Applying sine rule for the triangle ABC

$$\frac{AB}{\sin(\pi-\alpha)}=\frac{BC}{\sin(\pi-\beta)}=\frac{CA}{\sin(\pi-\gamma)}$$

$$\therefore \quad \frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$

It would be appropriate to mention that Lami's theorem can be used to resolve a given force along two lines not perpendicular to each other. With reference to Fig. 2.61. P_1 and P_2 are the components of force P and these components make angles α and β respectively on either side of P.

Then $P_1=\dfrac{P\sin\beta}{\sin(\alpha+\beta)}$ and $P_2=\dfrac{P\sin\alpha}{\sin(\alpha+\beta)}$

However, for the validity of Lami's theorem to hold good,

- the forces keep the body in equilibrium
- the three forces acting on the body are non-parallel
- the forces are concurrent, *i.e.*, act at a point on the body
- the forces are either directed towards or away from the point of concurrence Fig. 2.62
- the angle between any two forces is less than 180°.

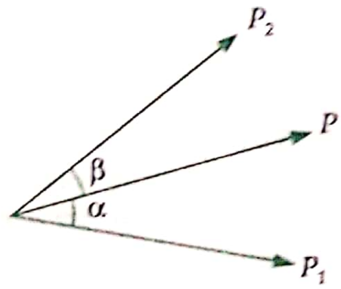


Fig. 2.61

It needs to be noted that the Lami's theorem is not meant to find the resultant of the three concurrent forces. It is only to provide the relation between a force and the angle between the other two forces.

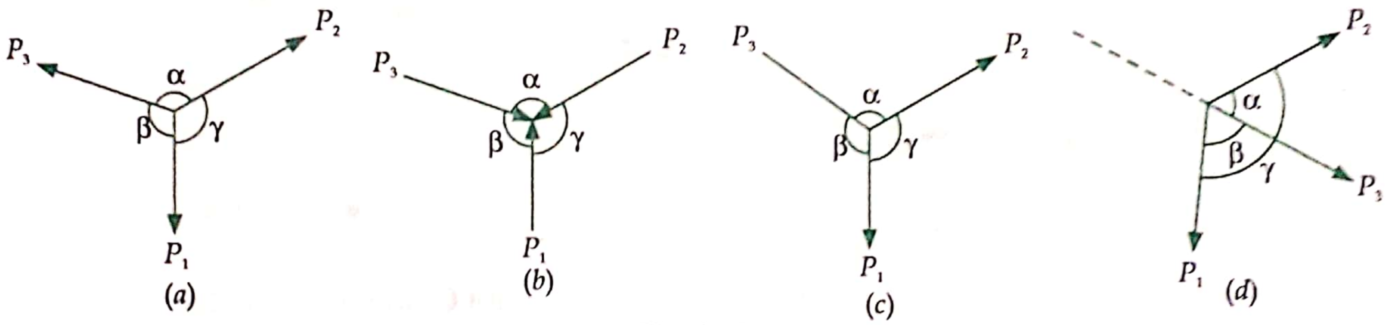


Fig. 2.62

To make the Lami's theorem applicable to the force system depicted in Fig. 2.62 *c*, the orientation of the forces is put as shown in Fig 2.62 *d*.

EXAMPLE 2.41

A weight of 2000 N is supported by two chains AC and BC as shown in Fig. 2.63. Determine the tension in each chain.

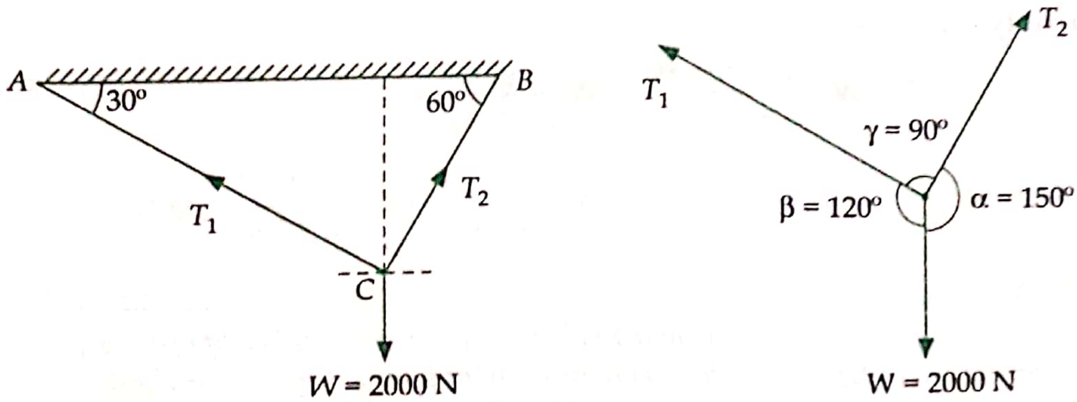


Fig. 2.63

Solution : Let T_1 and T_2 be the tensions in chains AC and BC respectively. Since the lines of action of these tensions and the weight W meet at a point, Lami's theorem can be applied. That is

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\gamma}$$

or
$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{2000}{\sin 90^\circ}$$

$\therefore$
$$T_1 = 2000 \times \frac{\sin 150^\circ}{\sin 90^\circ} = 2000 \times \frac{0.5}{1} = 1000 \text{ N}$$

$$T_2 = 2000 \times \frac{\sin 120^\circ}{\sin 90^\circ} = 2000 \times \frac{0.866}{1} = 1732 \text{ N}$$

EXAMPLE 2.42

A string 2 m long is tied to the ends of a uniform rod that weighs 60 N and is 1.6 m long. The string passes over a nail, so that the rod hangs horizontally. Make calculations for the tension in the string.

Solution :

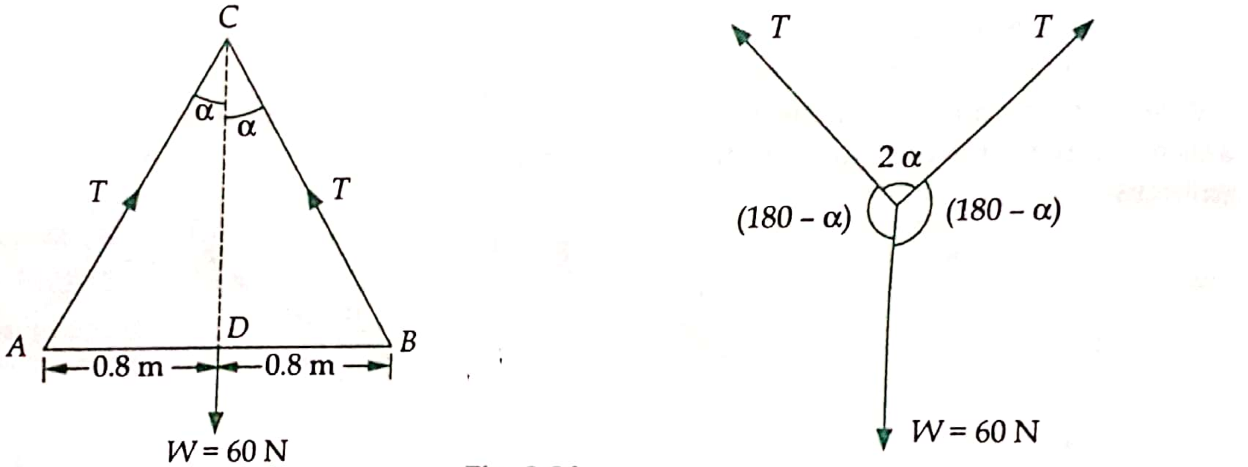


Fig. 2.64.

