EXAMPLE 5.5

A cantilever truss Wassen type is loaded as shown in the Fig. 5.12. Use the method of sections or the method of joints to find the forces in all members of the truss.

Solution : Consider the free body diagram of *joint C* with the direction of forces assumed as shown

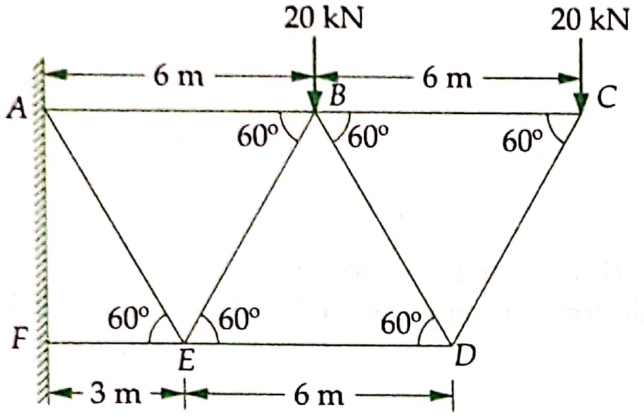
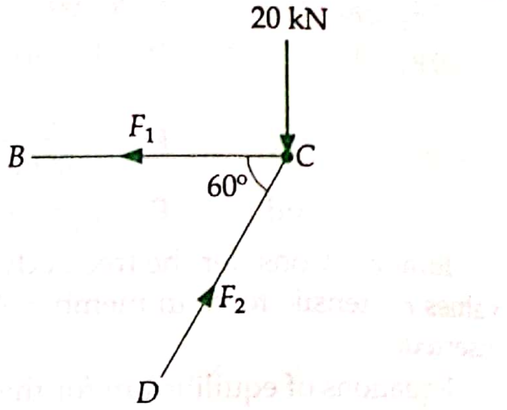


Fig. 5.12



Joint C

Equations of equilibrium are :

$\Sigma F_x = 0 : \quad -F_1 + F_2 \cos 60° = 0$

$\Sigma F_y = 0 : \quad -20 + F_2 \sin 60° = 0$

$$\therefore \quad F_2 = \frac{20}{\sin 60°} = 23.1 \text{ kN (compression)}$$

and $\quad F_1 = F_2 \cos 60° = 23.1 \cos 60° = 11.55$ kN (tension)

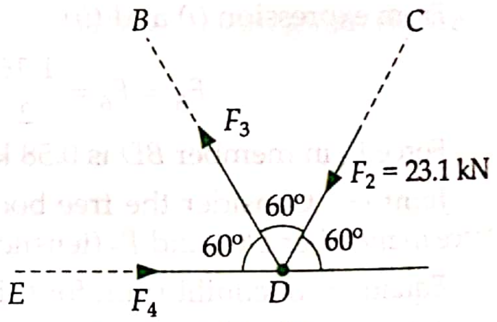
Joint D: From the equations of equilibrium,

$\Sigma F_x = 0 : \quad -F_2 \cos 60° - F_3 \cos 60° + F_4 = 0$

or $\quad 23.1 \times 0.5 + 0.5 F_3 = F_4$

$\Sigma F_y = 0 : \quad -F_2 \sin 60° + F_3 \sin 60° = 0$

or $\quad F_3 = F_2 = 23.1$ kN see below (tension)



Joint D

Substituting $F_3 = 23.1$ kN in expression for F_4, we get

$F_4 = 23.1 \times 0.5 + 0.5 \times 23.1 = 23.1$ kN (compression)

Joint B: From the equations of equilibrium,

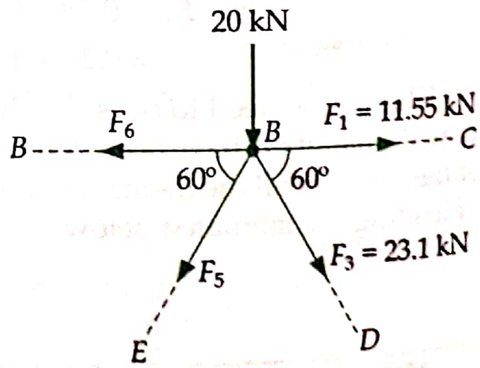
$\Sigma F_x = 0: \quad F_1 + F_3 \cos 60° - F_5 \cos 60 - F_6 = 0$

or $\quad 11.55 + 23.1 \times 0.5 - 0.5 F_5 = F_6$

$\Sigma F_y = 0: \quad -20 - F_3 \sin 60° - F_5 \sin 60° = 0$

or $\quad -20 - 23.1 \times 0.866 = 0.866 F_5$

$$\therefore \quad F_5 = \frac{-20 - 23.1 \times 0.866}{0.866} = -46.2 \text{ kN}$$



Joint B

The negative sign with the magnitude of force F_5 shows that a wrong choice has been made while assuming its direction. Obviously the assumed direction of force in member *BE* need to be reversed. Therefore $F_5 = 46.2$ kN (compression) and *BE* is a compressive member.

Substituting $F_5 = -46.2$ kN in expression for F_6, we get

$F_6 = 11.55 + 23.1 \times 0.5 - 0.5 \times (-46.2) = 46.2$ kN (tension)

Joint E: From the equations of equilibrium,

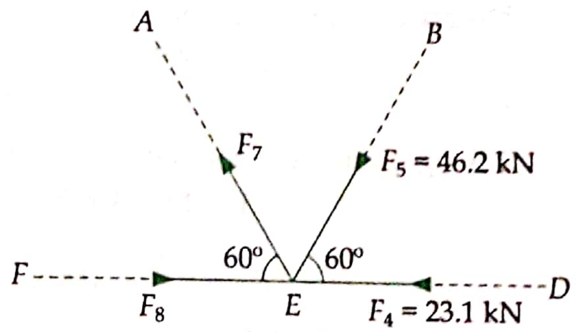
$\Sigma F_x = 0;\quad -F_4 - F_5 \cos 60° - F_7 \cos 60° + F_8 = 0$

or $\quad F_8 = F_4 + F_5 \cos 60° + F_7 \cos 60°$

$\quad = 23.1 + 46.2 \times 0.5 + 0.5\,F_7$

$\Sigma F_y = 0;\quad -F_5 \sin 60° + F_7 \sin 60°$

$\therefore\quad F_7 = F_5 = 46.2$ kN (tension)



Joint E

Substituting F_7 = 46.2 kN in expression for F_8,

we get $F_8 = 23.1 + 46.2 \times 0.5 + 0.5 \times 46.2$

$= 69.3$ kN (compression)

The forces in the various members have been tabulated below:

Member	*AB*	*BC*	*CD*	*DE*	*EF*	*AE*	*BE*	*BD*
Force (kN)	46.2	11.55	23.1	23.1	69.3	46.2	46.2	23.1
Nature	C	T	C	C	C	T	C	T

EXAMPLE 5.6

Determine the forces in all the members of the truss loaded and supported as shown in Fig. 5.13.

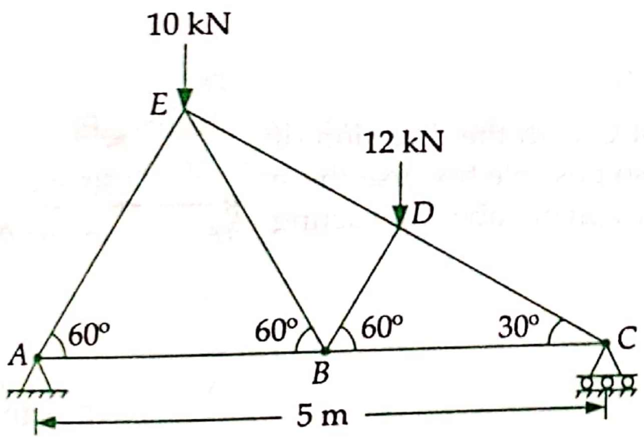
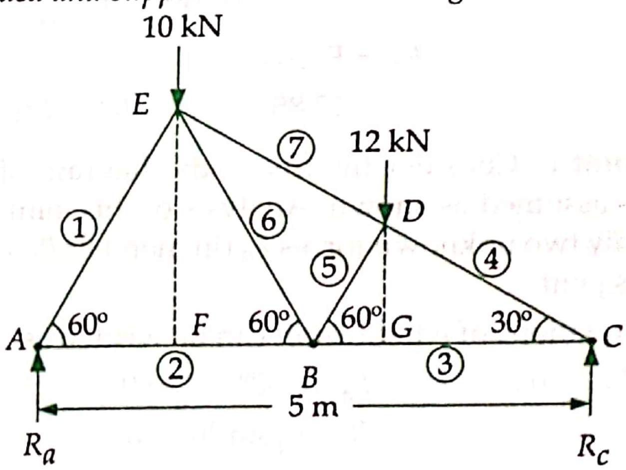


Fig. 5.13



Free body diagram for the whole truss

Solution : The reactions at the supports can be determined by considering equilibrium of the entire truss. Since both the external loads are vertical, only the vertical component of the reaction at the hinged end *A* need to be considered.

