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# Regularized Linear Regression and Bias vs. Variance

## I. Theory:

### 1. Multi-class Classification

#### Hypothesis:

$$h_{\theta}(x) = g(\theta^T x) = g(\theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n)$$

$$= g(z) = \frac{1}{1 + e^{-z}}$$

#### Linear regression cost function:

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

#### Gradient descent algorithm:

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad \text{for } j=0$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} \theta_j \quad \text{for } j \geq 1 \quad \text{The way to}$$

with  $m$  = number of samples

$j = 1 \dots n$  : number of features

implement by vectorization is quite similar to the homework 1 so that I won't mention it again in here.

## II. Results:

### 1. Linear regression and learning curve

- Calculate cost and gradient to fit linear regression with dataset (as shown in Figure 1)

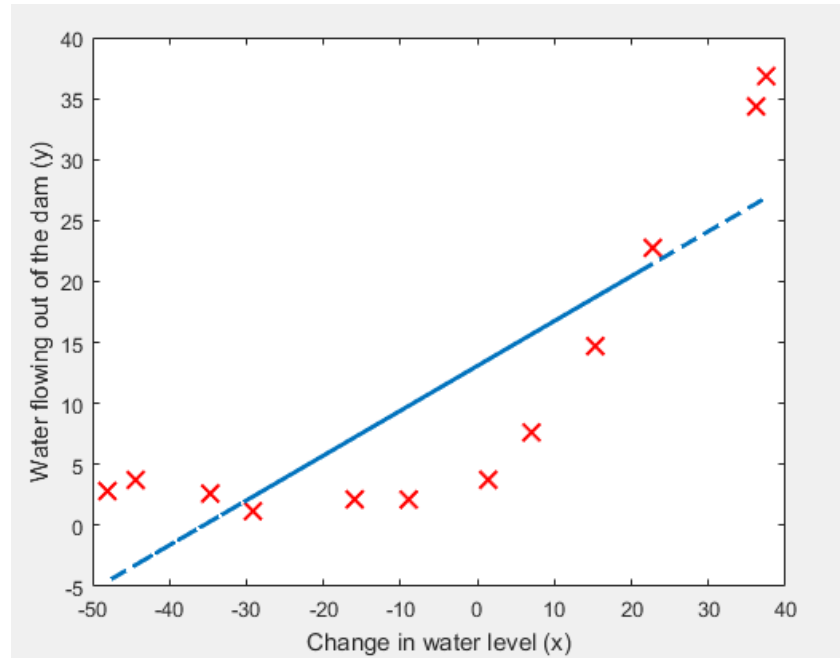


Figure 1 Visualize data and linear fit

- I use a loop from 1 to n (number of full training examples) to draw linear regression learning curve. The learning curve represented the cost function in Training and Cross Validation dataset base on the number of training examples. The result was illustrated in Fig.2. Note: we always calculate the cost for whole examples in cross validation set.

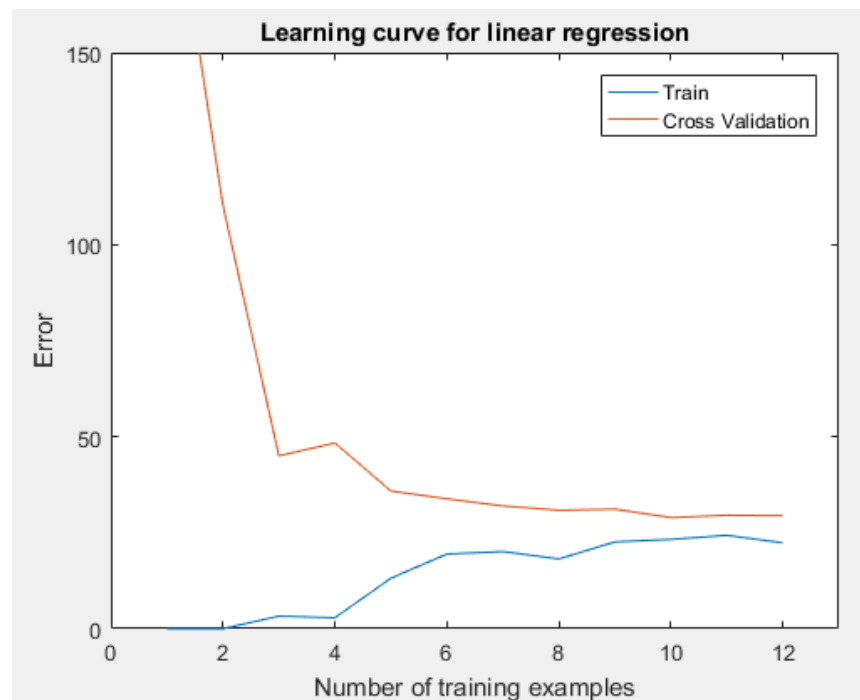


Figure 2 Linear regression learning curve

## 2. Polynomial regression

This part helps me to understand how to choose the best regularization term.

