

# Propositional Logic(Exercises:31-41)

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## Truth Table Rows Calculation

**31.**

Formula: The number of rows in a truth table is calculated as  $2^n$ , where n is the number of distinct variables.

- (a) Distinct variables: 1 (p), Number of rows:  $2^1 = 2$
- (b) Distinct variables: 4 (p, r, q, s), Number of rows:  $2^4 = 16$
- (c) Distinct variables: 6 (p, q, r, s, t, u), Number of rows:  $2^6 = 64$
- (d) Distinct variables: 4 (p, q, r, t), Number of rows:  $2^4 = 16$

**32.**

- (a) Distinct variables: 2 (p, q), Number of rows:  $2^2 = 4$
- (b) Distinct variables: 3 (p, s, t), Number of rows:  $2^3 = 8$
- (c) Distinct variables: 5 (p, r, s, t, u, v), Number of rows:  $2^6 = 64$
- (d) Distinct variables: 5 (p, q, r, s, t), Number of rows:  $2^5 = 32$

## Truth Tables

**33-41**

1. $p \wedge \neg p$		
p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Figure 1: Figure 33(a)

2.  $p \vee \neg p$

$p$	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Figure 2: Figure 33(b)

3.  $(p \vee \neg q) \rightarrow q$

$p$	$q$	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

Figure 3: Figure 33(c)

4.  $(p \vee q) \rightarrow (p \wedge q)$

$p$	$q$	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Figure 4: Figure 33(d)

5.  $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$p$	$q$	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
T	T	T	F	F	T	F
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

Figure 5: Figure 33(e)

6.  $(p \rightarrow q) \rightarrow (q \rightarrow p)$

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Figure 6: Figure 33(f)

1.  $p \rightarrow \neg p$

$p$	$\neg p$	$p \rightarrow \neg p$
T	F	F
F	T	T

Figure 7: Figure 34(a)

2.  $p \leftrightarrow \neg p$

$p$	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

Figure 8: Figure 34(b)

3.  $p \oplus (p \vee q)$

$p$	$q$	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

Figure 9: Figure 34(c)

4.  $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Figure 10: Figure 34(d)

5.  $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Figure 11: Figure 34(e)

6.  $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Figure 12: Figure 34(f)

1.  $(p \vee q) \rightarrow (p \oplus q)$

$p$	$q$	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Figure 13: Figure 35(a)

2.  $(p \oplus q) \rightarrow (p \wedge q)$

$p$	$q$	$p \oplus q$	$p \wedge q$	$(p \oplus q) \rightarrow (p \wedge q)$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Figure 14: Figure 35(b)

3.  $(p \vee q) \oplus (p \wedge q)$

$p$	$q$	$p \vee q$	$p \wedge q$	$(p \vee q) \oplus (p \wedge q)$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

Figure 15: Figure 35(c)

4.  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	F
F	F	T	T	F	T

Figure 16: Figure 35(d)

5.  $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

p	q	r	$p \leftrightarrow q$	$\neg p$	$\neg r$	$\neg p \leftrightarrow \neg r$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	T	F	F	T	F
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	F	F	T
F	T	F	F	T	T	T	F
F	F	T	T	T	F	F	T
F	F	F	T	T	T	T	F

Figure 17: Figure 35(e)

6.  $(p \oplus q) \rightarrow (p \oplus \neg q)$

p	q	$p \oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \rightarrow (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	T	F	T	T
F	F	F	T	F	T

Figure 18: Figure 35(f)

1.  $p \oplus p$

$p$	$p$	$p \oplus p$
T	T	F
F	F	F

Figure 19: Figure 36(a)

2.  $p \oplus \neg p$

$p$	$\neg p$	$p \oplus \neg p$
T	F	T
F	T	T

Figure 20: Figure 36(b)

3.  $p \oplus \neg q$

$p$	$q$	$\neg q$	$p \oplus \neg q$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

Figure 21: Figure 36(c)

4.  $\neg p \oplus \neg q$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \oplus \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

Figure 22: Figure 36(d)

5.  $(p \oplus q) \vee (p \oplus \neg q)$

$p$	$q$	$p \oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Figure 23: Figure 36(e)

6.  $(p \oplus q) \wedge (p \oplus \neg q)$

$p$	$q$	$p \oplus q$	$\neg q$	$p \oplus \neg q$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	T	F	F	F
F	F	F	T	T	F

Figure 24: Figure 36(f)

1.  $p \rightarrow \neg q$

$p$	$q$	$\neg q$	$p \rightarrow \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

Figure 25: Figure 37(a)



2.  $\neg p \leftrightarrow q$

p	q	$\neg p$	$\neg p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

Figure 26: Figure 37(b)

3.  $(p \rightarrow q) \vee (\neg p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow q$	$(p \rightarrow q) \vee (\neg p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Figure 27: Figure 37(c)

4.  $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow q$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Figure 28: Figure 37(d)

5.  $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

Figure 29: Figure 37(e)

6.  $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

Figure 30: Figure 37(f)

1.  $p \rightarrow \neg q$

p	q	$\neg q$	$p \rightarrow \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

Figure 31: Figure 37(a)

2.  $\neg p \leftrightarrow q$

p	q	$\neg p$	$\neg p \leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

Figure 32: Figure 37(b)

3.  $(p \rightarrow q) \vee (\neg p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow q$	$(p \rightarrow q) \vee (\neg p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Figure 33: Figure 37(c)

4.  $(p \rightarrow q) \wedge (\neg p \rightarrow q)$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow q$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Figure 34: Figure 37(d)

5.  $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

Figure 35: Figure 37(e)

6.  $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$p \leftrightarrow q$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

Figure 36: Figure 37(f)

1.  $p \rightarrow (\neg q \vee r)$

p	q	r	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	F	T

Figure 37: Figure 39(a)

2.  $\neg p \rightarrow (q \rightarrow r)$

$p$	$q$	$r$	$\neg p$	$q \rightarrow r$	$\neg p \rightarrow (q \rightarrow r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	F	F

Figure 38: Figure 39(b)

3.  $(p \rightarrow q) \vee (\neg p \rightarrow r)$

$p$	$q$	$r$	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow r$	$(p \rightarrow q) \vee (\neg p \rightarrow r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	F	F

Figure 39: Figure 39(c)

4.  $(p \rightarrow q) \wedge (\neg p \rightarrow r)$

p	q	r	$p \rightarrow q$	$\neg p$	$\neg p \rightarrow r$	$(p \rightarrow q) \wedge (\neg p \rightarrow r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	F
F	F	F	F	T	F	F

Figure 40: Figure 39(d)

5.  $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$

p	q	r	$p \leftrightarrow q$	$\neg q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Figure 41: Figure 39(e)

6.  $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	T	F
T	F	F	F	T	F	F	T
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

Figure 42: Figure 39(f)

Truth Table:

p	q	r	s	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$	$((p \rightarrow q) \rightarrow r) \rightarrow s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	F

Figure 43: Figure 40

Truth Table:

$p$	$q$	$r$	$s$	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Figure 44: Figure 41