CS325

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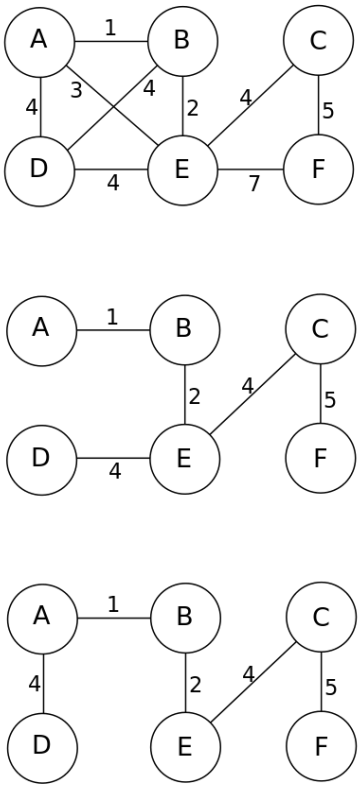
Problem1

(a) Find a graph that has multiple minimum spanning trees.




**Introduction:** Multiple Minimum Spanning. In some graphs, there may be more than one minimum spanning tree, and the trees all have the same total edge weight, but their structures may be different. In other words, if there is more than one set of edges in a weighted connected graph, these sets can form trees containing all vertices, and the sum of the edge weights of these trees is the smallest, then these trees are called multiple minimum spanning tree.

**Explain:** In the graph on the left, there are different connections connecting all the vertices, and the total weight of these connections is the same(4+1+2+4+5=16).

(From Wikipedia)

(b) Prove that any graph with distinct edge weights has a unique minimum spanning tree.

*Assume G is a connected, weighted graph with all distinct edge weights and
Suppose G has two different minimum spanning trees time 1 and time 2

1. **Graph G(V, E, W), MST T1 ,MST T2**

*There is at least one edge e in T1 but not in T2

2. **If T1 != T2:**

   **For each edge e in T1:**

   **If e is not in T2:**

*Choose an edge e' from T2 such that adding e' to T1 would form a cycle. In this cycle, all edges except for e' are from T1

3. **PutEdgeToMST(T2, e)**

**Cycle C = FindCycle(T2, e)**

*Since all edge weights of G are different, edge weights in the cycle are also different. Therefore, in this cycle, there must be an edge with the largest weight.

**4. maxWeightEdge = FindMaxWeightEdge(C)**

***If e' is not the edge with the highest weight in this cycle, then removing the edge with the highest weight in the cycle and adding e' will result in a spanning tree that is lighter than T1 (Contradiction)

**5. If maxWeightEdge != e:**

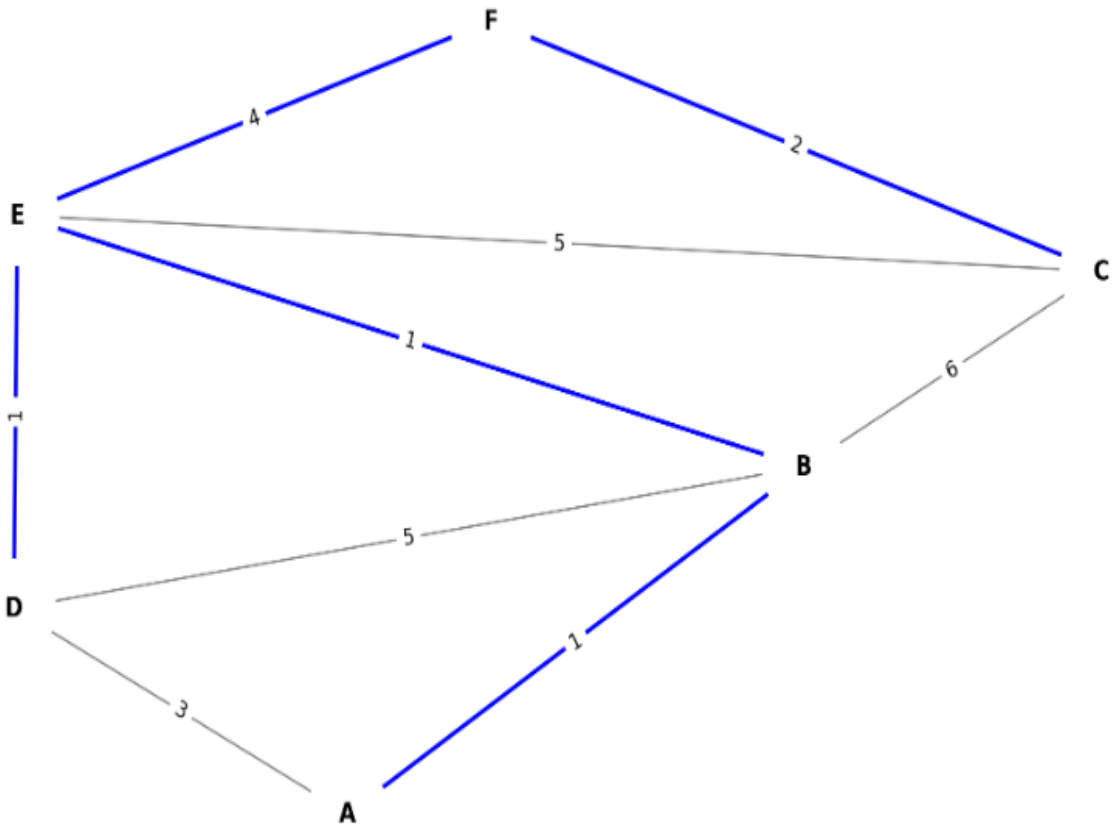
**RemoveEdgeFromMST(T1, maxWeightEdge)**

