ESE446: Robotics and Dynamics
Final Project: RRR Robotic Arm
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Abstract:

The RRR Robotic Arm project starts from developing dynamic equation for manipulator to implement general simulation of dynamics and path trajectory with MATLAB Simulink.

Derivation of Dynamic Equation for manipulator:

Initial parameters:

$\theta_{1\_initial} = 10\ \ degree$
$\theta_{2\_initial} = 20\ \ degree$
$\theta_{3\_initial} = 30\ \ degr\boxed{e}$
$L_1 = 4\, meters$
$L_2 = 3\, meters$
$L_3 = 2\ \ meters$
$m_1 = 20$ Kg
$m_2 = 15\ Kg$
$m_3 = 10\ Kg$
$I_1 = 0.5\ Kg - m^2$
$I_2 = 0.2\ Kg - m^2$
$I_3 = 0.1\ Kg - m^2$

$$I_{c1} = \begin{bmatrix} I_{1xx} & 0 & 0 \\ 0 & I_{1yy} & 0 \\ 0 & 0 & I_{1zz} \end{bmatrix}$$
$$I_{c2} = \begin{bmatrix} I_{2xx} & 0 & 0 \\ 0 & I_{2yy} & 0 \\ 0 & 0 & I_{2zz} \end{bmatrix}$$
$$I_{c3} = \begin{bmatrix} I_{3xx} & 0 & 0 \\ 0 & I_{3yy} & 0 \\ 0 & 0 & I_{3zz} \end{bmatrix}$$
$$J_{v1} = \begin{bmatrix} \frac{\partial P_1}{\partial q_1} & \frac{\partial P_1}{\partial q_2} & \frac{\partial P_1}{\partial q_3} \end{bmatrix}$$
$$J_{v2} = \begin{bmatrix} \frac{\partial P_2}{\partial q_1} & \frac{\partial P_2}{\partial q_2} & \frac{\partial P_2}{\partial q_3} \end{bmatrix}$$
$$J_{v3} = \begin{bmatrix} \frac{\partial P_3}{\partial q_1} & \frac{\partial P_3}{\partial q_2} & \frac{\partial P_3}{\partial q_3} \end{bmatrix}$$
$$J_{w1} = \begin{bmatrix} & 0 & 0 \\ {}^{0}\hat{z}_1 & 0 & 0 \\ & 0 & 0 \end{bmatrix}$$
$$J_{w2} = \begin{bmatrix} & & 0 \\ {}^{0}\hat{z}_1 & {}^{0}\hat{z}_2 & 0 \\ & & 0 \end{bmatrix}$$
$$J_{w3} = \begin{bmatrix} {}^{0}\hat{z}_1 & {}^{0}\hat{z}_2 & {}^{0}\hat{z}_3 \end{bmatrix}$$

$$M(q) = \sum_{i=1}^{3} \left[m_i J_{vi}^T J_{vi}^T + J_{wi}^T I_{ci}\, J_{wi} \right]$$

$$C(q,\dot{q}) = \begin{bmatrix} b_{111} & b_{122} & b_{133} \\ b_{211} & b_{222} & b_{233} \\ b_{311} & b_{323} & b_{333} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}m_{111} & m_{122} - \frac{1}{2}m_{211} & m_{133} - \frac{1}{2}m_{331} \\ m_{211} - \frac{1}{2}m_{112} & \frac{1}{2}m_{222} & m_{233} - \frac{1}{2}m_{332} \\ m_{311} - \frac{1}{2}m_{113} & \frac{1}{2}(m_{323} + m_{332} - m_{233}) & \frac{1}{2}m_{333} \end{bmatrix}$$

$$B(q,\dot{q}) = 2\begin{bmatrix} b_{112} & b_{113} & b_{123} \\ b_{212} & b_{213} & b_{223} \\ b_{312} & b_{313} & b_{323} \end{bmatrix} =$$

$$\begin{bmatrix} m_{112} & m_{113} & (m_{123} + m_{132} - m_{231}) \\ (m_{212} + m_{221} - m_{122}) & (m_{213} + m_{231} - m_{132}) & m_{223} \\ (m_{312} + m_{321} - m_{123}) & (m_{313} + m_{331} - m_{133}) & (m_{323} + m_{332} - m_{233}) \end{bmatrix}$$

$$F(\dot{q}) = c_1\dot{q} + c_2 sign(\dot{q})$$

$$G(q) = -\begin{bmatrix} J_{v1}^T\begin{bmatrix} 0 \\ m_1 g \\ 0 \end{bmatrix} & J_{v2}^T\begin{bmatrix} 0 \\ m_2 g \\ 0 \end{bmatrix} & J_{v3}^T\begin{bmatrix} 0 \\ m_3 g \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\ddot{q} = \begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \\ \ddot{q_3} \end{bmatrix} = M^{-1}(q)\begin{bmatrix} \tau - C(q,\dot{q})\begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} - B(q,\dot{q})\begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} - G(q) - F(\dot{q}) \end{bmatrix}$$

Demonstration for dynamics of the robot through simulation:
1: No actuator torque, no gravity. -> See 1_1.gif
2. No actuator torque, with gravity. -> See 1_2.gif
3. Joint-1, Joint-2, and Joint-3 actuator torque in order to provide equilibrium, with gravity. -> See 1_3.gif

Robot Simulation – with control partitioning:

With K_p: 10. $K_v = 2\sqrt{K_p}$.

