Queueing Theory HW3 鄭傑鴻 R10741015

1. By using the formula: $\underline{\lambda} = \underline{\gamma} + \underline{\lambda}\underline{R}$

$$[\lambda_A\ \lambda_B\ \lambda_C\ \lambda_D] = [30\ \ 0\ \ 0\ \ 0] + [\lambda_A\ \lambda_B\ \lambda_C\ \lambda_D] \begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0.1 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0.1 & 0.8 \\ 0 & 0.1 & 0 & 0.8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4×1 $\quad$ 4×4

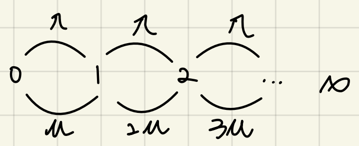
$\rightarrow \lambda_A = 30 + 0.1\lambda_A \qquad \rightarrow \lambda_A = \frac{30}{0.9} = \frac{100}{3} \approx 33.333$

$\lambda_B = 0.6\lambda_A + 0.1\lambda_C \qquad \rightarrow \lambda_B = 20 + 0.1\lambda_C \qquad \rightarrow \lambda_B = \frac{6200}{297} \approx 20.875$

$\lambda_C = 0.2\lambda_A + 0.1\lambda_B \qquad \rightarrow \lambda_C = \frac{20}{3} + 0.1\lambda_B \rightarrow \lambda_C = \frac{20}{3} + 2 + 0.01\lambda_C \rightarrow \lambda_C = \frac{2600}{297} \approx 8.754$

$\lambda_D = 0.8\lambda_B + 0.8\lambda_C \qquad \rightarrow \lambda_D = \frac{640}{27} \approx 23.704$

Node A: M/M/∞ :



$P_1 = \frac{\lambda}{\mu}P_0$

$P_2 = \frac{\lambda}{2\mu}P_1 = \left(\frac{\lambda}{\mu}\right)^2\frac{1}{2!}P_0$

$P_3 = \frac{\lambda}{3\mu}P_1 = \left(\frac{\lambda}{\mu}\right)^3\frac{1}{3!}P_0$

$\rightarrow$ $P_n = r^n\frac{1}{n!}P_0$, $P_0 = \left(\sum_{n=0}^{\infty}\frac{r^n}{n!}\right)^{-1} = e^{-r}$

$L = E(N) = \sum_{n=0}^{\infty} n\frac{r^n}{n!}e^{-r} = r\sum_{n=1}^{\infty}\frac{r^{n-1}}{n-1}e^{-r} = r\sum_{m=0}^{\infty}\frac{r^m}{m}e^{-r}$ $\quad e^r$

$\underline{L = r}$

$W = \frac{L}{\lambda} = \frac{r}{\lambda} = \frac{\frac{\lambda}{\mu}}{\lambda}$

$\underline{W = \frac{1}{\mu}}$

Therefore Node A: $L = r = \frac{\frac{100}{3}}{120} = \frac{5}{18}$

$W = \frac{1}{\mu} = \frac{1}{120}$

Node B, C, D: M/M/1 : $P_n = (1-\rho)\rho^n$

$L = \frac{\rho}{1-\rho}$

$W = \frac{\rho}{\lambda(1-\rho)}$

Therefore Node B: $\rho = \frac{\frac{6200}{297}}{30}$, $L = \frac{\rho}{1-\rho} = \frac{620}{271}$

$W = \frac{L}{\lambda} = \frac{297}{2710}$

Node C : $\rho = \frac{\frac{2600}{297}}{10}$, $L = \frac{260}{37}$

$W = \frac{297}{370}$

Node D: $\rho = \frac{\frac{640}{27}}{30}$, $L = \frac{64}{17}$

$W = \frac{27}{170}$

For the system

(a) $\underline{\underline{L = L_A + L_B + L_C + L_D \approx 13.357}}$

(b) $\underline{\underline{W = \frac{L}{\gamma_{total}} = \frac{13.357}{30} \approx 0.445 \text{ hour}}}$

