UNIT-2

FRICTION

Friction :-

The opposing force (resistance force) offered by one surface on other surface when they are relatively in movement.

OR

The opposing force, which acts in the opposite direction of the movement of the upper block, is called the force of friction or simply friction.

How to friction force arises between two surfaces :-

At microscopic level there we can see heights (crest's) and depths (valleys) interlocking of these crest's and valleys into each other will arises to friction.

Merits :-

1. Man walking on a road there should be friction.
2. Parts movement in machines will be possible by friction force.
3. Tightening of bolt or nut.

Demerits :-

1. Wear and tear.
2. Heat produced.

Types of friction : -

In general, the friction is of the following two types :

1. Static friction 2. Dynamic friction

1. Static friction :-

It is the friction, experienced by a body, when at rest.

2. Dynamic friction :-

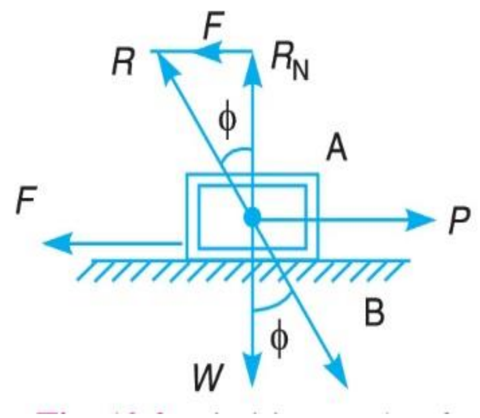
It is the friction, experienced by a body, when in motion. The dynamic friction is also called kinetic friction and is less than the static friction. It is of the following three types :

(a) Sliding friction:- It is the friction, experienced by a body, when it slides over another body.

(b) Rolling friction:- It is the friction, experienced between the surfaces which has balls or rollers interposed between them.

(c) Pivot friction:- It is the friction, experienced by a body, due to the motion of rotation as in case of foot step bearings.

Limiting Friction :-



Consider that a body A of weight W is lying on a rough horizontal body B as shown in Fig. In this position, the body A is in equilibrium under the action of its own weight W, and the normal reaction R_N (equal to W) of B on A. Now if a small horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F. in equilibrium under the action of the following three forces.

(i) Weight of the body (W)
(ii) Applied horizontal force (P), and
(iii) Reaction (R) between body A and the plane B.

" The maximum friction that arises, when the body is in a position of just to move. It is called limiting friction". It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction into play is called static friction which may have any value between zero and limiting friction.

Friction laws :-

1. The force of friction always acts in the opposite direction to that of movement of the body.
2. The force of friction is directly proportional to the normal reaction.

$$F \propto R_N$$
$$F = \mu R_N$$
$$\mu = \frac{F}{R_N}$$

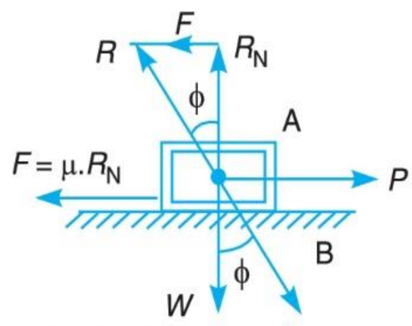
Co-efficient of friction (μ) :-

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = \frac{F}{R_N}$$

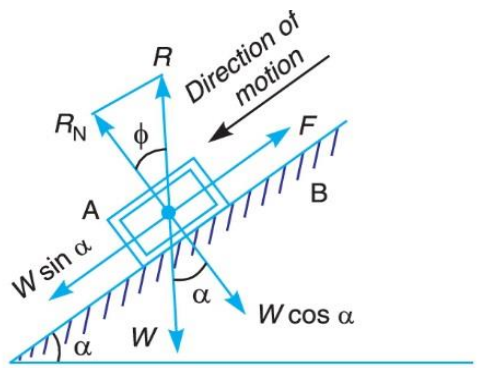
3. The force of friction is independent of area of the surface in contact.
4. The force of friction is independent of velocity of low speeds and dependent of velocity at higher speeds.
5. The force of friction depends upon the roughness of the surfaces. Which are in contact to each other.

Angle of friction :-



The angle between normal reaction (R_N) and resultant (R) of friction force and normal reaction is called angle of friction ($\emptyset$).

Angle of repose :-



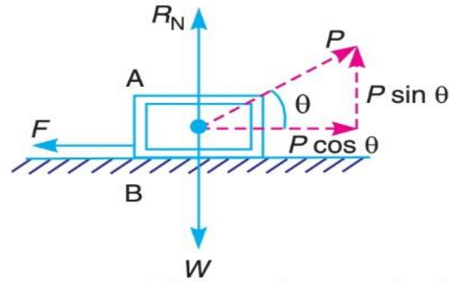
Angle of repose.

Consider that a body A of weight (W) is resting on an inclined plane B, as shown in Fig. If the angle of inclination α of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose.

A little consideration will show that the body will begin to move down the plane when the angle of inclination of the plane is equal to the angle of friction (i.e. α = ϕ).

Minimum Force Required to Slide a Body on a Rough Horizontal plane :-

Consider that a body A of weight (W) is lying on a horizontal plane B as shown in Fig. Let an effort P is applied at an angle θ to the horizontal such that the body A just moves. The various forces acting on the body are shown in Fig. Resolving the force P into two components, i.e. P sin θ acting upwards and P cos θ acting horizontally. Now for the equilibrium of the body A,



Minimum force required to slide a body.

