MATH 320 Lecture-1

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1 Cardinal Numbers

1.1 Sets and Logic (Rudin pp 24-30)

1.1.1 Russel's Paradox

We begin by classifying sets:

Definition

A set S is called

- Normal if $S \notin S$.
- Abnormal if $S \in S$.

Every set is one or the other.

Consider $\mathcal{N}$, the collection of all normal sets. To classify $\mathcal{N}$:

- Consider the case when $\mathcal{N}$ is normal. By choice of $\mathcal{N}$, it has to be abnormal, which is a contradiction.
- In the case where $\mathcal{N}$ is abnormal, i.e., $\mathcal{N} \in \mathcal{N}$, we see that $\mathcal{N}$ is also normal.

Clearly, this is a contradiction. The problem here is that the words used to set up $\mathcal{N}$ sound like maths, but fall out of the scope for safe logical reasoning. To get around this we will employ the ZFC (Zermelo–Fraenkel Choice) axiomatic system, which we will analyse more comprehensively as the course proceeds.

2 Mappings

2.1 Cartesian products

Definition

Given sets X and Y, a Cartesian product builds a new set of ordered pairs

$$X \times Y = \{(x, y) |\ x \in X, y \in Y\}.$$

We can extend this:

Definition

Given $X_1, X_2, \dots, X_n$, we define

$$\begin{aligned} X_1 \times X_2 \times \cdots \times X_n =& \{(x_1, x_2, \dots, x_n) | \ x_k \in X_k \text{ for all } k \in \{1, 2, \dots, n\}\} \\ =& \prod_{k=1}^{n} X_k. \end{aligned}$$

2.2 Multifunctions

Definition

Given any sets $X, \ Y$ any subset G of $X, \ Y$ defines a set-values mapping (ot multifunction) as: Given $x \in X$ we define

$$G(x) = \{y \in Y | \ (x, y) \in G\}.$$

G is called te graph of this mapping.

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