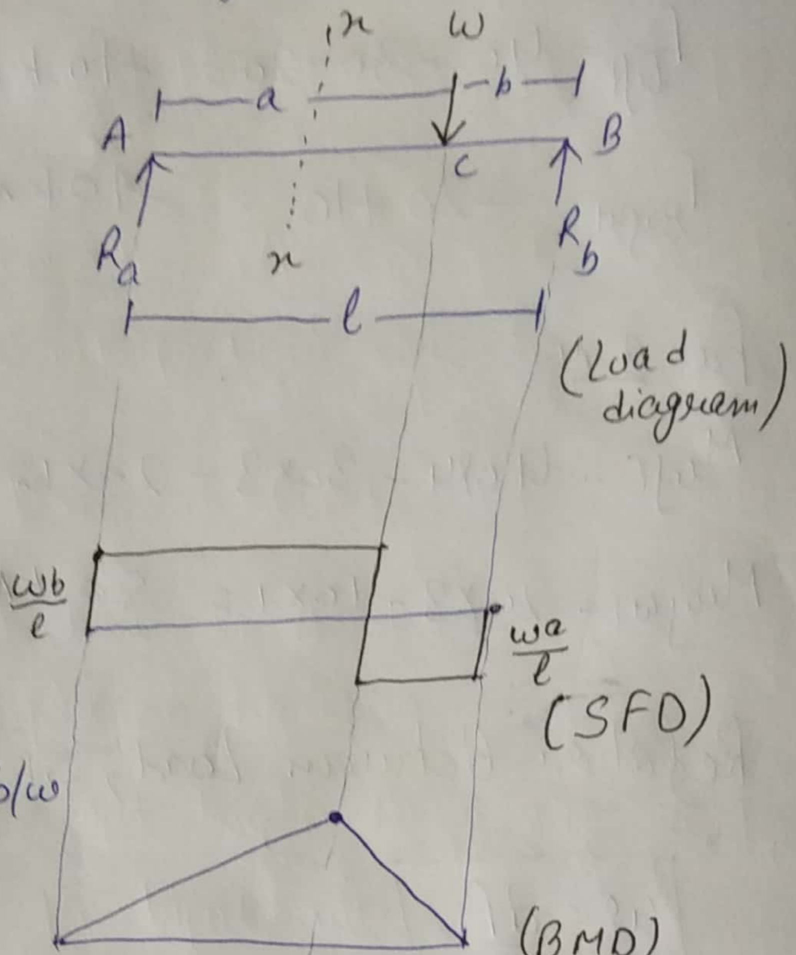
$$\boxed{w = \frac{dF}{dx}}$$

$$\boxed{F = \frac{dM}{dx}}$$

SF and BM diagram for some cases

(1) Simply supported beam subjected to

i) point load



$R_a = \frac{wb}{l}$; $R_b = \frac{wa}{l}$

<u>For</u> SF diagram

Consider a section x-x b/w point A and C

for left portion of beam

$SF = R_a = \frac{wb}{l}$

Now consider xx section b/w point C and B

for left portion of beam

when $x = a$

$SF = \frac{wb}{l} - w$

when $x = l$

$SF = \frac{wb}{l} - w + \frac{wa}{l} = 0$

For <u>BM</u> diagram

Consider a section xx b/w AC

$BM_x = R_a x$

when $x = 0$; $BM = 0$

$x = a$; $BM = \frac{wba}{l}$

Consider a section xx b/w CB

$BM_x = R_a x - w(x-a)$

at $x = a$

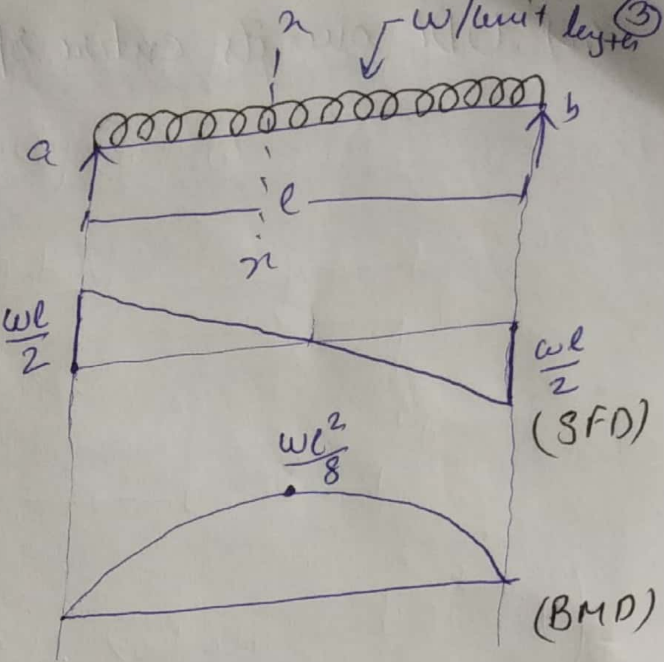
$BM_x = \frac{wba}{l}$

at $x = l$

$BM = 0$

ii) Udl

$R_a = R_b = \frac{wl}{2}$



$SF_x = R_a - wx$

at $x=0$; $SF = R_a = \frac{wl}{2}$

at $x=l$; $SF = -\frac{wl}{2}$

S.F will be zero at

$R_a - wx = 0$

$\frac{wl}{2} - wx = 0$

$\boxed{x = \frac{l}{2}}$

BM equation

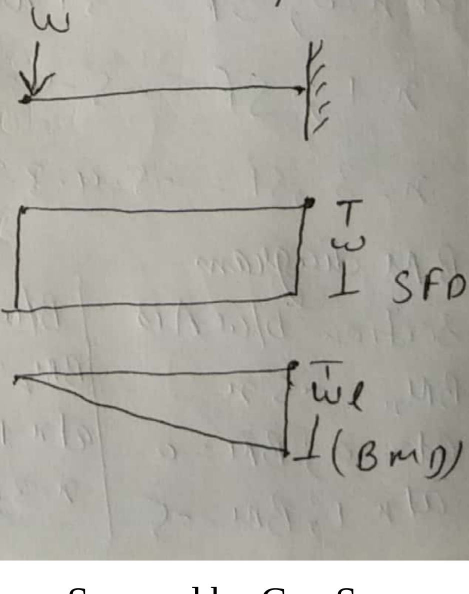
$BM_x = R_a x - wx \cdot \frac{x}{2}$

at $x=0$; $BM = 0$

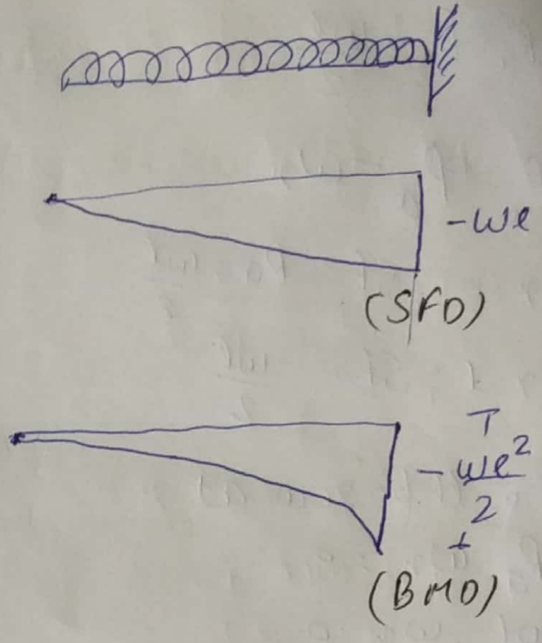
at $x=l$; $BM = 0$

$x = l/2$; $BM = \frac{wl^2}{8}$

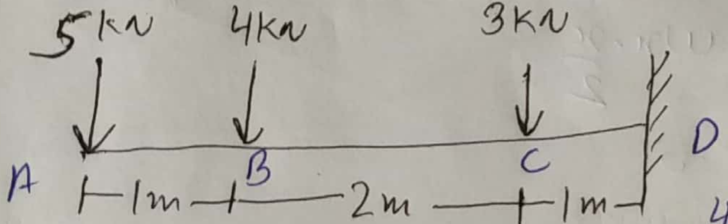
(2.) Cantilever subjected to (i) concentrated load at free end.

