8.9. FRICTION IN BEARINGS

A rotating shaft (whether horizontal or vertical) is supported in a bearing, which is stationary. When the shaft is rotating, there is a rubbing action between the surface of the shaft and bearing surface and resistance is offered by the bearing surface. This resistance is due to friction between shaft and bearing. Due to the resistance offered by bearing surface, some power of the rotating shaft is lost. This power lost should be minimum. Let us find the power lost in different types of bearing.

The important types of bearings are :

(*i*) Flat Bearing [Refer to Fig. 8.44 (*a*)]

(*ii*) Pivot Bearing [Refer to Fig. 8.44 (*b*) and (*c*)]

(*iii*) Collared Bearing [Refer to Fig. 8.44 (*d*)]

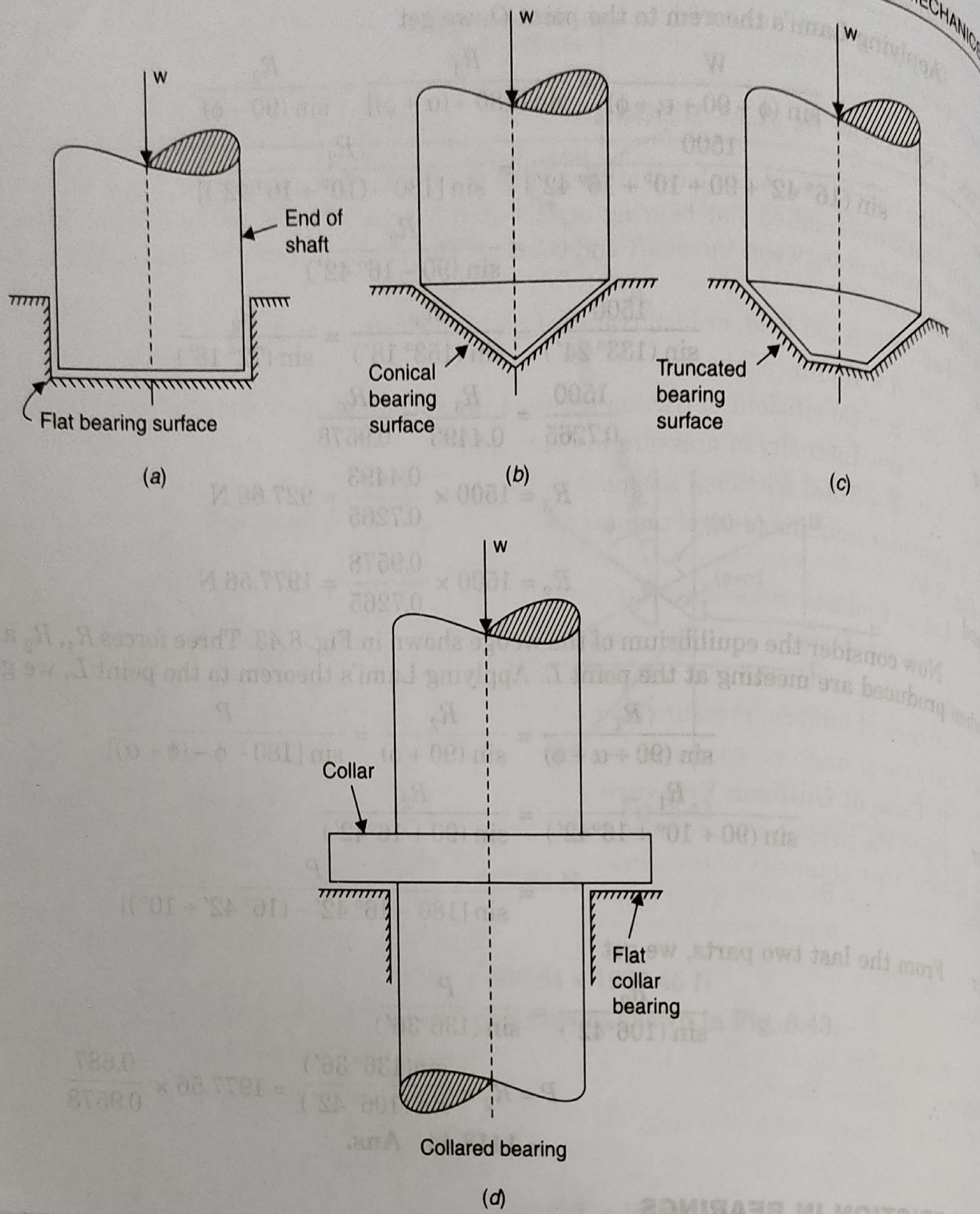

Fig. 8.44

For a **new bearing**, the contact between the shaft and bearing may be good over the whole surface. This means that the pressure over the rubbing surfaces may be assumed as uniformly distributed. But when **the bearing becomes old**, all parts of the rubbing surfaces will not move with the same velocity and hence the wear will be different at different radii. The pressure distribution will not be uniform. The rate of wear of surfaces depends upon the pressure and the rubbing velocities between the surfaces.

The design of bearings is based on the following assumptions though neither of them is strictly true :

(*i*) the pressure is uniformly distributed over the bearing surfaces, and

(*ii*) the wear is uniform over the bearing surface.

The power lost, due to friction in flat, pivot and collar bearings, are calculated on the above two assumptions.

8.10. FRICTION IN FLAT BEARING

