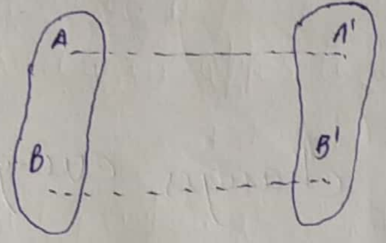
General plane motion

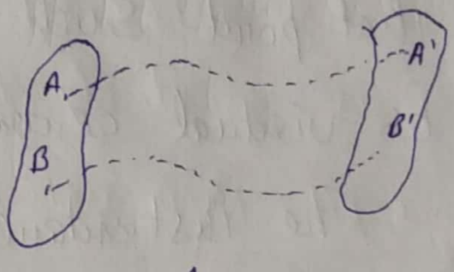
A body undergoes the three types of plane motions.

Translation- rigid body move in parallel planes and travel the same distance.

During translation, the particles have the same velocity and acceleration.



Rectilinear translation

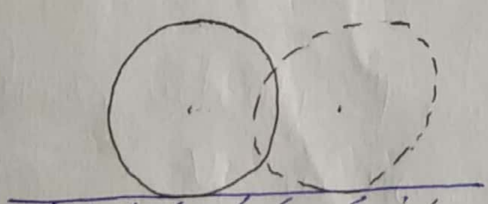


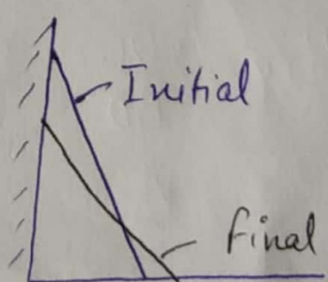
Curvilinear Translation

Rotation: The body rotates about a fixed point and all the particles constituting the body move in a circular path. The fixed point about which the body rotates is called the point of rotation and the axis passing through the point of rotation is called axis of rotation.

A point lying on the axis of rotation has a zero velocity and zero acceleration.

General plane motion: Combined motion of translation and rotation.





Instantaneous Centre: while analysing plane motion of a body, a point can be located in the plane which has zero velocity. The plane motion of all the particles constituting the body may be considered as pure rotation about that point. Such a point is called the instantaneous centre or virtual centre of body.

The instantaneous centre changes every moment and its locus is centrode. The surface generated by the instantaneous axis is called the axode.

The salient aspects in relation to IC.

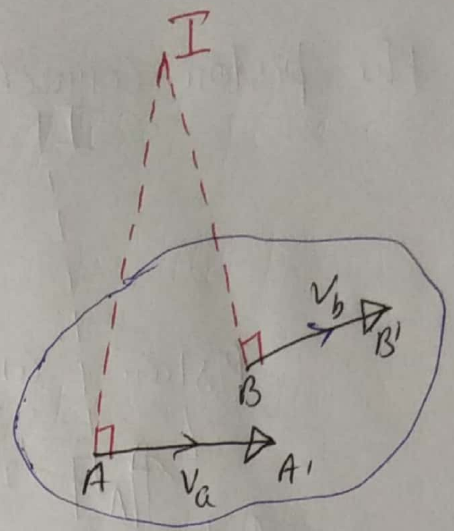
i) The IC is the point about which the body appears to rotate.

ii) The IC may may be inside or outside the body.

iii) IC is not a fixed point but changes from one instant to another as the body rotates.

iv) The velocity at IC is zero.