With reference to free body diagram of the whole truss, the triangle *AEC* is a right angled triangle with angle *AEC* = 90°. Then

$AE = AC \cos 60° = 5 \times 0.5 = 2.5$ m

$CE = AC \sin 60° = 5 \times 0.866 = 4.33$ m

Further, from the give geometry, $\Delta\, ABE$ is an equilateral triangle and therefore

$AB = BC = AE = 2.5$ m

Distance of line of action of force 10 kN from joint *A*,

$AF = AE \cos 60° = 2.5 \times 0.5 = 1.25$ m

Again, the traingle *BDC* is a right angled triangle with angle *BDC* = 90°. Also

$BC = AC - AB = 5 - 2.5 = 2.5$ m

$BD = BC \cos 60° = 2.5 \times 0.5 = 1.25$ m

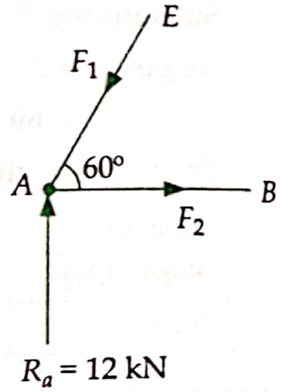
Distance of line of action of force 12 kN from joint A,

$$AG = AB + BG = AB + BD\cos 60^\circ$$
$$= 2.5 + 1.25 \times 0.5 = 3.125 \text{ m}$$

Taking moments about end A, we get

$$R_c \times 5 = 12 \times 3.125 + 10 \times 1.25 = 50$$

$$\therefore \quad R_c = \frac{50}{5} = 10 \text{ kN} \quad \text{and} \quad R_a = (10 + 12) - 10 = 12 \text{ kN}$$



Joint A

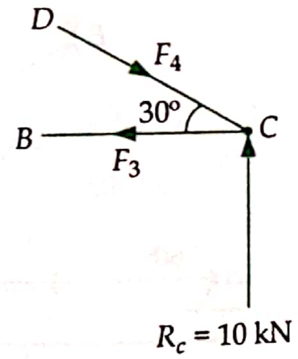
Joint A **:** Consider the free body diagram of joint A with the direction of forces assumed as shown. Start can be made with this joint because there are only two unknown forces F_1 (in member AE) and F_2 (in member AB) acting at this joint.

Equations of equilibrium can be written as

$$\Sigma F_x = 0: \quad F_2 - F_1\cos 60^\circ = 0$$
$$\Sigma F_y = 0: \quad R_a - F_1\sin 60^\circ = 0$$

$$\therefore \quad F_1 = \frac{R_a}{\sin 60^\circ} = \frac{12}{0.866}$$
$$= 13.85 \text{ kN (compression)}$$

$$F_2 = F_1\cos 60^\circ$$
$$= 13.85 \times 0.5 = 6.92 \text{ kN (tension)}$$



Joint C

Joint C**:** Consider the free body diagram of joint C with the direction of forces assumed as shown. Analysis of this joint is also possible because there are only two unknown forces F_3 (in member BC) and F_4 (in member CD) acting at this joint.

Equations of equilibrium can be written as

$$\Sigma F_x = 0: \quad F_4\cos 30^\circ - F_3 = 0$$
$$R_c - F_4\sin 30^\circ = 0$$

$$\therefore \quad F_4 = \frac{R_c}{\sin 30^\circ} = \frac{10}{0.5} = 20 \text{ kN (compression)}$$

$$\text{and} \quad F_3 = F_4\cos 30^\circ = 20 \times 0.866 = 17.32 \text{ kN (tension)}$$

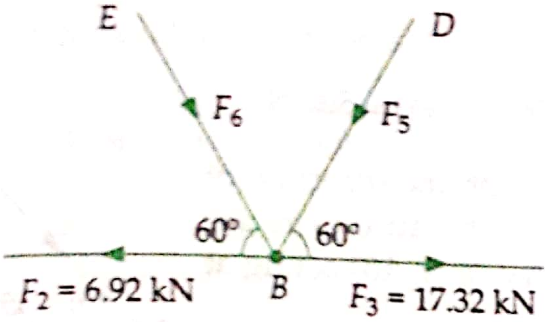
Joint B**:** Consider the free body diagram of joint B with the known values of forces F_2 (tensile force in member AB) and F_3 (tensile force in member BC) inserted.

Equations of equilibrium for this joint are:

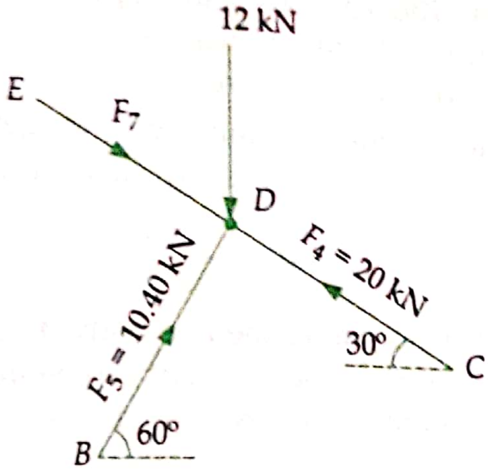
$$\Sigma F_x = 0: \quad 17.32 - 6.92 - F_5\cos 60^\circ - F_6\cos 60^\circ = 0$$
$$\text{or} \quad (F_5 + F_6) \times 0.5 = 10.40$$
$$\text{or} \quad F_5 + F_6 = 20.80 \quad \ldots(i)$$
$$\Sigma F_y = 0: \quad -F_5\sin 60^\circ + F_6\sin 60^\circ = 0$$
$$\text{or} \quad F_5 = F_6 \quad \ldots(ii)$$

From expressions (i) and (ii) : $F_5 = F_6 = 10.40$ kN

Force F_5 in member BD is 10.40 kN compressive and force F_6 in member BE is 10.40 kN tensile.



Joint B



Joint D

Joint D: Consider the free body diagram of joint D with known values of forces F_4 (compressive force in member DC) and F_5 (compressive force in member DB) inserted.

Equations of equilibrium for this joint are:

$\Sigma F_x = 0$; $\quad F_7 \cos 30° + F_5 \cos 60° - F_4 \cos 30° = 0$

or $\quad F_7 \cos 30° = -F_5 \cos 60° + F_4 \cos 30°$

$$= -10.40 \times 0.5 + 20 \times 0.866 = 12.12$$

$$\therefore \quad F_7 = \frac{12.12}{\cos 30°} = 13.99 \text{ kN (compressive)}$$

The magnitude and the nature of the forces in all the members of the given truss for the applied system of loading are tabulated below:

Member	AE	AB	BC	CD	BD	BE	DE
Force	F_1 = 13.85 kN	F_2 = 6.92 kN	F_3 = 17.32 kN	F_4 = 20 kN	F_5 = 10.40 kN	F_6 = 10.40 kN	F_7 = 13.99 kN
Nature	C	T	T	C	C	T	C

EXAMPLE 5.6

Determine the reactions and the forces in each member of a truss supporting two loads as shown in Fig. 5.14.

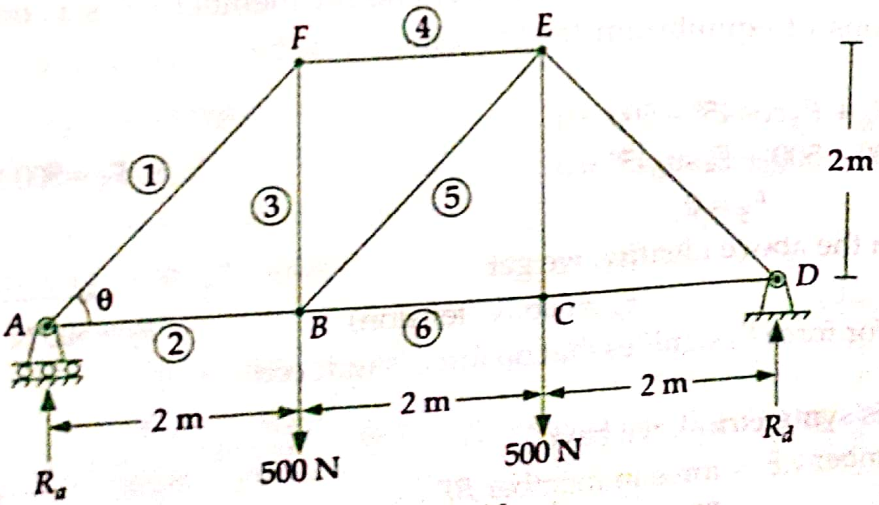


Fig. 5.14

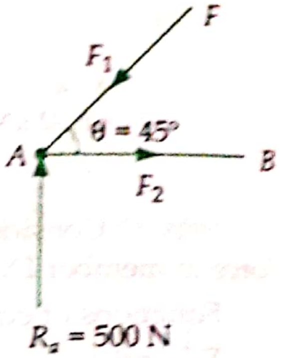
Solution : The reactions at the supports can be determined by considering equilibrium of the entire truss. Since both the external loads are vertical, only the vertical component of reaction at the hinged end D need to be considered.

Taking moments about end A, we get

$$R_d \times 6 = 500 \times 4 + 500 \times 2 = 3000$$

$$\therefore \quad R_d = \frac{3000}{6} = 500 \text{ N}; \quad R_a = (500 + 500) - 500 = 500 \text{ N}$$

Joint A: Consider the free body diagram of joint A with the direction of forces assumed as shown. Start can be made with this joint because there are only two unknown forces F_1 (in member AF) and F_2 (in member AB) acting at this joint.