*If e' is the edge with the largest weight in the ring, then removing e' and retaining the other edges in the ring, we will get a lighter spanning tree than T2 (Contradiction)

**6. RemoveEdgeFromMST(T2, e)**

Since the existence of T1 and T2 leads to a contradiction, the hypothesis is not valid. Therefore, the graph G cannot have two different minimum spanning trees. In other words, G must have a unique minimum spanning tree.

(c) Find a graph with non-distinct edge weights that has a unique minimum spanning tree (can you generalize (b)?).



**Explain**: In this graph, the blue edges represent the minimum spanning tree (MST). This MST includes edges A-B, A-D, B-E, D-E and E-F. This MST is unique because any other choice of edges that connect all vertices in the graph and do not form a cycle will have a greater overall weight. For example, if we choose edge B-C or C-F with a weight of 6 or 4, this will increase the total weight of the MST.

**can We generalize (b)?**
There is a contradiction in both minimum spanning trees. Therefore, we can conclude that any graph with different edge weights must have a unique minimum spanning tree.

Problem 2.

A number maze is an n × n grid of positive integers. A token starts in the upper left corner; your goal is to move the token to the lower-right corner. On each turn, you are allowed to move the token up, down, left, or right; the distance you may move the token is determined by the number on its current square. For example, if the token is on a square labeled 3, then you may move the token three steps up, three steps down, three steps left, or three steps right. However, you are never allowed to move the token off the edge of the board. Describe and analyze an efficient algorithm that either returns the minimum number of moves required to solve a given number maze, or correctly reports that the maze has no solution. For example, given the number maze in the figure below, your algorithm should return the integer 8.

Time complexity: In the worst case, we may need to visit each square in the maze once. Since each square is visited at most once, the time complexity is O(n^2)
*Use the breadth-first search (BFS) algorithm

Function to solve number maze:

```
n = number of rows (or columns) of the maze
Create a queue Q
Create a two-dimensional array visited, initialized to False
Q enqueue (0, 0, 0) # (starting row, starting column, number of moves)
```

```
visited[0][0] = True

When Q is not empty:
    (row, col, moves) = Q dequeue

    # If the end point is reached, return the number of moves
    If row == n-1 and col == n-1:
        Return moves

    # Check movement in four directions
    For each direction (dr, dc) in [(-1, 0), (1, 0), (0, -1), (0, 1)]:
        step = maze[row][col]
        new_row = row + dr * step
        new_col = col + dc * step

        # Check if the new location is valid
        If new_row is at [0, n-1] and new_col is at [0, n-1] and has not been accessed:
            Q joins the team (new_row, new_col, moves + 1)
            visited[new_row][new_col] = True

# If the end point cannot be reached, return no solution
Return -1
```