EXAMPLE 8.1

The moment of inertia of rectangular section beam about x-x and y-y axes passing though the centroid are 250×10^6 mm^4 and 40×10^6 mm^4 respectively. Calculate the size of the section.

Solution : Let b and d denote the breadth and depth respectively of the rectangular section beam. Then

$$I_{xx} = \frac{bd^3}{12} \; ; \quad 250 \times 10^6 = \frac{bd^3}{12} \qquad \text{...}(i)$$

and

$$I_{yy} = \frac{db^3}{12} \; ; \quad 40 \times 10^6 = \frac{db^3}{12} \qquad \text{...}(ii)$$

Dividing expression (i) by expression (ii)

$$5.25 = \left(\frac{d}{b}\right)^2 \quad \text{or} \quad \frac{d}{b} = 2.5$$

Substituting $d = 2.5\,b$ in expression (i), we get

$$\frac{b}{12}(2.5\,b)^3 = 250 \times 10^6$$

$$\text{or} \quad b^4 = \frac{250 \times 10^6 \times 12}{(2.5)^3} = 1.92 \times 10^8$$

That gives: $b = 117.7$ mm and $d = 2.5 \times 117.7 = 294.25$ mm

Therefore required size of the section is:

$$= 117.3 \text{ mm (breadth)} \times 294.25 \text{ mm (depth)}$$

EXAMPLE 8.2

Find the moment of inertia of a rolled steel joist girder of symmetrical I section shown in Fig. 8.16.

Solution : The areas of the three rectangles comprising the I-section are:

upper flange $A_1 = 6a \times a = 6a^2$

web $A_2 = 8a \times a = 8a^2$

lower flange $A_3 = 6a \times a = 6a^2$

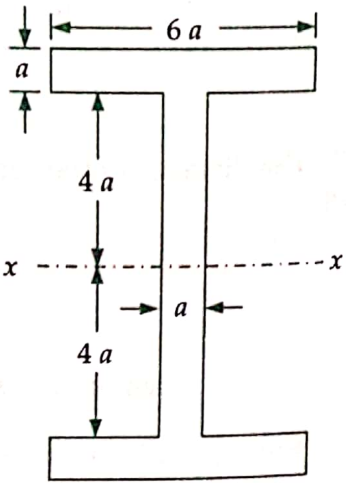


Fig. 8.16

MOI of upper flange about x-axis (using parallel axis theorem)

$$= \frac{6a \times a^3}{12} + 6a^2 \times \left(4a + \frac{a}{2}\right)^2$$

$$= \frac{a^4}{2} + \frac{243a^4}{2} = 122\,a^4$$

$$\textit{MOI} \text{ of web about } x\text{-axis} = \frac{a \times (8a)^3}{12} = \frac{128a^4}{3}$$

MOI of lower flange about x-axis (using parallel axis theorem)

$$= \frac{6a \times a^3}{12} + 4a^2\left(4a + \frac{a}{2}\right)^2$$

$$= \frac{a^4}{2} + \frac{243a^4}{2} = 122\,a^4$$

$\therefore$ Total *MOI* of the given I-section about x-axis

$$= 122\,a^4 + \frac{128 a^4}{3} + 122\,a^4$$

$$= \frac{860}{3} a^4$$

The *MOI* of the given I-section could also be worked out with reference to Fig. 8.17.

$$I_{xx} = I_{x1} - I_{x2}$$

$$= \frac{6a \times (10a)^3}{12} - \frac{5a \times (8a)^3}{12}$$

$$= 500\,a^4 - \frac{640}{3} a^4 = \frac{860}{3} a^4$$

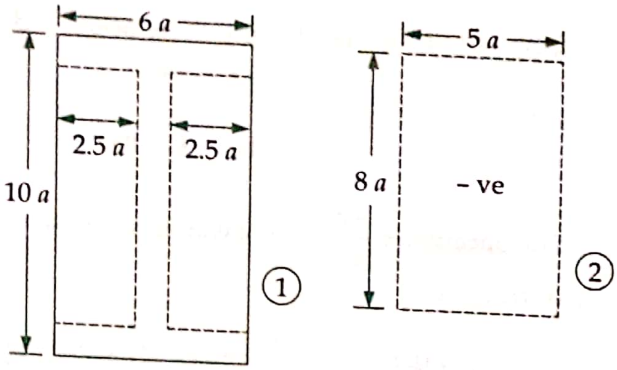


Fig. 8.17

EXAMPLE 8.3

Determine the moment of inertia of the T-section shown in Fig. 7.13 about an axis passing through the centroid and parallel to top most fibre of the section. Proceed to determine the moment of inertia about axis of symmetry and hence find out the radii of gyration.

Solution : From the calculations made in Example 7.10 the *CG* of the given T-section lies on the *y*-axis and at distance 43.71 mm from the top face of its flange

$$\bar{x} = 0 \quad \text{and} \quad \bar{y} = 43.71 \text{ mm}$$

Refering to this centroidal axis, the centroid of a_1 is (0.0, 38.71 mm) and that of a_2 is (0.0, 41.29 mm).