$$R_N + P \sin \theta = W$$

or $$R_N = W - P \sin \theta \quad \text{...}(i)$$

and $$P \cos \theta = F = \mu . R_N \quad \text{...}(ii)$$

$$\text{...}(\because F = \mu . R_N)$$

Substituting the value of R_N from equation *(i)*, we have

$$P \cos \theta = \mu (W - P \sin \theta) = \tan \phi (W - P \sin \theta) \quad \text{...}(\because \mu = \tan \phi)$$

$$= \frac{\sin \phi}{\cos \phi} (W - P \sin \theta)$$

$$P \cos \theta . \cos \phi = W \sin \phi - P \sin \theta . \sin \phi$$

$$P \cos \theta . \cos \phi + P \sin \theta . \sin \phi = W \sin \phi$$

$$P \cos (\theta - \phi) = W \sin \phi \quad \text{...}[\because \cos \theta . \cos \phi + \sin \theta . \sin \phi = \cos (\theta - \phi)]$$

$$P = \frac{W \sin \phi}{\cos (\theta - \phi)} \quad \text{...}(iii)$$

For P to be minimum, $\cos (\theta - \phi)$ should be maximum, *i.e.*

$$\cos (\theta - \phi) = 1 \quad \text{or} \quad \theta - \phi = 0° \quad \text{or} \quad \theta = \phi$$

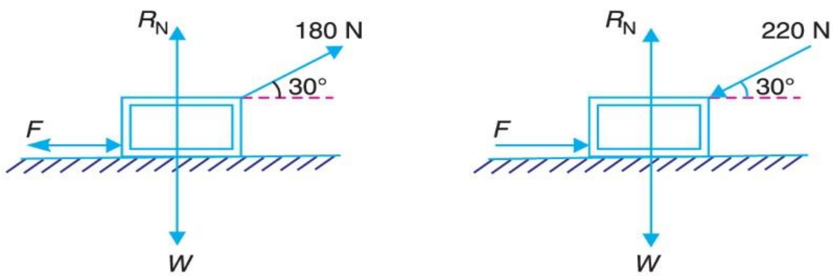
In other words, the effort P will be minimum, if its inclination with the horizontal is equal to the angle of friction.

$$\therefore \quad P_{min} = W \sin \theta \quad \text{...[From equation } (iii)]$$

Problem :- 1

1. A body, resting on a rough horizontal plane required a pull of 180 N inclined at 30º to the plane just to move it. It was found that a push of 220 N inclined at 30º to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

Ans:-



(*a*) (*b*)

Given : $\theta = 30°$

Let W = Weight of the body in newtons,

R_N = Normal reaction,

μ = Coefficient of friction, and

F = Force of friction.

First of all, let us consider a pull of 180 N. The force of friction (F) acts towards left as shown in Fig. 10.5 (*a*).

Resolving the forces horizontally,

$$F = 180 \cos 30° = 180 \times 0.866 = 156 \text{ N}$$

Now resolving the forces vertically,

$$R_N = W - 180 \sin 30° = W - 180 \times 0.5 = (W - 90) \text{ N}$$

We know that $F = \mu.R_N$ or $156 = \mu\,(W - 90)$ ...(*i*)

Now let us consider a push of 220 N. The force of friction (F) acts towards right as shown in Fig. 10.5 (*b*).

Resolving the forces horizontally,

$$F = 220 \cos 30° = 220 \times 0.866 = 190.5 \text{ N}$$

Now resolving the forces vertically,

$$R_N = W + 220 \sin 30° = W + 220 \times 0.5 = (W + 110) \text{ N}$$

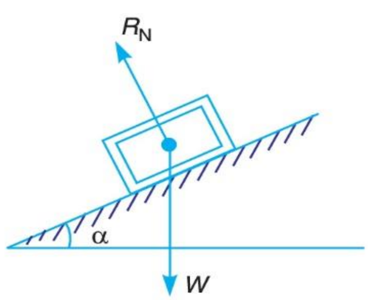
We know that $F = \mu.R_N$ or $190.5 = \mu\,(W + 110)$ ...(*ii*)

From equations (*i*) and (*ii*),

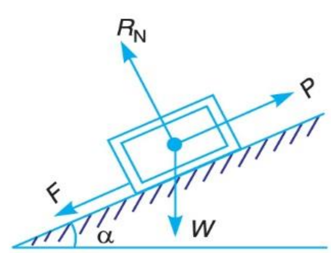
$W = 1000$ N, and $\mu = 0.1714$ **Ans.**

Friction of a Body Lying on a Rough Inclined Plane :-

Consider that a body of weight (W) is lying on a plane inclined at an angle α with the horizontal, as shown in Fig.



(*a*) Angle of inclination less than angle of friction.



(*b*) Angle of inclination more than angle of friction.

Let us now analyse the various forces which act on a body when it slides either up or down an inclined plane.

1. Considering the motion of the body up the plane:-

Let W = Weight of the body,

α = Angle of inclination of the plane to the horizontal,

ϕ = Limiting angle of friction for the contact surfaces,

P = Effort applied in a given direction in order to cause the body to slide with uniform velocity parallel to the plane, considering friction,

p_0 = Effort required to move the body up the plane neglecting friction,

θ = Angle which the line of action of P makes with the weight of the body W ,

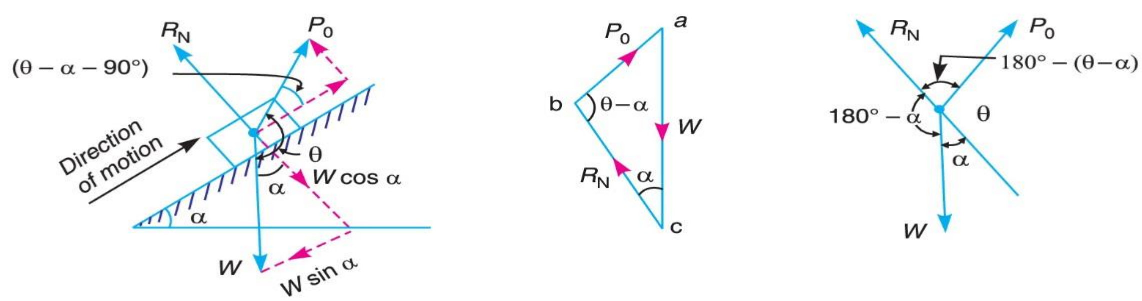
μ = Coefficient of friction between the surfaces of the plane and the body,

R_N = Normal reaction, and

R = Resultant reaction.