The lines of action of tensions T, T and weight W meet at point C, and therefore Lami's theorem is applicable. That is

$$\frac{W}{\sin 2\alpha} = \frac{T}{\sin(180-\alpha)} = \frac{T}{\sin(180-\alpha)} \qquad ...(i)$$

From the geometry of the arrangement

$$AC = BC = 1 \text{ m}; \quad CD = \sqrt{1^2 - 0.8^2} = 0.6 \text{ m}$$

$$\sin\alpha = 0.8 \quad \text{and} \quad \cos\alpha = 0.6$$

From identity (*i*), we get

$$T = W \frac{\sin(180-\alpha)}{\sin 2\alpha} = W \frac{\sin\alpha}{2\sin\alpha\cos\alpha}$$

$$= W \times \frac{1}{\cos\alpha} = 60 \times \frac{1}{2 \times 0.6} = 50 \text{ N}$$

EXAMPLE 2.43

In a jib crane, the jib and the tie rod are 5 m and 4 m long respectively. The height of crane post is 3 m and the tie rod remains horizontal. Determine the forces produced in the jib and tie rod when a load of 2 kN is suspended at the crane head.

Solution : Refer Fig. 2.65 *a* for the arrangement of the system.

$$\sin\theta = \frac{3}{5} = 0.6\,; \quad \theta = 36.87^\circ$$

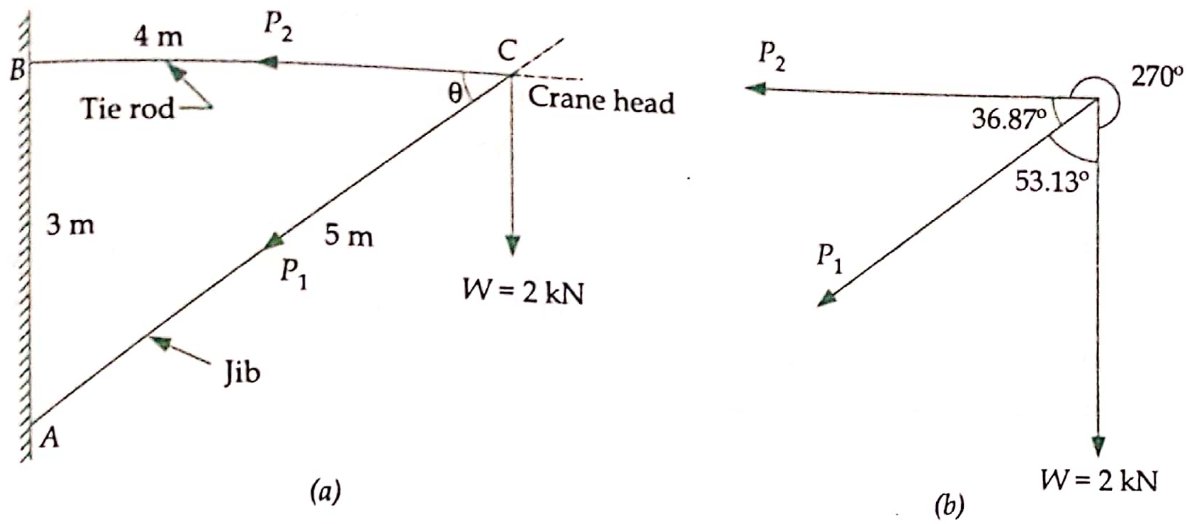


Fig. 2.65

Let P_1 and P_2 be the forces developed in the jib and tie rod respectively. The three forces P_1, P_2 and W are shown in Fig. 2.65 *b* with the angles between the forces calculated from the given directions. The lines of action of forces P_1, P_2 and weight W meet at the point C, and therefore Lami's theorem is applicable. That gives

$$\frac{P_1}{\sin 270^\circ} = \frac{P_2}{\sin 53.13^\circ} = \frac{2}{\sin 36.87^\circ}$$

$$\therefore \quad P_1 = 2 \times \frac{\sin 270^\circ}{\sin 36.87^\circ} = 2 \times \frac{1}{0.6} = -3.33 \text{ kN}$$

$$P_2 = 2 \times \frac{\sin 53.13^\circ}{\sin 36.87^\circ} = 2 \times \frac{0.8}{0.6} = 2.667 \text{ kN}$$

The – ve sign indicates that the direction of force P_1 is opposite to that shown in Fig. 2.71 (*a*). Obviously the tie rod will be under tension and the jib will in compression.

EXAMPLE 2.44

A machine weighing 5 kN is supported by two chains attached to some point on the machine. One chain goes to the hook in the ceiling and has an indication of 45° with the horizontal. The other chain goes to the eye bolt in the wall and is inclined at 30° to the horizontal. Make calculations for the tensions induced in the chain.

Solution : Refer Fig. 2.66 for the arrangement. The machine is acted upon by the following set of forces:

(*i*) weight of machine W = 5 kN acting vertically downwards,

(*ii*) tension T_1 in the chain OA which goes to hook in the ceiling,

(*iii*) tension T_2 in the chain OB which goes to the eye bolt.

These forces are concurrent and meet at point O. Applying Lami's theorem

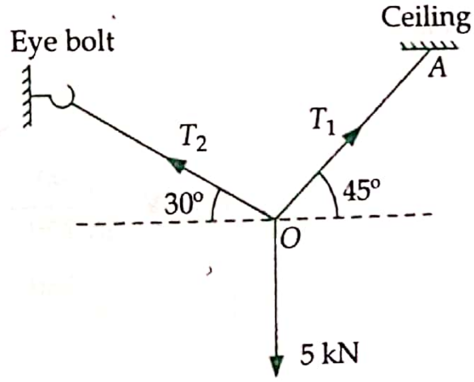


Fig. 2.66.

$$\frac{T_1}{\sin(90+30)} = \frac{T_2}{\sin(90+45)} = \frac{W}{\sin(180-30-45)}$$

or $$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{W}{\sin 105^\circ}$$

$$\therefore \quad T_1 = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 5 \times \frac{0.866}{0.966} = 4.48 \text{ kN}$$

$$T_2 = W \times \frac{\sin 135}{\sin 105} = 5 \times \frac{0.707}{0.966} = 3.66 \text{ N}$$

EXAMPLE 2.45

A smooth sphere of radius 15 cm and weight 2 N is supported in contact with a smooth vertical wall by a string whose length equals the radius of sphere. The string joins a point on the wall and a point on the surface of sphere. Workout inclination and the tension in the string and reaction of the wall.

Solution : Refer. Fig. 2.67 for the arrangement.