Moment of inertia of the section about centroid axis is

$$I_{xx} = MOI \text{ of area } a_1 \text{ about centroidal axis} + MOI \text{ of area } a_2 \text{ about centroidal axis}$$

$$= \left[\frac{160 \times 10^3}{12} + 1600 \times (38.71)^2\right] + \left[\frac{10 \times 150^3}{12} + 1500 \times (41.29)^2\right]$$

$$= 7780672 \text{ mm}^4$$

Similarly $$I_{yy} = \frac{10 \times 160^3}{12} + \frac{150 \times 10^3}{12} = 3425833 \text{ mm}^4$$

The radius of gyration is given by $k = \sqrt{\frac{I}{A}}$

$$\therefore \quad k_{xx} = \sqrt{\frac{7780672}{3100}} = 50.1 \text{ mm}$$

$$k_{yy} = \sqrt{\frac{3425833}{3100}} = 34.24 \text{ mm}$$

EXAMPLE 8.4

Determine the moment of inertia of the area shown shaded in Fig. 8.18 about axis *xx* which coincides with the base edge *AB*.

Solution : The given section comprises the full rectangle *ABCD* minus the semi-circle *DEC*.

Moment of cnertia of rectangle *ABCD* about *AB*

$$I_1 = I_{G1} + A_1\,h_1^2$$

$$= \frac{2 \times 2.5^3}{12} + (2 \times 2.5) \times 1.25^2$$
$$= 2.604 + 7.812 = 10.416 \text{ cm}^4$$

Moment of inertia of semi-circle about AB

$$I_2 = I_{G2} + A_2 h_2^2$$
$$= 0.11\,r^2 + \frac{1}{2}\pi r^2 \times \left(2.5 - \frac{4r}{3\pi}\right)^2$$

The parameter $\frac{4r}{3\pi}$ is the distance of centroid of semi-circle from DC.

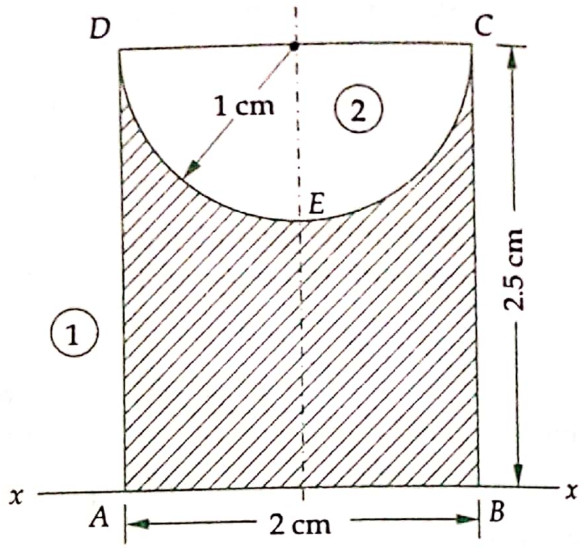


Fig. 8.18

$$\therefore \quad I_2 = 0.11 \times 1^2 + \frac{1}{2}\pi \times (1)^2 \times \left(2.5 - \frac{4 \times 1}{3\pi}\right)^2$$
$$= 0.11 + 6.76 = 6.87 \text{ cm}^4$$

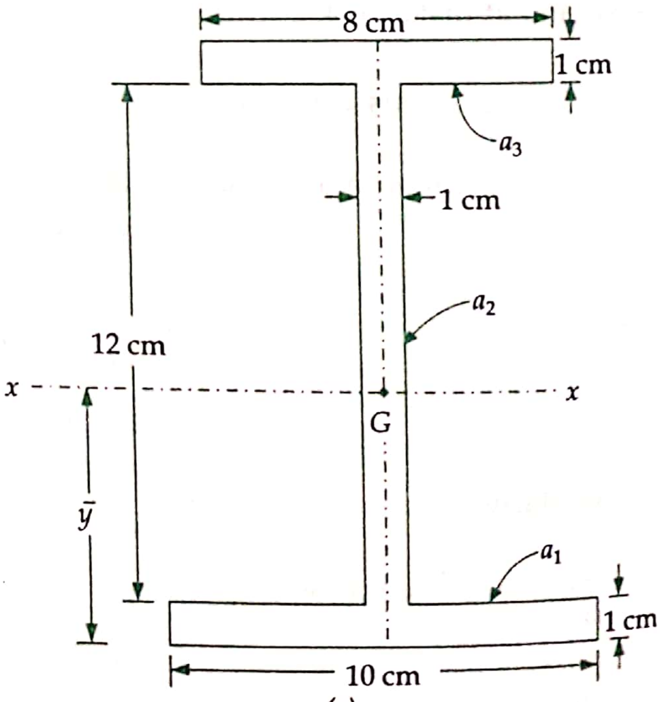
$\therefore$ Moment of inertia of shaded area bout AB = 10.416 – 6.87 = 3.546 cm^4

EXAMPLE 8.5

Determine the polar moment of inertia of the I-section shown in Fig. 8.19. Also make calculations for the radius of gyration with respect to x-axis and y-axis.

Solution : The I-section is symmetrical about y-axis and accordingly its CG lies at point G on the y-axis, i.e., $x = 0$. Further, the bottom fibre of lower flange has been chosen as reference axis to locate the centroid $\bar{y}$.

