Fig. 3.85

3.6. COUPLE

Two parallel forces equal in magnitude but opposite in direction, and separated by a finite distance are said to form a couple. If F and F' are two such forces, then the couple is denoted by (F, F'). A body acted upn by a couple spins round but remains at the same spot.

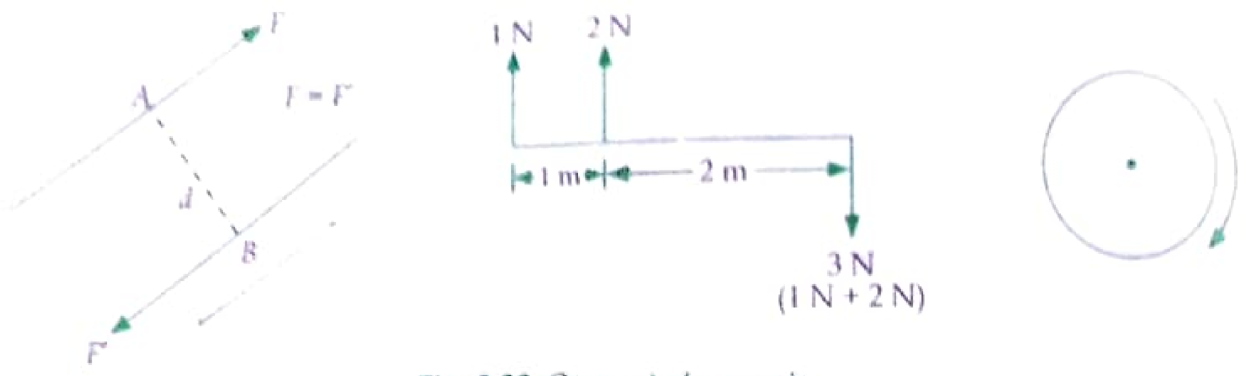


Fig. 3.86. Concept of a couple

With reference to Fig. 3.86, the moment of couple M is

$$M = F \times d$$

The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called *arm of the couple*.

Examples of a couple :

(*i*) Winding of a watch or a clock
(*ii*) Opening or closing a water tap
(*iii*) Unscrewing the cap of an ink bottle
(*iv*) Locking/unlocking of a lock with a key
(*v*) Turn of the cap of a pen
(*vi*) Forces applied to the handle of a screw

The salient aspects of a couple are :

(*i*) The algebraic sum of the vertical and horizontal components of the forces (F, F') forming a couple is zero, *i.e.*, the resultant of the forces (F, F') is zero.

(*ii*) A zero resultant of the forces forming the couple implies that a couple cannot produce translation. The couple causes only rotation and provides a turning effect, *i.e.*, tends to make the body turn.

(*iii*) The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called *arm of the couple*.

With reference to Fig. 3.86, the moment of couple is

$$M = F \times d$$

and $$M = 1 \times 3 + 2 \times 2 = 7 \text{ Nm}$$

(*iv*) The sum of the moments of the two forces forming a couple is equal to the moment of the couple.

Consider a point A (Fig. 3.87) lying in the plane of the forces F and F' that form a couple. Then the sum of the moments of the two forces about point A is

$$\begin{aligned} M &= F' d_2 - F d_1 \\ &= F d_2 - F d_1 \quad (\because F' = F) \\ &= F(d_2 - d_1) = F d \\ &= \text{moment of the couple} \end{aligned}$$

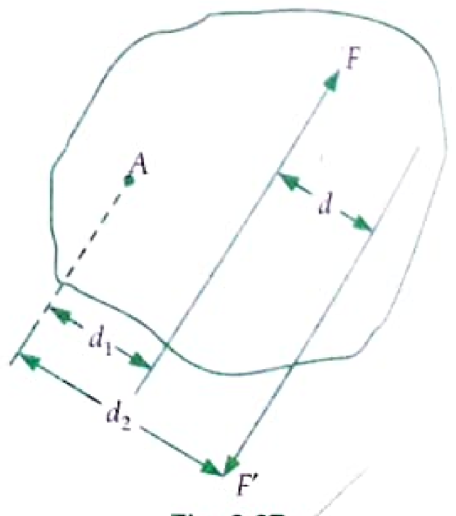


Fig. 3.87

The sum of the moments does not depend on the location of point A; it will have the same value and same sense.

Obviously a deduction can be made that

"the algebraic sum of the moments of two forces forming a couple about any point in their plane is constant and equal to the moment of the couple."