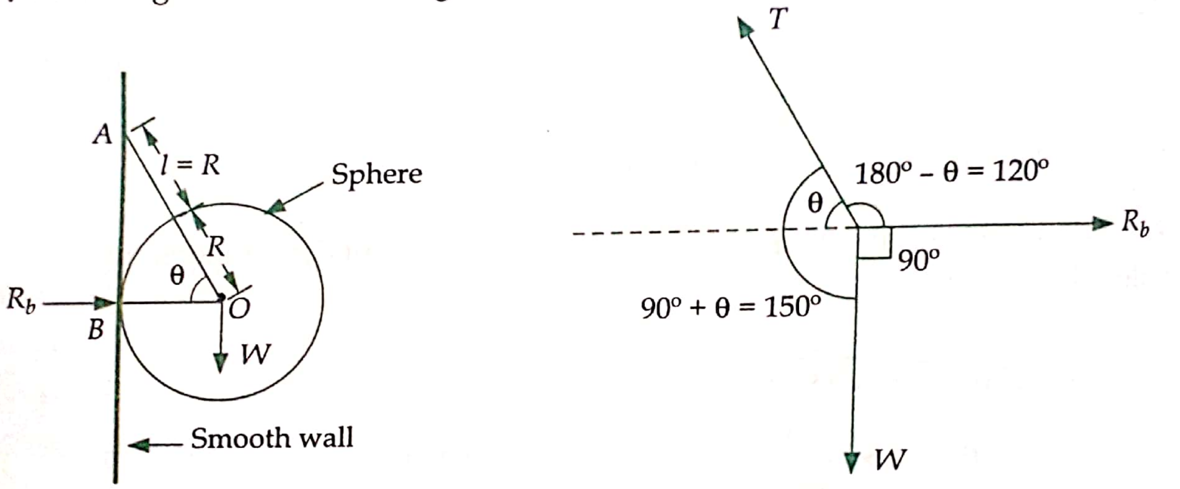


Fig. 2.67

In triangle AOB,

$$\cos\theta = \frac{OB}{OA} = \frac{R}{2R} = \frac{1}{2}; \quad \theta = 60^\circ$$

The sphere is in equilibrium under the action of following forces:

(*i*) Weight $W = 2$ N of the sphere which acts vertically downwards through the centre O.

(*ii*) Tension T in the string

(*iii*) Reaction R_b of the wall at the point of contact B. Since the wall is smooth, this reaction acts perpendicular to the wall.

These three forces are concurrent, *i.e.*, meet at point O and as such the Lami's theorem is applicable.

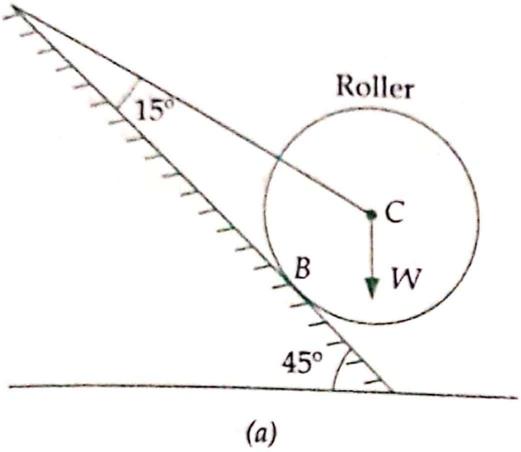
$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{R_b}{\sin 150^\circ}$$

$$\therefore \quad T = W\frac{\sin 90^\circ}{\sin 120^\circ} = 2 \times \frac{1}{0.866} = 2.31 \text{ N}$$

$$R_b = W\frac{\sin 150^\circ}{\sin 120^\circ} = 2 \times \frac{0.5}{0.866} = 1.15 \text{ N}$$

EXAMPLE 2.46

A roller of weight 500 N rests on a smooth inclined plane and is kept free from rolling down by a string as shown in Fig. 2.68 a. Work out tension in the string and reaction at the point of contact B.



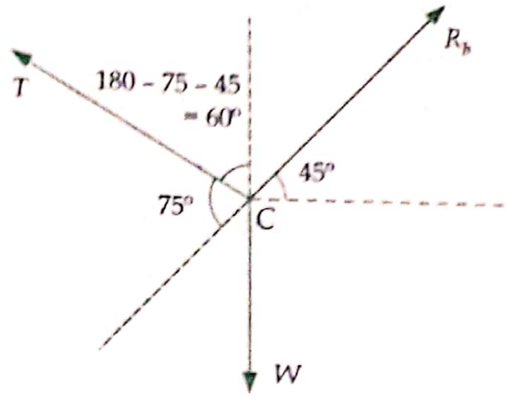


Fig. 2.68

Solution : The lines of action for tension T, weight W and reaction R_b at the contact point meet at C (the centre of the roller) and as such Lami's theorem is applicable. The angles between various segments around point C are as indicated in Fig. 2.68 b.

Invoking Lami's theorem,

$$\frac{W}{\sin(60+45)} = \frac{R_b}{\sin(75+45)} = \frac{T}{\sin(90+45)}$$

$$\therefore \quad R_b = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 500 \times \frac{0.866}{0.966} = 448.24 \text{ N}$$

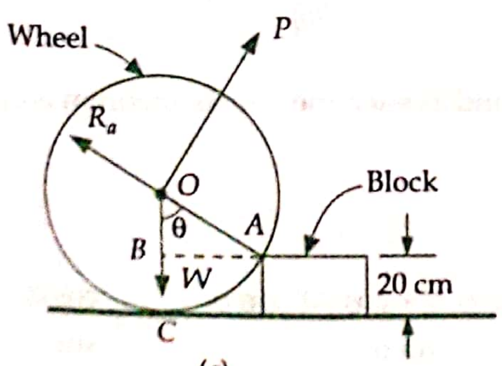
$$T = W \times \frac{\sin 135^\circ}{\sin 105^\circ} = 500 \times \frac{0.707}{0.966} = 365.94 \text{ N}$$

EXAMPLE 2.47

A uniform wheel of 50 cm diameter and 1 kN weight rests against a rigid rectangular block of thickness 20 cm (Fig. 2.69 a). Considering all surfaces smooth, determine :

(a) *least pull to be applied through the centre of wheel to just turn it over the corner of the block,*

(b) *reaction of the block*



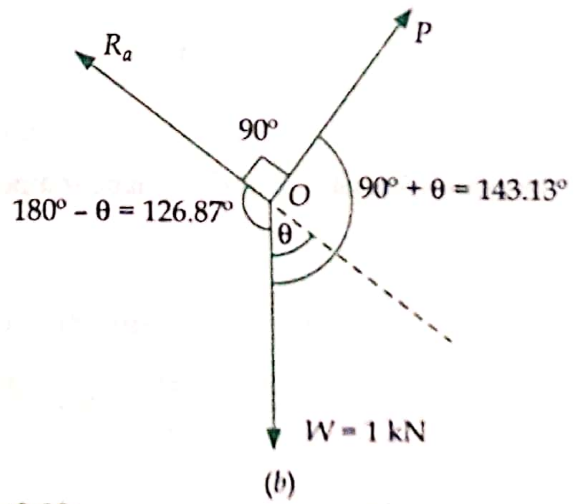


Fig. 2.69

Solution : The wheel is acted upon by the following forces when it is just about to turnover the block.

(*i*) Weight W = 1 kN of the wheel acting vertically downwards through centre of the sphere

(*ii*) Reaction R_a of the block

(*iii*) Pull P which must be applied normal to OA if it is to be minimum.

When the sphere is just about to turnover the block, it will lose contact with the floor and apparently, the reaction at the contact point C would be zero. Further the wheel is in equilibrium and as such the forces W, R_a and P meet at point O and the Lami's theorem applies.

In the right angled triangle AOB,

$$\cos\theta = \frac{OB}{OA} = \frac{50-20}{50} = 0.6\,; \quad \theta = 53.13^\circ$$

$$\frac{W}{\sin 90^\circ} = \frac{P}{\sin 126.87^\circ} = \frac{R_a}{143.13^\circ}$$

$$\therefore \quad P = W \times \frac{\sin 126.87^\circ}{\sin 90^\circ} = 1 \times \frac{0.8}{1} = 0.8 \text{ kN}$$

$$R_a = W \times \frac{\sin 143.13^\circ}{\sin 90^\circ} = 1 \times \frac{0.6}{1} = 0.6 \text{ kN}$$

EXAMPLE 2.48

Figure 2.70 a shows a weight W tied to the end of a cord of length l. Determine the magnitude of force F to pull the weight at an angle α as indicated in the figure. Proceed to find the tension in the cord.