1. we will address this overfitting problem by adding more features. In Figure 3, we can see that the linear fit is overfitting the dataset. Figure 4 shown the learning curve, we can see the big gap between cost function of training and cross validation set which addressed that our current model is overfitting (cost function in trainingset is too small while in cross validation dataset is too large).

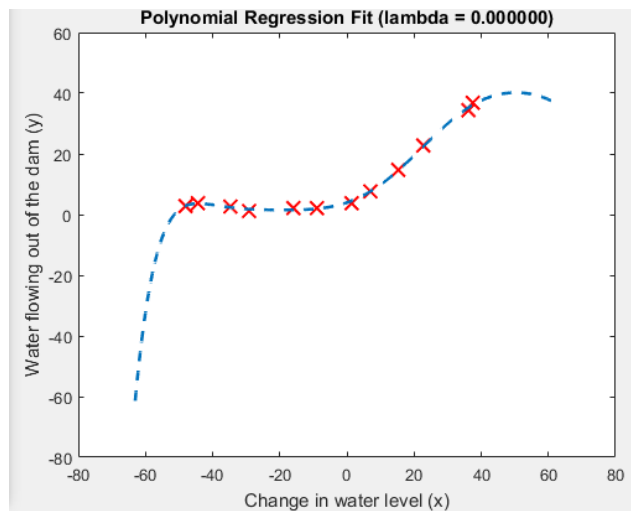


Figure 3 Linear fit with polynomial regression

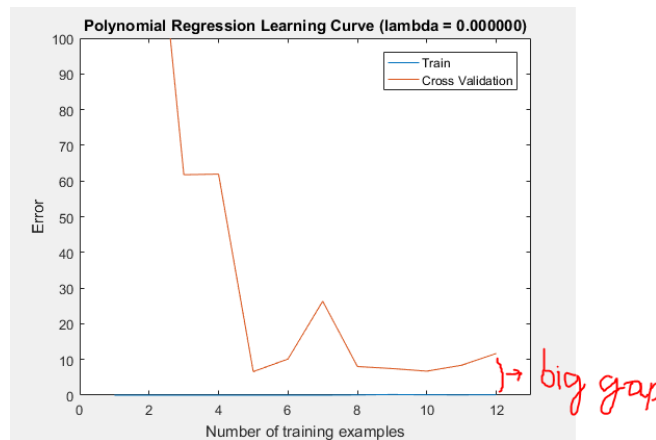
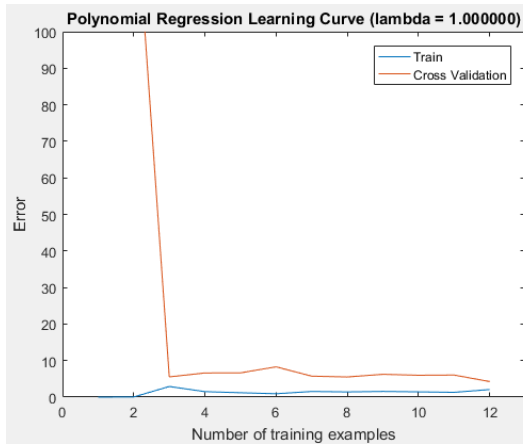
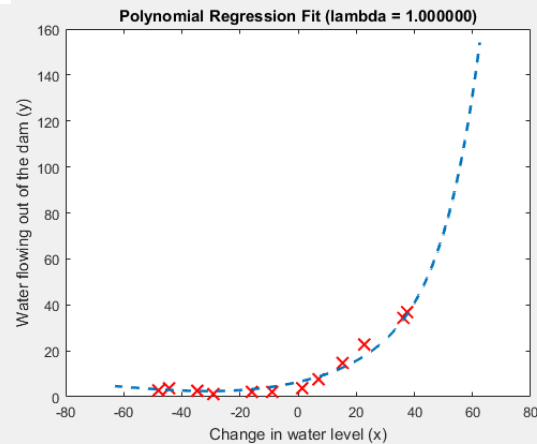
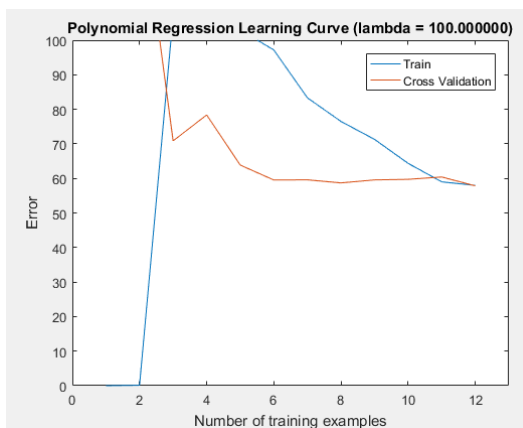
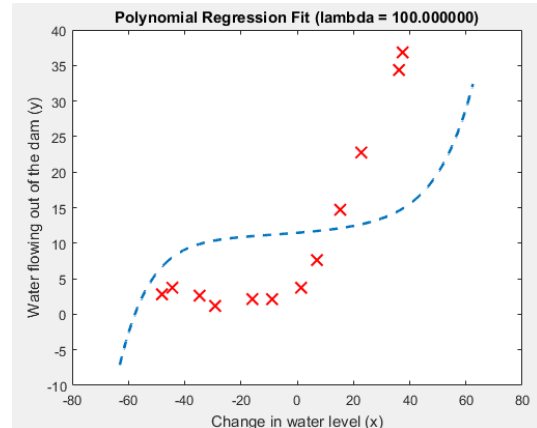
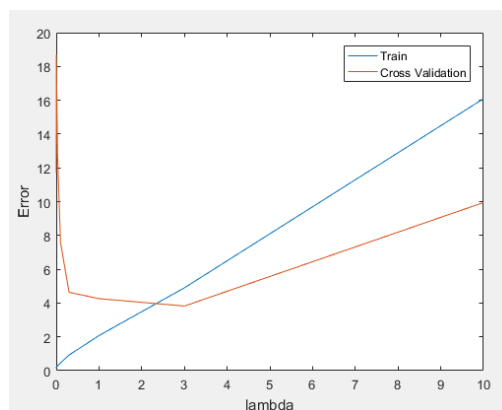