(*v*) Two caplanar couples, whose moments are equal and opposite, balance each other.

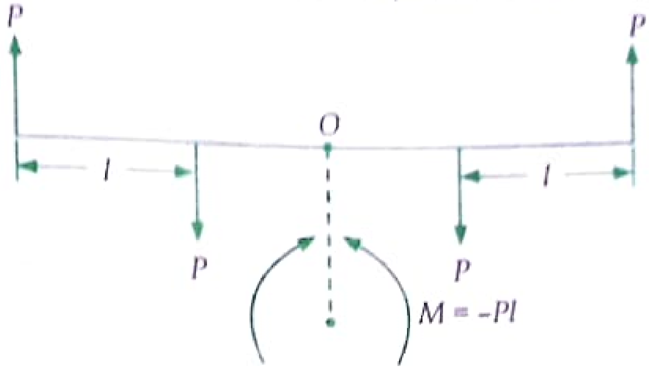


Fig. 3.88

With reference to Fig. 3.88, there act two couples whose moments are equal and opposite. These couples will balance each other and accordingly there will be no turning effect at point O.

(*vi*) Any two couples will be equivalent if their moments are equal, both in magnitude and direction.

Consider the system of forces depicted in Fig. 3.89

In Fig. 3.89 (*a*) : $M = P \times l = P\,l$ (anticlockwise)

In Fig. 3.89 (*b*) : $M = \dfrac{P}{2} \times 2\,l = P\,l$ (anticlockwise)

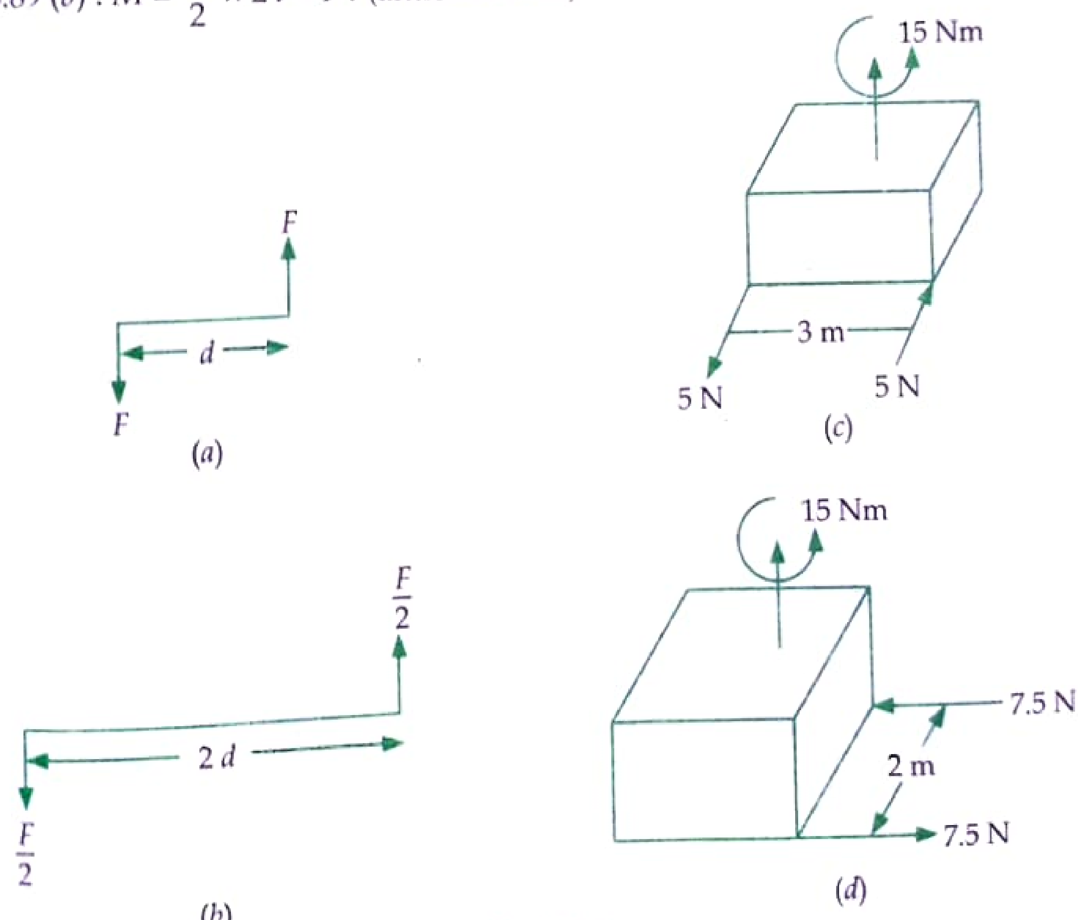


Fig. 3.89

Further, the two couples depicted in Fig. 3.89 *c* and *d* have a different force magnitude and direction but they have same moment magnitude and direction. They two represent an equivalent couple.