The areas and co-ordinates of centroids of the three rectangles comprising the given section are:



(a)

Fig. 8.19 (a)

Lower flange: $a_1 = 10 \times 1 = 10 \text{ cm}^2$

$$y_1 = \frac{1}{2} = 0.5 \text{ cm}$$

Web: $a_2 = 12 \times 1 = 12 \text{ cm}^2$

$$y_2 = 1 + \frac{12}{2} = 7 \text{ cm}$$

Upper flange: $a_3 = 8 \times 18 \text{ cm}^2$

$$y_3 = 1 + 12 + \frac{1}{2} = 13.5 \text{ cm}$$

Then:
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$
$$= \frac{10 \times 0.5 + 12 \times 7 + 8 \times 13.5}{10 + 12 + 8} = \frac{5 + 84 + 108}{30} = 5.57 \text{ cm}$$

With reference to the centroidal axes, the centroid of the lower flange, web and upper flange are (0, 5.07), (0, 1.43) and (0, 7.93) respectively.

Moment of inertia of the I-section about centroidal axis is

= MOI of area a_1 about centroidal axis + MOI of area a_2 about centroidal axis + MOI of area a_3 about centroidal axis.

$= (I_{G1} + A_1h_1^2) + (I_{G2} + A_2h_2^2) + (I_{G3} + A_3h_3^2)$

$$= \left[\frac{10 \times 1^3}{12} + 10 \times (5.07)^2\right] + \left[\frac{1 \times 12^3}{12} + 12 \times (1.43)^2\right] + \left[\frac{8 \times 1^3}{12} + 8 \times (7.93)^2\right]$$

$= (0.833 + 257.05) + (144 + 24.54) + (0.67 + 503.08)$
$= 930.17\ cm^4$

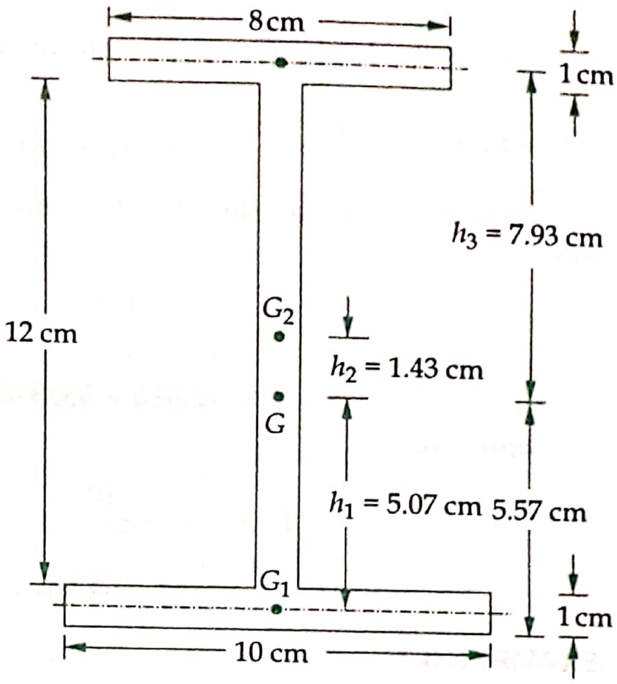


Fig. 8.19 (b)

and $$I_{yy} = \frac{1 \times 10^3}{12} + \frac{12 \times 1^3}{12} + \frac{1 \times 8^3}{12}$$

$= 83.33 + 1 + 42.64 = 127\ cm^4$

Polar moment of inertia $= I_{xx} + I_{yy} = 930.17 + 127$
$= 1057.17\ cm^4$

(b) The radius of gyration is given by: $k = \sqrt{\frac{I}{A}}$

$$\therefore \quad k_{xx} = \sqrt{\frac{930.17}{30}} = 5.567 \text{ cm}$$

$$k_{yy} = \sqrt{\frac{127}{30}} = 2.057 \text{ cm}$$

EXAMPLE 8.6

Determine the moment of inertia about centroidal axes x-x and y-y of the channel section shown in Fig. 8.20.

Solution : The section is divided into three rectangles with areas

$$A_1 = 10 \times 1.5 = 15\ cm^2$$
$$A_2 = (40 - 1.5 - 1.5) \times 1 = 37\ cm^2$$
$$A_3 = 10 \times 1.5 = 15\ cm^2$$
$$\Sigma A = A_1 + A_2 + A_3$$
$$= 15 + 37 + 15 = 67\ cm^2$$

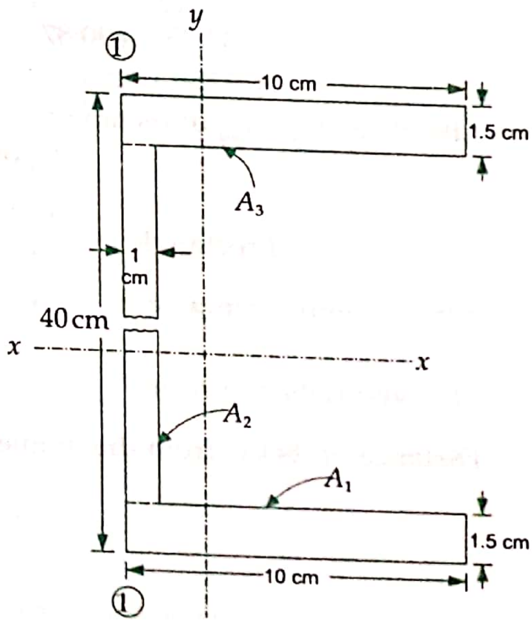


Fig. 8.20

The given section is symmetrical about the horizontal axis passing through the centroid of rectangle A_2.