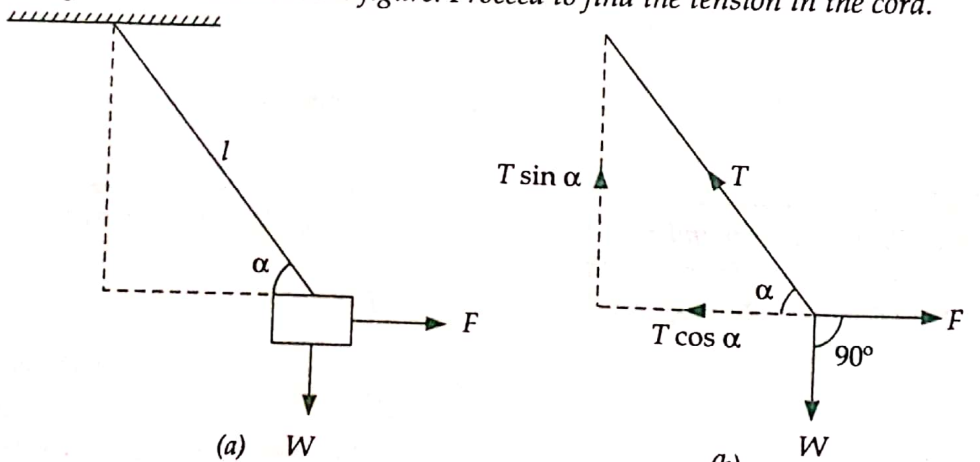


Fig. 2.70

Solution : The three forces F, T and W meet at a point and as such the Lami's theorem is applicable. Therefore

$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin(180^\circ - \alpha)} = \frac{F}{\sin(90^\circ + \alpha)}$$

$$\therefore \quad F = W \times \frac{\sin(90+\alpha)}{\sin(180-\alpha)} = W \times \frac{\sin 90^\circ \cos\alpha + \cos 90^\circ \sin\alpha}{\sin\alpha} = W \times \frac{\cos\alpha}{\sin\alpha} = W\cot\alpha$$

$$T = W \times \frac{\sin 90^\circ}{\sin(180^\circ - \alpha)} = W \times \frac{1}{\sin\alpha} = W \operatorname{cosec}\alpha$$

Alternatively :

Since the system is in equilibrium

$$\Sigma F_x = 0; \quad F = T\cos\alpha \qquad ...(i)$$

$$\Sigma F_y = 0; \quad W = T\sin\alpha \qquad ...(ii)$$

Upon division: $\dfrac{F}{W} = \dfrac{\cos\alpha}{\sin\alpha}$ or $F = W\cot\alpha$

Substituting $F = W\cot\alpha = W\dfrac{\cos\alpha}{\sin\alpha}$ in identity (*i*), we get

$$W\frac{\cos\alpha}{\sin\alpha} = T\cos\alpha$$

or $$T = \frac{W}{\sin\alpha} = W\,\text{cosec}\,\alpha$$

EXAMPLE 2.49

An electric light fixture weighing 50 N hangs from point C by two strings AC and BC as shown in Fig. 2.71 a. The string AC is inclined at 60° to the horizontal and string BC is 45° to the vertical. Using Lami's theorem or otherwise determine the forces in the strings AC and BC.

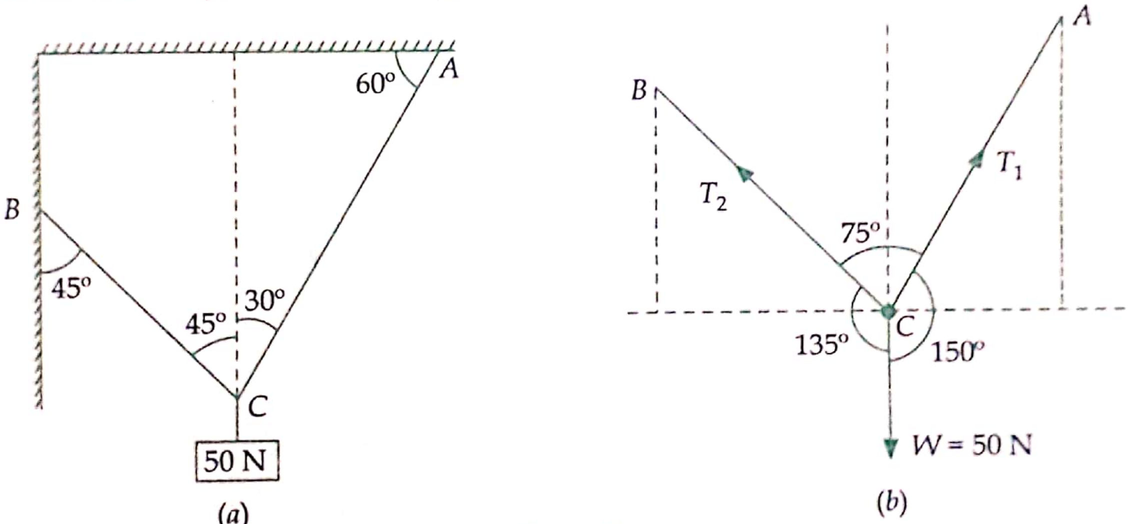


Fig. 2.71

Solution : Let T_1 and T_2 be the tensions in strings *AC* and *BC* respectively. The lines of action of tensions in *AC* and *BC* and weight *W* of the light fixture meet at point *C*, and therefore Lami's theorem is applicable. That gives

$$\frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{50}{\sin 75^\circ}$$

$$\therefore \quad T_1 = 50 \times \frac{\sin 135^\circ}{\sin 75^\circ} = 50 \times \frac{0.707}{0.966} = 36.594 \text{ N}$$

$$T_2 = 50 \times \frac{\sin 150^\circ}{\sin 75^\circ} = 50 \times \frac{0.50}{0.966} = 25.88 \text{ N}$$

Alternatively :

Since the light fixture is in equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Resolving all the forces horizontally, we get

$$T_1 \sin 30^\circ - T_2 \sin 45^\circ = 0$$
$$T_1 \times 0.5 - T_2 \times 0.707 = 0$$
$$T_2 = \frac{0.5}{0.707} T_1 = 0.707\, T_1$$

Resolving all the forces vertically, we get

$$T_1 \cos 30^\circ + T_2 \cos 45^\circ - 50 = 0$$
$$T_1 \times 0.866 + T_2 \times 0.707 = 50$$

Substituting $T_2 = 0.707\, T_1$ in the above identity, we get

$$0.866\, T_1 + 0.707 \times 0.707\, T_1 = 50$$

or

$$0.866\, T_1 + 0.5\, T_1 = 50$$

$$\therefore \quad T_1 = \frac{50}{1.366} = 36.6 \text{ N}$$

$$T_2 = 0.707\, T_1 = 0.707 \times 36.6 = 25.88 \text{ N}$$

EXAMPLE 2.50

A body acted upon by three forces P_1, P_2 and P_3, as shown in Fig. 2.72 a, is in equilibrium. If $P_2 = 300$ N, make calculations for the forces P_1 and P_3.