Two equivalent couples have the same effect on the body, and it does not matter

— where the two forces form the couple

— what magnitudes and directions the forces have

The magnitude and the direction of the moment of couple are only factors which decide the equivalence of couples.

(*vii*) Algebraic sum of moments of a number of couples is equal to the moment of a single couple

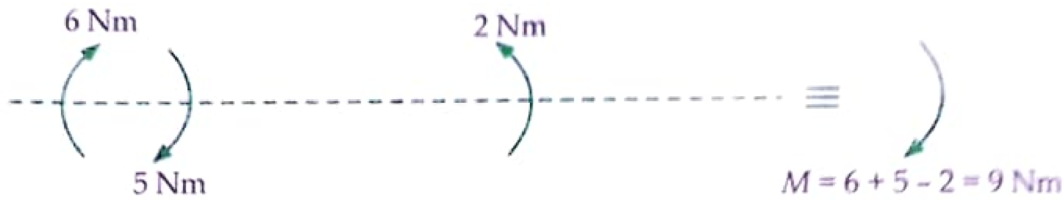


Fig. 3.90

(*viii*) A single force P and a couple M acting in the same plane on a body cannot balance each other. However, they are together equivalent to a single force at a distance $e = M/P$ from its original line of action.

Further :

- a couple cannot be balanced by a single force, but can be balanced only by a couple of opposite sense.
- any number of coplanar couples can be reduced to a single couple of moment equal to the algebraic sum of the moments of all the couples.
- the translatory effect of a couple on a body is zero.
- the effect of couple on a body remains unchanged if the couple is
  - (*a*) rotated through an angle,
  - (*b*) shifted to any other position,
  - (*c*) replaced by another pair of forces whose rotational effect is same

Resolution of force system into a force and a couple

The following procedure is adopted for resolution of a force system into a force and a couple :

(*i*) Determine the resultant of the given force system both in magnitude and location.

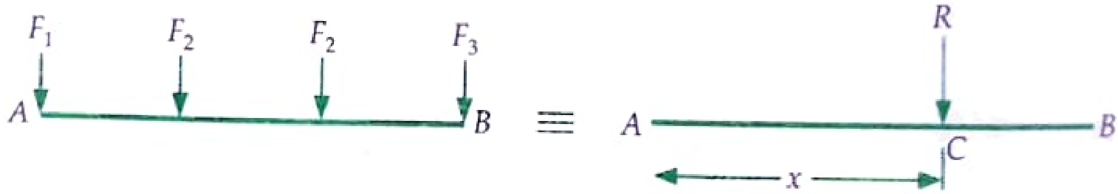


Fig. 3.91 a

With reference to (Fig. 3.91 *a*), R is the resultant of forces F_1, F_2, F_3 and F_4 and it acts at the distance x from a particular point A.

(*ii*) Introduce a force equivalent to the resultant at point A both in the upward and downward directions. This application of equal and opposite forces at point A gives zero net resultant at A and obviously there is no change in the given force system.

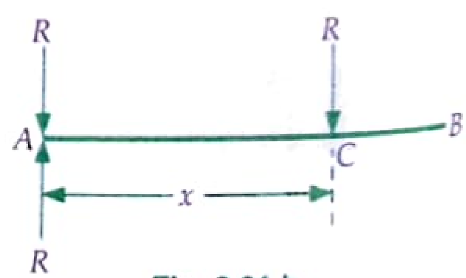


Fig. 3.91 b

(*iii*) The downward resultant R and the upward force R applied at A constitute a couple. The moment of this couple is in anti-clockwise direction and equals the product of R and the distance x between te forces. That is $M_A = R \times x$

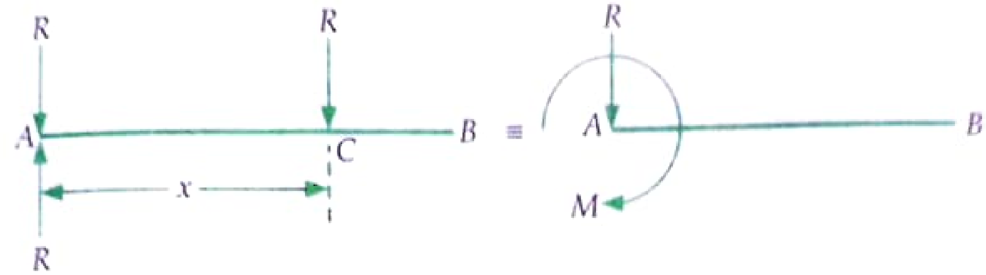


Fig. 3.91 c

The given force system thus gets replaced with a force and a couple at point A.