IC in diff. Cases



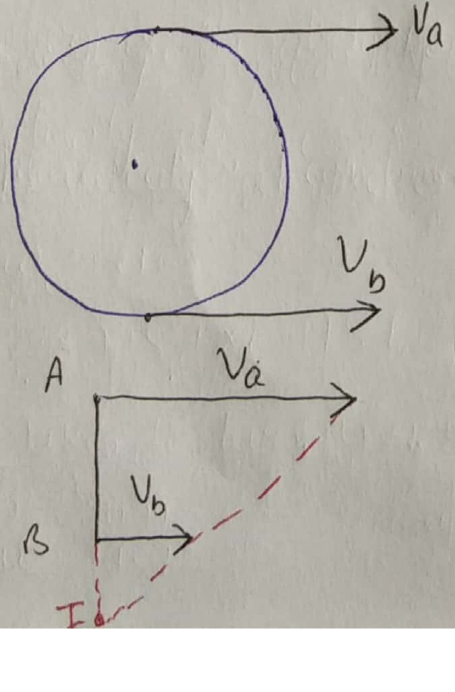
i) Let V_a and V_b be the linear velocities at points A and B on a rigid body. These velocities are directed along direction on AA' and BB'.

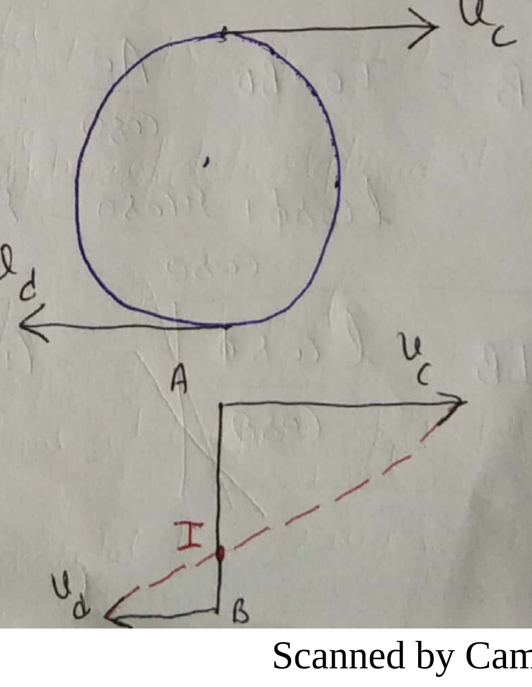
$$V_a = \omega \cdot IA \quad -(i)$$

$$V_b = \omega \cdot IB \quad -(ii)$$

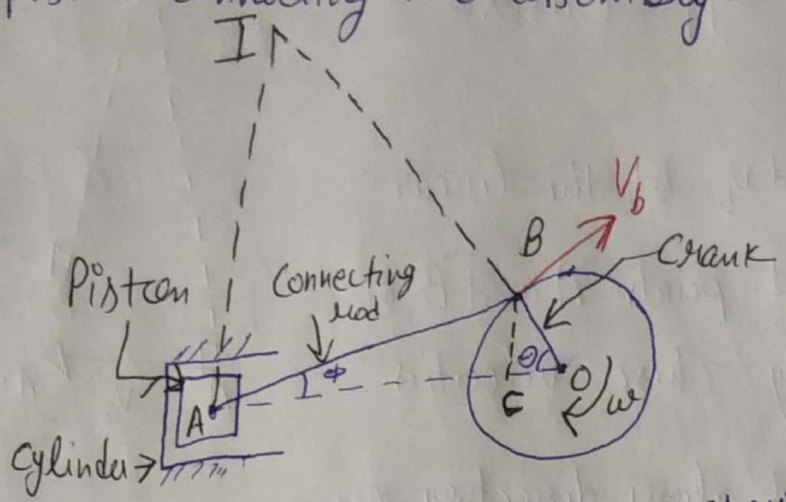
$$\boxed{\omega = \frac{V_a}{IA} = \frac{V_b}{IB}}$$

ii) Consider a solid circular cylinder in contact with two horizontal conveyer belts running with diff. velocities in same or opposite direction.





iii) For the piston-connecting rod assembly.