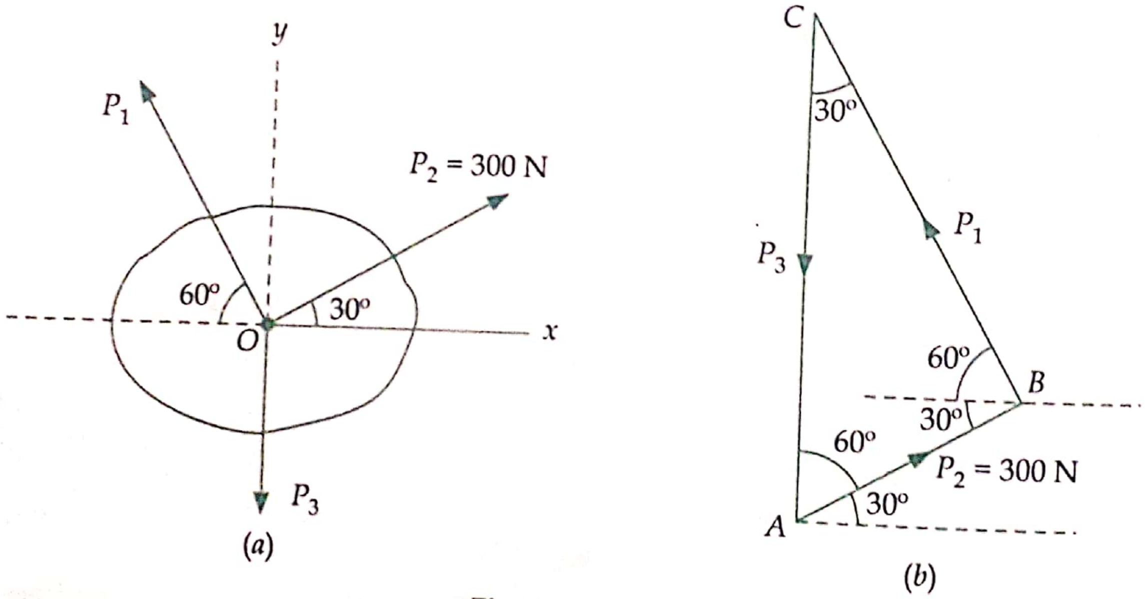


Fig. 2.72

Solution : The given forces can be represented by the sides AB, BC and CA of the triangle ABC (Fig. 2.72 *b*)

$$AB = P_2 = 300 \text{ N}; \quad BC = P_1 \text{ and } CA = P_3$$

Then applying the law of sines,

$$\frac{P_1}{\sin 60^\circ} = \frac{P_2}{\sin 30^\circ} = \frac{P_3}{\sin 90^\circ}$$

That gives: $P_1 = P_2 \dfrac{\sin 60^\circ}{\sin 30^\circ} = 300 \times \dfrac{0.866}{0.5} = 519.6 \text{ N}$

$$P_3 = P_2 \frac{\sin 90^\circ}{\sin 30^\circ} = 300 \times \frac{1}{0.5} = 600 \text{ N}$$

Alternatively :

Since the system of forces is in equilibrium

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

Resolution of forces along X-axis gives :

$$P_2 \cos 30^\circ + P_1 \cos 120^\circ + P_3 \cos 270^\circ = 0$$

All angles have been measured anticlockwise from axis O-X

$$0.866\, P_2 - 0.5\, P_1 + 0 = 0$$

$$\therefore \quad P_1 = \frac{0.866}{0.5} P_2 = \frac{0.866}{0.5} \times 300 = 519.6 \text{ N}$$

Resolution of all forces along Y-axis gives :

$$P_2 \sin 30^\circ + P_1 \sin 120^\circ + P_3 \sin 270^\circ = 0$$

$$0.5\, P_2 + 0.866\, P_1 - P_3 = 0$$

$$\therefore \quad P_3 = 0.5\, P_2 + 0.866\, P_1$$

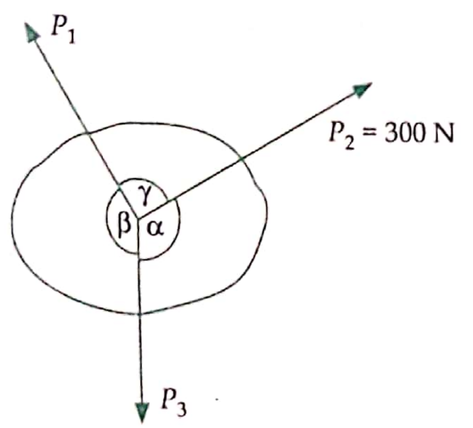
$$= 0.5 \times 300 + 0.866 \times 519.6 = 150 + 450 = 600 \text{ N}$$

Alternatively :

Since the three forces meet at a point, solution to this problem can be obtained by invoking the Lami's theorem.

$$\alpha = 120^\circ\ ;\ \beta = 150^\circ \text{ and } \gamma = 90^\circ$$

$$\frac{P_1}{\sin\alpha} = \frac{P_2}{\sin\beta} = \frac{P_3}{\sin\gamma}$$



$$\text{or} \quad \frac{P_1}{\sin 120^\circ} = \frac{300}{\sin 150^\circ} = \frac{P_3}{\sin 90^\circ}$$

$$\therefore \quad P_1 = 300 \times \frac{\sin 120^\circ}{\sin 150^\circ}$$

$$= 300 \times \frac{0.866}{0.5} = 519.6 \text{ N}$$

$$P_3 = 300 \times \frac{\sin 90^\circ}{\sin 150^\circ} = 300 \times \frac{1}{0.5} = 600 \text{ N}$$

EXAMPLE 2.51

A string ABCDE whose extremity A is fixed has weights W_1 and W_2 attached to it at B and C, and passes round a smooth peg at D carrying a weight of 800 N at the free end E (Fig. 2.73). If in a state of equilibrium, BC is horizontal and AB and CD make angles of 150° and 120° respectively with BC, make calculations for

(a) *the tensions in portions AB, BC, CD and DE of the string*

(b) *the value of weights W_1 and W_2*

(c) *the pressure on the peg D.*

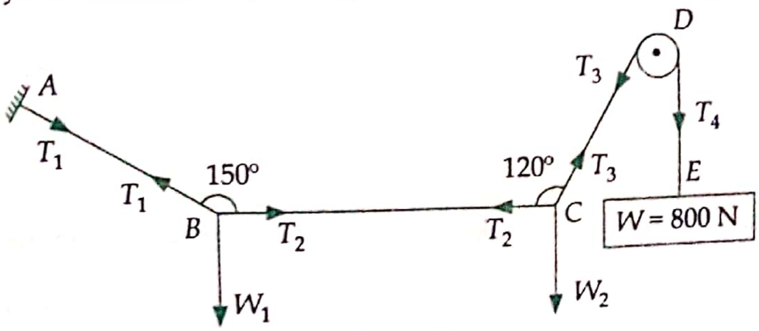


Fig. 2.73

Solution : Let T_1, T_2, T_3, T_4 be the tensions in segments AB, BC, CD and DE of the string.

Under equilibrium conditions,

$$T_3 = T_4 = 800 \text{ N}$$

Applying Lami's theorem at point C,

$$\frac{T_2}{\sin 150^\circ} = \frac{T_3}{\sin 90^\circ} = \frac{W_2}{\sin 120^\circ}$$

$$\therefore \quad T_2 = T_3 \frac{\sin 150^\circ}{\sin 90^\circ} = 800 \times \frac{0.5}{1} = 400 \text{ N}$$

$$W_2 = T_3 \frac{\sin 120^\circ}{\sin 90^\circ} = 800 \times \frac{0.866}{1} = 692.8 \text{ N}$$

Applying Lami's theorem at point B,

$$\frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{W_1}{\sin 150^\circ}$$

$$\therefore \quad T_1 = T_2 \frac{\sin 90^\circ}{\sin 120^\circ} = 400 \times \frac{1}{0.866} = 461.89 \text{ N}$$

$$W_1 = T_2 \frac{\sin 150^\circ}{\sin 120^\circ} = 400 \times \frac{0.5}{0.866} = 230.95 \text{ N}$$

(c) Pressure on the peg at D $= T_3 \sin 60^\circ + W$

$= 800 \sin 60^\circ + 800 = 692.82 + 800 = 1492.82$ N

EXAMPLE 2.52

A spherical ball of weight 100 N is attached to a string and is suspended from the ceiling as shown in Fig. 2.74 a. What tension would be induced in the string ?

