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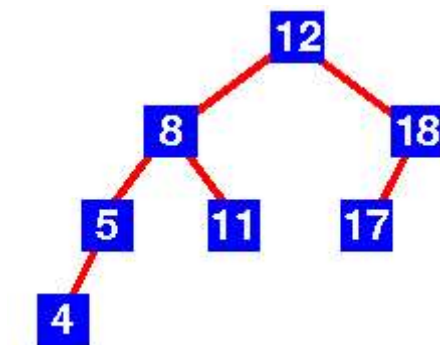Practice | GATE CS | Placements | Videos | Contribute

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AVL Tree | Set 1 (Insertion)

AVL tree is a self-balancing Binary Search Tree (BST) where the difference between heights of left and right subtrees cannot be more than one for all nodes.
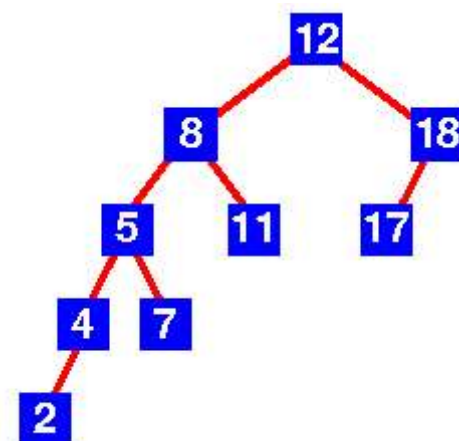
3.6

An Example Tree that is an AVL Tree

The above tree is AVL because differences between heights of left and right subtrees for every node is less than or equal to 1.

An Example Tree that is NOT an AVL Tree

The above tree is not AVL because differences between heights of left and right subtrees for 8 and 18 is greater than 1.
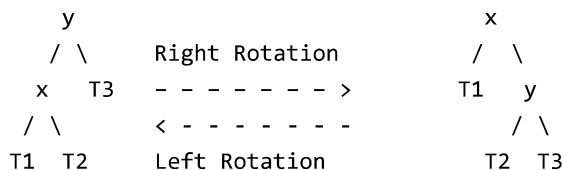
Images are taken from here.

Why AVL Trees?

Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take O(h) time where h is the height of the BST. The cost of these operations may become O(n) for a skewed Binary tree. If we make sure that height of the tree remains O(Logn) after every insertion and deletion, then we can guarantee an upper bound of O(Logn) for all these operations. The height of an AVL tree is always O(Logn) where n is the number of nodes in the tree (See this video lecture for proof).

Insertion

To make sure that the given tree remains AVL after every insertion, we must augment the standard BST insert operation to perform some re-balancing. Following are two basic operations that can be performed to re-balance a BST without violating the BST property (keys(left) < key(root) < keys(right)). 1) Left Rotation 2) Right Rotation

```
T1, T2 and T3 are subtrees of the tree rooted with y (on left side)
or x (on right side)
             y                               x
            / \     Right Rotation          /  \
           x   T3   - - - - - - - >        T1   y
          / \       < - - - - - - -            / \
        T1   T2     Left Rotation            T2   T3
Keys in both of the above trees follow the following order
      keys(T1) < key(x) < keys(T2) < key(y) < keys(T3)
So BST property is not violated anywhere.
```

Steps to follow for insertion

Let the newly inserted node be w

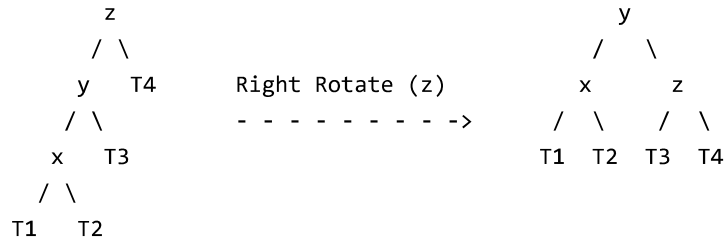**1)** Perform standard BST insert for w.

2) Starting from w, travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the child of z that comes on the path from w to z and x be the grandchild of z that comes on the path from w to z.