When the friction is neglected, the body is in equilibrium under the action of the three forces, i.e. p_0, W and R_N, as shown in Fig. 10.7 (a). The triangle of forces is shown in Fig. 10.7 (b). Now applying sine rule for these three concurrent forces,

$$\frac{P_0}{\sin\alpha} = \frac{W}{\sin(\theta - \alpha)} \quad \text{or} \quad {}^*P_0 = \frac{W \sin\alpha}{\sin(\theta - \alpha)} \qquad \ldots(i)$$

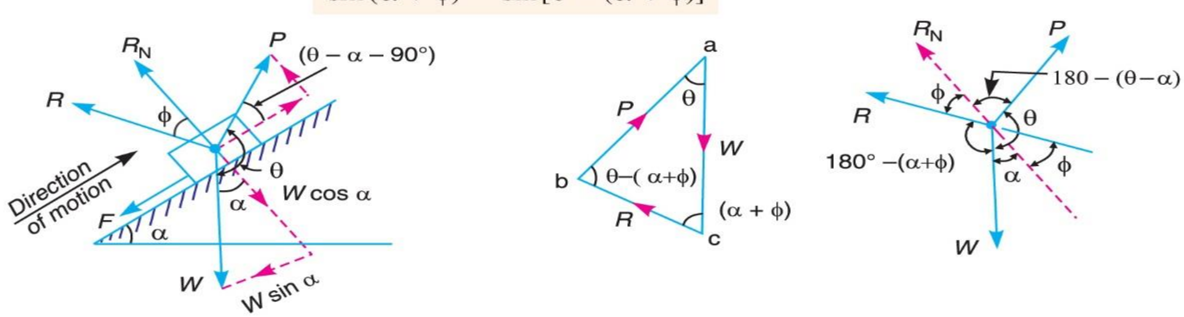


(a) (b) (c)

Fig. 10.7. Motion of the body up the plane, neglecting friction.

When friction is taken into account, a frictional force $F = \mu.R_N$ acts in the direction opposite to the motion of the body, as shown in Fig. 10.8 (*a*). The resultant reaction *R* between the plane and the body is inclined at an angle ϕ with the normal reaction R_N. The triangle of forces is shown in Fig. 10.8 (*b*). Now applying sine rule,

$$\frac{P}{\sin(\alpha + \phi)} = \frac{W}{\sin[\theta - (\alpha + \phi)]}$$



(a) (b) (c)

Fig. 10.8. Motion of the body up the plane, considering friction.

$$\therefore \qquad P = \frac{W \sin(\alpha + \phi)}{\sin[\theta - (\alpha + \phi)]} \qquad \text{...}(ii)$$

Notes : 1. When the effort applied is horizontal, then $\theta = 90°$. In that case, the equations (*i*) and (*ii*) may be written as

$$P_0 = \frac{W \sin \alpha}{\sin(90° - \alpha)} = \frac{W \sin \alpha}{\cos \alpha} = W \tan \alpha$$

and

$$P = \frac{W \sin(\alpha + \phi)}{\sin[90° - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos(\alpha + \phi)} = W \tan(\alpha + \phi)$$

2. When the effort applied is parallel to the plane, then $\theta = 90° + \alpha$. In that case, the equations (*i*) and (*ii*) may be written as

$$P_0 = \frac{W \sin \alpha}{\sin(90° + \alpha - \alpha)} = W \sin \alpha$$

and

$$P = \frac{W \sin(\alpha + \phi)}{\sin[(90° + \alpha) - (\alpha + \phi)]} = \frac{W \sin(\alpha + \phi)}{\cos \phi}$$

$$= \frac{W(\sin \alpha \cos \phi + \cos \alpha \sin \phi)}{\cos \phi} = W(\sin \alpha + \cos \alpha . \tan \phi)$$

$$= W(\sin \alpha + \mu \cos \alpha) \qquad \text{...}(\because \mu = \tan \phi)$$

2. *Considering the motion of the body down the plane*

Neglecting friction, the effort required for the motion down the plane will be same as for the motion up the plane, *i.e.*

$$P_0 = \frac{W \sin \alpha}{\sin(\theta - \alpha)} \qquad \text{...}(iii)$$

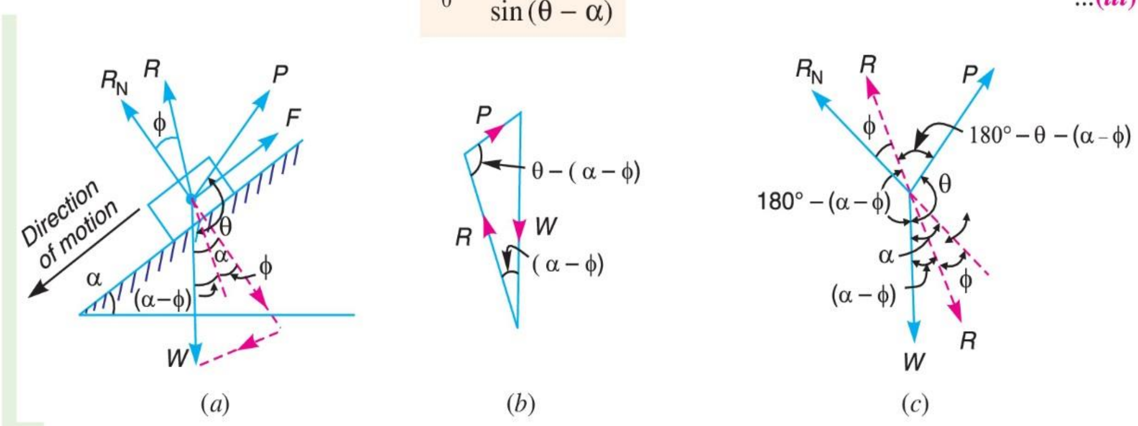


Fig. 10.9. Motion of the body down the plane, considering friction.