If the bearing surface placed at the end of the shaft is flat as shown in Fig. 8.45, then the bearing surface is called flat bearing or foot-step bearing. There will be friction along the surface of contact between the shaft and bearing. The power lost can be obtained by calculating the torque.

Let W = Axial load, or load transmitted to the bearing surface,

R = Radius of shaft

μ = Co-efficient of friction between the surface of shaft and surface of bearing,

p = Intensity of pressure in N/m^2, and

T = Total frictional torque.

Consider a circular ring of radius r and thickness dr as shown in Fig. 8.45.

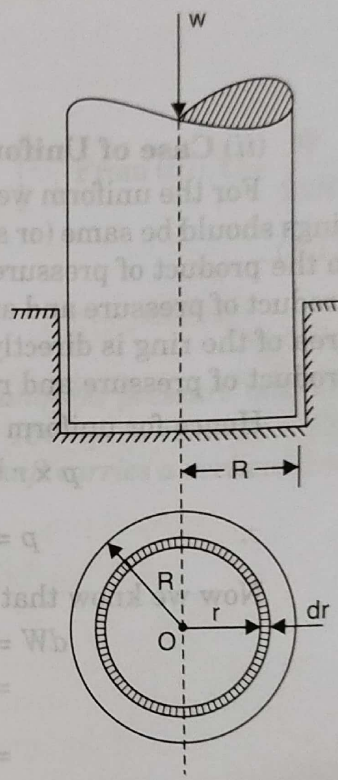

Fig. 8.45

Let us find the frictional torque on this ring.

$\therefore$ Area of ring $= 2\pi r.dr$

We will consider the two cases, namely

(*i*) case of uniform pressure over bearing surface and

(*ii*) case of uniform wear over bearing surface.

(*i*) Case of Uniform Pressure

When the pressure is assumed to be uniform over the bearing surface, then intensity of pressure (p) is given by

$$p = \frac{\text{Axial load}}{\text{Area of cross-section}}$$

$$= \frac{W}{\pi R^2} \quad \text{...(i)}$$

Now let us find the load transmitted to the ring and also frictional torque on the ring.

Load transmitted to the ring,

$$dW = \text{Pressure on the ring} \times \text{Area of ring}$$
$$= p \times 2\pi r dr$$

Frictional force* on the ring,

$$dF = \mu \times \text{load on ring} \qquad (\because dF = \mu \times dW)$$
$$= \mu \times p \times 2\pi r dr$$

$\therefore$ Frictional torque on the ring

$$= \text{Friction force} \times \text{Radius of ring} = dF \times r$$

$\therefore$ Frictional torque, $dT = \mu \times p \times 2\pi r dr \times r$

$$= 2\pi\mu p r^2 dr$$

The total frictional torque (T) will be obtained by integrating the above equation from 0 to R.