Joint A

$$\tan\theta = \frac{BF}{AB} = \frac{2}{2} = 1; \quad \theta = 45°$$

Equations of equilibrium can be written as,

$$\Sigma F_x = 0: \qquad F_2 - F_1 \cos 45° = 0$$

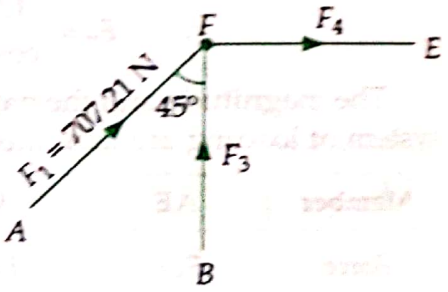
$$\Sigma F_y = 0: \qquad R_a - F_1 \sin 45° = 0$$

$$\therefore \quad F_1 = \frac{R_a}{\sin 45°} = \frac{500}{0.707} = 707.21 \text{ N (compression)}$$

$$F_2 = F_1 \cos 45°$$

$$= 707.21 \times 0.707 = 500 \text{ N (tension)}.$$

Joint F: Equations of equilibrium for the joint F can be written as



Joint F

$$\Sigma F_x = 0: \qquad F_4 + 707.21 \sin 45° = 0$$

$$\therefore \quad F_4 = -707.21 \times 0.707 = -500 \text{ N}$$

$$\Sigma F_y = 0: \qquad F_3 + 707.21 \cos 45° = 0$$

$$\therefore \quad F_3 = -707.21 \times 0.707 = -500 \text{ N}$$

The – ve sign with the magnitude of forces F_3 and F_4 shows that a wrong choice has been made while assuming their directions. Obviously the assumed direction of the forces in members FB and FE need to be reversed.

Therefore the member FB is a tension member and the member FE is a compression member.

Joint B: Equations of equilibrium for the joint B can be written as

$$\Sigma F_x = 0: \qquad F_6 + F_5 \cos 45° - 500 = 0$$

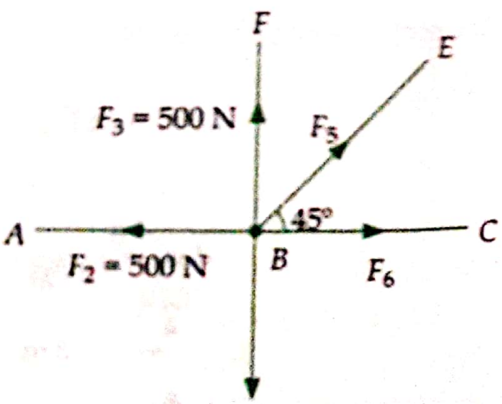
$$\Sigma F_y = 0: \qquad 500 - 500 + F_5 \sin 45° = 0$$

$$\therefore \quad F_5 = 0$$

Putting $F_5 = 0$ in the above identity, we get

$$F_6 = 500 \text{ N (tension)}$$

The zero value for force F_5 signifies that no force is induced in the member BE.



Joint B

Since the truss is symmetrical, we have

$$\text{Force in member } CE = \text{force in member } BF$$

$$= 500 \text{ N (tension)}$$

Force in member CD = force in member AB
= 500 N (tension)

Force in member DE = force in member AF
= 707.21 (compression)

The forces in all the members of the given truss for the applied system of loading are tabulated below:

Member	AF, DE	AB, CD	BF, CE	FE	BE	BC
Force	707.21 N	500 N	500 N	500 N	0	500 N
Nature	C	T	T	C	—	T

EXAMPLE 5.7

A truss has been loaded and supported as shown in Fig. 5.15. Make calculations for the reactions at the supports and the forces in the members of the truss.

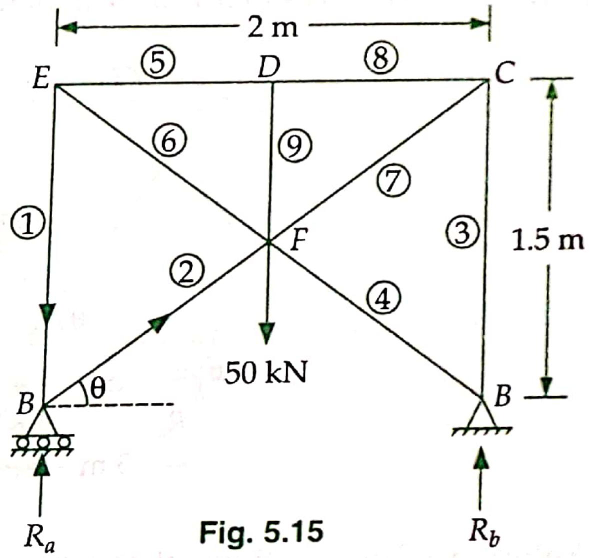


Fig. 5.15

Solution : Both the supports will have only vertical reactions as the external load of 50 kN at the joint F is vertical. Further, due to symmetry

$$R_a = R_b = \frac{50}{2} = 25 \text{ kN}$$

From the geometry of the truss, the inclination of the inclined members with the horizontal is

$$\tan\theta = \frac{0.75}{1} = \frac{3}{4}$$

That gives: $\sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$

Joint A: From the equations of equilibrium,

$\Sigma F_x = 0$: $\quad F_2 \cos\theta = 0$;

$\therefore \quad F_2 = 0$ (no force)

$\Sigma F_y = 0$: $\quad R_a - F_1 = 0$;

$\therefore \quad F_1 = R_a =$ **25 kN** (compressive)

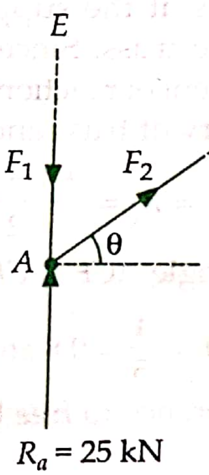
Joint B: From the equations of equilibrium,

$\Sigma F_x = 0$: $\quad F_4 \cos\theta = 0$;

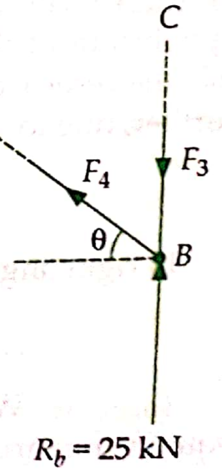
$\therefore \quad F_4 = 0$ (no force)

$\Sigma F_y = 0$: $\quad R_b - F_3 = 0$;

$\therefore \quad F_3 = R_b =$ **25 kN** (compressive)



Joint A



Joint B

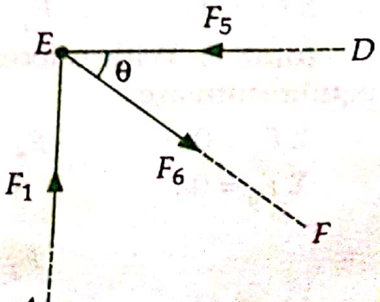
Joint C: From the equations of equilibrium,

$\Sigma F_x = 0$: $\quad F_6 \cos\theta - F_5 = 0$

$\Sigma F_y = 0$: $\quad F_1 - F_6 \sin\theta = 0$

$$\therefore \quad F_6 = \frac{F_1}{\sin\theta} = \frac{25}{3/5} = 41.67 \text{ kN (tensile)}$$

and $\quad F_5 = F_6 \cos\theta = 41.67 \times \dfrac{4}{5} = 33.34$ kN (compression)



Joint E

Likewise considering equilibrium of joint C,

$$F_7 = 41.67 \text{ kN (tension)}$$

and $$F_8 = 33.34 \text{ kN (compression)}$$

Joint *D*: Considering equilibrium of forces in vertical direction,

$$F_9 = 0 \text{ (no force)}$$

The forces in various members of the truss are tabulated below:

Members	AE, BC	DE, DC	FE, FC	FD, FA, FB
Force	25 kN	· 33.34 kN	41.67 kN	0 (no force)
Nature	compressive	compressive	tensile	---

EXAMPLE 5.8

Determine the forces in all the members of the truss loaded and supported as shown in Fig. 5.16.