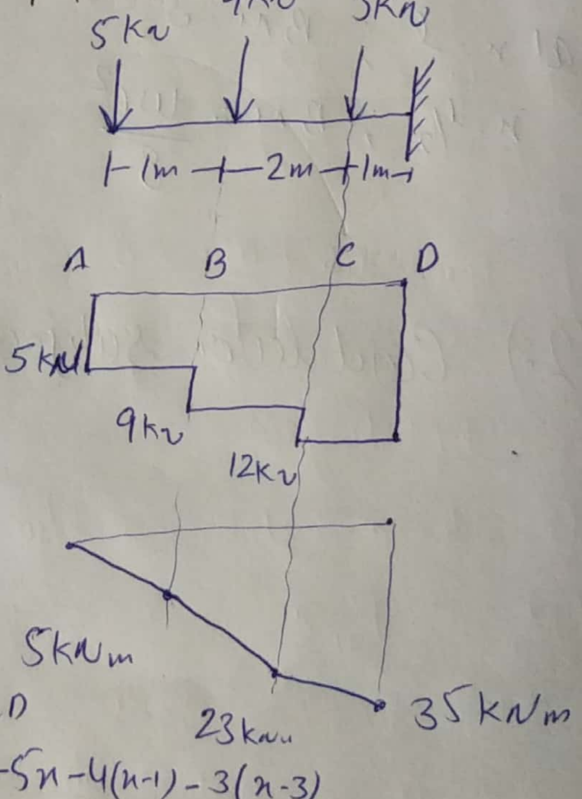


(ii) UDL over its entire span



Q Construct the shear force and bending moment diagram for the Cantilever beam as shown in fig