Simulated result:

$L = 13.5416$

$W = 0.4377$

3. (ii) Same $\rho = \frac{\lambda}{\mu} = \frac{0.8}{1} = 0.8$ $(c=1)$

For M/M/1 : $P_n = (1-\rho)\rho^n$

$$L = \frac{\rho}{1-\rho} = 4 \qquad L_q = \frac{\rho^2}{1-\rho} = 3.2$$

$$W = \frac{\rho}{\lambda(1-\rho)} = 5 \qquad W_q = \frac{\rho^2}{\lambda(1-\rho)} = 4$$

For M/Ek/1 : $W_q = \frac{1+\frac{1}{k}}{2} \frac{\rho}{\mu(1-\rho)}$ $\rightarrow$ $W = W_q + \frac{1}{\mu}$

$$L_q = \frac{1+\frac{1}{k}}{2} \frac{\rho^2}{1-\rho} \qquad L = L_q + \rho$$

M/E2/1 : $k=2 \rightarrow W_q = 3$ $W = 4$

$L_q = \frac{12}{5}$ $L = \frac{16}{5}$

M/E10/1 : $k=10 \rightarrow W_q = \frac{11}{5}$ $W = \frac{16}{5}$

$L_q = \frac{44}{25}$ $L = \frac{64}{25}$

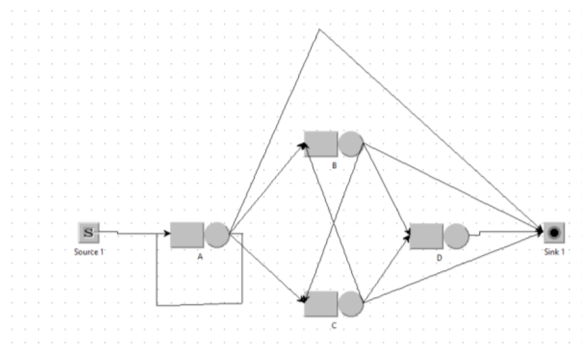
M/D/1 : $k \rightarrow \infty$ $W_q = 2$ $W = 3$

$L_q = \frac{8}{5}$ $L = \frac{12}{5}$

	M/M/1	M/E2/1	M/E10/1	M/D/1
L	4	3.2	2.56	2.4
W	5	4	3.2	3
Simulated	3.9804	3.7724	2.5432	2.4471
	5.0344	3.9691	3.1635	3.0371

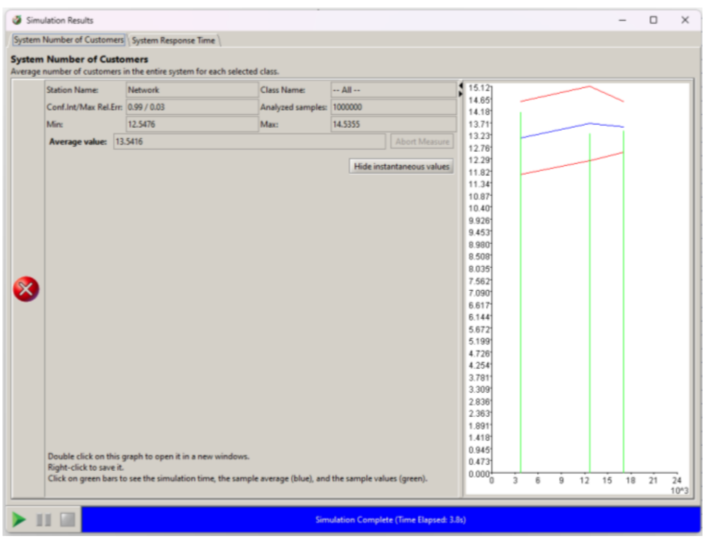
More deterministic (less random) $\rightarrow$ better performance on L, W