3) Re-balance the tree by performing appropriate rotations on the subtree rooted with z. There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways. Following are the possible 4 arrangements:

a) y is left child of z and x is left child of y (Left Left Case)

b) y is left child of z and x is right child of y (Left Right Case)

c) y is right child of z and x is right child of y (Right Right Case)

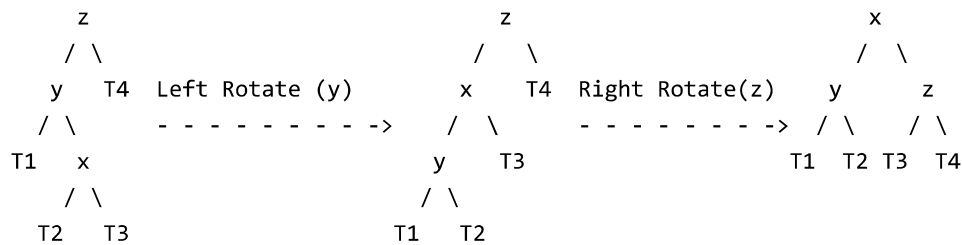d) y is right child of z and x is left child of y (Right Left Case)

Following are the operations to be performed in above mentioned 4 cases. In all of the cases, we only need to re-balance the subtree rooted with z and the complete tree becomes balanced as the height of subtree (After appropriate rotations) rooted with z becomes same as it was before insertion. (See this video lecture for proof)
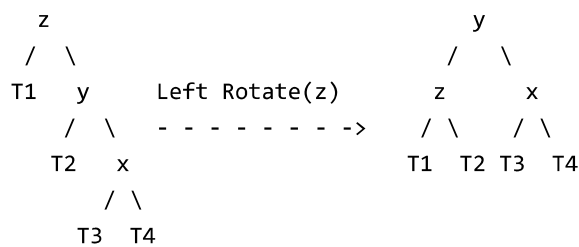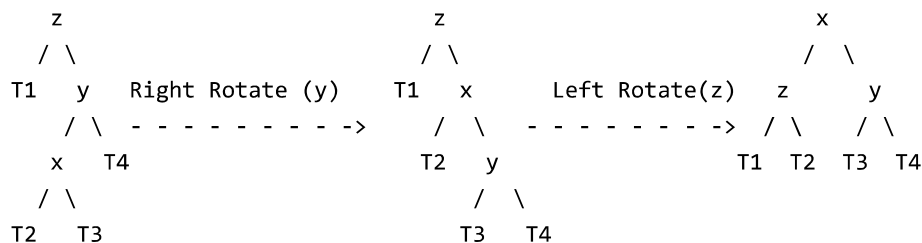
a) Left Left Case

```
T1, T2, T3 and T4 are subtrees.
       z                                      y
      / \                                   /   \
     y   T4       Right Rotate (z)         x     z
    / \          - - - - - - - - ->      / \   / \
   x   T3                               T1  T2 T3  T4
  / \
 T1  T2
```
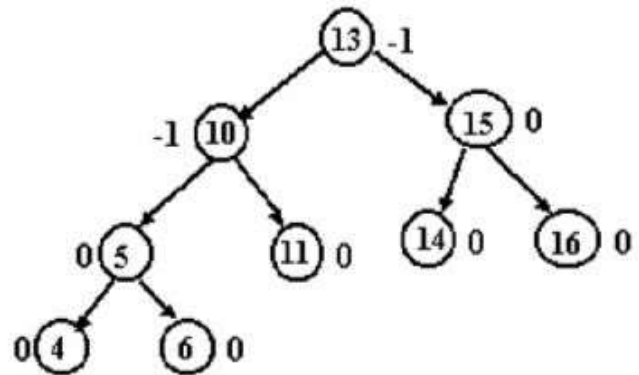
b) Left Right Case

```
    z                            z                             x
   / \                          /   \                         /  \
  y   T4  Left Rotate (y)      x    T4  Right Rotate(z)      y      z
 / \      - - - - - - - - ->  / \       - - - - - - - ->   / \    / \
T1  x                        y   T3                       T1  T2 T3  T4
   / \                      / \
  T2  T3                   T1  T2
```

c) Right Right Case

```
  z                                y
 /  \                             /   \
T1   y      Left Rotate(z)       z     x
    / \     - - - - - - - ->    / \   / \
   T2  x                       T1 T2 T3  T4
      / \
     T3  T4
```