Sol SF diagram

Consider any section at distance x from A

at $x=0$; $SF = -5\,kN$

$x=1$; $SF = -5-4 = -9\,kN$

$x=3$; $SF = -5-4-3 = -12$

BM diagram

Section b/w AB

$BM_x = -5x$

at $x=0$; $BM = 0$

at $x=1$; $BM = -5$

B/w BC

$BM_x = -5x - 4(x-1)$

at $x=1$; $BM_1 = -5$

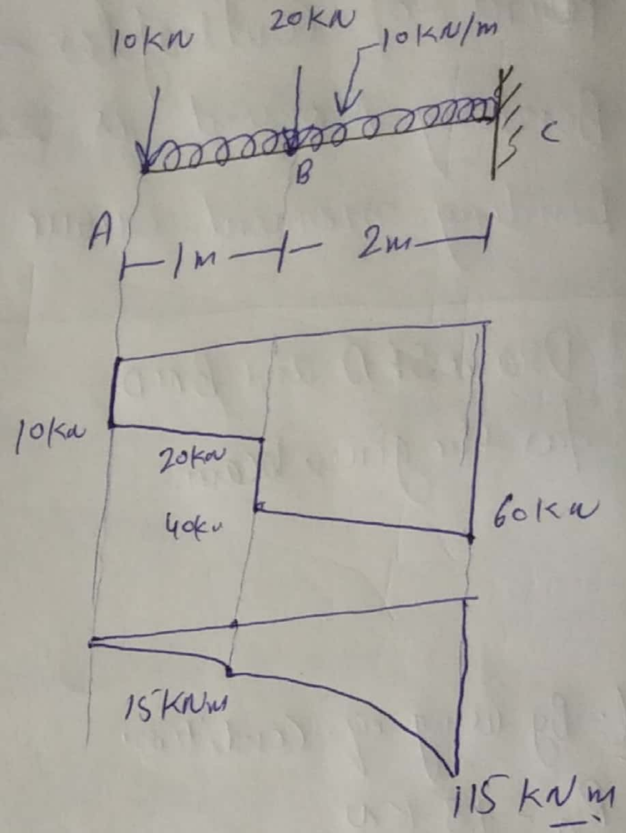
$x=3$; $BM_x = -23$

