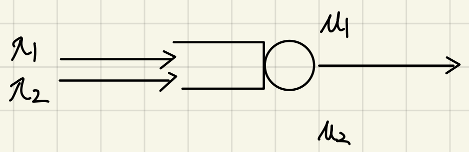
Queueing Theory HW4 鄭傑鴻 R10741015

FCFS, M/D/1, 2 class customer



$$\rho_1 = \frac{\lambda_1}{\mu_1} < 1$$

$$\rho_1 + \rho_1 = \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} > 1$$

(i) E(Remaining Service time)

Since the server is always busy $\rightarrow$ $P(\text{server busy}) = 1 = \underbrace{\rho_1}_{\text{probability serving class-1}} + \overbrace{(1-\rho_1)}^{\text{probability serving class-2}}$

$$E(R) = P(\text{serving class-1})\frac{1}{\mu_1} + P(\text{serving class-2})\frac{1}{\mu_2}$$

$$E(R) = \rho_1 \frac{1}{\mu_1} + (1-\rho_1)\frac{1}{\mu_2}$$

(ii) Mean waiting time before service $W_{q\text{-}1}$

$$W_{q\text{-}1} = \cancel{P(\text{server idle}) \times 0} + P(\text{server busy})\, E(T_{bp}|R)$$

$$= 1 \times \frac{\frac{\rho_1}{\mu_1} + \frac{1-\rho_1}{\mu_2}}{1-\rho_1}$$

$$W_{q\text{-}1} = \frac{\frac{\rho_1}{\mu_1} + \frac{1-\rho_1}{\mu_2}}{1-\rho_1}$$

Any M/G/1:

$P_0 = 1-\rho \rightarrow 1-P_0 = \rho$

$E(T_{bp}|I) = \dfrac{E(I)}{1-\rho}$

(iii) Yes, since class-1 customers are within server limit, the overall system is just class-1 customers occupying ρ_1 of the overall time portion, while the others given to class-2 customers to fill in the gap.

$$\rho_1 = \frac{\lambda_1}{\mu_1} \quad : \quad L_{q1} = \lambda_1 W_{q1} = \lambda_1 \frac{\frac{\rho_1}{\mu_1} + \frac{1-\rho_1}{\mu_2}}{1-\rho_1}$$

$\downarrow$

$$L_1 = L_{q1} + \rho_1 = \lambda_1 \frac{\frac{\rho_1}{\mu_1} + \frac{1-\rho_1}{\mu_2}}{1-\rho_1} + \rho_1$$