When the friction is taken into account, the force of friction $F = \mu . R_N$ will act up the plane and the resultant reaction R will make an angle ϕ with R_N towards its right as shown in Fig. 10.9 (*a*). The triangle of forces is shown in Fig. 10.9 (*b*). Now from sine rule,

$$\frac{P}{\sin(\alpha - \phi)} = \frac{W}{\sin[\theta - (\alpha - \phi)]}$$

or

$$P = \frac{W \sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \qquad \text{...}(iv)$$

Notes : 1. The value of P may also be obtained either by applying Lami's theorem to Fig. 10.9 (*c*), or by resolving the forces along the plane and perpendicular to the plane and then using $\Sigma H = 0$ and $\Sigma V = 0$ (See Art. 10.18 and 10.19).

2. When P is applied horizontally, then $\theta = 90°$. In that case, equation (*iv*) may be written as

$$P = \frac{W \sin(\alpha - \phi)}{\sin[90° - (\alpha - \phi)]} = \frac{W \sin(\alpha - \phi)}{\cos(\alpha - \phi)} = W \tan(\alpha - \phi))$$

3. When P is applied parallel to the plane, then $\theta = 90° + \alpha$. In that case, equation **(*iv*)** may be written as

$$P = \frac{W \sin(\alpha - \phi)}{\sin[90° + \alpha) - (\alpha - \phi)]} = \frac{W \sin(\alpha - \phi)}{\cos \phi}$$

$$= \frac{W(\sin\alpha\cos\phi - \cos\alpha\sin\phi)}{\cos\phi} = W(\sin\alpha - \tan\phi\cos\alpha)$$

$$= W(\sin\alpha - \mu\cos\alpha) \qquad \ldots(\because \tan\phi = \mu)$$

10.15. Efficiency of Inclined Plane

The ratio of the effort required neglecting friction (*i.e.* P_0) to the effort required considering friction (*i.e.* P) is known as efficiency of the inclined plane. Mathematically, efficiency of the inclined plane,

$$\eta = P_0 / P$$

Let us consider the following two cases :

1. *For the motion of the body up the plane*

Efficiency,
$$\eta = \frac{P_0}{P} = \frac{W\sin\alpha}{\sin(\theta - \alpha)} \times \frac{\sin[\theta - (\alpha + \phi)]}{W\sin(\alpha + \phi)}$$

$$= \frac{\sin\alpha}{\sin\theta\cos\alpha - \cos\theta\sin\alpha} \times \frac{\sin\theta\cos(\alpha + \phi) - \cos\theta\sin(\alpha + \phi)}{\sin(\alpha + \phi)}$$

Multiplying the numerator and denominator by $\sin(\alpha + \phi)\sin\theta$, we get

$$\eta = \frac{\cot(\alpha + \phi) - \cot\theta}{\cot\alpha - \cot\theta}$$

Notes : 1. When effort is applied horizontally, then $\theta = 90°$.

$$\therefore \qquad \eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

2. When effort is applied parallel to the plane, then $\theta = 90° + \alpha$.

$$\therefore \qquad \eta = \frac{\cot(\alpha + \phi) - \cot(90° + \alpha)}{\cot\alpha - \cot(90° + \alpha)} = \frac{\cot(\alpha + \phi) + \tan\alpha}{\cot\alpha + \tan\alpha} = \frac{\sin\alpha\cos\phi}{\sin(\alpha + \phi)}$$

2. *For the motion of the body down the plane*

Since the value of P will be less than P_0, for the motion of the body down the plane, therefore in this case,

$$\eta = \frac{P}{P_0} = \frac{W\sin(\alpha - \phi)}{\sin[\theta - (\alpha - \phi)]} \times \frac{\sin(\theta - \alpha)}{W\sin\alpha}$$

$$= \frac{\sin(\alpha - \phi)}{\sin\theta\cos(\alpha - \phi) - \cos\theta\sin(\alpha - \phi)} \times \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\sin\alpha}$$

Multiplying the numerator and denominator by $\sin(\alpha - \phi)\sin\theta$, we get

$$\eta = \frac{\cot\alpha - \cot\theta}{\cot(\alpha - \phi) - \cot\theta}$$

Notes : 1. When effort is applied horizontally, then $\theta = 90°$.

$$\therefore \qquad \eta = \frac{\cot\alpha}{\cot(\alpha - \phi)} = \frac{\tan(\alpha - \phi)}{\tan\alpha}$$

2. When effort is applied parallel to the plane, then $\theta = 90° + \alpha$.

$$\therefore \qquad \eta = \frac{\cot\alpha - \cot(90° + \alpha)}{\cot(\alpha - \phi) - \cot(90° + \alpha)} = \frac{\cot\alpha + \tan\alpha}{\cot(\alpha - \phi) + \tan\alpha} = \frac{\sin(\alpha - \phi)}{\sin\alpha\cos\phi}$$

Problem :- 2

2. An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12º, force acting parallel to the plane. If the angle of inclination is increased to 15º, then the effort required is 1720 N. Find the weight of the body and the coefficient of friction.

Solution. Given : $P_1 = 1500$ N ; $\alpha_1 = 12°$; $\alpha_2 = 15°$; $P_2 = 1720$ N

Let W = Weight of the body in newtons, and

μ = Coefficient of friction.

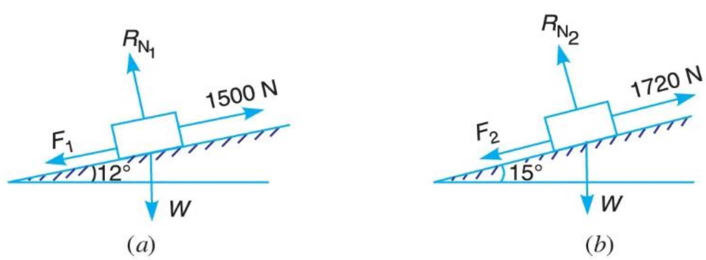


Fig. 10.10

First of all, let us consider a body lying on a plane inclined at an angle of 12° with the horizontal and subjected to an effort of 1500 N parallel to the plane as shown in Fig. 10.10 (*a*).

Let R_{N_1} = Normal reaction, and

F_1 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_1),

$$1500 = W(\sin\alpha_1 + \mu\cos\alpha_1) = W(\sin 12° + \mu\cos 12°) \qquad ...(i)$$

Now let us consider the body lying on a plane inclined at an angle of 15° with the horizontal and subjected to an effort of 1720 N parallel to the plane as shown in Fig. 10.10 (*b*).