*Load on the ring is vertically downward. Hence frictional force on the ring will be equal to $\mu \times$ normal reaction *i.e.*, $\mu \times$ load on ring. Here normal reaction on the ring is equal to load on the ring. Hence frictional force on ring = $\mu \times dW$.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi\mu p r^2\, dr$

$$= 2\pi\mu p \int_0^R r^2 dr \qquad (\because\ \mu \text{ and } p \text{ are constant})$$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu p \left[\frac{R^3}{3}\right] = \frac{2}{3}\pi\mu p R^3$$

$$= \frac{2}{3}\pi \times \mu \times \frac{W}{\pi R^2} \times R^3 \qquad \left(\because \text{ From } (i),\ p = \frac{W}{\pi R^2}\right)$$

$$= \frac{2}{3}\mu W R \qquad ...(8.8)$$

$\therefore$ Power lost in friction $= T \times \omega$

$$= T \times \frac{2\pi N}{60} \qquad \left(\because\ \omega = \frac{2\pi N}{60}\right)$$

$$= \frac{2\pi N T}{60} \qquad ...(8.9)$$

(*ii*) Case of Uniform Wear

For the uniform wear of the bearing surface, the load transmitted to the various circular rings should be same (or should be constant). But load transmitted to any circular ring is equal to the product of pressure and area of the ring, (*i.e.*, $p \times 2\pi r\, dr$). Hence for uniform wear, the product of pressure and area of ring should be constant. For a ring of constant thickness dr the area of the ring is directly proportional to the radius of the ring. Hence for uniform wear, the product of pressure and radius should be constant or $p \times r$ = constant.

Hence for uniform wear, we have

$$p \times r = \text{Constant (say } C) \qquad ...(8.10)$$

$$\therefore \quad p = \frac{C}{r} \qquad ...(i)$$

Now we know that load transmitted to the ring,

$$dW = \text{Pressure} \times \text{Area of ring}$$

$$= p \times 2\pi r dr$$

$$= \frac{C}{r} \times 2\pi r dr \qquad \left[\because \text{ From } (i),\ p = \frac{C}{r}\right]$$

$$= 2\pi C\, dr \qquad ...(ii)$$

Total load transmitted to the bearing, is obtained by integrating the above equation from 0 to R.

$\therefore$ Total load transmitted to the bearing

$$= \int_0^R 2\pi C dr = 2\pi C \int_0^R dr = 2\pi C\, [r]_0^R$$

$$= 2\pi C R$$

Let us find the value of constant C.

The total load transmitted to the bearing is also equal to W

$$\therefore \quad 2\pi C R = W$$

or

$$C = \frac{W}{2\pi R} \qquad ...(iii)$$

Now frictional force on the ring,

$$dF = \mu \times \text{Load on ring}$$
$$= \mu \times dW$$
$$= \mu \times 2\pi C dr$$

[From (*ii*), load on ring = $2\pi C dr$]

Hence frictional torque on the ring,

$$dT = \text{Frictional force on ring} \times \text{radius} = dF \times r$$
$$= \mu \times 2\pi C dr \times r$$

$\therefore$ Total frictional torque,

$$T = \int_0^R dT$$
$$= \int_0^R \mu \times 2\pi C r\, dr$$
$$= 2\pi\mu C \int_0^R r\, dr \qquad [\mu \text{ and } C \text{ are constant}]$$
$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_0^R = 2\pi\mu C \times \frac{R^2}{2}$$
$$= 2\pi\mu \times \frac{W}{2\pi R} \times \frac{R^2}{2} \qquad \left[\because \text{ From } (iii),\ C = \frac{W}{2\pi R}\right]$$

or

$$T = \frac{1}{2} \times \mu W R \qquad \text{...(8.11)}$$

$\therefore$ Power lost in friction $= T \times \omega = \dfrac{2\pi N T}{60}$.