The distance of the centroid of the section with reference to section 1 – 1 is

$$\frac{\Sigma Ax}{\Sigma A} = \frac{(15 \times 5) + \left(37 \times \frac{1}{2}\right) + (15 \times 5)}{67} = 2.51 \text{ cm}$$

With reference to the centroidal axes x-x and y-y, the centroids of the rectangles are:

$\left[(5 - 2.51), \left(\frac{40}{2} - \frac{1.5}{2}\right)\right]$ or $(2.49, 19.25)$ for rectangle A_1

$\left[\left(2.51-\frac{1}{2}\right), 0.0\right]$ or (2.01, 0.0) for rectangle A_2

$\left[(5-2.51), \left(\frac{40}{2}-\frac{1.5}{2}\right)\right]$ or (2.49, 19.25) for rectangle A_3

Then invoking parallel axis theorem, the moment of inertia of areas A_1, A_2, and A_3 about x-x,

$$I_{xx} = \left[\frac{10\times1.5^3}{12}+15\times19.25^2\right]+\left[\frac{1\times37^3}{12}\right]+\left[\frac{10\times1.5^3}{12}+15\times19.25^2\right]$$

$$= (2.812 + 5558.437) + (4221.083) + (2.812 + 558.437) = 155343.58 \text{ cm}^4$$

Similarly,

$$I_{yy} = \left[\frac{1.5\times10^3}{12}+15\times2.49^2\right]+\left[\frac{37\times1^3}{12}\right]+\left[\frac{1.5\times10^3}{12}+15\times2.49^2\right]$$

$$= (125 + 93.00) + 3.08 + (125 + 93.00) = 439.08 \text{ cm}^4$$

EXAMPLE 8.7

Determine I_{xx} and I_{yy} of the cross-section of a cast iron beam shown in Fig. 8.21.

Solution : The *MOI* of the given sections can be worked out by looking it as a rectangle minus two semi-circles.

$\therefore$ $I_{xx} = I_{xx}$ of rectangle $- I_{xx}$ of circular part

$$= \frac{bd^3}{12} - \frac{\pi r^4}{4}$$

$$= \frac{12\times15^3}{12} - \frac{\pi\times5^4}{4}$$

$$= 33.75 - 490.87$$

$$= 2884.13 \text{ cm}^4$$

Likewise : $I_{yy} = I_{yy}$ of rectangle $- I_{yy}$ of semi-circular parts

$$I_{yy} \text{ of rectangle} = \frac{15\times12^3}{12} = 2160 \text{ cm}^4$$

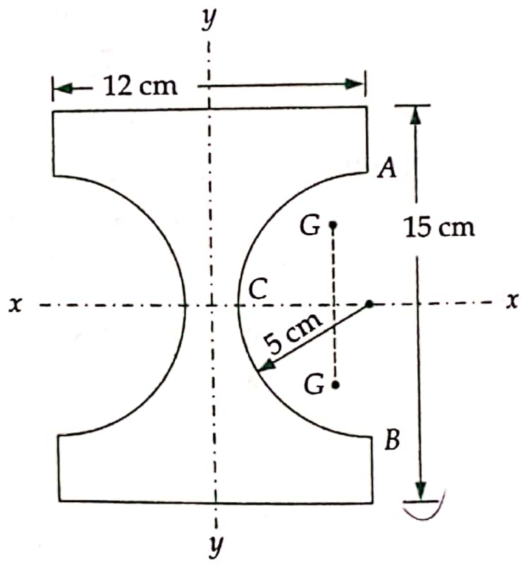


Fig. 8.21

For the semi-circular part ACB;

MOI about its diameter, $I_{AB} = \frac{1}{2}\times\frac{\pi\times5^4}{4} = 245.43 \text{ cm}^4$

Distance of its *CG* from the diameter,

$$h = \frac{4r}{3\pi} = \frac{4\times5}{3\pi} = 2.12 \text{ cm}$$

$$\text{Area } A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\times5^2 = 39.27 \text{ cm}^2$$

From the correlation, $I_{AB} = I_{GG} + Ah^2$, the moment of inertia of semi-circular part about its centroidal axis

$$I_{GG} = 245.43 - 39.27\times(2.12)^2 = 68.94 \text{ cm}^4$$

Again from the parallel axis theorem,

$$I_{yy} = I_{GG} + Ah_1^2$$

where h_1 = distance between axis and G-axis, = 6 – 2.12 = 3.88 cm

$$\therefore \quad I_{yy} = 68.94 + 39.27 \times 3.88^2 = 660.13 \text{ cm}^4$$

Since there are two semi-circular parts,

I_{yy} for two semi-circular parts = 2 × 660.13 = 1320.26 cm^4

∴ I_{yy} for the section = 2160 – 1320.26 = 839.74 cm^4

EXAMPLE 8.8

Determine the moments of inertia about the x and y centroidal axis of a beam whose cross-sectional area is as shown in Fig. 8.22. All dimensions are in cm.