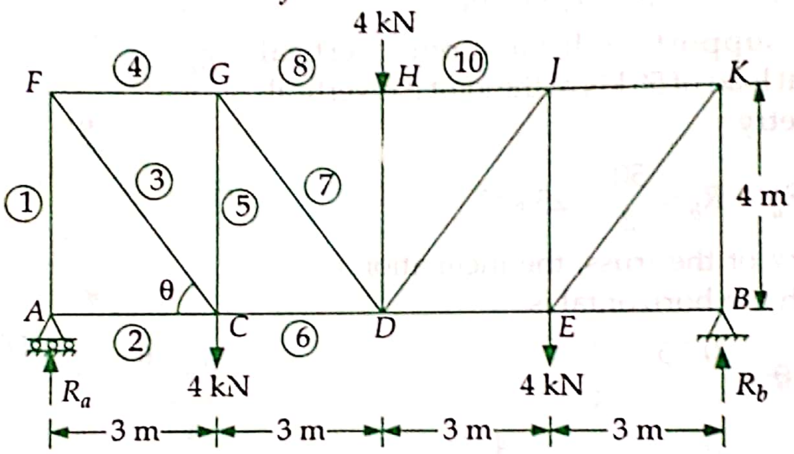
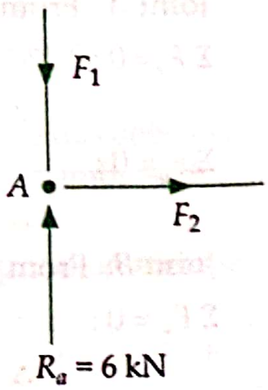


Fig. 5.16

Solution : The reactions at the supports can be determined by considering equilibrium of the entire truss. Since all the three external loads are vertical, only the vertical component of reaction at the hinged end B need to be considered. Further, due to symmetry of truss and loading



Joint A

$$R_a = R_b = \frac{4+4+4}{2} = 6 \text{ kN}$$

In right angled triangle *ACF*, $CF = \sqrt{3^2 + 4^2} = 5$ m

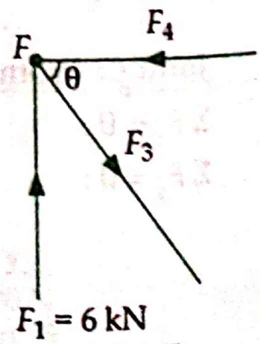
$$\therefore \quad \sin\theta = \frac{4}{5} = 0.8 \text{ and } \cos\theta = \frac{3}{5} = 0.6$$

Joint *A*: With reference to free body diagram of joint *A*, the equations of equilibrium are:

$\Sigma F_x = 0$: $\quad F_2 = 0$

$\Sigma F_y = 0$: $\quad F_1 = 6$ **kN** (compression)

Joint *F*: With reference to free body diagram of joint *F*, the equations of equilibrium are :



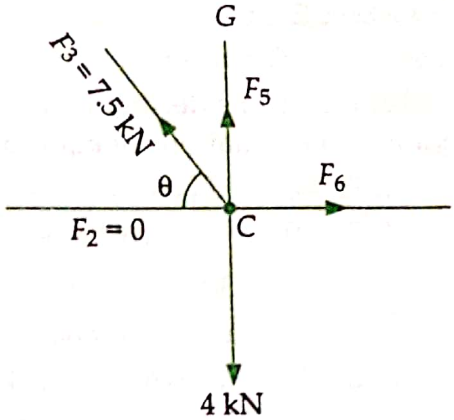
Joint F

$\Sigma F_x = 0$: $\quad -F_4 + F_3 \cos\theta = 0$

$\Sigma F_y = 0$: $\quad 6 - F_3 \sin\theta = 0$

$$\therefore \quad F_3 = \frac{6}{\sin\theta} = \frac{6}{0.8} = 7.5 \text{ kN (tension)}$$

and $F_4 = F_3 \cos\theta = 7.5 \times 0.6$
$= 4.5$ **kN** (compression)



Joint C

Joint C: With reference to free body diagram of joint C, the equations of equilibrium are:

$F_x = 0$: $\quad F_6 - F_3\cos\theta = 0$

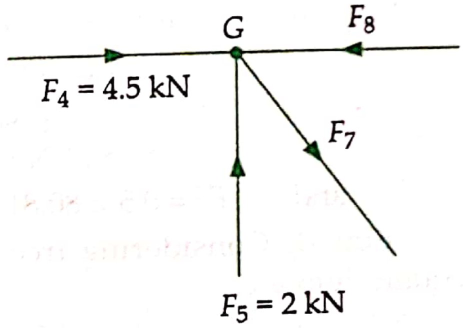
$\therefore \quad F_6 = F_3 \cos\theta = 7.5 \times 0.6$
$= 4.5$ **kN** (tension)

$F_y = 0$: $\quad F_5 + F_3 \sin\theta - 4 = 0$

$\therefore \quad F_5 = 4 - F_3 \sin\theta$
$= 4 - 7.5 \times 0.8 = -2$ **kN**

The negative sign with the magnitude of force F_5 shows that a wrong choice has been made while assuming its direction. Obviously the assumed direction of force in member CG need to be reversed.

Therefore, $F_5 = 2$ kN (compression) and CG is a compression member.



Joint G

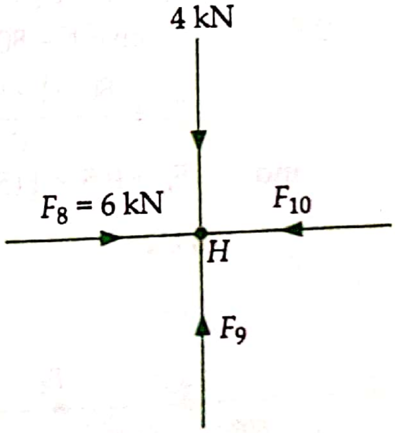
Joint G: With reference to free body diagram of joint G, the equations of equilibrium are :

$\Sigma F_x = 0$: $\quad 4.5 - F_8 + F_7 \cos\theta = 0$

$\Sigma F_y = 0$: $\quad 2 - F_7 \sin\theta = 0$

$$\therefore \quad F_7 = \frac{2}{\sin\theta} = \frac{2}{0.8} = 2.50 \text{ kN (tension)}$$

and $F_8 = 4.5 + F_7 \cos\theta$
$= 4.5 + 2.50 \times 0.6 = 6$ **kN** (compression)



Joint H

Joint H: With reference to free body diagram of joint H, the equations of equilibrium are :

$\Sigma F_x = 0$: $\quad F_{10} - 6 = 0$

$\therefore \quad F_{10} = 6$ **kN** (compression)

$\Sigma F_y = 0$: $\quad F_9 - 4 = 0$

$\therefore \quad F_9 = 4$ **kN** (compression)

The truss is symmetrical and as such, there is no need to consider equilibrium of the joints on the left. Forces in all the members of the given truss for the applied system of loading are tabulated below:

Member	Force	Nature
AF, BK	6 kN	compressive
AC, BE	0	—
CF, EK	7.5 kN	tensile
FG, JK	4.5 kN	compressive
GC, JE	2 kN	compressive
CD, DE	4.5 kN	tensile
DG, DJ	2.5 kN	tensile
GH, HJ	6 kN	compressive
HD	4 kN	compressive

EXAMPLE 5.9

Determine the forces in each member of the cantilever truss loaded as shown in Fig. 5.17.

Solution : Considering the free body diagram of joint A, the equations of equilibrium are :

$$\Sigma F_x = 0: \quad F_1 \cos 60^\circ - F_2 = 0$$
$$\Sigma F_y = 0: \quad F_1 \sin 60^\circ - 30 = 0$$
$$\therefore \quad F_1 = \frac{30}{\sin 60^\circ} = \frac{30}{0.866} = \mathbf{34.64\ kN}\ \text{(tension)}$$
and $F_2 = F_1 \cos 60^\circ = 34.64 \times 0.5$
$$= \mathbf{17.32\ kN}\ \text{(compression)}$$

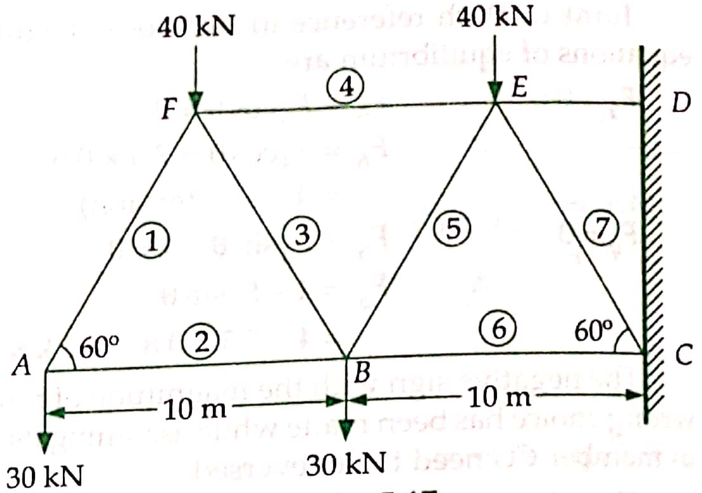
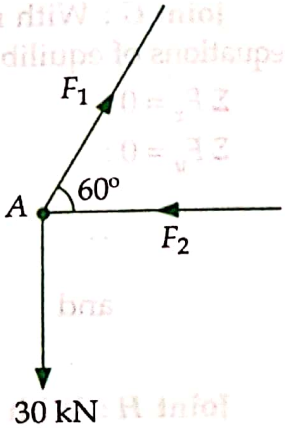


Fig. 5.17

Joint F : With reference to free body diagram of joint F, the equations of equilibrium are :