8.12 (A) FRICTION IN COLLARED BEARING

The bearing surface provided at any position along the shaft (but not at the end of the shaft), to carry axial thrust is known as collar which may be flat or conical. If the surface is flat, then bearing surface is known as flat collar as shown in Fig. 8.47. The collar bearings are also known as *thrust bearings*. The power lost in friction can be obtained by calculating the torque.

Let r_1 = External radius of collar

r_2 = Internal radius of collar

p = Intensity of pressure

W = Axial load or total load transmitted to the bearing surface
μ = Co-efficient of friction
T = Total frictional torque

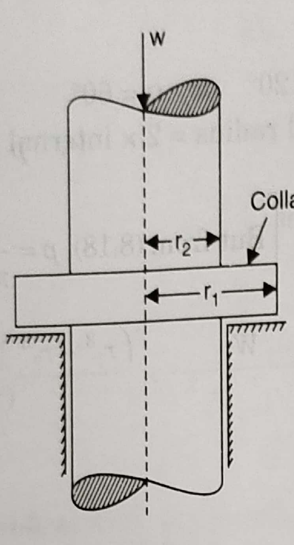

Fig. 8.47

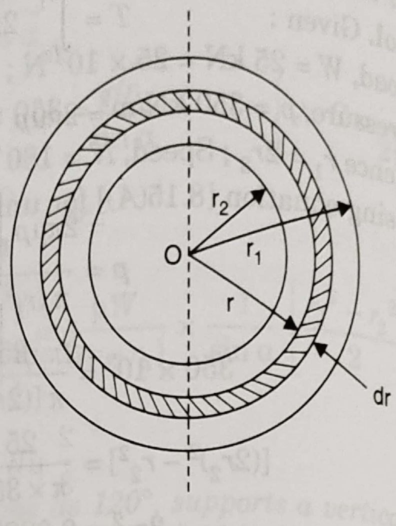

Fig. 8.47(*a*)

Consider a circular ring of radius r and thickness dr as shown in Fig. 8.47 (*a*).

$\therefore$ Area of ring $= 2\pi r\, dr$.

Load on the ring = Pressure × Area of ring

$= p \times 2\pi r\, dr$ ...(*i*)

Frictional force on the ring = μ × Load on ring

$= \mu \times p \times 2\pi r\, dr$

Frictional torque on the ring, dT = Frictional force × Radius

$= (\mu \times p \times 2\pi r\, dr) \times r$

$= 2\pi\mu p r^2\, dr$

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} dT$$

$$= \int_{r_2}^{r_1} 2\pi\mu p r^2 dr \quad \text{...(ii)}$$

(*i*) **Uniform Pressure**

p = Constant

Total load transmitted to the bearing

$$= \int_{r_2}^{r_1} \text{Load on ring}$$

or $$W = \int_{r_2}^{r_1} p \times 2\pi r\, dr \qquad (\because \text{ Load on ring from } (i) = p \times 2\pi r\, dr)$$

$$= p \times 2\pi \int_{r_2}^{r_1} r dr \qquad (\because\ p \text{ is constant})$$

$$= 2\pi p \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi p \left[\frac{r_1^{\,2} - r_2^{\,2}}{2}\right] = \pi \times p\,[r_1^{\,2} - r_2^{\,2}]$$

$$\therefore \quad p = \frac{W}{\pi[r_1^2 - r_2^2]} \qquad ...(8.18)$$

Total frictional torque is given by equation (*ii*),

$$\therefore \quad T = \int_{r_2}^{r_1} 2\pi\mu p r^2\, dr$$

$$= 2\pi\mu p \int_{r_2}^{r_1} r^2\, dr \qquad (\because \; p \text{ is constant})$$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} \qquad \left[\text{But from (8.18)}, \; p = \frac{W}{\pi(r_1^2 - r_2^2)}\right]$$

$$= 2\pi\mu p \left[\frac{r_1^3 - r_2^3}{3}\right] = 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \left(\frac{r_1^3 - r_2^3}{3}\right)$$

$$= \frac{2}{3}\mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \qquad ...(8.19)$$

