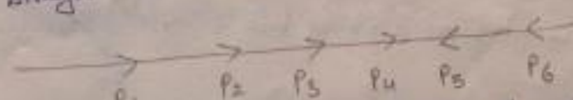
FORECE SYSTEM

System of Forces
- Coplanar Forces
  - Concurrent
  - Parallel
    - Like
    - Unlike
  - Non-Concurrent Non Parallel
- Non-coplanar or Spatial force
  - Concurrent
  - Parallel
  - Non Concurrent Non Parallel

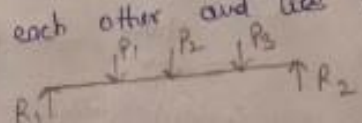
Considering the plane in which forces are applied and depending upon the position of line of action, forces may be classified above.

Collinear forces :- The lines of action of all forces lies along the same straight line and in the same plane.



Example - Force on a rope in a tug of war.

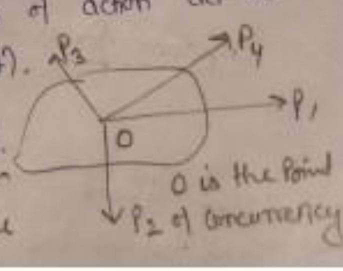
Coplanar Parallel forces :- The lines of action of all forces are parallel to each other and lies in a single plane.



Example - vertical load acting on a beam.

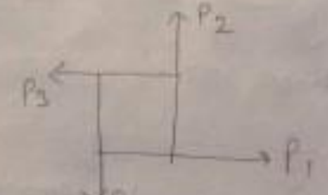
Coplanar Concurrent forces :- All forces lies in the same plane, have different directions but their lines of action act at a one point (Pass through a single point).

Example - forces on a rod resting against a wall.



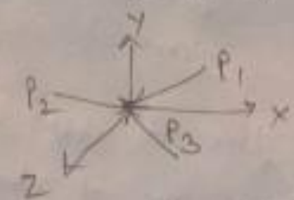
Coplanar non concurrent force :-

Coplanar → Same Plane
non concurrent → don't Pass through a single Point



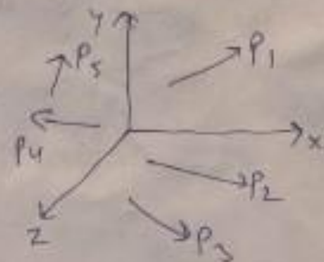
Example - Forces on a ladder resting against wall and a Person standing on a rung which is not at its centre of gravity.

Non Coplanar Concurrent force :-



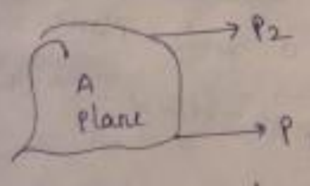
Example - forces on a tripod carrying a camera.

Non - coplanar and non- concurrent forces

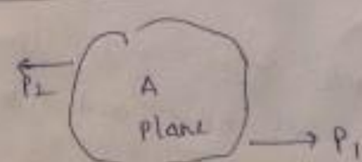


Example - Forces acting on a moving bus

Coplanar like & unlike Parallel force System

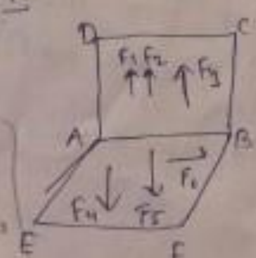


Coplanar like Parallel force



Coplanar unlike parallel force System

Coplanar Forces :-



F_1, F_2, F_3 are coplaner contain in ABCD Plane

F_4 F_5 F_6 are coplanar contain in ABEF

but F_1 F_2 F_3 F_4 F_5 and F_6 are not coplanar.

After that

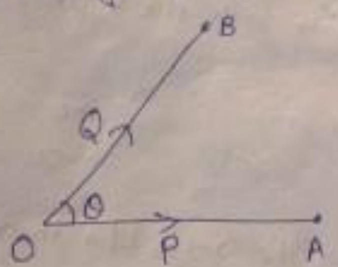
⇒ Parallelogram law

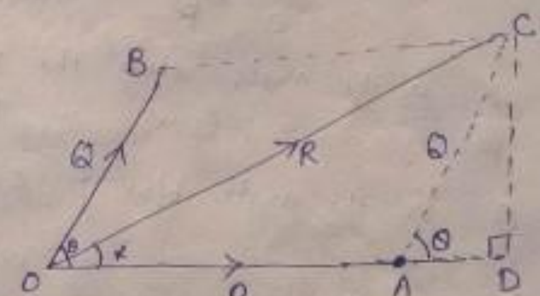
to the body to brings it in equilibrium state that single force is known as equilibrant.

Equilibrant is equal and opposite to the resultant of several forces acting on the body.