Solution : The given section has been divided into three segments marked 1, 2 and 3

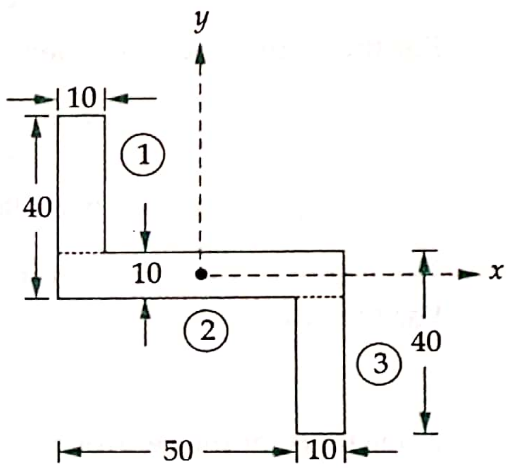


Fig. 8.22

$$(I_{xx})_1 = I_{G_1} + A_1h_1^2 = I_{G_1} + A_1(\bar{y} - y_1)^2$$

$$= \frac{1}{12} \times 10 \times 30^3 + (30 \times 10)(35 - 15)^2$$

$$= 1.425 \times 10^5 \text{ cm}^4$$

$$(I_{xx})_2 = I_{G_2} + A_2h_2^2 = I_{G_2} + A_2(\bar{y} - y_2)^2$$

$$= \frac{1}{12} \times 10^3 \times 60 + (60 \times 10) \times 0$$

$$= 0.05 \times 10^5 \text{ cm}^4$$

$$(I_{xx})_3 = I_{G_3} + A_3h_3^2 = I_{G_3} + A_3(\bar{y} - y_3)^2$$

$$= \frac{1}{12} \times 10 \times 30^3 + (30 \times 10)(35 - 15)^2 = 1.425 \times 10^5 \text{ cm}^4$$

$$\therefore \quad I_{xx} = 1.425 \times 10^5 + 0.05 \times 10^5 + 1.425 \times 10^5 = 2.90 \times 10^5 \text{ cm}^4$$

$$(I_{yy})_1 = I_{G_1} + A_1h_1^2 = I_{G_1} + A_1(\bar{x} - x_1)^2$$

$$= \frac{1}{12} \times 30 \times 10^3 + (30 \times 10) \times (30 - 5)^2 = 1.9 \times 10^5 \text{ cm}^4$$

$$(I_{yy})_2 = I_{G_2} + A_2h_2^2 = I_{G_2} + A_2(\bar{x} - x_2)^2$$

$$= \frac{1}{12} \times 60^3 \times 10 + (60 \times 10) \times 0 = 1.8 \times 10^5 \text{ cm}^4$$

$$(I_{yy})_3 = I_{G_3} + A_3h_3^2 = I_{G_3} + A_3(\bar{x} - x_3)^2$$

$$= \frac{1}{12} \times 30 \times 10^3 + (30 \times 10) \times (30 - 5)^2$$

$$= 1.9 \times 10^5 \text{ cm}^4$$

$$\therefore \quad I_{yy} = 1.9 \times 10^5 + 1.8 \times 10^5 + 1.9 \times 10^5 = 5.6 \times 10^5 \text{ cm}^4$$

EXAMPLE 8.9

Find the moment of inertia about the centroid horizontal axis of the area shown shaded in Fig. 8.23. The section consists of triangle ABC, semi-circle on BC as diameter, and a circular hole of diameter 4 cm with its centre on BC.

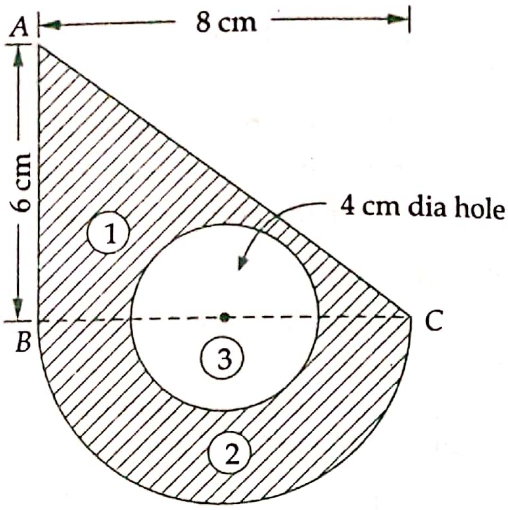


Fig. 8.23

Solution : The shaded area can be considered as a triangle (1), semicircle (2) and a circular hole (3)

Location of Centroid : For the triangular element,

$$a_1 = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

y_1 (distance of centroid from BC)

$$= \frac{6}{3} = 2 \text{ cm}$$

For the semi-circular element,

$$a_2 = \frac{1}{2} \pi r^2 = \frac{1}{2} \pi \times 4^2 = 25.12 \text{ cm}^2$$

$$y_2 (\text{distance of centroid from } BC) = \frac{-4r}{3\pi} = \frac{-4 \times 4}{3\pi} = -1.7 \text{ cm}$$

The negative sign stems from the fact that it lies below BC.