Moment and couple

The differences between a moment and a couple may be stated as :

(*i*) Two parallel forces having the same magnitude but acting in opposite direction form a couple. The moment of couple is the product of either of the forces and the perpendicular distance between them.

Moment about an axis : $F \times r$

Couple with moment : $F \times d$

Fig. 3.92

The product of a force and the perpendicular distance of the line of action of the force from a point or axis is referred to as the moment of force about that point or axis.

(*ii*) Both the moment and couple have sense of direction (clockwise or anti-clockwise) and are expressed in the units Nm.

(*iii*) Moment of a force varies where as the moment of a couple is always constant.

The moment of force about any point depends upon the perpendicular distance of that point from the line of action of force. However, the moment of couple is independent of the distance of the centre of couple from any outside point; it depends only upon the distances between the forces constituting the couple.

Further, the moment of couple has the same value and same sense irrespective of the location of moment centre. The magnitude and the sense of the moment of force does change with change in the location of moment centre.

(*iv*) Both the moment of force and couple tend to cause a rotational effect on the body.

(*v*) Moment of force about a point can be translated into moment of force about another point and a couple.

EXAMPLE 3.65

(a) A body is acted upon by the following two couples:

(i) a clockwise couple of 30 Nm on the x-axis with its centre 4 m from the origin, and

(ii) a counter clockwise couple of 50 Nm along the y-axis with its centre 2 m from the origin.

Calculate the resultant effect at the origin.

(b) Find the moment of three couples acting on a L-bar as shown in the adjoining figure.

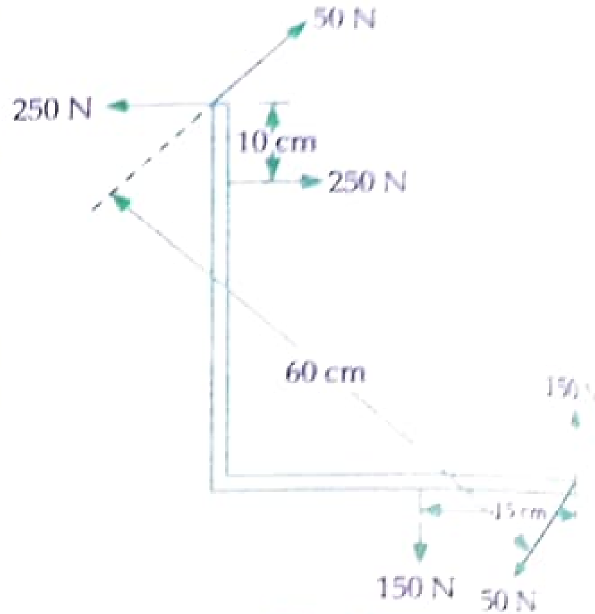


Fig. 3.93

Solution : The moment of a couple has the same value and sense irrespective of the location of moment centre.

∴ Resultant moment

$$= 30 - 50 = -20 \text{ Nm (anti-clockwise)}$$

(*b*) Resultant moment on the bar due to the three given couples is

$$= -(250 \times 0.1) + (50 \times 0.6) - (150 \times 0.15)$$
$$= -25 + 30 - 22.5$$
$$= -17.5 \text{ Nm} \quad \text{(anticlockwise)}$$

EXAMPLE 3.66.

A cantilever beam is acted upon by two vertical forces and a couple of moment 1250 Nm as shown in the figure given below:

Determine: (i) resultant of the system and (ii) and equivalent system through the fixed and A.