Let R_{N_2} = Normal reaction, and

F_2 = Force of friction.

We know that for the motion of the body up the inclined plane, the effort applied parallel to the plane (P_2),

$$1720 = W(\sin\alpha_2 + \mu\cos\alpha_2) = W(\sin 15° + \mu\cos 15°) \qquad ...(ii)$$

Coefficient of friction

Dividing equation (*ii*) by equation (*i*),

$$\frac{1720}{1500} = \frac{W(\sin 15° + \mu\cos 15°)}{W(\sin 12° + \mu\cos 12°)}$$

$$1720 \sin 12° + 1720\,\mu \cos 12° = 1500 \sin 15° + 1500\,\mu \cos 15°$$

$$\mu\,(1720 \cos 12° - 1500 \cos 15°) = 1500 \sin 15° - 1720 \sin 12°$$

$$\therefore \qquad \mu = \frac{1500 \sin 15° - 1720 \sin 12°}{1720 \cos 12° - 1500 \cos 15°} = \frac{1500 \times 0.2588 - 1720 \times 0.2079}{1720 \times 0.9781 - 1500 \times 0.9659}$$

$$= \frac{388.2 - 357.6}{1682.3 - 1448.5} = \frac{30.6}{233.8} = 0.131 \textbf{ Ans.}$$

Weight of the body

Substituting the value of μ in equation **(*i*)**,

$$1500 = W\,(\sin 12° + 0.131 \cos 12°)$$

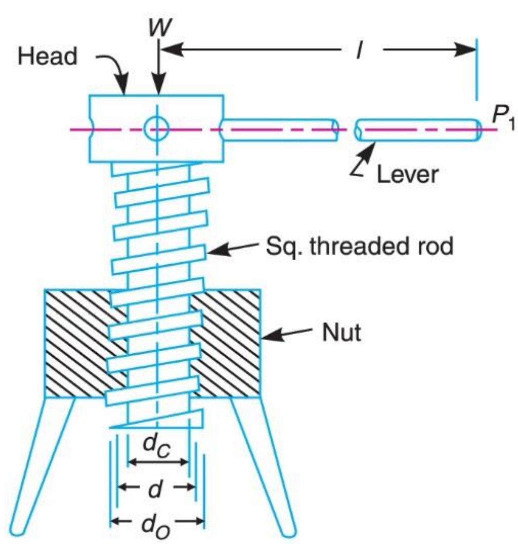
$$= W\,(0.2079 + 0.131 \times 0.9781) = 0.336\,W$$

$$\therefore \qquad W = 1500/0.336 = 4464 \text{ N } \textbf{Ans.}$$

Screw jack :-

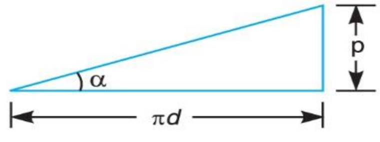
The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings.

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works is similar to that of an inclined plane.

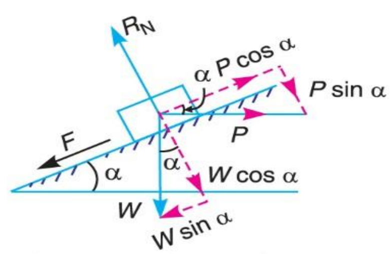


(*a*) Screw jack.

Torque Required to Lift the Load by a Screw Jack :-



(*a*) Development of a screw.



(*b*) Forces acting on the screw.

Let p = Pitch of the screw, d = Mean diameter of the screw, α = Helix angle, P = Effort applied at the circumference of the screw to lift the load, W = Load to be lifted, and μ = Coefficient of friction, between the screw and nut = tan ϕ, where ϕ is the friction angle.

From the geometry of the Fig (a), we find that

$$\tan \alpha = p/\pi d$$

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . R_N \ldots \quad \text{(i)}$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \ldots \quad \text{(ii)},$$

Substituting this value of R_N in equation (i),

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$P \cos \alpha = W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

$$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

$$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by cos ϕ,

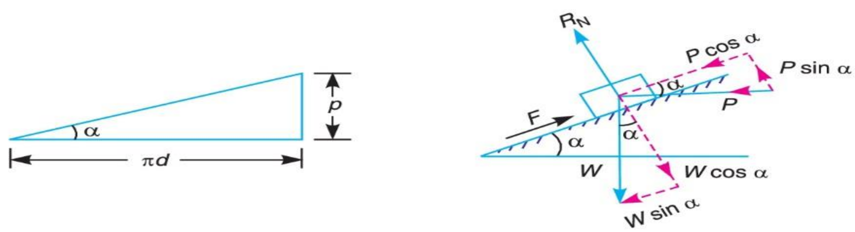
$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$$= W \tan (\alpha + \phi)$$

∴ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

Torque Required to Lower the Load by a Screw Jack :-



(a) (b)

$$P = W \tan(\phi - \alpha)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha)\,\frac{d}{2}$$

Efficiency of a Screw Jack :-

The efficiency of a screw jack may be defined as the ratio between the ideal effort (i.e. the effort required to move the load, neglecting friction) to the actual effort (i.e. the effort required to move the load taking friction into account).

We know that the effort required to lift the load (W) when friction is taken into account,

$$P = W \tan(\alpha + \phi) \qquad ...(i)$$

where α = Helix angle,

ϕ = Angle of friction, and

μ = Coefficient of friction, between the screw and nut = $\tan \phi$.

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W \tan\alpha \qquad \text{[}i.e.\text{ Putting } \phi = 0 \text{ in equation } (i)\text{]}$$

$$\therefore \text{ Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan\alpha}{W \tan(\alpha+\phi)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

Problem:-3

3. A 150 mm diameter valve, against which a steam pressure of 2 MN/m2 is acting, is closed by means of a square threaded screw 50 mm in external diameter with 6 mm pitch.If the coefficient of friction is 0.12 find the torque required to turn the handle.