For the circular hole

$$a_3 = \pi r^2 = \pi \times 2^2 = 12.56 \text{ cm}^2 \text{ (this area is removed)}$$

$$y_3 = 0 \text{ (centroid lies on } BC)$$

$\therefore$ Distance of the centroid of the shaded area from BC

$$= \frac{\Sigma ay}{\Sigma a} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{24 \times 2 + 25.12 \times (-1.7) - 12.56 \times 0}{24 + 25.12 - 12.56} = 0.145 \text{ cm}$$

Moment of Inertia

I_1 = moment of inertia of triangle ABC about base BC

$$= \frac{1}{12} bh^3 = \frac{1}{12} \times 8 \times 6^3 = 144 \text{ cm}^4$$

I_2 = moment of inertia of semi-circle about BC

$$= \frac{1}{128} \pi d^4 = \frac{1}{128} \times \pi \times 8^4 = 100.48 \text{ cm}^4$$

I_3 = moment of inertia of circular hole about BC

$$= \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 4^4 = 12.56 \text{ cm}^2$$

$\therefore$ Moment of inertia of the shaded area about BC

$$= 144 + 100.48 - 12.56 = 231.92 \text{ cm}^4$$

Area of the shaded portion = 24 + 25.12 − 12.56 = 36.56 cm

Invoking parallel axis theorem,

Moment of inertia of shaded area about centroidal axis

$$I_G = I_{BC} - A h^2 = 231.92 - 36.56 \times 0.145^2 = 231.15 \text{ cm}^4$$

EXAMPLE 8.10

Find the horizontal centroidal moment of inertia of the lamina ABCDEFG shown in Fig. 8.24.

Solution : The composite figure is divided into the following simple figures:

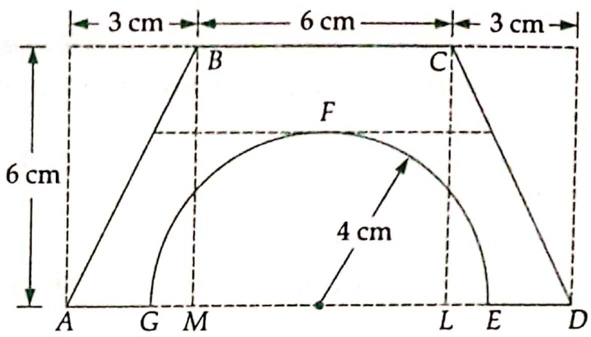


Fig. 8.24

1. A triangle ABM: $A_1 = \frac{1}{2} bh$

$$= \frac{1}{2} \times 3 \times 6 = 9 \text{ cm}^2$$

$$I_{G_1} = \frac{bh^3}{36} = \frac{3 \times 6^3}{36} = 18 \text{ cm}^4$$

y_1 (centroidal distance from line AD) $= \frac{6}{3} = 2$ cm

2. A square $BCLM$: $A_2 = 6 \times 6 = 36 \text{ cm}^2$

$$I_{G_2} = \frac{6 \times 6^3}{12} = 108 \text{ cm}^4$$

y_2 (centroidal distance from line AD) $= \frac{6}{2} = 3$ cm

3. A triangle CDL: $A_3 = \frac{1}{2} bh = \frac{1}{2} \times 3 \times 6 = 9 \text{ cm}^2$

$$I_{G_3} = \frac{bh^3}{36} = \frac{3 \times 6^3}{36} = 18 \text{ cm}^4$$

y_3 (centroidal distance from line AD) $= \frac{6}{3} = 2$ cm

4. A semi-circle GFE to be subtracted: $A_4 = \frac{\pi r^2}{2} = \frac{\pi \times 4^2}{2} = 25.12 \text{ cm}^2$ (– ve)

$$I_{G_4} = 0.11\, r^4 = 0.11 \times 4^4 = 28.16 \text{ cm}^4$$

y_4 (centroidal distance from line AD) $= \frac{4r}{3\pi} = \frac{4 \times 4}{3\pi} = 1.698$ cm

For the composite section

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{(9 \times 2) + (36 \times 3) + (9 \times 2) - (25.12 \times 1.698)}{9 + 36 + 9 - 25.12}$$

$$= \frac{18 + 108 + 18 - 42.65}{28.88} = 3.51 \text{ cm}$$

Then
$$I_{xx} = I_{xx1} + I_{xx2} + I_{xx3} - I_{xx4}$$
$$= [18 + 9 \times (3.51 - 2)^2] + [108 + 36 \times (3.51 - 3)^2]$$
$$+ [18 + 9 \times (3.51 - 2)^2] - [28.16 + 25.12\,(3.51 - 1.698)^2]$$

The above relation has been written by applying the parallel axis theorem:

$$I_{xx} = I_{GG} + Ah^2$$

$\therefore \quad I_{xx} = 38.52 + 117.36 + 38.52 - 110.64 = 83.76 \text{ cm}^4$

EXAMPLE 8.11

For the shaded area shown in Fig. 8.25, determine :

(a) co-ordinates of the centroid, and

(b) moment of inertia about the centroidal axes

The removed area is semicircular and all the given dimensions are in cm.