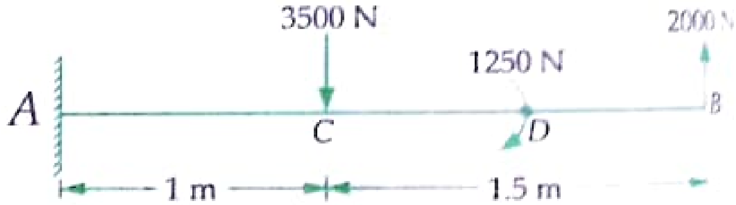


Fig. 3.94

Solution : Resultant force $R = 3500 - 2000 = 1500$ N acting downward

Resultant moment about A (clockwise moment positive)

$$= 3500 \times 1 + 1250 - 2000 \times 2.5$$
$$= 3500 + 1250 - 5000 = -250 \text{ Nm} \quad \text{(anticlockwise)}$$

The resultant force acts downward and it will give the anti-clockwise moment if it acts towards left of A. Let x be the distance of resultant force (1500 N) from A. Then

$$1500\,x = 250 \qquad \therefore \quad x = \mathbf{0.166\ m}$$

Hence the resultant of the system is 1500 N downward and it acts at a distance of 0.166 m left of fixed and A.

(*ii*) This means that we have to determine a single resultant force and a single moment through A

Single resultant force = 1500 N (downward)

Single resultant moment = 250 N (anticlockwise)

EXAMPLE 3.67

A rigid bar is subjected to a system of parallel forces as shown in Fig. 3.95.

Reduce this system to

(a) a single force

(b) a single force-moment system at A

(c) a single force moment system at B

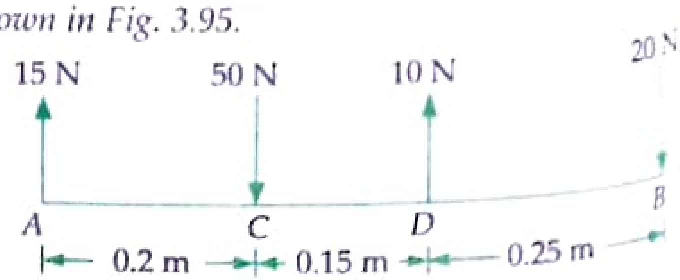


Fig. 3.95

Solution : (*a*) **A single force or resultant**

Taking downward forces as positive, the resultant of the given force system is

$$R = -15 + 50 - 10 + 20 = 45 \text{ N}$$

Thus the resultant has a magnitude of 45 N, its line of action is parallel to that of given forces and it acts vertically downwards.

Let this resultant act at a distance x from the end A.

Taking moments about A (clockwise moments positive) and applying the principle of moments,

Moment of the resultant = sum of the moments of its components

$$45x = (50 \times 0.2) - (10 \times 0.35) + (20 \times 0.6) = 10 - 3.5 + 12 = 18.5$$

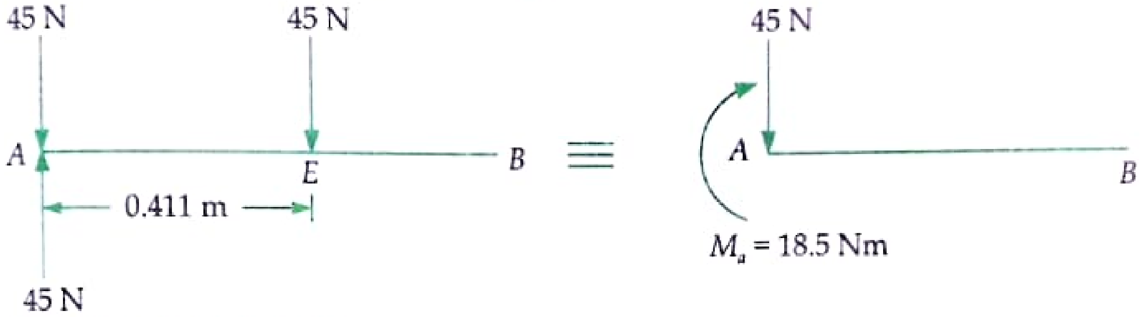
$$\therefore \quad x = \frac{18.5}{45} = 0.411 \text{ m}$$

Thus the resultant lies at a distance of 0.411 from A.