Figure 4 Polynomial learning curve, lambda=0

2. We can see the effect of regularization term by adjusting lambda (as shown in Figure 5, 6, 7, 8)

Figure 5 Polynomial learning curve,  $\lambda=1$ Figure 6 Polynomial fit,  $\lambda=1$ Figure 5 Polynomial learning curve,  $\lambda=100$ Figure 6 Polynomial fit,  $\lambda=100$ 

3. In order to choose the best  $\lambda$ , we will draw the learning curve when we adjust the value of  $\lambda$ . The best suitable  $\lambda$  is the value that let the cost function in Cross Validation set smallest. As we can see in Figure 9, the best  $\lambda$  we should choose is 3 where the cost function of Cross Validation set is smallest.

Figure 7 Selecting  $\lambda$  using a cross validation test

4. After we chosen the best  $\lambda=3$ , We can evaluate our result on test set and the result obtained : test error = 3.85988

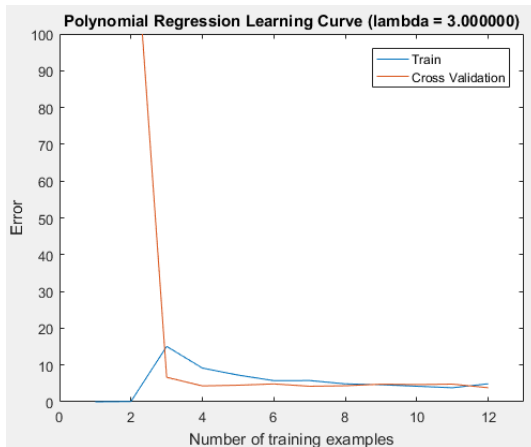


Figure 10 Polynomial learning curve,  $\lambda=3$

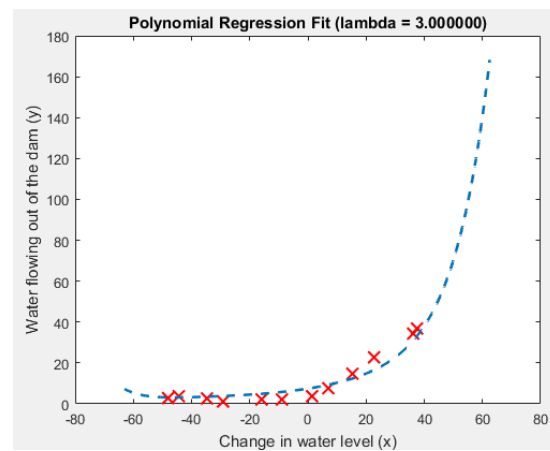


Figure 81 Polynomial fit,  $\lambda=3$

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test error 3.859888
Polynomial Regression (lambda = 3.000000)
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