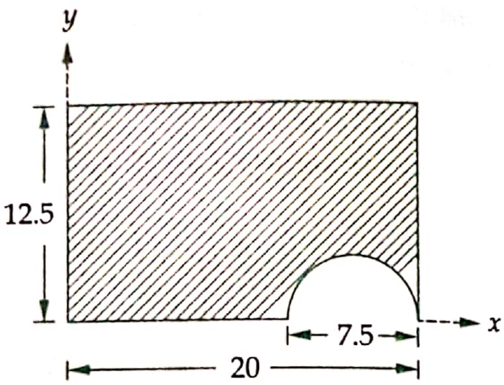


Fig. 8.25

Solution : The shaded area is the area of the rectangle 20 × 12.5 cm minus the area of semicircle of radius 3.75 cm.

For the rectangle :

$$\text{area } a_1 = 20 \times 12.5 = 250 \text{ cm}^2$$

$$y_1 = \frac{12.5}{2} = 6.25 \text{ cm}; \quad x_1 = \frac{20}{2} = 10 \text{ cm}$$

For the semicircle :

$$\text{area } a_2 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 3.75^2 = 22.08 \text{ cm}^2$$

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 3.75}{3\pi} = 1.59 \text{ cm}$$

$$x_2 = (20 - 3.75) = 16.25 \text{ cm}$$

∴ For the shaded section,

$\bar{x}$ (distance of centroid from axis oy)

$$= \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{250 \times 10 - 22.08 \times 16.25}{250 - 22.08} = 9.39 \text{ cm}$$

$\bar{y}$ (distance of centroid from axis ox)

$$= \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{250 \times 6.25 - 22.08 \times 1.59}{250 - 22.08} = 6.70 \text{ cm}$$

Thus the co-ordinates of the centroid are: $\bar{x}$ = 9.39 cm and $\bar{y}$ = 6.70 cm

Moment of inertia of the shaded section = MOI of rectangle – MOI of semicircle

(*i*) With reference to centroidal x-axis

$$\text{MOI of rectangle} = \frac{20 \times 12.5^3}{12} + (20 \times 12.5) \times (6.7 - 6.25)^2$$

$$= 3255.21 + 50.62 = 3305.83 \text{ cm}^4$$

$$\text{MOI of semicircle} = 0.11 r^4 + \frac{1}{2}\pi r^2 (\bar{y} - y_2)^2$$

$$= 0.11 \times 3.75^4 + \frac{1}{2}\pi (3.75)^2 \times (6.70 - 1.59)^2$$

$$= 21.75 + 576.50 = 598.25 \text{ cm}^4$$

∴ MOI of shaded section about centroidal x-axis

$$= 3305.83 - 598.25 = 2707.58 \text{ cm}^4$$

(ii) With reference to centroidal y-axis

$$\text{MOI of rectangle} = \frac{12.5 \times 20^3}{12} + (20 \times 12.5)(10 - 9.39)^2$$

$$= 8333.33 + 93.02 = 8426.35 \text{ cm}^4$$

$$\text{MOI of semicircle} = \frac{\pi r^4}{8} + \frac{1}{2}\pi r^2 (20 - 3.75 - 9.39)^2$$

$$= \frac{\pi \times 3.75^4}{8} + \frac{1}{2}\pi (3.75)^2 \times (6.86)^2$$

$$= 77.62 + 1038.98 = 1116.6 \text{ cm}^4$$

$\therefore$ MOI of shaded section about centroidal y-axis

$$= 8426.35 - 1116.6 = 7309.75 \text{ cm}^4$$

EXAMPLE 8.12

Determine the moment of inertia of the plane area shown in Fig. 8.26, about its centroidal axis.

Solution : The given section has been divided into three segments marked as 1, 2 and 3.

1. Rectangle

$$a_1 = 8 \times 12 = 96 \text{ cm}^2$$

$$x_1 = 4 + \frac{8}{2} = 8 \text{ cm}$$

$$y_1 = \frac{12}{2} = 6 \text{ cm}$$

2. Triangle

$$a_2 = \frac{1}{2} \times 4 \times 12 = 24 \text{ cm}^2$$

$$x_2 = \frac{2}{3} \times 4 = 2.67 \text{ cm}$$

$$y_2 = \frac{12}{3} = 4 \text{ cm}$$

3. Semicircle

$$a_3 = \frac{1}{2} \times \left[\frac{\pi}{4} \times (4)^2\right] = 6.28 \text{ cm}^2$$