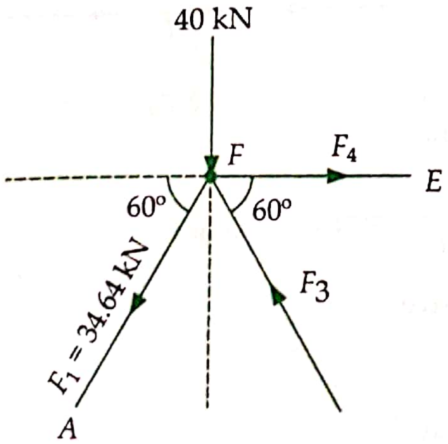
$$\Sigma F_x = 0: \quad F_4 - F_3 \cos 60^\circ - 34.64 \cos 60^\circ$$
or $$F_4 = 0.5\, F_3 + 17.32$$
$$\Sigma F_y = 0: \quad F_3 \sin 60 - 34.64 \sin 60^\circ - 40 = 0$$
$$\therefore \quad F_3 = \frac{(34.64 \times 0.866) + 40}{0.866}$$
$$= \mathbf{80.81\ kN}\ \text{(compression)}$$
and $F_4 = 0.5 \times 80.81 + 17.32 = \mathbf{57.72\ kN}$ (tension)



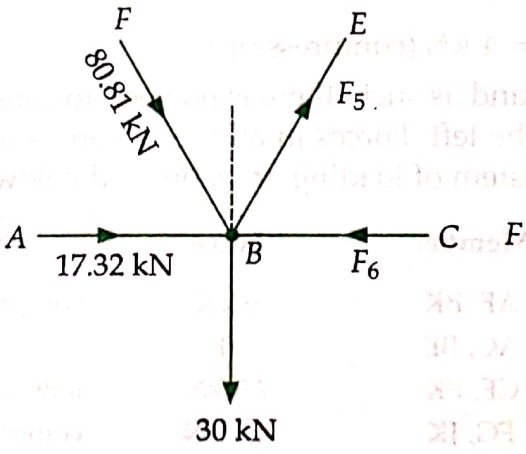
Joint A

Joint B: Considering free body diagram of joint B, the equations of equilibrium are:

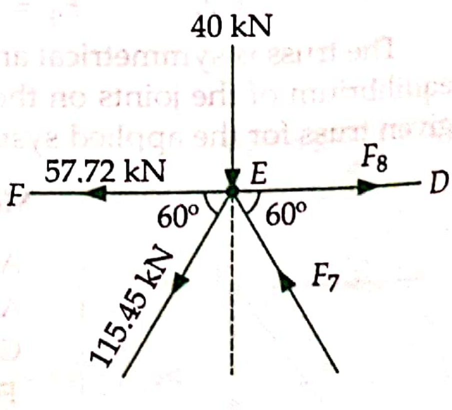
$$\Sigma F_x = 0: \quad F_5 \cos 60^\circ + 80.81 \cos 60^\circ + 17.32 - F_6 = 0$$
$$F_6 = 0.5\, F_5 + 80.81 \times 0.5 + 17.32 = 0.5\, F_5 + 57.72$$
$$\Sigma F_y = 0: \quad F_5 \sin 60^\circ - 80.81 \sin 60^\circ - 30 = 0$$
$$\therefore \quad F_5 = \frac{80.81 \times 0.866 + 30}{0.866} = \mathbf{115.45\ kN}\ \text{(tension)}$$
and $F_6 = 0.5 \times 115.45 + 57.72 = \mathbf{115.44}$ (compression)



Joint F



Joint B



Joint E

Joint E: Considering joint E, the equations of equilibrium are:

$$\Sigma F_x = 0: \quad F_8 - 57.72 - F_7 \cos 60^\circ - 115.45 \cos 60^\circ = 0$$
or $$F_8 = 57.72 + 0.5\, F_7 + 115.45 \times 0.5 = 0.5\, F_7 + 115.44$$

$\Sigma F_y = 0: \quad F_7 \sin 60° - 115.45 \sin 60° - 40 = 0$

$$\therefore \quad F_7 = \frac{115.45 \times 0.866 + 40}{0.866} = \mathbf{161.62\ kN} \text{ (compression)}$$

and $\quad F_8 = 0.5 \times 161.62 + 115.44 = \mathbf{196.25\ kN}$ (tension)

The magnitude and the nature of forces in the various members of the truss have been tabulated below:

Member	AF	AB	BF	FE	BE	BC	CE	ED
Force kN	34.64	17.32	80.81	57.72	115.45	115.44	161.62	196.25
Nature	tensile	compressive	compressive	tensile	tensile	compressive	compressive	tensile

EXAMPLE 5.10

Determine the forces in the various members of the cantilever truss loaded and supported as shown in Fig. 5.18.

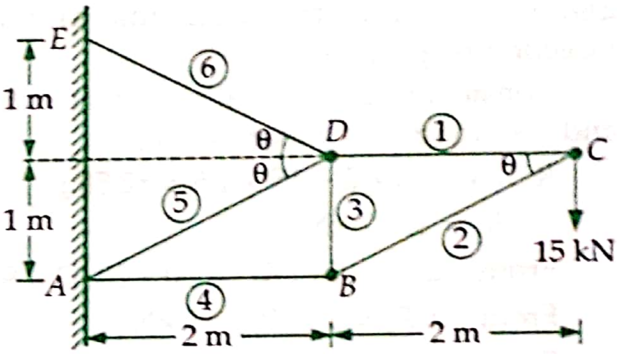


Fig. 5.18

Solution : From the geometrical configuration of the truss,

$$BC = \sqrt{2^2 + 1^2} = 2.236 \text{ m}$$

$$\sin\theta = \frac{1}{2.236} = 0.447$$

$$\cos\theta = \frac{2}{2.236} = 0.894$$

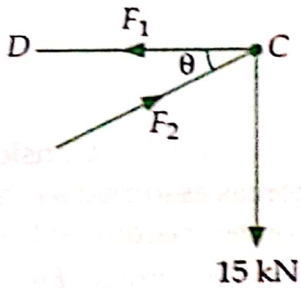
Joint C: Considering the free body diagram of joint C, the equation of equilibrium are:

$\Sigma F_x = 0: \quad F_2 \cos\theta - F_1 = 0$

$\Sigma F_y = 0: \quad F_2 \sin\theta - 15 = 0$

$$\therefore \quad F_2 = \frac{15}{\sin\theta} = \frac{15}{0.447} = \mathbf{33.56\ kN} \text{ (compression)}$$

and $\quad F_1 = F_2 \cos\theta = 33.56 \times 0.894 = \mathbf{30.00\ kN}$ (tension)



Joint C

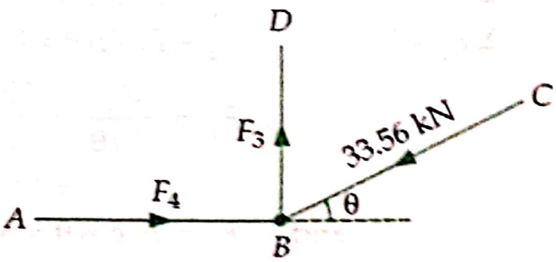
Joint B: With reference to free body diagram of joint B, the equations of equilibrium are:

$\Sigma F_x = 0: \quad F_4 - 33.56 \cos\theta$

$\therefore \quad F_4 = 33.56 \times 0.894 = \mathbf{30.00\ kN}$ (compression)

$\Sigma F_y = 0: \quad F_3 - 33.56 \sin\theta = 0$

$\therefore \quad F_3 = 33.56 \times 0.447 = \mathbf{15.00\ kN}$ (tension)



Joint B

Joint D: Considering joint D, the equations of equilibrium are:

$\Sigma F_x = 0: \quad 30 - F_5 \cos\theta - F_6 \cos\theta = 0$

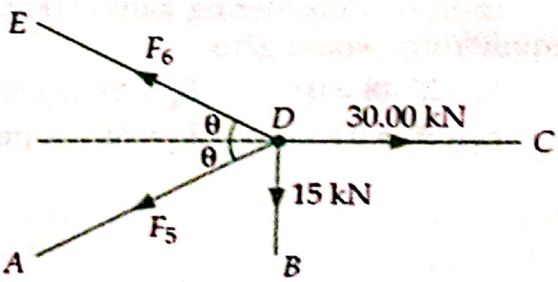
or $\quad F_5 + F_6 = \dfrac{30}{0.894} = 33.56 \quad ...(i)$

$\Sigma F_y = 0: \quad F_6 \sin\theta - F_5 \sin\theta - 15 = 0$

or $\quad F_6 - F_5 = \dfrac{15}{0.447} = 33.56 \quad ...(ii)$

From identities (*i*) and (*ii*),

$F_6 = \mathbf{33.56\ kN}$ (tension) $\quad$ and $\quad F_5 = \mathbf{0}$ (no force)



Joint D

The magnitude and the nature of forces in the various members of the truss have been tabulated below:

Member	CD	CB	BD	BA	AD	DE
Force	30 kN	33.56 kN	15 kN	30 kN	0	33.56 kN
Nature	tensile	compressive	tensile	compressive	—	tensile

EXAMPLE 5.11

Determine the magnitude and nature of forces in the various members of the triangular truss loaded and supported as shown in Fig. 5.19.

Solution : The roller support (joint A) is frictionless and provides a reaction R_a at right angles to the roller base. The reaction at the hinged support (joint B) will have two components R_{bx} and R_{by} acting in the horizontal and vertical direction respectively.