d) Right Left Case

```
   z                           z                              x
  / \                         / \                            / \
 T1  y    Right Rotate (y)   T1  x      Left Rotate(z)       z     y
    / \   - - - - - - - - -> / \        - - - - - - - ->    / \   / \
   x   T4                   T2  y                          T1 T2 T3  T4
  / \                          / \
 T2  T3                       T3  T4
```
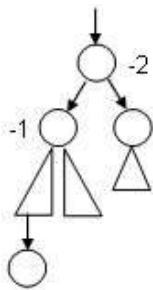
Insertion Examples:
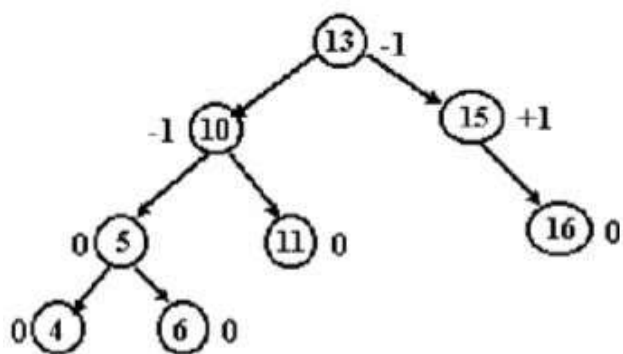
Insert 14
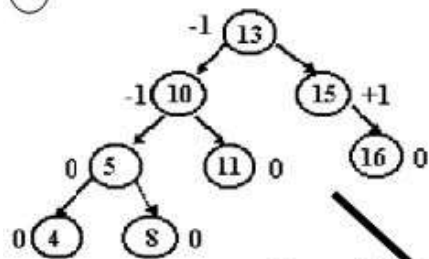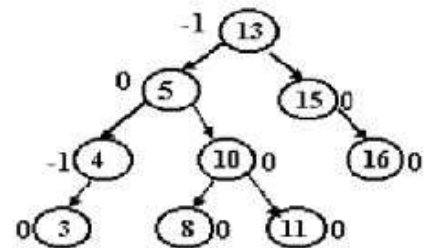
Insert 3
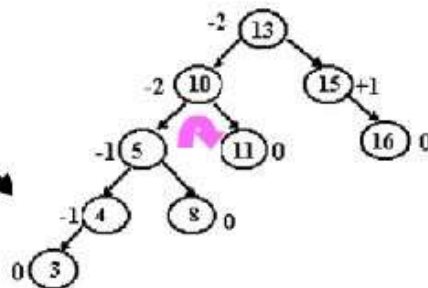
right rotation, with node 10 as pivot
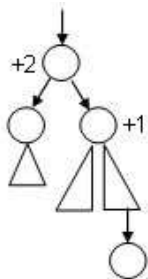
Insert 45