2. Construct the queueing network:

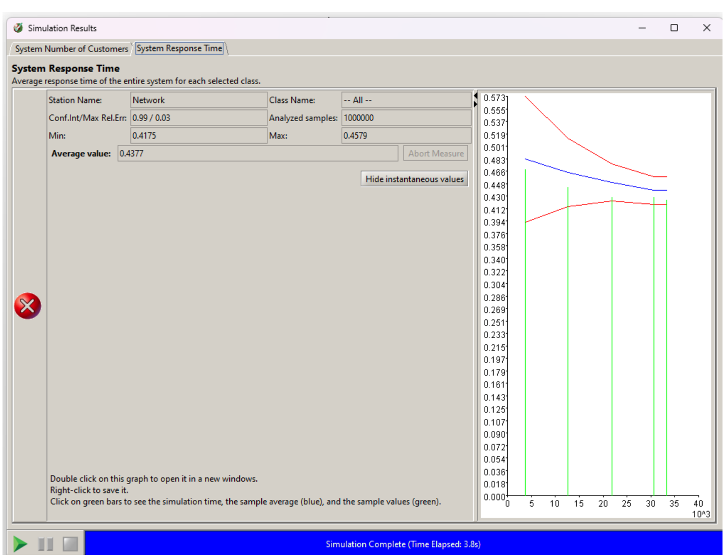


Run the simulation:

- Simulated system number of customers (L): 13.5416 (Calculated: 13.357)
- Reference: Second trial 13.471

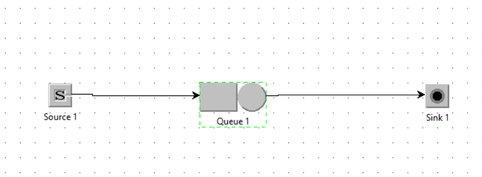


- Simulated system response time (W): 0.4377 (Calculated: 0.445)
- Reference: Second Trial: 0.486



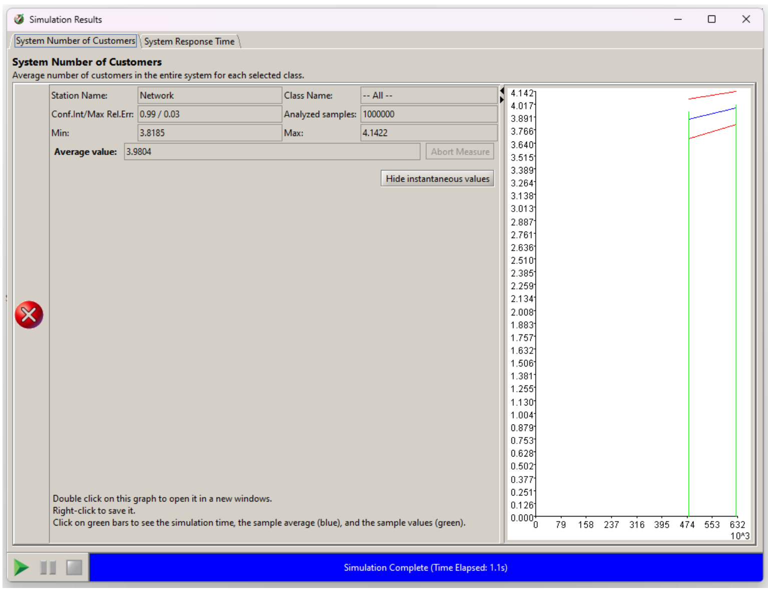
3. (ii)