Angular velocity ω_{ab} of the connecting rod about I is

$$\boxed{\omega_{ab} = \frac{v_a}{IA} = \frac{V_b}{IB} = \frac{\omega r}{IB}}$$

$[\because v_b = \omega r]$

ω: angular velocity of crank

Further

$$AO = AC + CO$$
$$= AB\cos\phi + BO\cos\theta$$
$$\boxed{AO = l\cos\phi + r\cos\theta} \quad -(i)$$

$\left[\begin{array}{l} l = \text{length of conn. rod} \\ r = \text{radius of crank} \end{array}\right]$

$$IA = AO\tan\theta$$
$$\boxed{IA = (l\cos\phi\tan\theta + r\sin\theta)} \quad -(ii)$$

$[\because AO = l\cos\phi + r\cos\theta]$

$$IB = IO - BO = \frac{AO}{\cos\theta} - r$$
$$= \frac{l\cos\phi + r\cos\theta}{\cos\theta} - r$$
$$\boxed{IB = l\,\frac{\cos\phi}{\cos\theta}} \quad -(iii)$$

from (i)

$$V_a = \frac{IA}{IB} \cdot V_b$$

$$= \frac{l\cos\phi\tan\theta + r\sin\theta}{\frac{l\cos\phi}{\cos\theta}} \cdot \omega r$$

$$= \left(\cos\theta\tan\theta + \frac{r}{l}\frac{\sin\theta.\cos\theta}{\cos\phi}\right)\omega r \quad -(iv)$$

Applying sine rule to ΔOAB

$$\frac{OB}{\sin\phi} = \frac{AB}{\sin\theta} \Rightarrow \frac{r}{\sin\phi} = \frac{l}{\sin\theta} \Rightarrow \frac{r}{l} = \frac{\sin\phi}{\sin\theta} \quad -(V)$$

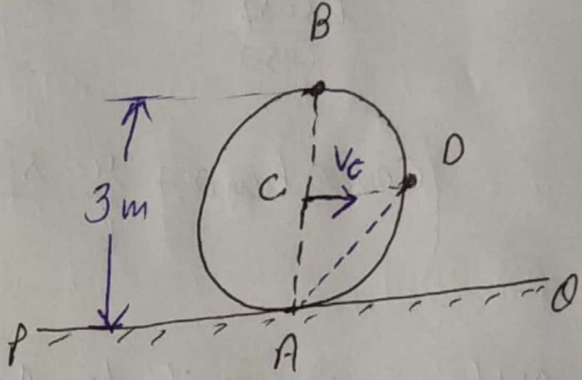
from (iv) and (V)

$$V_a = \left(\cos\theta.\tan\theta + \frac{\sin\phi}{\sin\theta} \cdot \frac{\sin\theta\cos\theta}{\cos\phi}\right)\omega r$$

$$= (r\sin\theta + r\cos\theta\tan\phi)\,\omega$$

$$\boxed{V_a = \omega(l\sin\phi + r\cos\theta\tan\phi)}$$

Q A cylinder of dia 3m rolls without slipping along a horizontal surface PQ as shown in fig. If its centre has a uniform velocity of 30 m/s, determine the velocities of points B and D lying on the rim of the cylinder.



Sol.

When the cylinder rolls without slipping, its point of contact with the horizontal surface at any instant has zero velocity.

Thus the point A is IC.

$V_C = \omega \cdot AC$

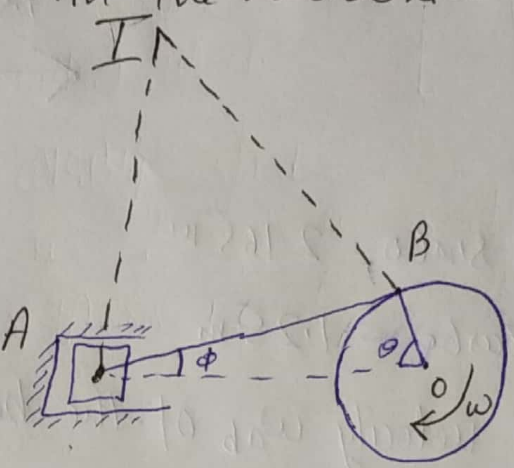
$\omega = \frac{30}{1.5} = 20$ rad/s

$V_B = \omega \cdot AB = \omega \cdot 2 \cdot r = 60$ m/s

$V_D = \omega \cdot AD = \omega \cdot \sqrt{AC^2 + DC^2} = 42.42$ m/s

Q In a reciprocating engine mechanism, the lengths of cranks OB and Conn. rod AB are 30 cm and 1 cm resp. If the Crank is rotating at a constant angular velocity of 200 rev/min Determine (a) angular velocity of Conn. rod