left rotation, with node 30 as the pivot

Insert 7

left rotation, with node 5 as the pivot

right rotation, with node 10 as the pivot

Insert 15

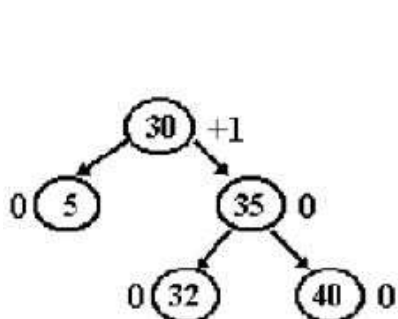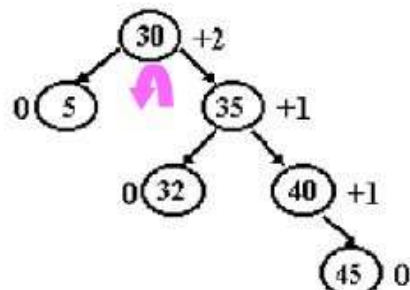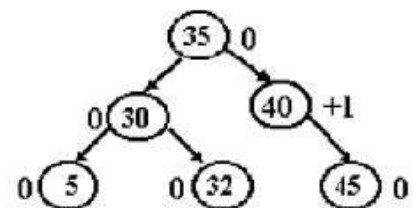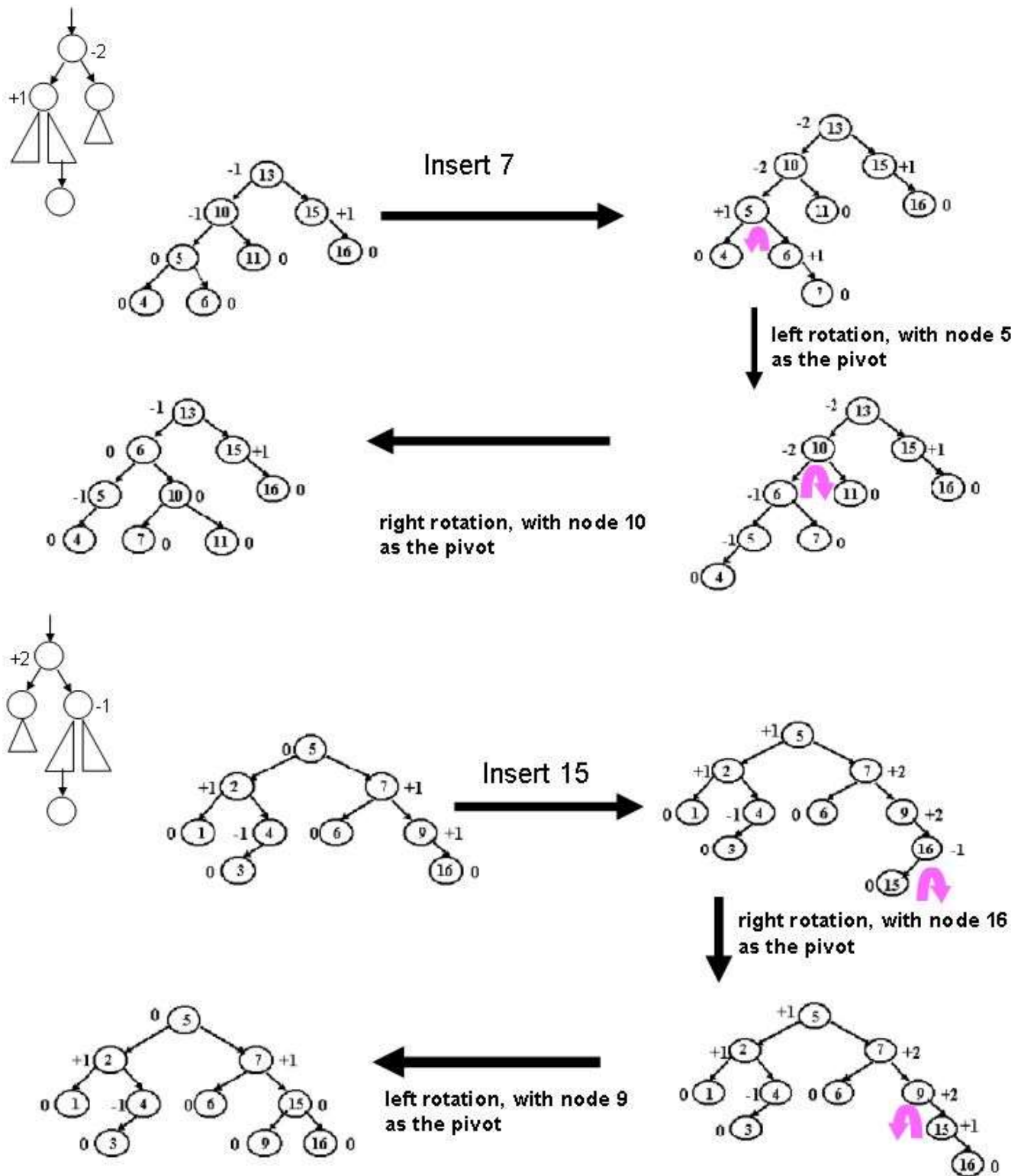right rotation, with node 16 as the pivot

left rotation, with node 9 as the pivot

The above images are taken from here.