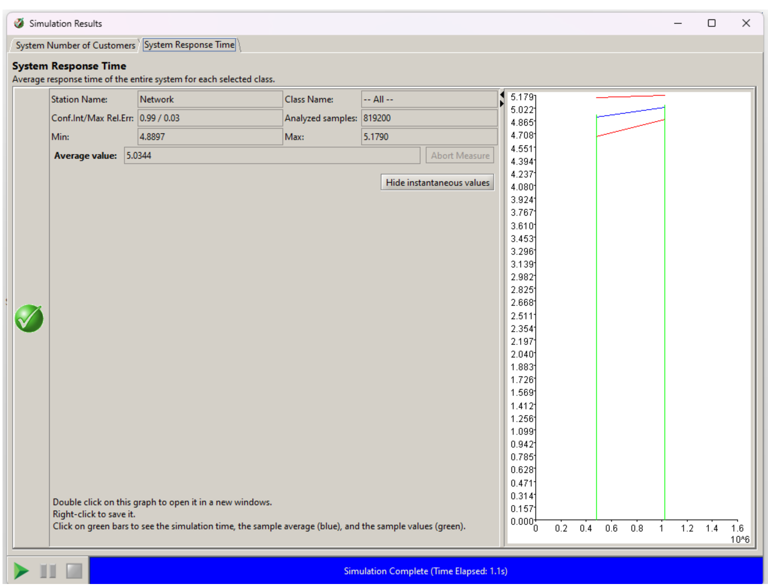
Construct the model:



For M/M/1:

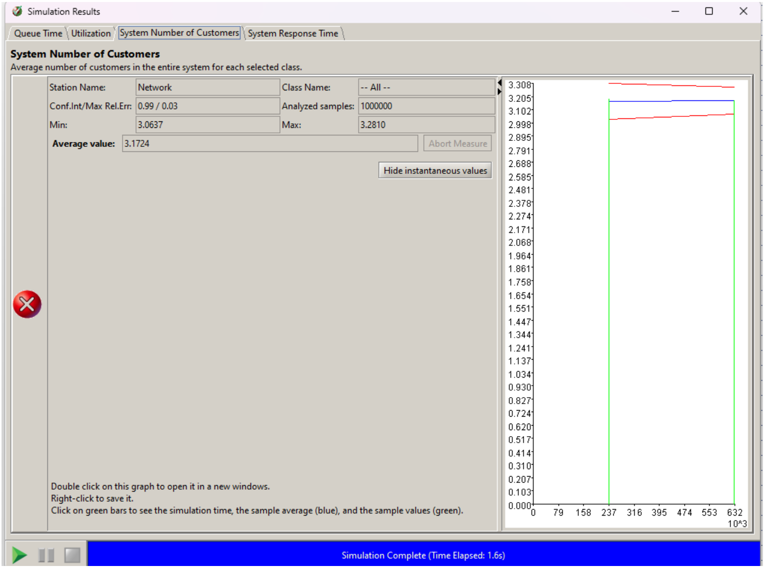
- Simulated system number of customers (L): 3.9804 (Calculated: 4)
- Simulated system response time (W): 5.0344 (Calculated: 5)

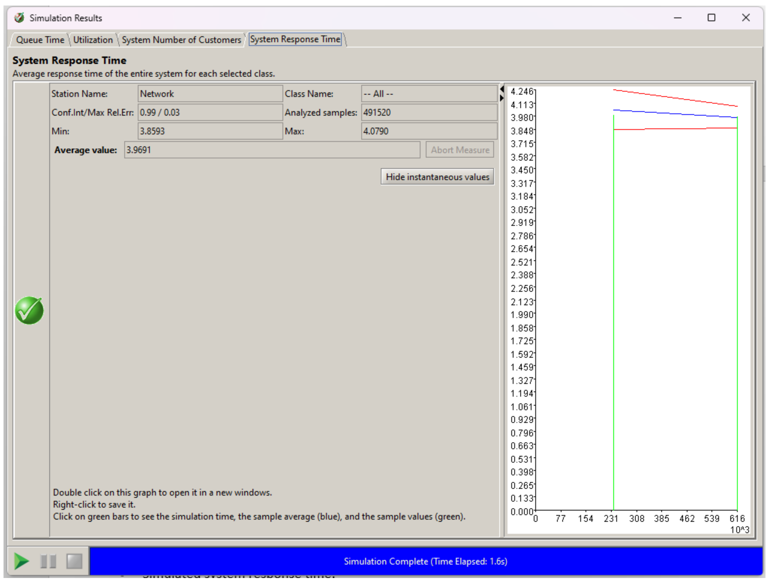




For M/E_2/1:

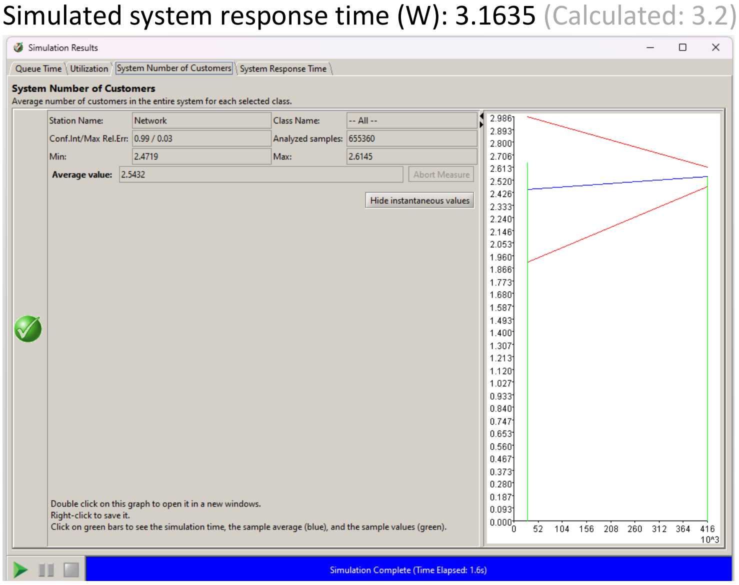
- Set the service time distribution to Erlang-k: λ(each stage)=2, k=2, such that mean service rate μ=1
- Simulated system number of customers (L): 3.1724 (Calculated: 3.2)
- Simulated system response time (W): 3.9691 (Calculated: 4)

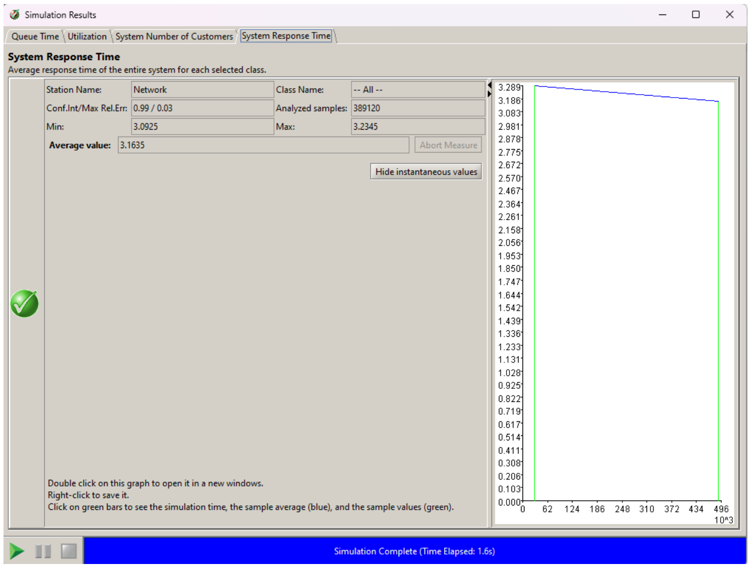




For M/ E_{10}/1:

- Set the service time distribution to Erlang-k: λ(each stage)=10, k=10, such that mean service rate μ=1
- Simulated system number of customers (L): 2.5432 (Calculated: 2.56)
- Simulated system response time (W): 3.1635 (Calculated: 3.2)





For M/D/1:

- Set the service distribution to deterministic with service rate μ=1
- Simulated system number of customers (L): 2.4471 (Calculated: 2.4)
- Simulated system response time (W): 3.0371 (Calculated: 3)