B/w CD

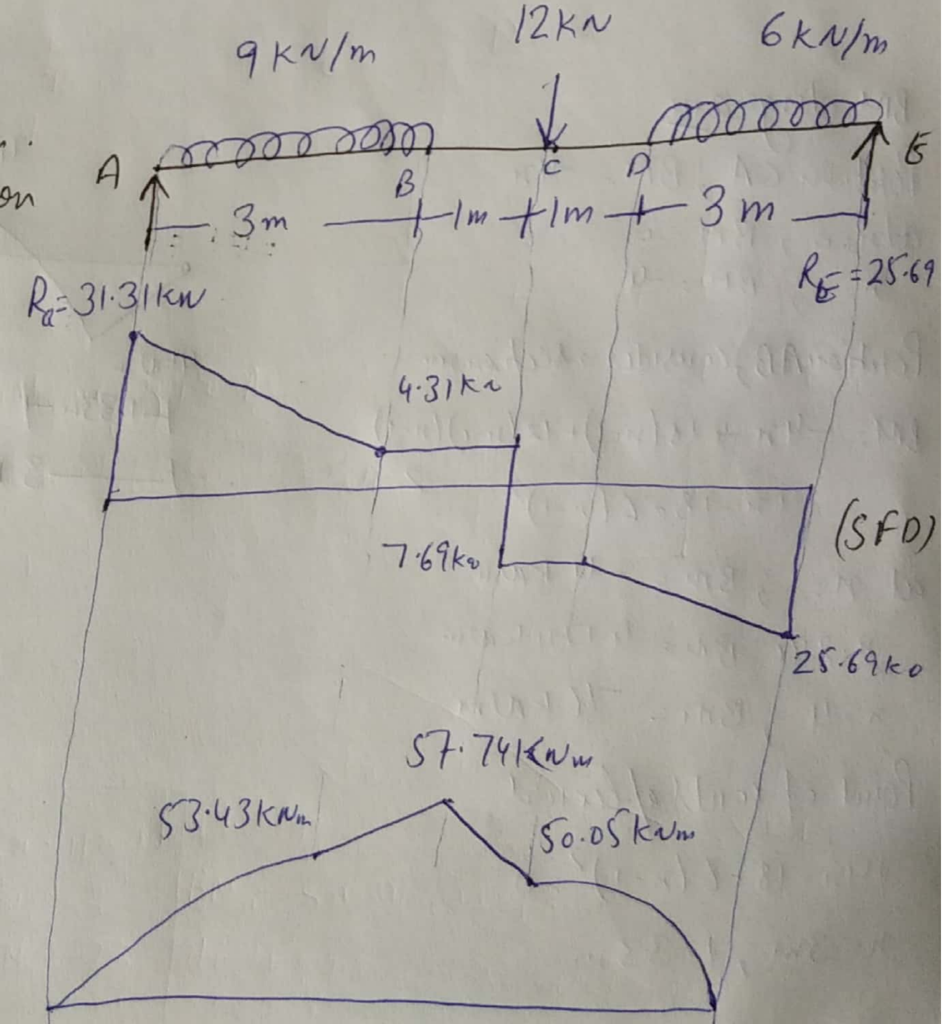
$BM_x = -5x - 4(x-1) - 3(x-3)$

at $x=4$; $BM = -35\,kNm$

Q Construct SFD and BMD



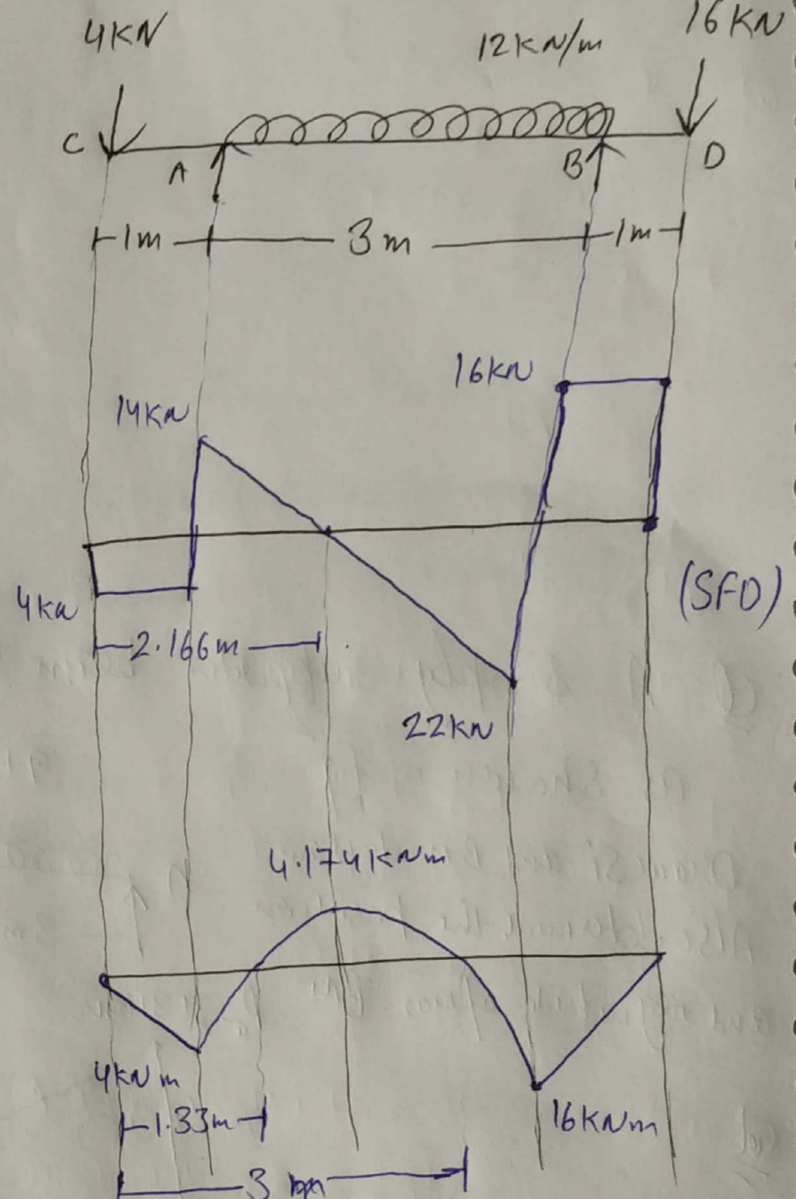
Q A simply supported beam with 8m span is loaded as shown in fig. Draw SF and BM diagram. Also determine the position and magnitude of max. BM.