Recommended: Please solve it on "*PRACTICE*" first, before moving on to the solution.

implementation

Following is the implementation for AVL Tree Insertion. The following implementation uses the

recursive BST insert to insert a new node. In the recursive BST insert, after insertion, we get pointers to all ancestors one by one in bottom up manner. So we don't need parent pointer to travel up. The recursive code itself travels up and visits all the ancestors of the newly inserted node.

1) Perform the normal BST insertion.

2) The current node must be one of the ancestors of the newly inserted node. Update the height of the current node.

3) Get the balance factor (left subtree height – right subtree height) of the current node.

4) If balance factor is greater than 1, then the current node is unbalanced and we are either in Left Left case or left Right case. To check whether it is left left case or not, compare the newly inserted key with the key in left subtree root.

5) If balance factor is less than -1, then the current node is unbalanced and we are either in Right Right case or Right Left case. To check whether it is Right Right case or not, compare the newly inserted key with the key in right subtree root.

C

```c
// C program to insert a node in AVL tree
#include<stdio.h>
#include<stdlib.h>

// An AVL tree node
struct Node
{
    int key;
    struct Node *left;
    struct Node *right;
    int height;
};

// A utility function to get maximum of two integers
int max(int a, int b);

// A utility function to get height of the tree
int height(struct Node *N)
{
    if (N == NULL)
        return 0;
    return N->height;
}

// A utility function to get maximum of two integers
int max(int a, int b)
{
    return (a > b)? a : b;
}

/* Helper function that allocates a new node with the given key and
   NULL left and right pointers. */
struct Node* newNode(int key)
{
    struct Node* node = (struct Node*)
                            malloc(sizeof(struct Node));
    node->key    = key;
    node->left   = NULL;
    node->right  = NULL;
    node->height = 1;  // new node is initially added at leaf
    return(node);
```

```c
}

// A utility function to right rotate subtree rooted with y
// See the diagram given above.
struct Node *rightRotate(struct Node *y)
{
    struct Node *x = y->left;
    struct Node *T2 = x->right;

    // Perform rotation
    x->right = y;
    y->left = T2;

    // Update heights
    y->height = max(height(y->left), height(y->right))+1;
    x->height = max(height(x->left), height(x->right))+1;

    // Return new root
    return x;
}

// A utility function to left rotate subtree rooted with x
// See the diagram given above.
struct Node *leftRotate(struct Node *x)
{
    struct Node *y = x->right;
    struct Node *T2 = y->left;

    // Perform rotation
    y->left = x;
    x->right = T2;

    //  Update heights
    x->height = max(height(x->left), height(x->right))+1;
    y->height = max(height(y->left), height(y->right))+1;

    // Return new root
    return y;
}

// Get Balance factor of node N
int getBalance(struct Node *N)
{
    if (N == NULL)
        return 0;
    return height(N->left) - height(N->right);
}

// Recursive function to insert key in subtree rooted
// with node and returns new root of subtree.
struct Node* insert(struct Node* node, int key)
{
    /* 1.  Perform the normal BST insertion */
    if (node == NULL)
        return(newNode(key));

    if (key < node->key)
        node->left  = insert(node->left, key);
    else if (key > node->key)
        node->right = insert(node->right, key);
    else // Equal keys are not allowed in BST
        return node;

    /* 2. Update height of this ancestor node */
    node->height = 1 + max(height(node->left),
                           height(node->right));

    /* 3. Get the balance factor of this ancestor
          node to check whether this node became
          unbalanced */
```

```c
    int balance = getBalance(node);

    // If this node becomes unbalanced, then
    // there are 4 cases

    // Left Left Case
    if (balance > 1 && key < node->left->key)
        return rightRotate(node);

    // Right Right Case
    if (balance < -1 && key > node->right->key)
        return leftRotate(node);

    // Left Right Case
    if (balance > 1 && key > node->left->key)
    {
        node->left =  leftRotate(node->left);
        return rightRotate(node);
    }

    // Right Left Case
    if (balance < -1 && key < node->right->key)
    {
        node->right = rightRotate(node->right);
        return leftRotate(node);
    }

    /* return the (unchanged) node pointer */
    return node;
}

// A utility function to print preorder traversal
// of the tree.
// The function also prints height of every node
void preOrder(struct Node *root)
{
    if(root != NULL)
    {
        printf("%d ", root->key);
        preOrder(root->left);
        preOrder(root->right);
    }
}

/* Drier program to test above function*/
int main()
{
  struct Node *root = NULL;

  /* Constructing tree given in the above figure */
  root = insert(root, 10);
  root = insert(root, 20);
  root = insert(root, 30);
  root = insert(root, 40);
  root = insert(root, 50);
  root = insert(root, 25);

  /* The constructed AVL Tree would be
            30
           /  \
         20    40
        /  \     \
      10   25     50
  */

  printf("Preorder traversal of the constructed AVL"
          " tree is \n");
  preOrder(root);

  return 0;
```