$\therefore$ Power lost in friction,

$$P = \frac{2\pi NT}{60}.$$

(*ii*) **Uniform Wear**

$$p \times r = \text{constant} \qquad (\text{say } C)$$

$$\therefore \quad p = \frac{C}{r}$$

Total load transmitted to the bearing

$$= \int_{r_2}^{r_1} \text{Load on ring} = \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$\therefore \quad W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi r\, dr \qquad \left(\because \; p = \frac{C}{r}\right)$$

$$= 2\pi C \int_{r_2}^{r_1} dr \qquad (\because \; C \text{ is constant})$$

$$= 2\pi C\,[r]_{r_2}^{r_1} = 2\pi C\,[r_1 - r_2]$$

$$\therefore \quad C = \frac{W}{2\pi(r_1 - r_2)} \qquad ...(iii)$$

Total frictional torque is given by equation (*ii*),

$$T = \int_{r_2}^{r_1} 2\pi\mu p r^2\, dr$$

$$= 2\pi\mu \int_{r_2}^{r_1} p r^2\, dr \qquad \left(\text{Here } p \text{ is not constant it is } = \frac{C}{r}\right)$$

$$= 2\pi\mu \int_{r_2}^{r_1} \frac{C}{r} r^2\, dr$$

$$= 2\pi\mu \int_{r_2}^{r_1} Cr\,dr = 2\pi\mu C \int_{r_2}^{r_1} r\,dr \qquad (C \text{ is constant})$$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C \left[\frac{r_2^2 - r_1^2}{2}\right]$$

$$= 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \left[\frac{r_2^2 - r_1^2}{2}\right] \qquad \left[\because C = \frac{W}{2\pi(r_1 - r_2)} \text{ from (iii)}\right]$$

$$= \frac{\mu W}{2}(r_1 + r_2) \qquad \ldots(8.20)$$

$\therefore$ Power lost in friction

$$P = \frac{2\pi NT}{60}$$

If the axial load on the bearing is too great, then the bearing pressure on the collar will become more than the limiting bearing pressure which is approximately equal to 400 kN/m^2. Hence to reduce the intensity of pressure on collar, two or more collars are used (or multi-collars are used) as shown in Fig. 8.48.

If n = number of collars in multi-collar bearing, then

(*i*) $$n = \frac{\text{Total load}}{\text{Load permissible on one collar}}$$

(*ii*) p = Intensity of the uniform pressure

$$= \frac{\text{Load}}{\text{No. of collars} \times \text{Area of one collar}}$$

$$= \frac{W}{n \times \pi[r_1^2 - r_2^2]}$$

(*iii*) Total torque transmitted remains constant *i.e.,*

$$T = \frac{2}{3} \times \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

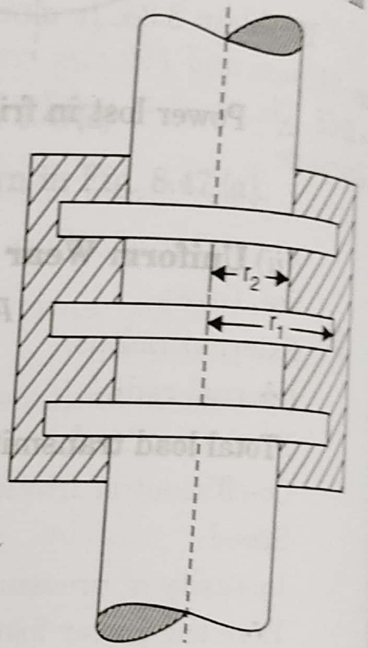

Fig. 8.48