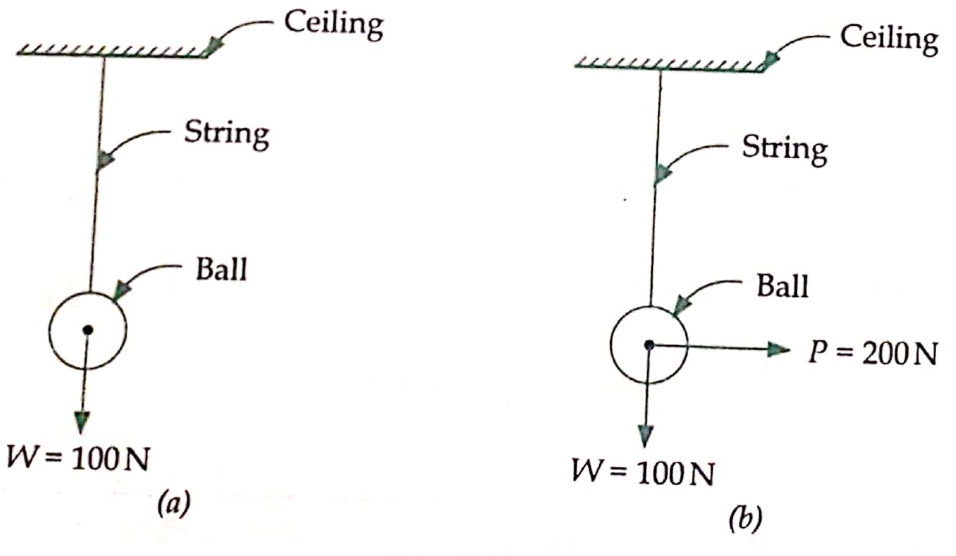


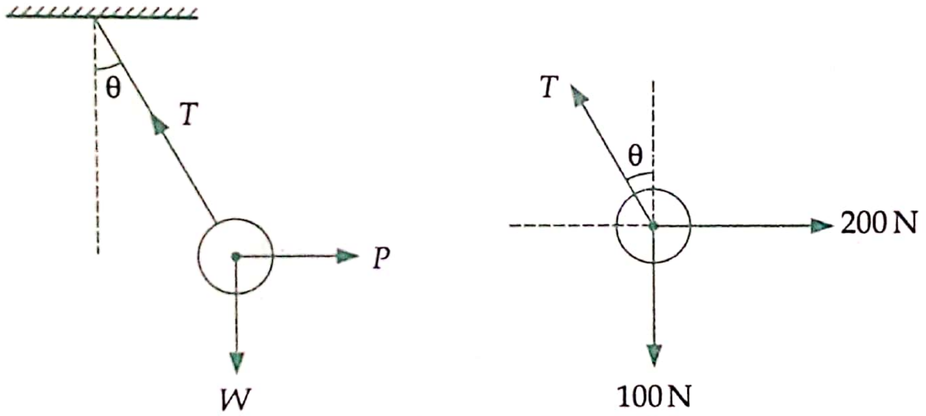
Fig. 2.74

Subsequently a horizontal force of 200 N is applied to the ball as shown in Fig. 2.74 b. Calculate the resultant tension in the string and the angle which the string makes with the vertical.

Solution : The ball is in equilibrium under the action of two forces namely, the weight W of the ball and the tension T induced in the string

$$\therefore \quad T = W = 80 \text{ N}$$

(ii) When the horizontal force P is applied the ball, it will be in equilibrium under the action of three forces namely weight of the ball, horizontal force applied and the tension induced in the string.



With reference to free body diagram, we have under equilibrium state

$$\Sigma F_x = 0: \quad -T \sin\theta + 200 = 0 \qquad \ldots(i)$$

$$\Sigma F_y = 0: \quad T\cos\theta - 100 = 0 \qquad \ldots(ii)$$

From expressions (*i*) and (*ii*)

$$\tan\theta = \frac{200}{100} = 2\,; \quad \theta = 63.55°$$

$$T = \frac{200}{\sin 63.55} = 223.46 \text{ N}$$

Alternatively

Invoking Lami's theorem

$$\frac{T}{\sin 90°} = \frac{100}{\sin(90+\theta)} = \frac{200}{\sin(180-\theta)}$$

or $$\frac{T}{\sin 90} = \frac{100}{\cos\theta} = \frac{200}{\sin\theta}$$

and hence $$\tan\theta = \frac{200}{100} = 2\,; \quad \theta = \tan^{-1}(2) = 63.55°$$

$$T = \frac{\sin 90°}{\cos 63.55°} \times 100 = \frac{1}{0.4454} \times 100 = 224.52 \text{ N}$$

$\sin(\alpha + \beta)$

EXAMPLE 2.58

Two rollers of the same diameter are supported by an inclined plane and a vertical wall as shown in Fig. 2.81. The upper and the lower rollers are respectively 200 N and 250 N in weight. Assuming smooth surfaces, find the reactions induced at the points of support A, B, C and D.

Solution : The upper cylinder is kept in equilibrium by the following set of concurrent forces:

(*i*) weight 200 N acting vertically downward through its centre O_2

(*ii*) reaction R_c acting perpendicular to the inclined plane

(*iii*) pressure R_d from the lower cylinder in the direction O_1O_2

Since these three forces are concurrent, the Lami's theorem applies. That gives

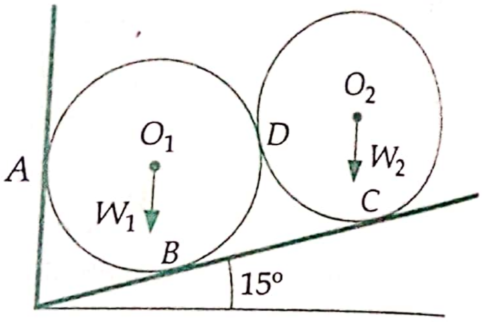
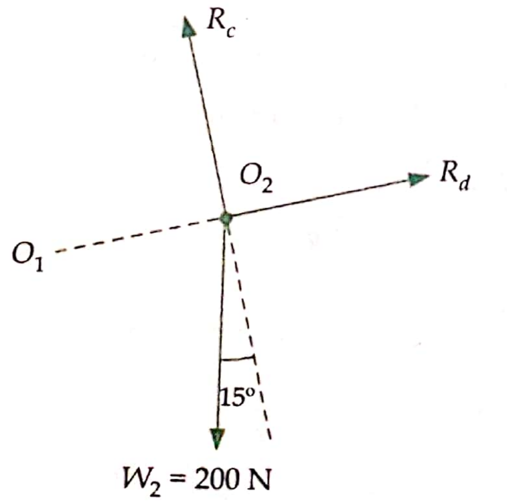


