ESE 441 – Spring 2017 – Homework 9

1. For the following system

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 5 \\ 0 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} -1 & 1 \end{bmatrix} \mathbf{x}$$

   Design a full order state observer with both poles at -5.

2. For the second order transfer function

$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

   (a) Show that the maximum of $|H(j\omega)|$ is located at

$$\omega_r = \omega_0\sqrt{1 - 2\zeta^2}$$

   (b) Show that the height of the maximum is given by

$$|H(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

   (c) Where is the maximum of $|H(j\omega)|$ when $\zeta > \sqrt{2}/2$?

   (d) For small damping ratio, $\omega_r \approx \omega_0$. Show that

$$|H(j\omega_0)| = \frac{1}{2\zeta}$$

3. Sketch the Bode plots and estimate phase and gain margins (if they exist) for the following transfer functions:

   (a) $G(s) = \dfrac{\sqrt{10}}{s + 0.1}$

   (b) $G(s) = 0.1\dfrac{s + 10\sqrt{10}}{s + 0.1}$

   (c) $G(s) = \dfrac{100}{s(s + 10)}$

4. For the open loop transfer function $G(s)$, use Matlab to create the Bode plot for $K = 1$, then determine from the plot for what values of K the closed loop system is stable.

$$G(s) = \frac{K}{s(s+10)(s+100)}$$

5. For the inverted pendulum state feedback control you designed and modeled in Simulink for homework 8, design a full-order linear state observer so that the state feedback will work without direct measurement of x_2. Try three options for the observer poles, both poles at -5, both poles at -10, and both poles at -50, and compare the results. Use the following parameters:

```
len = 1;
Jt = 0.1;
mg = 10;
gamma = 0.1;
theta0 = 0.1;
omega0 = 0;
```

Do not use the initial conditions, `theta0` and `omega0`, in your observer.

Model your design in Simulink. How has the output changed relative to what it was for homework 8?

Hand in your design work, a printout of the modified system, and a plot of the system output. Also, hand in a short paragraph discussing the effect of observer pole placement.

HINT: First, add the observer to the Simulink model, while keeping the state feedback connected to the true system state. Plot the observer output error, $y - \hat{y}$, and confirm that the error converges to zero. Then, and only then, connect the state feedback to the estimated state.