$$x_3 = 4 + 2 + \frac{4}{2} = 8 \text{ cm}$$

$$y_3 = 12 - \left(\frac{4 \times 2}{3\pi}\right) = 11.15 \text{ cm}$$

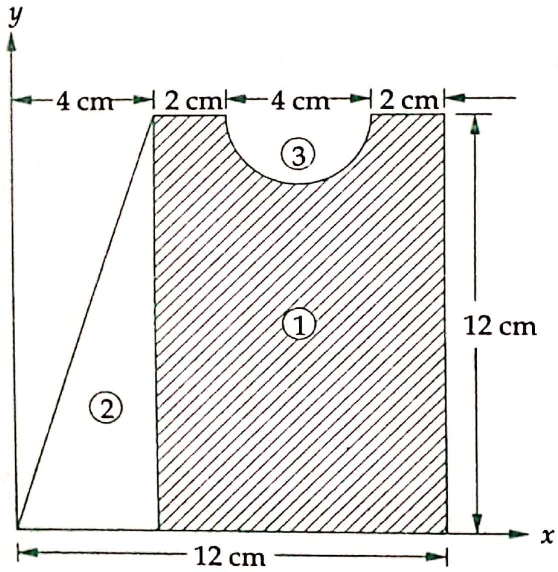


Fig. 8.26

Then the coordinates of the centroid of the given plane area are :

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{96 \times 8 + 24 \times 2.67 - (6.28 \times 8)}{96 + 24 - 6.28} = \frac{768 + 64.08 - 50.24}{113.72}$$

$$= 6.87 \text{ cm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{96 \times 6 + 24 \times 4 - 6.28 \times 11.15}{96 + 24 - 6.28} = \frac{576 + 96 - 70.02}{113.72}$$

$$= 5.29 \text{ cm}$$

The centroid of the given lamina have been shown in Fig. 8.27.

Moment of inertia about horizontal centroidal axis:

$$I_{xx} = I_1 + I_2 - I_3$$

where

$$I_1 = I_{G_1} + A_1 h_1^2$$
$$= I_{G_1} + A_1(\bar{y} - y_1)^2$$
$$= \frac{8 \times 12^3}{12} + (8 \times 12)(5.29 - 6)^2$$
$$= 1200.39 \text{ cm}^4$$

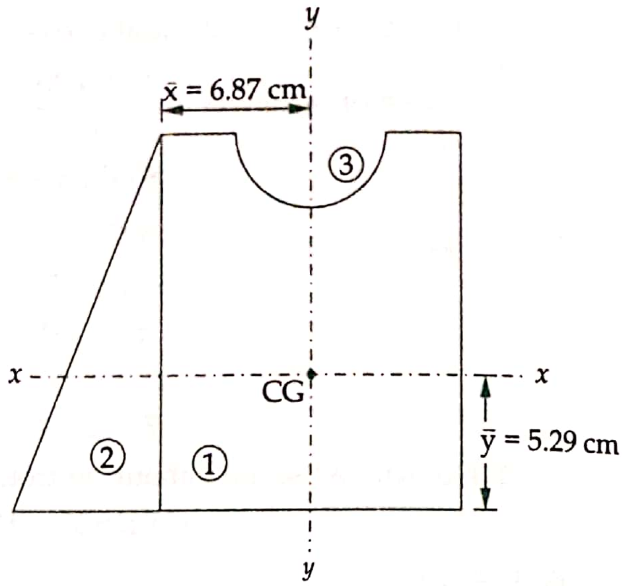


Fig. 8.27

$$I_2 = I_{G_2} + A_2 h_2^2$$
$$= I_{G_2} + A_2(\bar{y} - y_2)^2$$
$$= \frac{4 \times 12^3}{36} + \left(\frac{1}{2} \times 4 \times 12\right)(5.29 - 4)^2 = 231.94 \text{ cm}^4$$
$$I_3 = I_{G_3} + A_3 h_3^2$$
$$= I_{G_3} + A_3(\bar{y} - y_3)^2$$
$$= 0.11 r^4 + \frac{1}{2}\pi r^2 (\bar{y} - y_3)^2$$
$$= 0.11 \times 2^4 + \frac{1}{2}\pi \times 2^2 (5.29 - 11.15)^2 = 217.41 \text{ cm}^4$$

$\therefore \quad I_{xx} = 1200.39 + 231.94 - 217.4 = 1214.93 \text{ cm}^4$

Moment of inertia about vertical centroidal axis

$$I_{yy} = I_1 + I_2 - I_3$$

where

$$I_1 = I_{G_1} + A_1 h_1^2$$
$$= I_{G_1} + A_1(\bar{x} - x_1)^2$$
$$= \frac{12 \times 8^3}{12} + (12 \times 8) \times (6.87 - 8)^2 = 634.58 \text{ cm}^4$$
$$I_2 = I_{G_2} + A_2 h_2^2$$
$$= I_{G_2} + A_2(\bar{x} - x_2)^2$$
$$= \frac{12 \times 4^3}{36} + \left(\frac{1}{2} \times 12 \times 4\right) \times \left(6.87 - \frac{8}{3}\right)^2 = 444.69 \text{ cm}^4$$
$$I_3 = I_{G_3} + A_3 h_3^2$$
$$= \frac{\pi r^4}{8} + \frac{1}{2}\pi r^2 (\bar{x} - x_3)^2$$
$$= \frac{\pi \times 2^4}{8} + \frac{1}{2}\pi(2)^2(6.87 - 8)^2 = 14.3 \text{ cm}^4$$

$\therefore \quad I_{yy} = 634.58 + 444.69 - 14.3 = 1064.97 \text{ cm}^4$