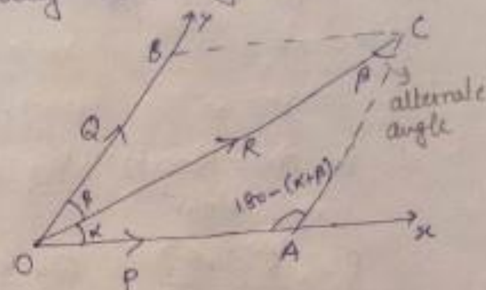
Resolution of Forces System

Resolution of Forces :-

Finding the components of a given force in a two given directions is called resolution. These component forces will have the same effect on the body as the given single force.

$\angle BOC = \angle OCA = \beta$



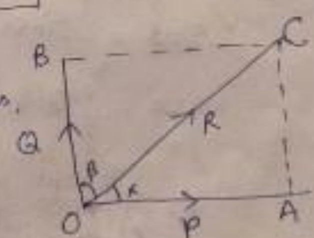
Apply Sine rule ΔOAC

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin\{180-(\alpha+\beta)\}}$$

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin(\alpha+\beta)}$$

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

$$\boxed{P = \frac{R\sin\beta}{\sin(\alpha+\beta)}} \quad , \quad \boxed{Q = \frac{R\sin\alpha}{\sin(\alpha+\beta)}}$$



If R is to be resolved along ⊥ directions,

$\alpha+\beta = 90^\circ \Rightarrow \beta = 90^\circ-\alpha$

$P = R\sin\beta \quad , \quad Q = R\sin\alpha$

$P = R\sin(90-\alpha)$

$\boxed{P = R\cos\alpha} \quad \boxed{Q = R\sin\alpha}$

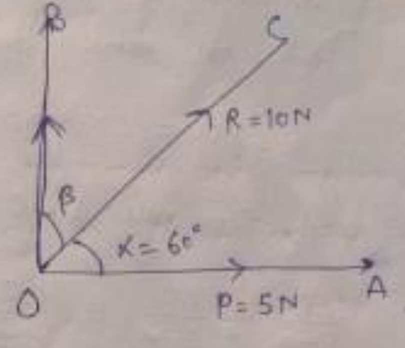
Numerical :- The resultant of two forces is 10N and its is inclined at 60° to one of the forces whose magnitude is 5N. Determine the magnitude and direction of the other force

Sol^n :- $OA = P = 5N$

$OB =$

$\alpha = 60°$

$OC = R = 10\,N$



$$P = \frac{R \sin\beta}{\sin(\alpha+\beta)} \qquad Q = \frac{R \sin\alpha}{\sin(\alpha+\beta)}$$

$$\cancel{5} = \frac{\cancel{10}^{\,2} \sin\beta}{\sin(60°+\beta)}$$

$$\cancel{2 \sin\beta = \sin 60° \cos\beta + \cos 60° \sin\beta}$$

$$\cancel{= 0.866\ \beta}$$

$$2 \sin\beta = \sin 60 \cos\beta + \cos 60° \sin\beta$$

$$2 \sin\beta - \frac{1}{2} \sin\beta = 0.866 \cos\beta$$

$$1.5 \sin\beta = 0.866 \cos\beta$$

$$\tan\beta = \frac{0.866}{1.5} \Rightarrow \beta = \tan^{-1}(0.5773) = 30°$$

$$\boxed{\beta = 30°}$$

$$Q = \frac{R \sin\alpha}{\sin(\alpha+\beta)}$$

$$Q = \frac{10 \sin 60}{\sin(60+30°)} = 10 \times 0.866 = 8.66\,N$$

$$\boxed{Q = 8.66\,N}$$

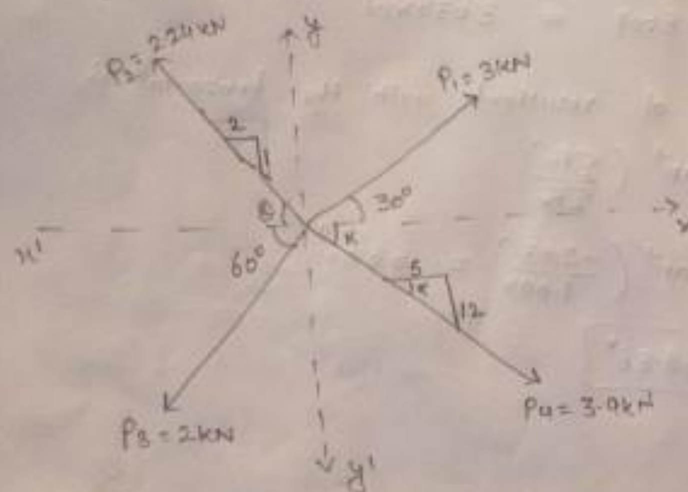
Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

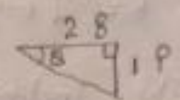
and its inclination θ to x-axis is given by

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Numerical

Determine the resultant both in magnitude and direction, of the four forces acting on the body as shown in the figure given below





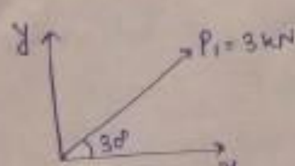
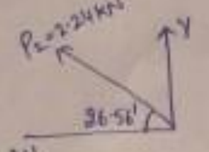
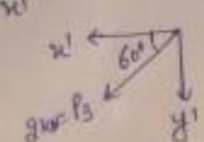
PBP
HHB

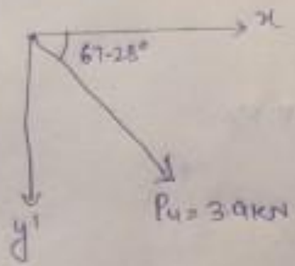
$\tan\theta = \frac{1}{2}$

$\tan\alpha = \frac{12}{5} = 67.28°$

Inclination of force 2.24kN with $Ox' = \tan^{-1}\left(\frac{1}{2}\right) = 26.56°$

Inclination of force 3.9 kN with $Ox = \tan^{-1}\left(\frac{12}{5}\right) = 67.28°$

	Horizontal Component F_x	Vertical Component F_y
	$3\cos 30°$	$3\sin 30°$
	$-2.24\cos 26.56$	$2.24\sin 26.56°$
	$-2\cos 60°$	$-2\sin 60°$



	F_x	F_y
	$3.9 \cos 67.28^\circ$	$-3.9 \sin 67.28^\circ$

Magnitude of the resultant force is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(1.094)^2 + (-2.83)^2}$$

$$R = \sqrt{1.197 + 8.009} = 3.034\,kN$$

and the Inclination of resultant with the horizontal is

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$\alpha = \tan^{-1}\left(\frac{-2.83}{1.094}\right) = 68.86^\circ$$

$$\boxed{\alpha = 68.86^\circ}$$