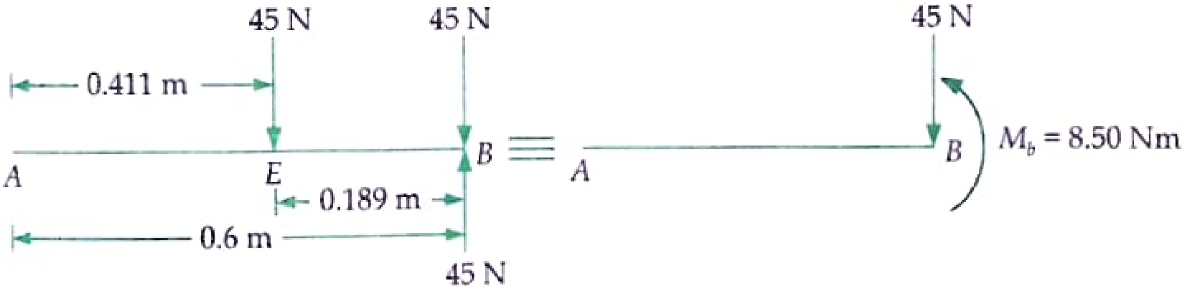
(*b*) **Single force-moment at A**

When a force of 45 N acting at E is shifted to A, it is a accompanied by a moment

$$M_a = (45 \times 0.411) = 18.5 \text{ Nm}$$



(*c*) **Single force - moment at B**



When the force of 45 N acting at E is moved to B, it is accompanied by a anticlock wise moment

$$M_b = -(45 \times 0.189) = -8.50 \text{ Nm}$$

EXAMPLE 3.68

Check whether the parallel force system depicted in Fig. 3.96 is in equilibrium. If not, can this system be brought to equilibrium by applying a single force.

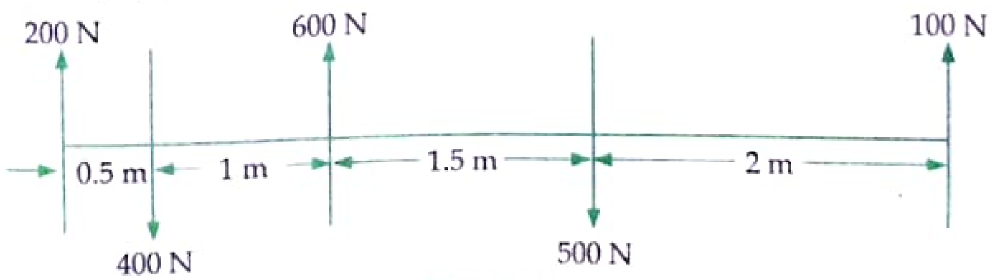


Fig. 3.96

Solution: Considering downward forces as positive, the resultant of the given force system is

$$R = -200 + 400 - 600 + 500 - 100 = 0$$

Thus the resultant force is zero.

Taking moments (clockwise moments positive) about a point on the line of action of force 200 N at the left end

$$M = (400 \times 0.5) - (600 \times 1.5) + (500 \times 3) - (100 \times 5)$$

$$= 200 - 900 + 1500 - 500 = 300 \text{ Nm } (\circlearrowright)$$

Obviously the given force system of parallel forces reduces to a clockwise couple of 300 Nm. It may be recalled that the conditions of equilibrium of coplanar parallel system are

(*a*) $\Sigma F = 0$ and (*b*) $\Sigma M = 0$

The given system is thus not in equilibrium.

It may be further recalled that a couple cannot be balanced by a single force. Therefore, the given force system cannot be brought to equilibrium by applying a single force.

EXAMPLE 3.69

Apply the laws of equilibrium to state the conditions under which the three force system given in the adjoining figure will be in equilibrium.

Solution: Apply the three conditions of equilibrium, *i.e.*,

(i) $\Sigma F_x = 0$. This is true as no horizontal force is acting on the body.

(ii) $\Sigma F_y = 0$. For this condition to hold good,

$$P_1 + P_3 = P_2$$

(iii) $\Sigma M = 0$. Taking moments of all the forces about point A, we get

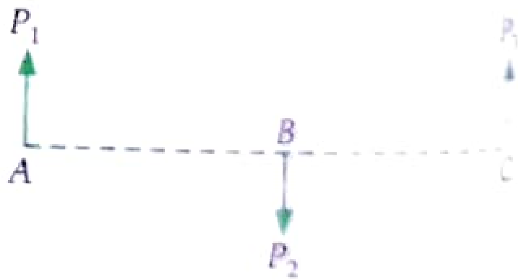


Fig. 3.97

$$\Sigma M_A = P_2 \times AB - P_3 \times AC$$

and for equilibrium, ΣM_A should be zero. That gives

$$P_2 \times AB - P_3 \times AC = 0$$

Apparently the distances AB and AC have to be such that the above identity is satisfied.

3.7. GENERAL CONDITI