Sol

Point of Contraflex – The point at which Bending Moment is zero. At this point the bending moment curve intersects with the zero line.

Q Draw SFD and BMD for the given beam.



Sol: By using eq. condition

$R_a = 18$ KN

$R_b = 38$ KN

BM diagram

Portion CA ; $BM = -4x$

at $x = 0$; $BM = 0$

$x = 1$; $BM = -4$

Portion AB, consider a section mm

$BM = -4x + 18(x-1) + 12(x-1)\frac{(x-1)}{2}$

$= 14x - 18 - 6(x-1)^2$

at $x = 1$; $BM = -4$ KNm

$x = 2.166$; $BM = 4.174$ KNm

$x = 4$; $BM = -16$ KNm

Point of contraflexure

$14x - 18 - 6(x-1)^2 = 0$

$x = 3$m, 1.33m

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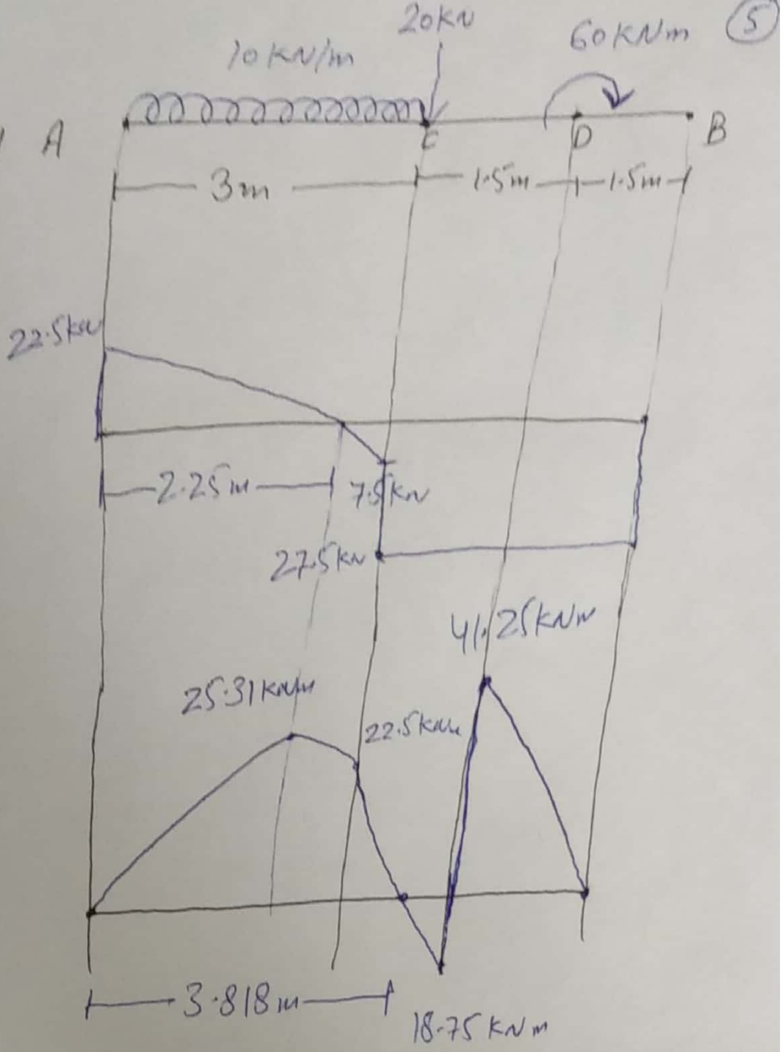
Q. Draw SFD & BMD.
Also locate the point of contraflexure, if any



Sol. By using eq. condition

$R_A = 22.5\,kN$

$R_B = 27.5\,kN$

BM equation

Portion AC

$BM = 22.5x - 10 \cdot \frac{x^2}{2}$

Portion CD

$BM = 22.5x - 10 \cdot 3(x-1.5) - 20(x-3)$

$= -27.5x + 105$

Portion DB - Bending moment has linear variation from D to B.