IAAC Qualification Round 2026 - Solution

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Problem A: Cosmic Distances

Task: Match the astronomical images to the object names and their corresponding positions on the logarithmic distance ruler.

Analysis of Objects and Distances

Using standard astronomical data and the logarithmic scale provided (10^1 to 10^9 light-years):

1. **Vega:** A bright, nearby star located approximately 25 light-years from Earth.
2. **Aldebaran:** A red giant star located approximately 65 light-years from Earth.
3. **Pleiades:** An open star cluster located approximately 444 light-years away (Confirmed as image 1/Position C in the problem statement).
4. **Eagle Nebula:** A star-forming region (Pillars of Creation) located approximately 5,700 light-years away. This fits the range $10^3 - 10^4$ (Position D).
5. **Sagittarius A*:** The supermassive black hole at the Galactic Center, located approximately 27,000 light-years away. This fits the range $10^4 - 10^5$ (Position E).
6. **Large Magellanic Cloud (LMC):** A satellite galaxy of the Milky Way, approximately 160,000 light-years away. This fits the range $10^5 - 10^6$ (Position F).
7. **Andromeda Galaxy:** The nearest major spiral galaxy, approximately 2.5 million light-years away. This fits the range $10^6 - 10^7$ (Position G).
8. **Coma Cluster:** A large cluster of galaxies located approximately 320 million light-years away. This fits the range $10^8 - 10^9$ (Position H).

Final Solution Matching

Based on the visual appearance of the images in the problem file:

- **1. Pleiades (C)** (Given in problem)
- **2. Large Magellanic Cloud (F)** (Irregular cloud structure)
- **3. Vega (A)** (Bright blue-white star)
- **4. Coma Cluster (H)** (Dense field of distant galaxies)
- **5. Sagittarius A* (E)** (Black hole shadow/accretion disk image)
- **6. Aldebaran (B)** (Bright orange/red star)

- **7. Eagle Nebula (D)** (Nebulosity/Pillars of Creation)
- **8. Andromeda Galaxy (G)** (Distinct spiral galaxy)

Problem B: Low Earth Orbit

(a) Total volume of space within Low Earth Orbit (LEO)

Given:

- Earth Radius (R_E) $\approx 6,370$ km.
- LEO Altitude range: $h_{min} = 160$ km to $h_{max} = 2,000$ km.

Calculation: We treat the LEO region as a spherical shell. The volume V is the difference between the volume of the outer sphere (R_{outer}) and the inner sphere (R_{inner}).

$$R_{inner} = R_E + h_{min} = 6,370 + 160 = 6,530\,\text{km}$$
$$R_{outer} = R_E + h_{max} = 6,370 + 2,000 = 8,370\,\text{km}$$

The volume formula for a sphere is $V = \frac{4}{3}\pi R^3$. The volume of the shell is:

$$V_{LEO} = \frac{4}{3}\pi(R_{outer}^3 - R_{inner}^3)$$

Calculating the cubes:

$$R_{outer}^3 = (8,370)^3 \approx 5.863 \times 10^{11}\,\text{km}^3$$
$$R_{inner}^3 = (6,530)^3 \approx 2.784 \times 10^{11}\,\text{km}^3$$

$$V_{LEO} = \frac{4}{3}\pi(3.079 \times 10^{11})$$
$$V_{LEO} \approx 1.289 \times 10^{12}\,\text{km}^3$$

Answer: The total volume is approximately 1.29×10^{12} **km**^{3}.

(b) Average volume available to each satellite

Given:

- Number of satellites (N) $\approx 13,500$.

Calculation:

$$V_{avg} = \frac{V_{LEO}}{N} = \frac{1.289 \times 10^{12}}{13,500}$$
$$V_{avg} \approx 95,481,481\,\text{km}^3$$

Answer: The average volume per satellite is approximately 9.55×10^7 **km**^{3}.

Problem C: Total Solar Eclipse

(a) Eclipse Types

- **Total Solar Eclipse:** The Moon completely covers the Sun's disk. This occurs when the Moon is close enough to Earth to appear larger than the Sun in the sky.
- **Partial Solar Eclipse:** The Moon, Sun, and Earth are not perfectly aligned, so the Moon only covers a fraction of the Sun.
- **Annular Solar Eclipse:** The Moon passes centrally in front of the Sun but is too far from Earth to cover it completely, leaving a "ring of fire" visible.

(b) Next Total Solar Eclipse

The Qualification Round deadline is April 2026. The next total solar eclipse will occur on **August 12, 2026**.

(c) Condition for Totality

For a total eclipse, the angular size of the Moon (θ_M) must be greater than or equal to the angular size of the Sun (θ_S). Using the small-angle approximation $\theta \approx 2R/d$:

$$\frac{2R_M}{d_M} \geq \frac{2R_S}{d_S} \implies \frac{R_M}{d_M} \geq \frac{R_S}{d_S}$$

Problem D: The Last Eclipse

Objective: Determine when (t) the condition derived in Problem C will no longer hold.
Given:

- $R_M = 1,740$ km, $R_S(0) = 696,000$ km.
- $d_M(0) = 356,000$ km (Perigee), $d_S = 152,000,000$ km (Aphelion).
- Recession rate $v = 3.8$ cm/yr $= 3.8 \times 10^{-5}$ km/yr.
- $t_0 = 4.6 \times 10^9$ years.

Calculation: The limiting condition is equality:

$$\frac{R_M}{d_M(0) + vt} = \frac{R_S(t)}{d_S}$$

Substitute the solar expansion formula $R_S(t) = R_S(0)\sqrt{\frac{1}{1-0.4(t/t_0)}}$:

$$\frac{R_M}{d_M(0) + vt} = \frac{R_S(0)}{d_S} \cdot \sqrt{\frac{1}{1 - \frac{0.4t}{t_0}}}$$

Rearranging to isolate t:

$$\left(\frac{R_M d_S}{R_S(0)(d_M(0) + vt)}\right)^2 = \frac{1}{1 - \frac{0.4t}{t_0}}$$

$$\left(\frac{d_M(0) + vt}{K}\right)^2 = 1 - \frac{0.4t}{t_0}$$

Where constant $K = \frac{R_M d_S}{R_S(0)} = \frac{1740 \cdot 152 \times 10^6}{696 \times 10^3} \approx 380,000$ km.

Substituting values (v in km/yr):

$$\left(\frac{356,000 + 3.8 \times 10^{-5} t}{380,000}\right)^2 = 1 - \frac{0.4t}{4.6 \times 10^9}$$

Approximating the solution: LHS $\approx (0.9368)^2 +$ linear term $\approx 0.8777 +$ linear term. Solving for t yields approximately:

$$t \approx 447,000,000 \, \text{years}$$

Answer: The last total solar eclipse will occur in approximately **447 million years**.

Problem E: Testing Relativity

The 1919 total solar eclipse allowed scientists to test Einstein's General Theory of Relativity. The theory predicted that massive objects like the Sun warp spacetime, causing light passing nearby to bend. During the eclipse, the Sun's glare was blocked, allowing astronomers to photograph stars (the Hyades cluster) near the Sun's limb. The observed positions of these stars were shifted by approximately 1.75 arcseconds compared to their true positions, confirming Einstein's prediction of gravitational lensing.