Solution. Given : D = 150 mm = 0.15 mm = 0.15 m ; Ps = 2 MN/m² = 2 × 10⁶ N/m² ; d_0 = 50 mm ; p = 6 mm ; $\mu = \tan\phi = 0.12$

We know that load on the valve,

$$W = \text{Pressure} \times \text{Area} = p_S \times \frac{\pi}{4} D^2 = 2 \times 10^6 \times \frac{\pi}{4}(0.15)^2 \text{ N}$$

$$= 35\,400 \text{ N}$$

Mean diameter of the screw,

$$d = d_0 - p/2 = 50 - 6/2 = 47 \text{ mm} = 0.047 \text{ m}$$

$$\therefore \qquad \tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 47} = 0.0406$$

We know that force required to turn the handle,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 35400\left[\frac{0.0406 + 12}{1 - 0.0406 \times 0.12}\right] = 5713 \text{ N}$$

$\therefore$ Torque required to turn the handle,

$$T = P \times d/2 = 5713 \times 0.047/2 = 134.2 \text{ N-m} \quad \textbf{Ans.}$$

4. The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $\mu = \tan\phi = 0.13$; $W = 25$ kN $= 25 \times 10^3$ N

We know that, $\tan\alpha = \dfrac{p}{\pi\, d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and force required on the screw to raise the load,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13}\right] = 5305 \text{ N}$$

Torque required on the screw

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 5305 \times 50/2 = 132\,625 \text{ N-mm} \quad \textbf{Ans.}$$

Ratio of the torques required to raise and lower the load

We know that the force required on the screw to lower the load,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 25 \times 10^3\left[\frac{0.13 + 0.08}{1 + 0.13 \times 0.08}\right] = 1237 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1237 \times 50/2 = 30\,905 \text{ N-mm}$$

$\therefore$ Ratio of the torques required,

$$= T_1/T_2 = 132625/30925 = 4.3 \quad \textbf{Ans.}$$

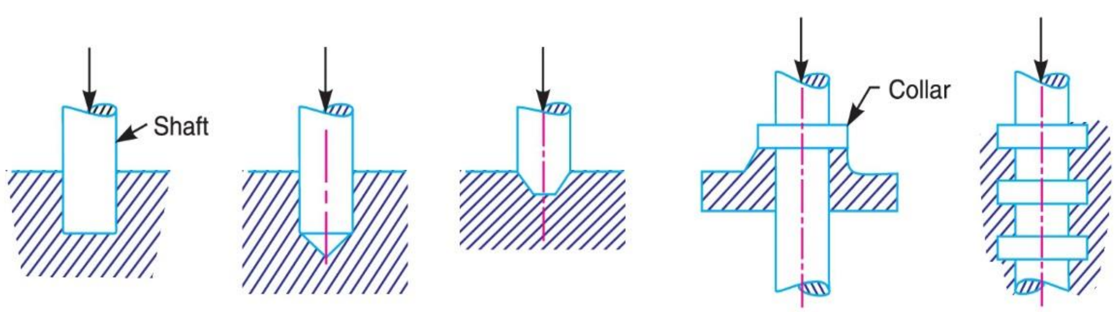
Efficiency of the machine

We know that the efficiency,

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\tan\alpha(1 - \tan\alpha.\tan\phi)}{\tan\alpha + \tan\phi} = \frac{0.08(1 - 0.08 \times 0.13)}{0.08 + 0.13}$$

$$= 0.377 = 37.7\% \quad \textbf{Ans.}$$

Friction of Pivot and Collar Bearing :-



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.

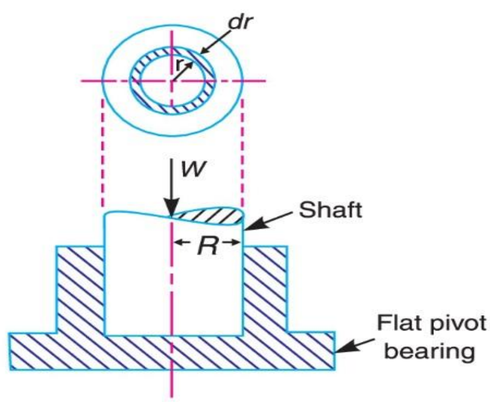
The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Fig (a) and (b) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig (c).

If the load or thrust force is bearded by a flat plate or collar which is attached on the shaft is known as collar bearing. If one collar is used then it is known as single flat collar bearing. If more than one collar used it is known as multiple collar bearing.

There are two cases two understand the friction in the above all possibilities.

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

Flat Pivot Bearing:-



Flat pivot or footstep bearing.

When a vertical shaft rotates in a flat pivot bearing (known as foot step bearing), as shown in Fig, the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W = Load transmitted over the bearing surface, R = Radius of bearing surface, p = Intensity of pressure per unit area of bearing surface between rubbing surfaces, and μ = Coefficient of friction.

We will consider the following two cases :

1. When there is a uniform pressure; and 2. When there is a uniform wear.

1. Considering uniform pressure :-

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area.

$\therefore$ Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\,\pi\, r.dr \qquad ...(i)$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu\, p \times 2\pi\, r.dr = 2\pi\, \mu.p.r.dr$$

$\therefore$ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi\, \mu\, p\, r.dr \times r = 2\,\pi\,\mu\, p\, r^2\, dr \qquad ...(ii)$$

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi\mu\, p\, r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad ...\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi\, N/60 \qquad ...(\because\ \omega = 2\pi N/60)$$

where N = Speed of shaft in r.p.m.

Considering uniform wear :-

We have already discussed that the rate of wear depends upon the intensity of pressure (p) and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (i.e. p.v..). Since the velocity of rubbing surfaces increases with the distance (i.e. radius r) from the axis of the bearing, therefore for uniform wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C/r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad ...[\text{From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\, dr = 2\pi\mu \times \frac{C}{r} \times r^2\, dr \qquad ...\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\, dr \qquad ...(iii)$$

$\therefore$ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\,\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \ldots\left(\because\ C = \frac{W}{2\pi R}\right)$$

Conical Pivot Bearing :-

The conical pivot bearing supporting a shaft carrying a load W is shown in Fig. 10.18.