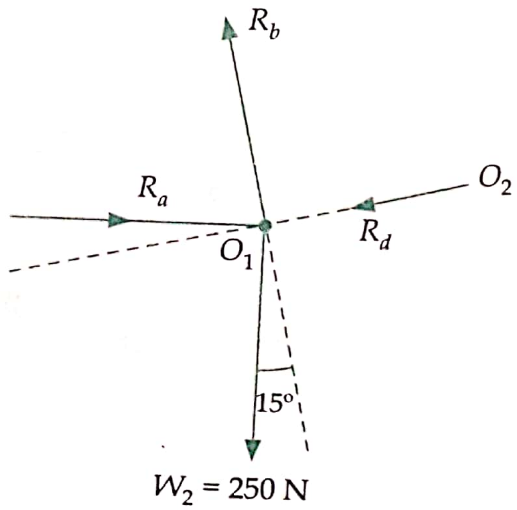
Fig. 2.81

$$\frac{R_d}{\sin(180-15)} = \frac{R_c}{\sin(90+15)} = \frac{W}{\sin 90}$$

$$\therefore \quad R_d = W \times \frac{\sin 165^\circ}{\sin 90^\circ} = 200 \times \frac{0.2588}{1} = 51.76 \text{ N}$$

$$R_c = W \times \frac{\sin 105^\circ}{\sin 90^\circ} = 200 \times \frac{0.9659}{1} = 193.18 \text{ N}$$





The following set of forces keep the lower cylinder in equilibrium :

(i) weight 250 N acting vertically downward through its centre O_1

(ii) reaction R_a acting perpendicular to the vertical wall

(iii) reaction R_b acting perpendicular to the inclined plane

(iv) pressure R_d from the upper cylinder in the direction O_2O_1

Resolving the forces along O_1O_2, we get

$$R_a \cos 15° - W_1 \sin 15° - R_d = 0$$

$$R_a \times 0.966 - 250 \times 0.259 - 51.76 = 0$$

$$\therefore \quad R_a = \frac{(250 \times 0.259) + 51.76}{0.966} = \frac{116.51}{0.966} = 120.61 \text{ N}$$

Resolving the forces perpendicular to O_1O_2, we get

$$R_b - R_a \sin 15° - W_1 \cos 15° = 0$$

$$R_b = R_a \sin 15° + W_1 \cos 15°$$

$$= (120.61 \times 0.2588) + (250 \times 0.9659)$$

$$= 31.21 + 241.47 = 272.68 \text{ N}$$

EXAMPLE 2.59

Two smooth spheres P, Q each of radius 25 cm and weighing 500 N, rest in a horizontal channel having vertical walls (Fig. 2.82). If the distance between the walls is 90 cm, make calculations for the pressure exerted on the wall and floor at points of contact A, B and C.

Solution : The following points need consideration

(i) the spheres are smooth and as such the pressures at various points of contact would be normal to the surface.

(ii) at the point of contact between the two spheres, the reactions would act along the line joining their centres.

With reference to the adjoining figure the line C_1C_2 makes an angle α with the horizontal line passing through centre C_1 of sphere P.

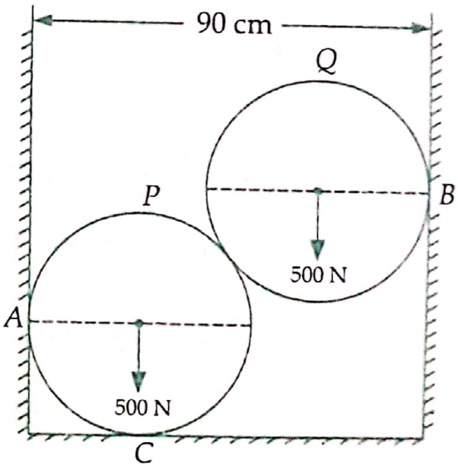
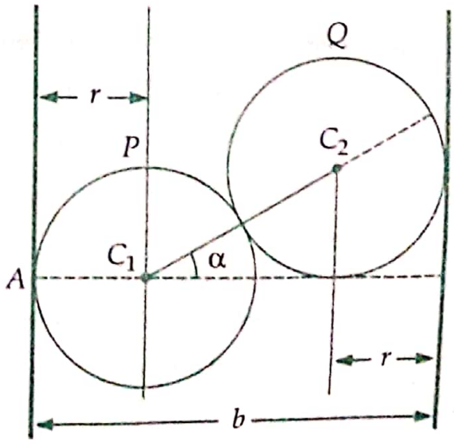


Fig. 2.82



$$\cos\alpha = \frac{b - r - r}{2r}$$

$$= \frac{90 - 25 - 25}{50} = \frac{40}{50}$$

$$\therefore \quad \alpha = \cos^{-1}(0.8) = 36.87°$$

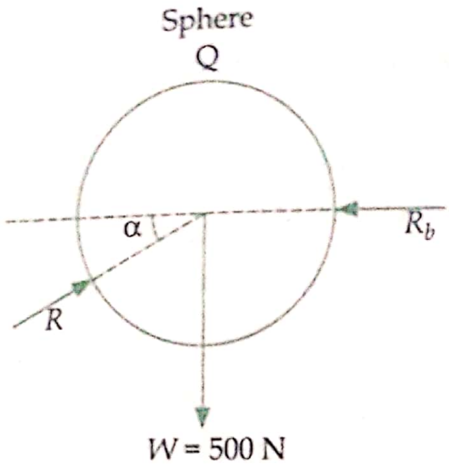
Considering the equilibrium of sphere Q

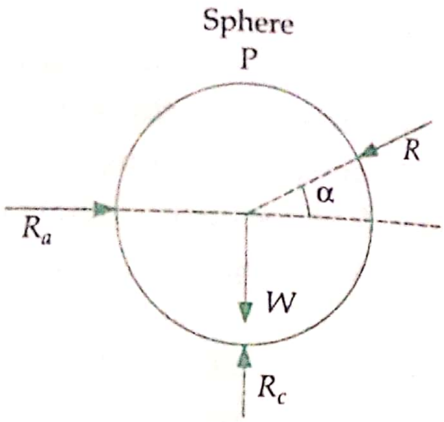
$$\Sigma F_x = 0; \quad R_b - R\cos\alpha = 0$$

$$\Sigma F_y = 0; \quad R\sin\alpha - 500 = 0$$

$$\therefore \quad R = \frac{500}{\sin\alpha} = \frac{500}{\sin 36.87} = 833.33 \text{ N}$$

$$R_b = R\cos\alpha = 833.33 \times \cos 36.87 = 666.66 \text{ N}$$





Considering equilibrium of sphere P,

$$\Sigma F_x = 0; \quad R_a - R\cos\alpha = 0$$
$$R_a = R\cos\alpha = 833.33 \times \cos 36.87 = 666.67 \text{ N}$$
$$\Sigma F_y = 0; \quad R_c - W - R\sin\alpha = 0$$
$$R_c = W + R\sin\alpha = 500 + 833.33 \times \sin 36.87 = 1000 \text{ N}$$

EXAMPLE 2.60

Refer to the system of cylinders arranged as depicted in Fig. 2.83. The cylinders A and B weigh 1000 N each and the weight of cylinder C is 2000 N. Determine the forces exerted at the contact points.

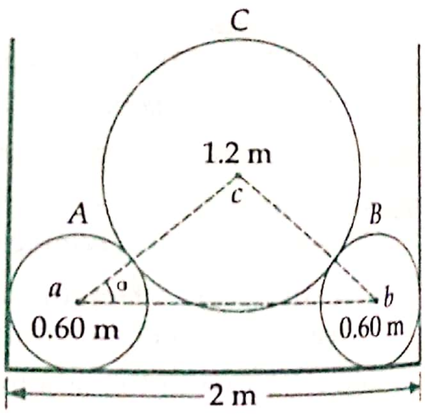


Fig. 2.83

Solution : a, b and c are centres of spheres.