Run on IDE

Java

```java
// Java program for insertion in AVL Tree
class Node {
    int key, height;
    Node left, right;

    Node(int d) {
        key = d;
        height = 1;
    }
}

class AVLTree {

    Node root;

    // A utility function to get height of the tree
    int height(Node N) {
        if (N == null)
            return 0;

        return N.height;
    }

    // A utility function to get maximum of two integers
    int max(int a, int b) {
        return (a > b) ? a : b;
    }

    // A utility function to right rotate subtree rooted with y
    // See the diagram given above.
    Node rightRotate(Node y) {
        Node x = y.left;
        Node T2 = x.right;

        // Perform rotation
        x.right = y;
        y.left = T2;

        // Update heights
        y.height = max(height(y.left), height(y.right)) + 1;
        x.height = max(height(x.left), height(x.right)) + 1;

        // Return new root
        return x;
    }

    // A utility function to left rotate subtree rooted with x
    // See the diagram given above.
    Node leftRotate(Node x) {
        Node y = x.right;
        Node T2 = y.left;

        // Perform rotation
        y.left = x;
        x.right = T2;

        //  Update heights
        x.height = max(height(x.left), height(x.right)) + 1;
        y.height = max(height(y.left), height(y.right)) + 1;

        // Return new root
```

```java
        return y;
    }

    // Get Balance factor of node N
    int getBalance(Node N) {
        if (N == null)
            return 0;

        return height(N.left) - height(N.right);
    }

    Node insert(Node node, int key) {

        /* 1.  Perform the normal BST insertion */
        if (node == null)
            return (new Node(key));

        if (key < node.key)
            node.left = insert(node.left, key);
        else if (key > node.key)
            node.right = insert(node.right, key);
        else // Duplicate keys not allowed
            return node;

        /* 2. Update height of this ancestor node */
        node.height = 1 + max(height(node.left),
                              height(node.right));

        /* 3. Get the balance factor of this ancestor
              node to check whether this node became
              unbalanced */
        int balance = getBalance(node);

        // If this node becomes unbalanced, then there
        // are 4 cases Left Left Case
        if (balance > 1 && key < node.left.key)
            return rightRotate(node);

        // Right Right Case
        if (balance < -1 && key > node.right.key)
            return leftRotate(node);

        // Left Right Case
        if (balance > 1 && key > node.left.key) {
            node.left = leftRotate(node.left);
            return rightRotate(node);
        }

        // Right Left Case
        if (balance < -1 && key < node.right.key) {
            node.right = rightRotate(node.right);
            return leftRotate(node);
        }

        /* return the (unchanged) node pointer */
        return node;
    }

    // A utility function to print preorder traversal
    // of the tree.
    // The function also prints height of every node
    void preOrder(Node node) {
        if (node != null) {
            System.out.print(node.key + " ");
            preOrder(node.left);
            preOrder(node.right);
        }
    }

    public static void main(String[] args) {
```

```java
        AVLTree tree = new AVLTree();

        /* Constructing tree given in the above figure */
        tree.root = tree.insert(tree.root, 10);
        tree.root = tree.insert(tree.root, 20);
        tree.root = tree.insert(tree.root, 30);
        tree.root = tree.insert(tree.root, 40);
        tree.root = tree.insert(tree.root, 50);
        tree.root = tree.insert(tree.root, 25);

        /* The constructed AVL Tree would be
             30
            /  \
          20   40
         /  \    \
        10  25   50
        */
        System.out.println("Preorder traversal" +
                        " of constructed tree is : ");
        tree.preOrder(tree.root);
    }
}
// This code has been contributed by Mayank Jaiswal
```