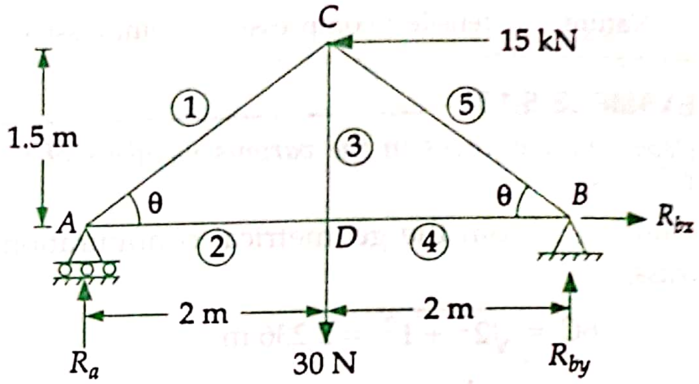


Fig. 5.19

Consider free body diagram of the entire truss and take moments about joint B;

$R_a \times 4 = 30 \times 2 + 15 \times 1.5 = 82.50$;

$R_a = 20.62$ kN

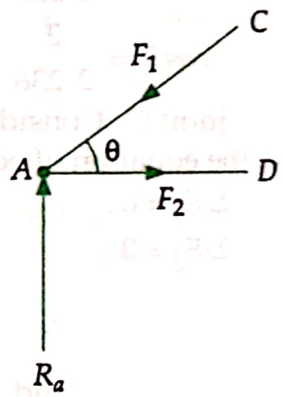
From $\Sigma F_y = 0$: $R_{by} = 30 - 20.62 = 9.38$ kN

From $\Sigma F_x = 0$: R_{bx} 15 kN

Further, from the geometrical configuration of the truss

$$AC = \sqrt{(AD)^2 + (CD)^2} = \sqrt{2^2 + 1.5^2} = 2.5 \text{ m}$$

$$\sin\theta = \frac{CD}{AC} = \frac{1.5}{2.5} = 0.6 \quad \text{and} \quad \cos\theta = \frac{AD}{AC} = \frac{2}{2.5} = 0.8$$



Joint A

Joint A : Consider the free body diagram of joint A with the direction of forces assumed as shown. Start can be made with this joint because of the three forces meeting at this point, only two are unknown.

Equations of equilibrium can be written as

$\Sigma F_x = 0$: $F_2 - F_1 \cos\theta = 0$

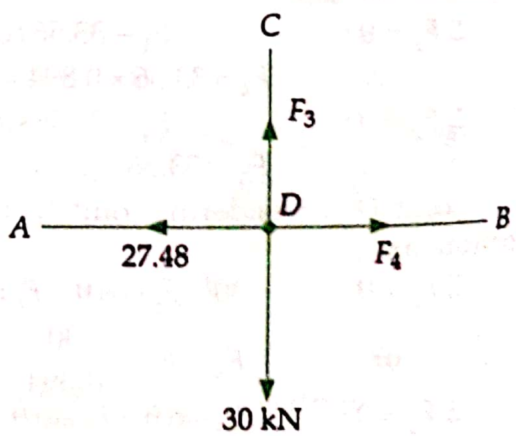
$\Sigma F_y = 0$: $R_a - F_1 \sin\theta = 0$

$$\therefore \quad F_1 = \frac{R_a}{\sin\theta} = \frac{20.62}{0.6}$$

= 34.36 kN (compression)

and $F_2 = F_1 \cos\theta = 34.36 \times 0.8$

= 27.48 kN (tensile)



Joint D

Joint D: Considering joint D (Fig. 5.52), the equations of equilibrium would give

$F_4 - 27.48 = 0$: $F_4 = 27.48$ kN (tensile)

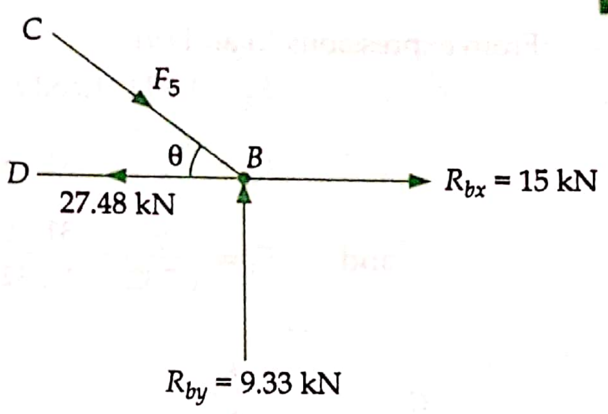
$F_3 - 30 = 0$: $F_3 = 30$ kN (tensile)

Joint B: Considering joint B, the equations of equilibrium are :

$\Sigma F_x = 0$: $15 - 27.48 + F_5 \cos\theta = 0$

$$\therefore \quad F_5 = \frac{27.48 - 15}{\cos\theta}$$

$$= \frac{12.48}{0.8} = 15.60 \text{ kN (compression)}$$



Joint B

Check:

$\Sigma F_y = 0$: $F_5 \sin\theta - R_{by} = 0$

$$F_5 = \frac{R_{by}}{\sin\theta} = \frac{9.38}{0.6} = 15.63 \text{ kN (compression)}$$

The magnitude and the nature of forces in the various members of the truss have been tabulated below:

Member	AD	AC	DC	DB	BC
Force	27.48 kN	34.36 kN	30 kN	27.48 kN	15.60 kN
Nature	T	C	T	T	C

EXAMPLE 5.12

A truss has been loaded and supported as shown in Fig. 5.20. Make calculations for the reactions at the supports and the forces in the members of the truss.

Solution : The reaction at the hinged support (joint A) will have two R_{ax} and R_{ay} components acting in the horizontal and vertical directions respectively. The roller support (joint B) is frictionless and provides a reaction R_b at right angles to the roller base.

Height of the truss $= 2\tan 60° = 3.464$ m

Consider free body diagram of the entire truss and take moments about joint A;

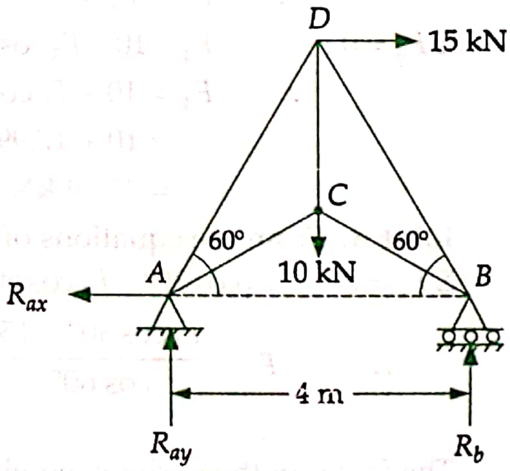


Fig. 5.20

$$R_b \times 4 = (10 \times 2) + (15 \times 3.464)$$
$$= 20 + 51.96 = 71.96$$

$$\therefore \quad R_b = \frac{71.96}{4} = 17.99 \text{ kN}$$

From $\Sigma F_y = 0$;

$$R_{ay} = 10 - 17.99 = -7.99 \text{ kN}$$

The negative sign indicates that the reaction R_{ay} is acting downwards.

From $\Sigma F_x = 0$; $R_{ax} = 15$ kN

Joint B : Consider free body diagram of joint B with the direction of forces assumed as shown. Equations of equilibrium can be written as :

$\Sigma F_x = 0$: $F_2 \cos 60° - F_1 \cos 30° = 0$

or $0.5\, F_2 - 0.866\, F_1$; $F_2 = 1.732\, F_1$ ...(*i*)

$\Sigma F_y = 0$: $-F_2 \sin 60° + F_1 \sin 30° + R_b = 0$

or $-0.866\, F_2 + 0.5\, F_1 + 17.99 = 0$

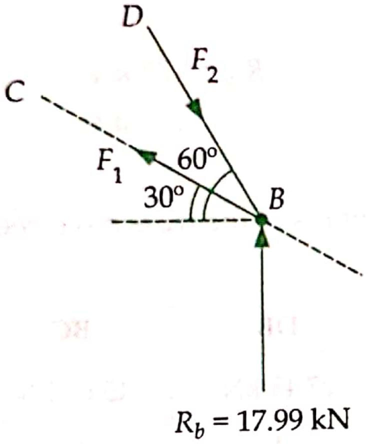
or $F_1 = 1.732\, F_2 - 35.98$ ...(*ii*)

From expressions (i) and (ii)

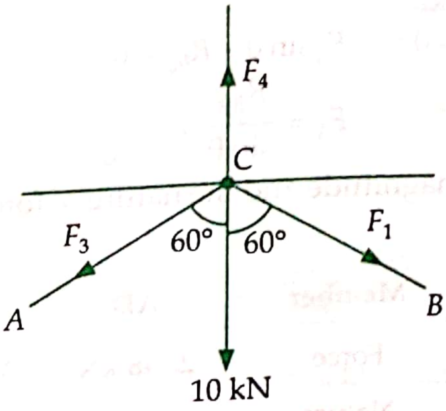
$$F_2 = 1.732\,(1.732\,F_2 - 35.98) = 3F_2 - 62.32$$

$$\therefore \quad F_2 = \frac{62.32}{2} = 31.16 \text{ kN (compressive)}$$

and $$F_1 = \frac{F_2}{1.732} = \frac{31.16}{1.732} = 17.99 \text{ kN (tensile)}$$