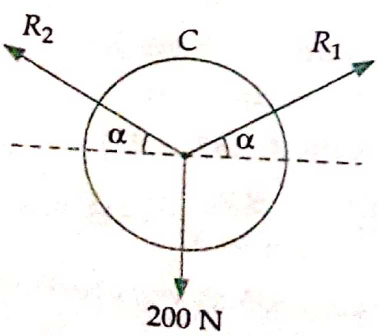
$$ab = 2 - \frac{0.6}{2} - \frac{0.6}{2} = 1.4 \text{ m}$$
$$ac = 0.3 + 0.6 = 0.9 \text{ m}$$
$$\cos\alpha = \frac{1.4/2}{0.9} = 0.7777; \quad \alpha = 38.94^\circ$$

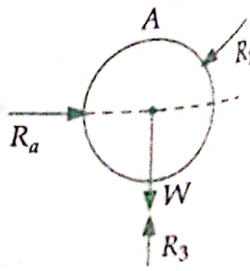
Applying Lami's theorem to the forces acting on sphere C,

$$\frac{R_1}{\sin(90+\alpha)} = \frac{R_2}{\sin(90+\alpha)} = \frac{2000}{\sin(180-2\alpha)}$$
$$R_1 = R_2 = 2000 \times \frac{\sin(90+\alpha)}{\sin(180-2\alpha)} = 2000 \times \frac{\sin(90+38.94)}{\sin(180-2\times 38.94)}$$
$$= 2000 \times \frac{0.7777}{0.9777} = 1590.87 \text{ N}$$

Considering the free body diagram of cylinder A,

$$\Sigma F_x = 0; \quad R_a - R_1\cos\alpha = 0$$
$$R_a = R_1\cos\alpha = 1590.87 \times \cos 38.94$$
$$= 1237.38 \text{ N}$$
$$\Sigma F_y = 0; \quad R_1\sin\alpha + W - R_3 = 0$$
$$R_3 = R_1\sin\alpha + W$$
$$= 1590.87 \sin 38.94 + 1000$$
$$= 1999.87 \text{ N}$$





EXAMPLE 2.62

Two cylinders P and Q rest in a channel as shown in Fig. 2.85. The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N. If the bottom width of the channel is 180 mm and with one side vertical and the other inclined at 60°, determine the pressures at all the four points of contact.

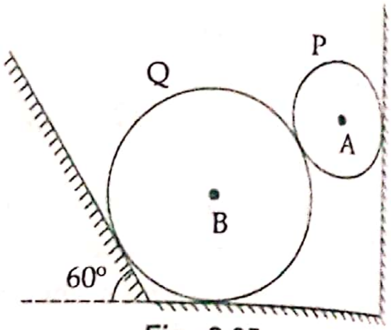


Fig. 2.85

Solution : The cylinder P is in equilibrium under the action of following forces which pass through its centre A

(*i*) weight 200 N of the cylinder acting downwards

(*ii*) reaction R_1 at the vertical side

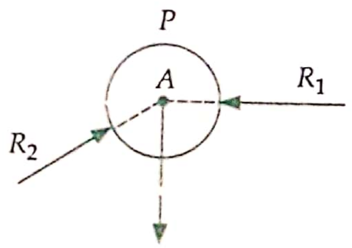
(*iii*) reaction R_2 at the point of contact with cylinder Q

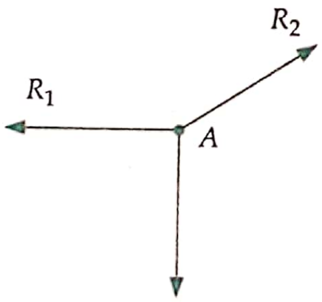
From geometrical configuration, we have

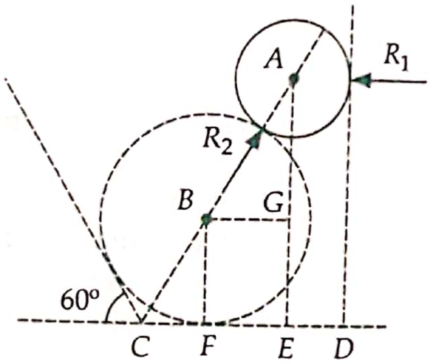
$$\angle BCF = 60°; CF = BF \cot 60° = \frac{180}{2} \times 0.577 = 52 \text{ mm}$$

$$BG = FE = CD - CF - ED = 180 - 52 - 50 = 78 \text{ mm}$$

$$AB = 50 + 90 = 140 \text{ mm}$$







$$\cos\angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.557; \angle ABG = 56.15°$$

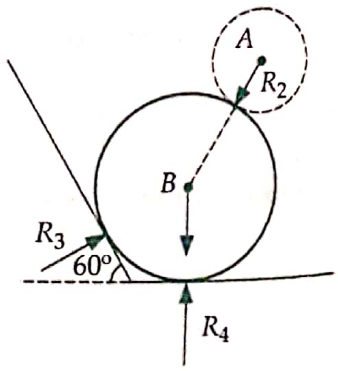
Applying Lami's equation to the system of forces acting at point A

$$\frac{R_1}{\sin(90 + 56.15)} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180 - 56.15)}$$

$$\text{or} \quad \frac{R_1}{0.557} = \frac{R_2}{1} = \frac{200}{0.830}$$

$$\therefore \quad R_1 = \frac{0.557 \times 200}{0.830} = 134.22 \text{ N}$$

$$\text{and} \quad R_2 = \frac{1 \times 200}{0.830} = 240.96 \text{ N}$$



(*a*)

(*b*) The cylinder Q is in equilibrium under the action of following forces which pass through its centre B.

(*i*) weight 500 N of the cylinder acting downwards

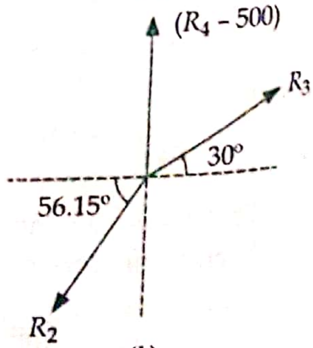
(*ii*) reaction R_2 = 240.96 N of cylinder P

(*iii*) reaction R_3 at the inclined surface

(*iv*) reaction R_4 at the base of channel

The lines of action of reaction R_4 (acting upwards) and weight of the cylinder (acting downwards) coincide with each other.

$\therefore$ Net upward force = $(R_4 - 500)$



(*b*)

Applying Lami's equation to the system of forces acting at point B

$$\frac{R_2}{\sin 60} = \frac{R_3}{\sin(90 + 56.15)} = \frac{R_4 - 500}{\sin(180 + 30 - 56.15)}$$

or

$$\frac{R_2}{0.866} = \frac{R_3}{0.557} = \frac{R_4 - 500}{0.4407}$$

$$\therefore \quad R_3 = \frac{R_2 \times 0.557}{0.866} = \frac{240.96 \times 0.557}{0.866} = 154.98 \text{ N}$$

and

$$R_4 - 500 = \frac{R_2 \times 0.4407}{0.866} = \frac{240.96 \times 0.4407}{0.866} = 112.62$$

$$\therefore \quad R_4 = 500 + 122.62 = 622.62 \text{ N}$$

EXAMPLE 2.63