(b) Velocity of piston when crank makes an angle $\theta = 45'$ with the horizontal



Sol.

Angular velocity of crank $\omega = \frac{2\pi N}{60} = \boxed{20.93 \text{ rad/sec}}$

$V_b = \omega . r = 20.93 \times 0.3 = \boxed{6.279 \text{ m/s}}$

Angular velocity ω_{ab} of Conn. rod about I is

$\omega_{ab} = \frac{V_a}{IA} = \frac{V_b}{IB} = \frac{\omega r}{IB}$

Apply sine rule in ΔOAB

$\frac{l}{\sin\theta} = \frac{r}{\sin\phi} \Rightarrow \boxed{\phi = 12.25^\circ}$

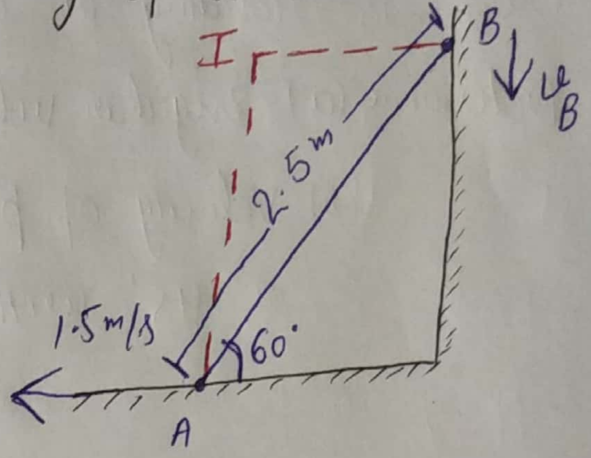
$IA = l\cos\phi\tan\theta + r\sin\theta = \boxed{1.1843 \text{ m}}$

$IB = \frac{l\cos\phi}{\cos\theta} = \boxed{1.382 \text{ m}}$

Angular velocity of Conn. rod $\omega_{ab} = \frac{V_b}{IB} = \boxed{4.543 \text{ rad/s}}$

Velocity of piston $V_a = \omega_{ab} . IA = \boxed{5.38 \text{ m/s}}$

Q Length of beam is 2.5 m and placed as shown in fig. If the end A has a velocity of 1.5 m/s, determine the angular velocity of the beam and the velocity of its end B at the position shown in fig



Sol.

$IA = AB \sin 60 = 2.165\,m$

$IB = AB \cos 60 = 1.25\,m$

Then angular velocity ω_{ab} of the beam is given by

$$\omega_{ab} = \frac{V_a}{IA} = 0.693\ rad/s$$

Velocity of end B is

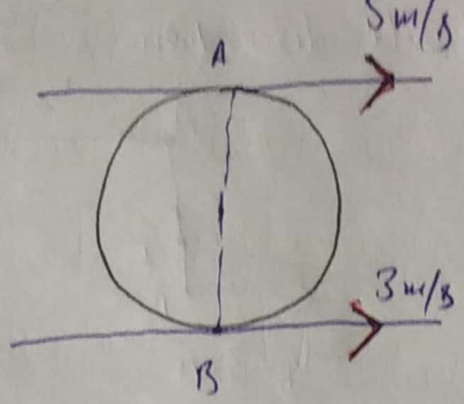
$$V_b = \omega_{ab} \cdot IB = 0.866\ m/s$$