Joint B



Joint C

Joint C: From the equations of equilibrium,

$\Sigma F_x = 0:$ $\quad F_1 \sin 60° - F_3 \sin 60° = 0$

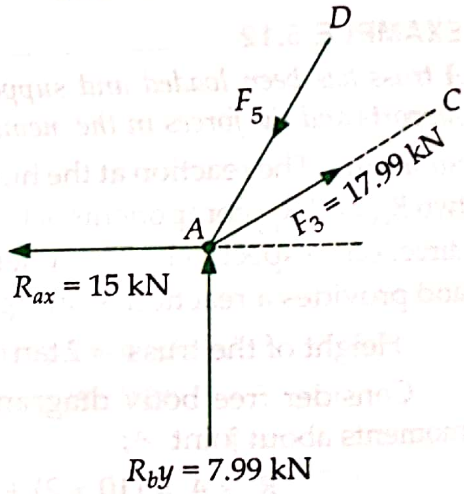
$\therefore \quad F_3 = F_1 = 17.99$ **kN** (tensile)

$\Sigma F_y = 0:$ $\quad F_4 - 10 - F_3 \cos 60° - F_1 \cos 60° - 0$

$\therefore \quad F_4 = 10 + F_3 \cos 60° + F_1 \cos 60°$

$\qquad = 10 + 17.99 \times 0.5 + 17.99 \times 0.5$

$\qquad = 27.99$ **kN** (tensile)



Joint A

Joint A: From the equations of equilibrium

$\Sigma F_x = 0:$ $\quad F_3 \cos 30° - F_5 \cos 60° - 15 = 0$

$$\therefore \quad F_5 = \frac{F_3 \cos 30° - 15}{\cos 60°} = \frac{17.99 \times 0.866 - 15}{0.5}$$

$$= 1.16 \text{ kN (compressive)}$$

The forces in the various members of the truss are tabulated below :

Member	AC	AD	CD	BC	BD
Force	17.99 kN	1.16 kN	27.99 kN	17.99 kN	31.16 kN
Nature	tensile	compressive	tensile	tensile	compressive

EXAMPLE 5.13

Make calculations for the forces in each member of the truss loaded and supported as shown in Fig. 5.21. All members of the truss are 2 m in length.

Solution : Let T be the tension in rope ; R_{bx} and R_{by} be the reactions at the hinged support as shown in the figure.

Consider equilibrium of the entire truss

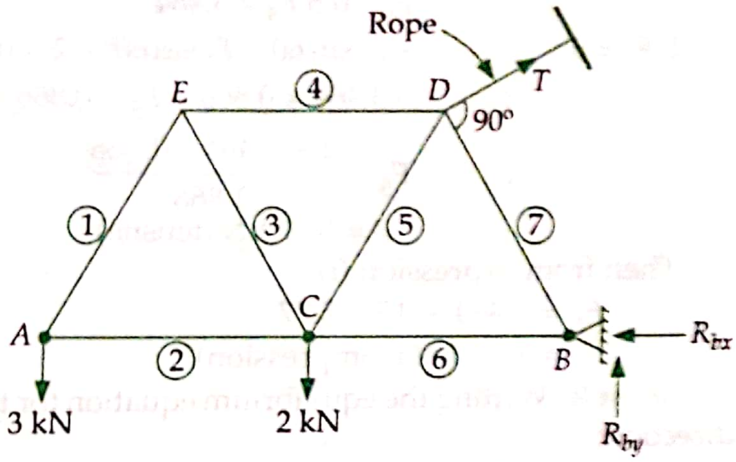


Fig. 5.21

$M_b = 0$: $\quad T \times DB - 2 \times CB - 3 \times AB = 0$

$\therefore \quad T = \dfrac{2 \times CB + 3 \times AB}{DB}$

$= \dfrac{2 \times 2 + 3 \times 4}{2} = 8 \text{ kN}$

$\Sigma F_x = 0$: $\quad T\cos 30 - R_{bx} = 0$

$\therefore \quad R_{bx} = T\cos 30 = 8 \times 0.866$

$= 6.93 \text{ kN}$

$\Sigma F_y = 0$: $\quad R_{by} + T\sin 30^\circ - 2 - 3 = 0$

$\therefore \quad R_{by} = 2 + 3 - 8\sin 30^\circ$

$= 2 + 3 - 8 \times 0.5 = 1 \text{ kN}$

Joint *A*: From the equations of equilibrium

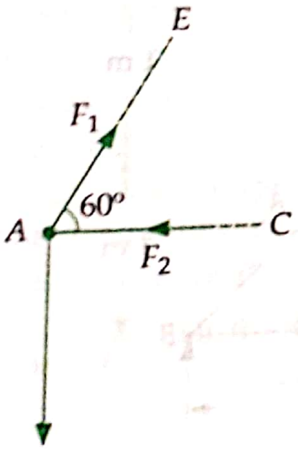
$\Sigma F_x = 0$: $\quad F_1 \cos 60^\circ - F_2 = 0$

$\Sigma F_y = 0$: $\quad F_1 \sin 60^\circ - 3 = 0$

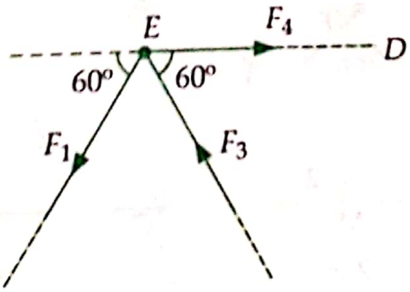
$\therefore \quad F_1 = \dfrac{3}{\sin 60^\circ} = 3.464 \text{ kN}$ (tensile)

and $\quad F_2 = F_1 \cos 60^\circ = 3.464 \times 0.5$

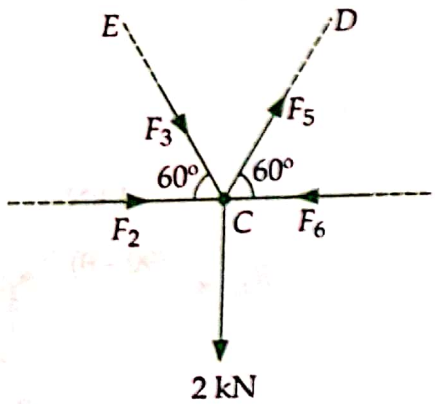
$= 1.732 \text{ kN}$ (compression)



Joint A



Joint E



Joint C

Joint *E*: From the equations of equilibrium

$\Sigma F_x = 0$: $\quad F_4 - F_1 \cos 60^\circ - F_3 \cos 60^\circ = 0$

$\Sigma F_y = 0$: $\quad F_1 \sin 60^\circ + F_3 \sin 60^\circ = 0$

$\therefore \quad F_3 = F_1 = 3.464 \text{ kN}$ (compression)

and $\quad F_4 = (F_1 + F_3)\cos 60^\circ$

$= (3.464 + 3.464) \times 0.5 = 3.464 \text{ kN}$ (tensile)

Joint *C*: Writing the equations of equilibrium,

$\Sigma F_x = 0$: $\quad F_2 - F_6 + F_3 \cos 60^\circ + F_5 \cos 60^\circ = 0$

or $\quad 1.732 - F_6 + 3.464 \times 0.5 + 0.5\, F_5 = 0$

$$F_6 - 0.5\,F_5 = 3.464 \qquad \ldots(i)$$

$\Sigma F_y = 0:$ $\quad -F_3 \sin 60 + F_5 \sin 60^\circ - 2 = 0$

or $\quad -3.464 \times 0.866 + F_5 \times 0.866 = 2$

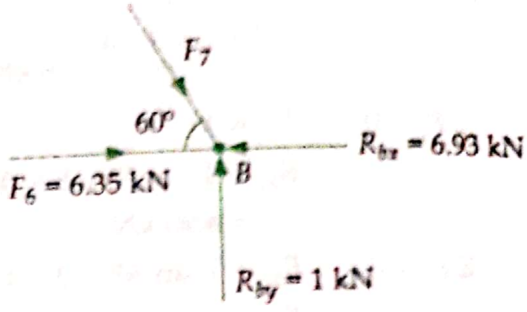
$$\therefore \quad F_5 = \frac{2 + 3.464 \times 0.866}{0.866}$$

$$= 5.77 \text{ kN (tensile)}$$

Then from expression (*i*)

$$F_6 = 3.464 + 0.5 \times 5.77$$

$$= 6.35 \text{ kN (compression)}$$



Joint B

Joint B: Writing the equilibrium equation for the vertical direction:

$\Sigma F_y = 0: \quad F_7 \sin 60^\circ - 1 = 0$

$$\therefore \quad F_7 = \frac{1}{0.866} = 1.155 \text{ kN (compression)}$$

EXAMPLE 5.14

For the truss loaded as shown in Fig. 5.22, make calculations for the reactions at the supports and forces set up in each member of the truss.