Parallelogram Law of forces :-

if ~~the~~ two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a Parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point





The resultant R of P and Q is given by

$$R = OC = \sqrt{OD^2 + CD^2} = \sqrt{(OA + AD)^2 + CD^2}$$

$OA = P$

$AD = AC\cos\theta \Rightarrow AD = Q\cos\theta$

$CD = AC\sin\theta \Rightarrow CD = Q\sin\theta$

$$R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2}$$

$$R = \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$$

$$R = \sqrt{P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta}$$

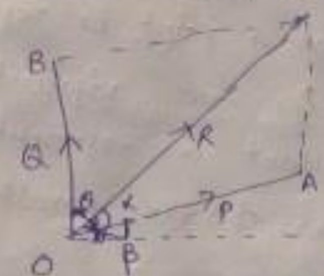
$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

The inclination of the resultant R to the direction of force P is given by

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\boxed{\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P+Q\cos\theta}\right]}$$

Note :-



α is the angle which the resultant makes with the direction of P.

β " " " " " force P makes with the x axis.

θ is the angle b/w P & Q.

Special cases :-

(i) When two forces are equal and θ is the angle b/w them.

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = \sqrt{P^2+P^2+2P^2\cos\theta} = \sqrt{2P^2(1+\cos\theta)}$$

$$R = \sqrt{2P^2\times 2\cos^2\left(\tfrac{\theta}{2}\right)} = \sqrt{4P^2\cos^2\tfrac{\theta}{2}} = 2P\cos\tfrac{\theta}{2}$$

$$\boxed{R = 2P\cos\tfrac{\theta}{2}} \qquad \alpha = \tan^{-1}\left[\frac{P\sin\theta}{P+P\cos\theta}\right] = \tan^{-1}\left[\frac{P\sin\theta}{P(1+\cos\theta)}\right]$$

$$\alpha = \tan^{-1}\frac{\cancel{2}\sin\theta/2\,\cos\theta/2}{\cancel{2}\cos^2\theta/2} = \tan^{-1}(\tan\theta/2)$$

$$\boxed{\alpha = \frac{\theta}{2}}$$

$\alpha = \frac{\theta}{2}$

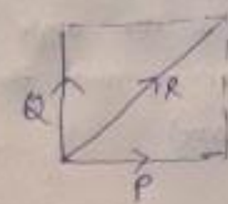
resultant bisects the angle b/w the forces.

(ii) When the two forces act at right angles $\theta = 90$.

$\underline{\theta = 90^\circ}$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$R = \sqrt{P^2 + Q^2}$$



$$\alpha = \tan^{-1}\left[\frac{Q\sin 90}{P + Q\cos 90}\right]$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

(iii) When $\theta = 0$, when two forces act in the same line and same sense.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0}$$

$$R = P + Q$$

Apparently the resultant is maximum when the forces are collinear and act in the same direction.

(iv) When the two forces have the same line of action but opposite senses i.e. $\theta = 180^\circ$.

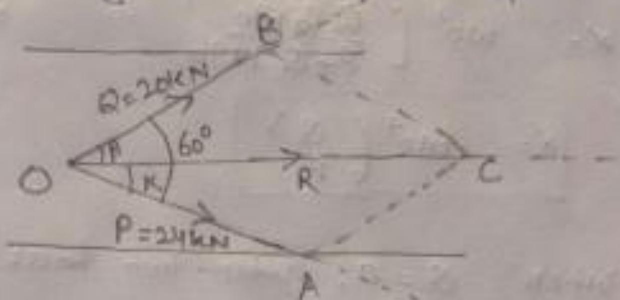
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ}$$

$$\boxed{R = P - Q}$$

obviously the resultant is minimum, when the two forces are collinear but act in opposite direction.

Numerical :- Two locomotives on opposite banks of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes have been measured to be 20kN and 24kN while the angle b/w them is 60°. Find the resultant pull on the vessel and the angle b/w each of the ropes and the sides of the canal. (12)

Sol^n :-



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{24^2 + 20^2 + 2\times 24\times 20\times \cos 60^\circ}$$

$$R = \sqrt{576 + 400 + 960/2} = \sqrt{1456} \Rightarrow \boxed{R = 38.16N}$$

The Inclination of resultant with direction of force P is

$$\tan\kappa = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{20\times\sin 60^\circ}{24 + 20\cos 60^\circ} = \frac{20\times 0.866}{24 + 20\times 0.5}$$

$$\tan\kappa = 0.5094$$

$$\kappa = \tan^{-1}(0.5094) \Rightarrow \boxed{\kappa = 27^\circ}$$

$$\beta = 60 - \kappa$$

$$\beta = 60 - 27^\circ$$

$$\boxed{\beta = 33^\circ}$$