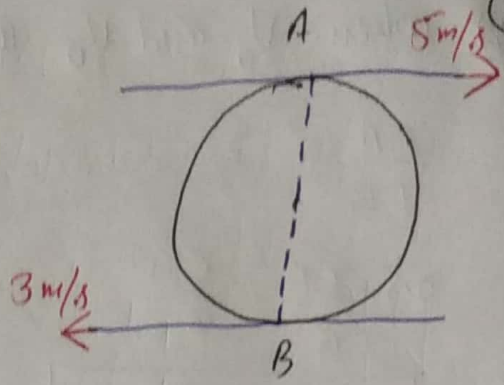
Q A cylindrical roller, 50 cm in dia, is in contact with two horizontal conveyer belts running at uniform speeds of 5 m/s and 3 m/s as shown in fig

Assuming there is no slip at the points of contact Determine

a) position of IC of roller.

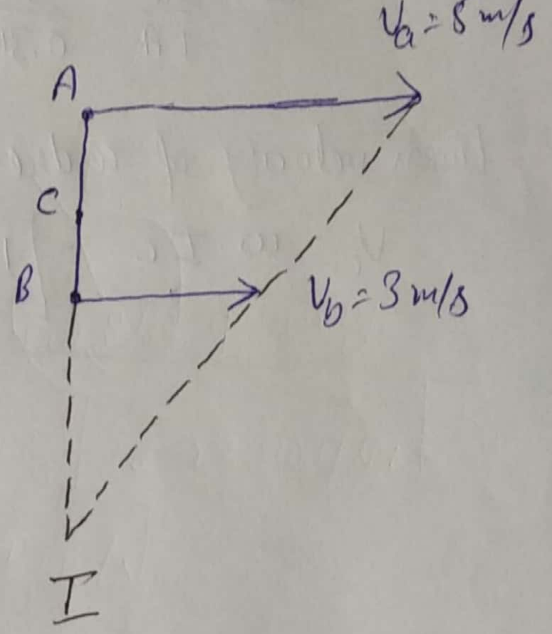
b) linear velocity of Centre C.





c) the angular velocity of roller.

d) How these parameters would be affected if the velocities of the belts are in opposite direction.



Sol. Case I when V_a and V_b are in same direction.

$V_a = \omega_{ab} \cdot IA$; $V_b = \omega_{ab} \cdot IB$

$$\frac{IA}{IB} = \frac{V_a}{V_b} = \frac{5}{3}$$

$$\frac{IB + AB}{IB} = \frac{5}{3}$$

$$\boxed{IB = 75\,cm}$$

$$\boxed{IA = 125\,cm}$$

Angular velocity of roller = angular velocity of A about I

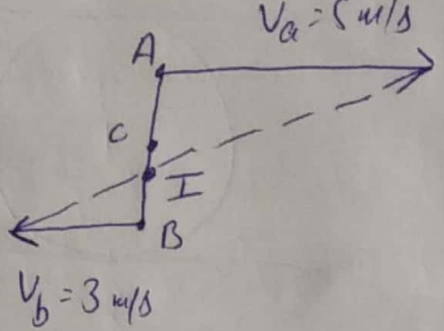
$$\omega = \frac{V_a}{IA} = \boxed{4\ rad/s}$$

Linear velocity of centre of roller.

$$\boxed{V_c = \omega \cdot IC = 4\,m/s}$$

(b) when V_a and V_b are in opposite direction.

$$\frac{IA}{IB} = \frac{5}{3}$$



$$\frac{0.5 - IB}{IB} = \frac{5}{3}$$

$$\boxed{\begin{array}{l} IB = 18.75\ cm \\ IA = 31.25\ cm \end{array}}$$

Angular velocity of roller

$$\omega = \frac{V_a}{IA} = \frac{5}{0.3125} = \boxed{6\ m/s}$$

Linear velocity of centre of roller

$$V_c = \omega \cdot IC = \boxed{1\ m/s}$$