Let P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing, and

R = Radius of the shaft.

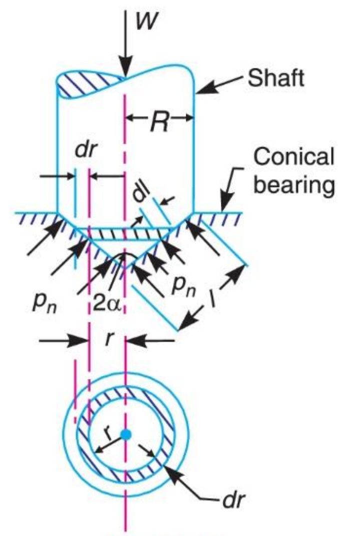


Fig. 10.18.
Conical pivot bearing.

Consider a small ring of radius r and thickness dr. Let dl is the length of ring along the cone, such that

$$dl = dr \operatorname{cosec} \alpha$$

$\therefore$ Area of the ring,

$$A = 2\pi r.dl = 2\pi r.dr \operatorname{cosec} \alpha \qquad \ldots(\because\ dl = dr \operatorname{cosec} \alpha)$$

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area}$$

$$= p_n \times 2\pi r.dr \operatorname{cosec} \alpha$$

and vertical load acting on the ring,

$$^*\delta W = \text{Vertical component of } \delta W_n = \delta W_n.\sin\alpha$$

$$= p_n \times 2\pi r.dr \operatorname{cosec} \alpha.\ \sin\alpha = p_n \times 2\pi\ r.dr$$

$\therefore$ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\,p_n\left[\frac{r^2}{2}\right]_0^R = 2\pi\,p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

or $$p_n = W / \pi R^2$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n.2\pi r.dr \operatorname{cosec} \alpha = 2\pi\mu.p_n.\operatorname{cosec} \alpha.r.dr$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = 2\pi\mu.p_n.\operatorname{cosec} \alpha.r.dr \times r = 2\pi\mu.p_n \operatorname{cosec}\alpha.r^2.dr$$

Integrating the expression within the limits from 0 to R for the total frictional torque on the conical pivot bearing.

∴ Total frictional torque,

$$T = \int_0^R 2\,\pi\,\mu.p_n \text{ cosec } \alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec } \alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec } \alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec } \alpha \qquad ...(i)$$

Substituting the value of p_n in equation (i),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec } \alpha = \frac{2}{3} \times \mu.W.R.\text{ cosec } \alpha$$

Note : If slant length (l) of the cone is known, then

$$T = \frac{2}{3} \times \mu.W.l \qquad ...(\because\ l = R \text{ cosec } \alpha)$$

2. *Considering uniform wear*

In Fig. 10.18, let p_r be the normal intensity of pressure at a distance r from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

∴ $p_r.r = C$ (a constant) or $p_r = C/r$

and the load transmitted to the ring,

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C\ [r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu.p_r.\text{cosec } \alpha.r^2.dr = 2\pi\mu \times \frac{C}{r} \times \text{cosec}\,\alpha.r^2.dr$$

$$= 2\pi\mu.C.\text{cosec } \alpha.r.dr$$

∴ Total frictional torque acting on the bearing,

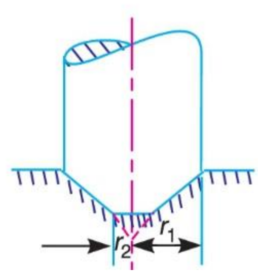
$$T = \int_0^R 2\pi\ \mu.C.\text{cosec } \alpha.r.dr = 2\pi\mu.C.\text{cosec } \alpha\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\ \mu.C.\text{cosec } \alpha \times \frac{R^2}{2} = \pi\mu.C.\text{cosec } \alpha.R^2$$

Substituting the value of C, we have

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec } \alpha.R^2 = \frac{1}{2} \times \mu.W.R \text{ cosec } \alpha = \frac{1}{2} \times \mu.W.l$$

Trapezoidal or Truncated Conical Pivot Bearing :-



Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

∴ Intensity of uniform pressure,

$$p_n = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad ...(i)$$

1. *Considering uniform pressure*

The total torque acting on the bearing is obtained by integrating the value of T_r (as discussed in Art. 10.27) within the limits r_1 and r_2.

∴ Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu . p_n \text{ cosec } \alpha . r^2 . dr = 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{r^3}{3} \right]_{r_2}^{r_1}$$

$$= 2\pi\mu . p_n . \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

Substituting the value of p_n from equation ***(i)***,

$$T = 2\pi . \mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3} \right]$$

$$= \frac{2}{3} \times \mu . W . \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

2. *Considering uniform wear*

We have discussed in Art. 10.26 that the load transmitted to the ring,

$$\delta W = 2\pi C . dr$$

∴ Total load transmitted to the ring,

$$W = \int_{r_2}^{r_1} 2\pi \, C . dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

or

$$C = \frac{W}{2\pi (r_1 - r_2)} \qquad \text{...}(ii)$$

We know that the torque acting on the ring, considering uniform wear, is

$$T_r = 2\pi \, \mu . C \text{ cosec } \alpha . r . dr$$

∴ Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi \, \mu . C \text{ cosec } \alpha . r . dr = 2\pi \, \mu . C . \text{cosec } \alpha \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= \pi \, \mu . C . \text{cosec } \alpha \left[(r_1)^2 - (r_2)^2 \right]$$

Substituting the value of C from equation ***(ii)***, we get

$$T = \pi\mu \times \frac{W}{2\pi (r_1 - r_2)} \times \text{cosec } \alpha \, [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu . W (r_1 + r_2) \text{ cosec } \alpha = \mu . W . R \text{ cosec } \alpha$$

where R = Mean radius of the bearing $= \dfrac{r_1 + r_2}{2}$

Problem :- 5

5. A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution. Given : $W = 20$ kN $= 20 \times 10^3$ N ; $2\alpha = 120°$ or $\alpha = 60°$; $p_n = 0.3$ N/mm^2 ; $N = 200$ r.p.m. or $\omega = 2\pi \times 200/60 = 20.95$ rad/s ; $\mu = 0.1$

Outer and inner radii of the bearing surface

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm.