Run on IDE

Python3

```python
# Python code to insert a node in AVL tree

# Generic tree node class
class TreeNode(object):
    def __init__(self, val):
        self.val = val
        self.left = None
        self.right = None
        self.height = 1

# AVL tree class which supports the
# Insert operation
class AVL_Tree(object):

    # Recursive function to insert key in
    # subtree rooted with node and returns
    # new root of subtree.
    def insert(self, root, key):

        # Step 1 - Perform normal BST
        if not root:
            return TreeNode(key)
        elif key < root.val:
            root.left = self.insert(root.left, key)
        else:
            root.right = self.insert(root.right, key)

        # Step 2 - Update the height of the
        # ancestor node
        root.height = 1 + max(self.getHeight(root.left),
                           self.getHeight(root.right))

        # Step 3 - Get the balance factor
        balance = self.getBalance(root)

        # Step 4 - If the node is unbalanced,
        # then try out the 4 cases
        # Case 1 - Left Left
        if balance > 1 and key < root.left.val:
```

```python
        return self.rightRotate(root)

    # Case 2 - Right Right
    if balance < -1 and key > root.right.val:
        return self.leftRotate(root)

    # Case 3 - Left Right
    if balance > 1 and key > root.left.val:
        root.left = self.leftRotate(root.left)
        return self.rightRotate(root)

    # Case 4 - Right Left
    if balance < -1 and key < root.right.val:
        root.right = self.rightRotate(root.right)
        return self.leftRotate(root)

    return root

def leftRotate(self, z):

    y = z.right
    T2 = y.left

    # Perform rotation
    y.left = z
    z.right = T2

    # Update heights
    z.height = 1 + max(self.getHeight(z.left),
                     self.getHeight(z.right))
    y.height = 1 + max(self.getHeight(y.left),
                     self.getHeight(y.right))

    # Return the new root
    return y

def rightRotate(self, z):

    y = z.left
    T3 = y.right

    # Perform rotation
    y.right = z
    z.left = T3

    # Update heights
    z.height = 1 + max(self.getHeight(z.left),
                     self.getHeight(z.right))
    y.height = 1 + max(self.getHeight(y.left),
                     self.getHeight(y.right))

    # Return the new root
    return y

def getHeight(self, root):
    if not root:
        return 0

    return root.height

def getBalance(self, root):
    if not root:
        return 0

    return self.getHeight(root.left) - self.getHeight(root.right)

def preOrder(self, root):

    if not root:
        return
```
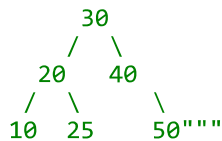
```python
            print("{0} ".format(root.val), end="")
            self.preOrder(root.left)
            self.preOrder(root.right)


# Driver program to test above function
myTree = AVL_Tree()
root = None

root = myTree.insert(root, 10)
root = myTree.insert(root, 20)
root = myTree.insert(root, 30)
root = myTree.insert(root, 40)
root = myTree.insert(root, 50)
root = myTree.insert(root, 25)

"""The constructed AVL Tree would be
            30
           /  \
         20    40
        /  \     \
      10   25     50"""

# Preorder Traversal
print("Preorder traversal of the",
      "constructed AVL tree is")
myTree.preOrder(root)
print()

# This code is contributed by Ajitesh Pathak
```

Run on IDE

Output:

```
Preorder traversal of the constructed AVL tree is
30 20 10 25 40 50
```

Time Complexity: The rotation operations (left and right rotate) take constant time as only few pointers are being changed there. Updating the height and getting the balance factor also take constant time. So the time complexity of AVL insert remains same as BST insert which is O(h) where h is height of the tree. Since AVL tree is balanced, the height is O(Logn). So time complexity of AVL insert is O(Logn).

Comparison with Red Black Tree

The AVL tree and other self balancing search trees like Red Black are useful to get all basic operations done in O(Log n) time. The AVL trees are more balanced compared to Red Black Trees, but they may cause more rotations during insertion and deletion. So if your application involves many frequent insertions and deletions, then Red Black trees should be preferred. And if the insertions and deletions are less frequent and search is more frequent operation, then AVL tree should be preferred over Red Black Tree.