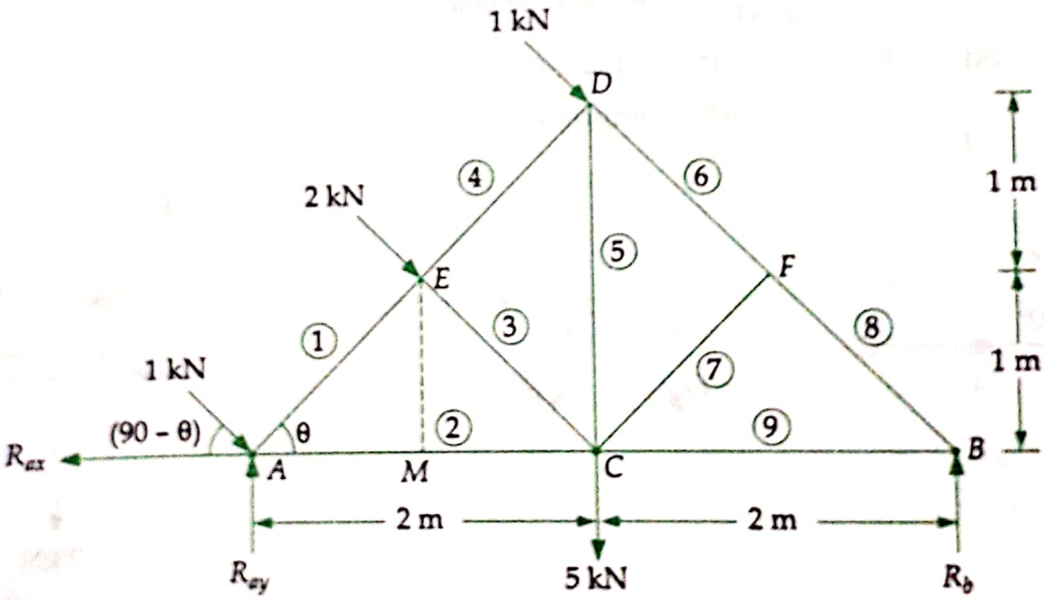


Fig. 5.22

Solution : $\quad AD = \sqrt{(AC)^2 + (CD)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2.828$ m

$$AE = \frac{AD}{2} = 1.414 \text{ m}$$

$$\sin\theta = \frac{EM}{AE} = \frac{1}{1.414} = 0.707$$

$$\cos\theta = \frac{AM}{AE} = \frac{1}{1.414} = 0.707$$

Applying equilibrium conditions for the entire truss,

$\Sigma M_a = 0: \quad -2 \times AE - 1 \times AD + R_b \times AB - 5 \times AC = 0$

or $\quad -2 \times 1.414 - 1 \times 2.828 + R_b \times 4 - 5 \times 2$

$$\therefore \quad R_b = \frac{2\times 1.414 + 1\times 2.828 + 10}{4} = 3.914 \text{ kN}$$

$\Sigma F_x = 0: \quad 1\cos(90-\theta) + 2\cos(90-\theta) + 1\cos(90-\theta) - R_{ax} = 0$

or $\quad R_{ax} = (1+2+1)\sin\theta = 4\times 0.707 = 2.828$ kN

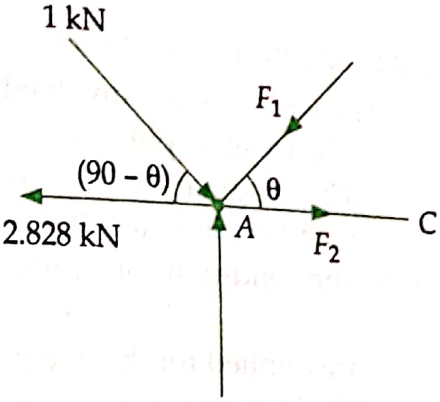
$\Sigma F_y = 0: \quad R_{ay} - 5 + R_b - 1\sin(90-\theta)$

$\qquad -2\sin(90-\theta) - 1\sin(90-\theta)$

or $\quad R_{ay} = 5 - R_b + (1+2+1)\cos\theta$

$\qquad = 5 - 3.914 + 4\times 0.707$

$\qquad = 3.914$ kN



Joint A

Joint *A*: Consider the free body diagram of joint *A* with the direction of forces assumed as shown.

Equations of equilibrium are:

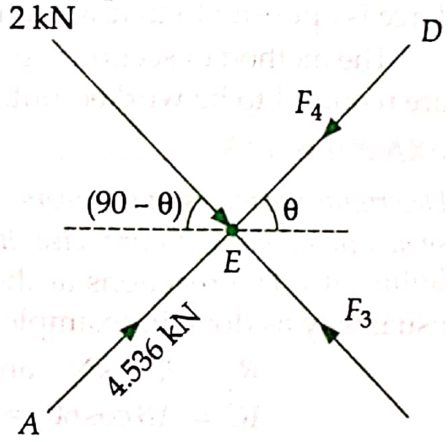
$\Sigma F_x = 0: \quad F_2 - F_1\cos\theta + 1\cos(90-\theta) - 2.828 = 0$

or $\quad F_2 = F_1\cos\theta - \sin\theta + 2.828$

$\Sigma F_y = 0: \quad -F_1\sin\theta - 1\sin(90-\theta) + 3.914$

$$\therefore \quad F_1 = \frac{3.914 - \cos\theta}{\sin\theta} = \frac{3.914 - 0.707}{0.707} = 4.536 \text{ kN (compression)}$$

and $\quad F_2 = 4.536\times 0.707 - 0.707 + 2.828 = 5.328$ (tension)



Joint E

Joint *E*: With reference to free body diagram of joint *E*, the equations of equilibrium along the direction *AD* and perpendicular to it are :

$F_4 - 4.536 = 0; \quad F_4 = 4.536$ kN (compression)

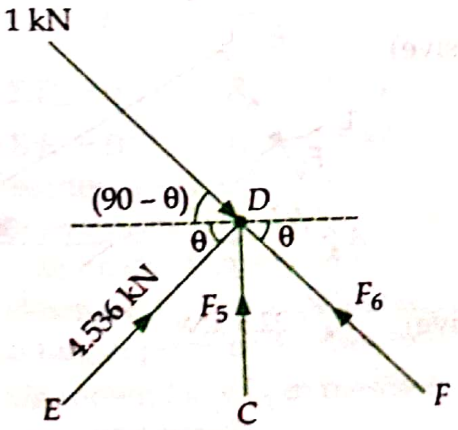
$F_3 - 2 = 0; \quad F_2 = 2$ kN (compression)

Joint *D*: With reference to free body diagram of joint *D*, the equations of equilibrium are:

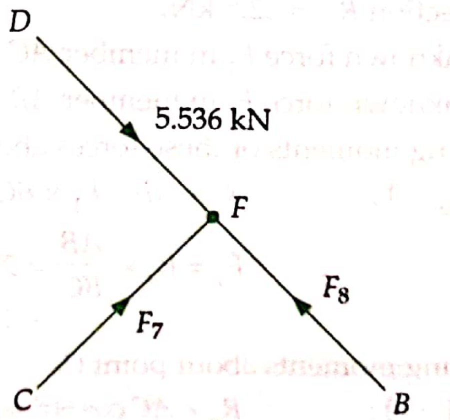
$\Sigma F_x = 0: \quad 1\cos(90-\theta) + 4.536\cos\theta - F_6\cos\theta$

$$\text{or} \quad F_6 = \frac{\sin\theta + 4.536\cos\theta}{\cos\theta} = \frac{0.707 + 4.536\times 0.707}{0.707}$$

$$= 5.536 \text{ kN (compression)}$$



Joint D



Joint F

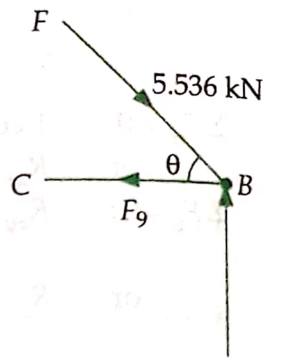
It is apparent from free body diagram of joint F, that for its static equilibrium

$$F_8 = 5.536 \text{ kN (compression)} \quad \text{and} \quad F_7 = 0 \text{ (no force)}$$

Applying conditions of equilibrium to point B, we have

$\Sigma F_x = 0:$ $\quad F_9 - 5.536 \cos\theta = 0$

$\therefore$ $\quad F_9 = 5.536 \times 0.707 = 3.914 \text{ kN (tension)}$



Joint B

5.3.2. Method of sections

The various steps involved are:

(*i*) The truss is split into two parts by passing an imaginary section.

(*ii*) The imaginary section has to be such that it does not cut more than three members in which the forces are to be determined.

(*iii*) The conditions of equilibrium

$$\Sigma F_x = 0; \quad \Sigma F_y = 0 \text{ and } \Sigma M = 0$$

are applied for the one part of the truss and the unknown force in the member is determined.

(*iv*) While considering equilibrium, the nature of force in any member is chosen arbitrarily to be tensile or compressive.

If the magnitude of a particular force comes out positive, the assumption in respect of its direction is correct. However, if the magnitude of the forces comes out negative, the actual direction of the force is opposite to that what has been assumed.

The method of section is particularly convenient when the forces in a few members of the frame are required to be worked out.

EXAMPLE 5.15