Since the external diameter is twice the internal diameter, therefore

$$r_1 = 2\,r_2$$

We know that intensity of normal pressure (p_n),

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$\therefore \quad (r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3$ or $r_2 = 84$ mm **Ans.**

and $\quad r_1 = 2\,r_2 = 2 \times 84 = 168$ mm **Ans.**

Power absorbed in friction

We know that total frictional torque (assuming uniform pressure),

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec}\,60° = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

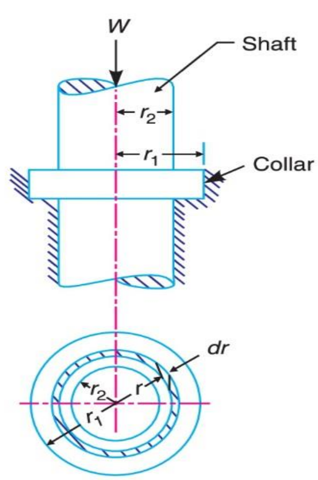
$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

$\therefore$ Power absorbed in friction,

$$P = T.\omega = 301.76 \times 20.95 = 6322 \text{ W} = 6.322 \text{ kW}$$ **Ans.**

Flat Collar Bearing :-

The collar bearings are also known as thrust bearings.



Consider a single flat collar bearing supporting a shaft as shown in Fig.

Let r_1 = External radius of the collar, and r_2 = Internal radius of the collar.

∴ Area of the bearing surface,

$$A = \pi\,[(r_1)^2 - (r_2)^2]$$

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \text{...}(i)$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation ***(i)***,

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering unifrom wear*

We have seen in Art. 10.25 that the load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \text{...}(ii)$$

We also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

∴ Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation ***(ii)***,

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

Problem:-6

6. A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine 90 r.p.m., find the power absorbed in friction at the thrust block, assuming l. uniform pressure ; and 2. uniform wear.

Solution. Given : $n = 6$; $d_1 = 600$ mm or $r_1 = 300$ mm ; $d_2 = 300$ mm or $r_2 = 150$ mm ; $W = 100$ kN $= 100 \times 10^3$ N ; $\mu = 0.12$; $N = 90$ r.p.m. or $\omega = 2\pi \times 90/60 = 9.426$ rad/s

1. Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

$$T = \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

$$= \frac{2}{3} \times 0.12 \times 100 \times 10^3 \left[\frac{(300)^3 - (150)^3}{(300)^2 - (150)^2} \right] = 2800 \times 10^3 \text{ N-mm}$$

$$= 2800 \text{ N-m}$$

$\therefore$ Power absorbed in friction,

$$P = T.\omega = 2800 \times 9.426 = 26\,400 \text{ W} = 26.4 \text{ kW} \quad \textbf{Ans.}$$

2. Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3\ (300 + 150) \text{ N-mm}$$

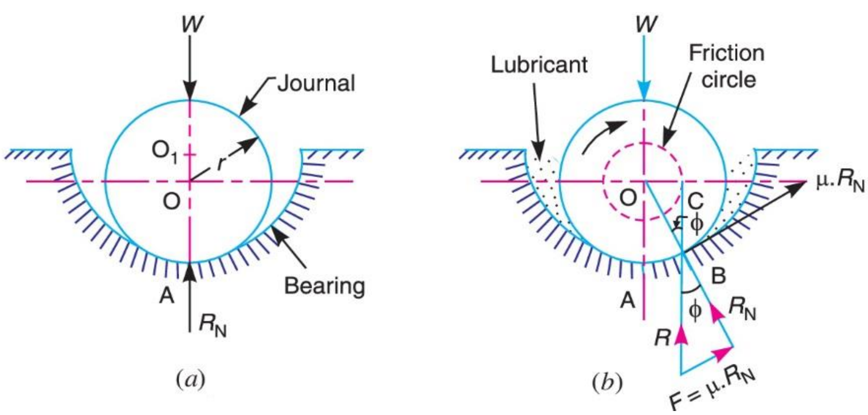
$$= 2700 \times 10^3 \text{ N-mm} = 2700 \text{ N-m}$$

$\therefore$ Power absorbed in friction,

$$P = T.\omega = 2700 \times 9.426 = 25\,450 \text{ W} = 25.45 \text{ kW} \quad \textbf{Ans.}$$

Friction in Journal Bearing-Friction Circle :-

A journal bearing forms a turning pair as shown in Fig (a). The fixed outer element of a turning pair is called a bearing and that portion of the inner element (i.e. shaft) which fits in the bearing is called a journal.



(a) (b)

Let ϕ = Angle between R (resultant of F and R_N) and R_N, μ = Coefficient of friction between the journal and bearing, T = Frictional torque in N-m, and r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin \phi = W.r \sin \phi$$

Since ϕ is very small, therefore substituting $\sin \phi = \tan \phi$

$$\therefore \quad T = W.r \tan \phi = \mu.W.r \qquad \ldots(\because \mu = \tan \phi)$$

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

where N = Speed of the shaft in r.p.m.

Notes : **1.** If a circle is drawn with centre O and radius $OC = r \sin \phi$, then this circle is called the ***friction circle*** of a bearing.

2. The force R exerted by one element of a turning pair on the other element acts along a tangent to the friction circle.

Example 10.15. *A 60 mm diameter shaft running in a bearing carries a load of 2000 N. If the coefficient of friction between the shaft and bearing is 0.03, find the power transmitted when it runs at 1440 r.p.m.*

Solution. Given : $d = 60$ mm or $r = 30$ mm $= 0.03$ m ; $W = 2000$ N ; $\mu = 0.03$; $N = 1440$ r.p.m. or $\omega = 2\pi \times 1440/60 = 150.8$ rad/s

We know that torque transmitted,

$$T = \mu.W.r = 0.03 \times 2000 \times 0.03 = 1.8 \text{ N-m}$$

$\therefore$ Power transmitted, $P = T.\omega = 1.8 \times 150.8 = 271.4$ W **Ans.**