$\theta_{1\_initial} = 10\ degree$
$\theta_{2\_initial} = 20\ degree$
$\theta_{3\_initial} = 30\ degr$□

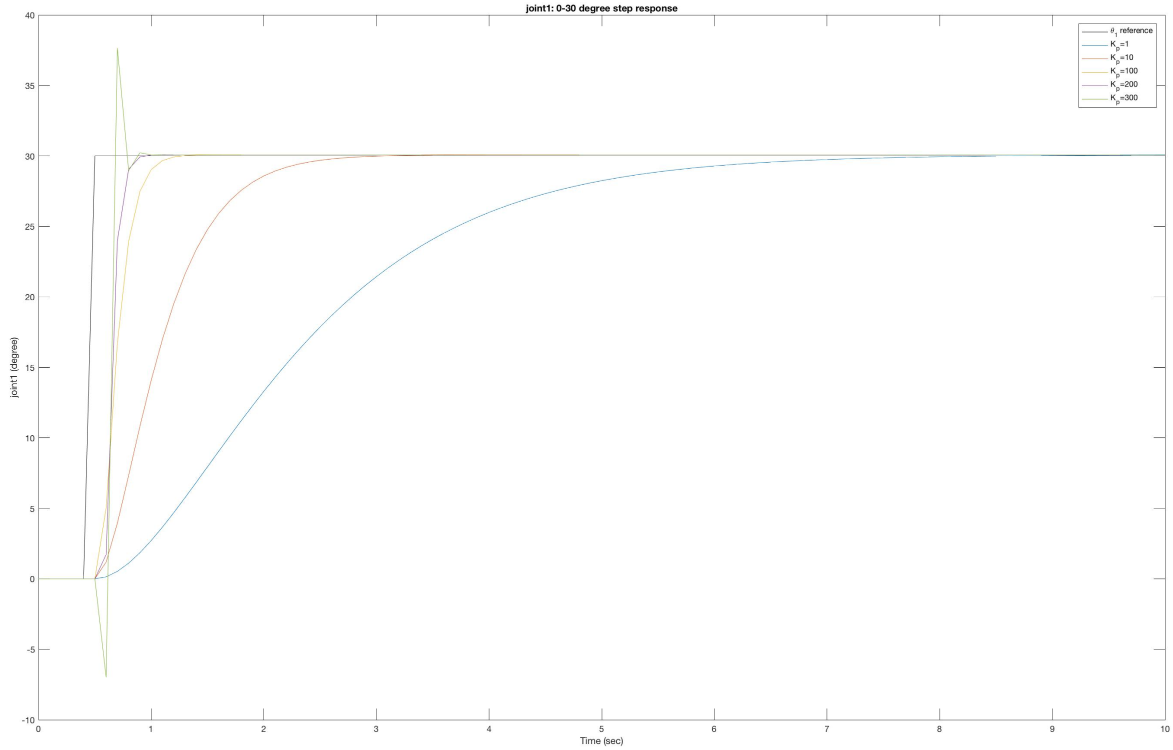
$\theta_{1\_desired} = 30\ degree$
$\theta_{2\_desired} = -20\ degree$
$\theta_{3\_desired} = -10\ degr$□

Simulation -> see 2_1.gif

Experiment with various $K_p: 1, 10, 100, 200, 500, 1000$. $K_v = 2\sqrt{K_p}$.

K_p	1	10	100	200	500	1000
gif	3_1.gif	3_2.gif	3_3.gif	3_4.gif	3_5.gif	3_6.gif
Rise Time: T_r	$\cong 6\ sec$	$\cong 2\ sec$	$\cong 0.7\ sec$	$\cong 0.5\ sec$	$\cong 1.5\ sec$	N/A
Situation	Critical Damped	Critical Damped	Critical Damped	Critical Damped	Underdamped / Overshoot	Underdamped / Blows up

Overshoot begins when $K_p \geq 225$. The system can be critical damped with single K_p values for each joint. The following figure of joint1 "0 to 30 degree" step response shows the response behaves like a second order system:

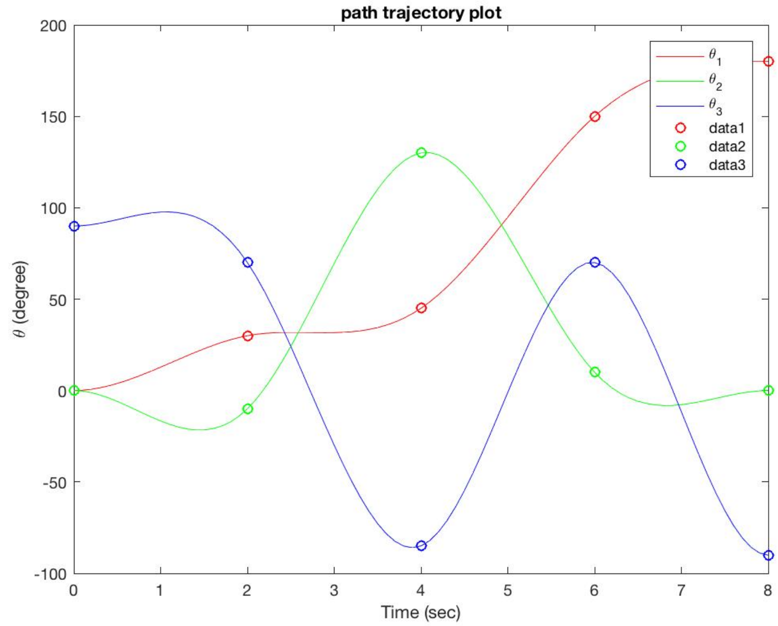


Robot Simulation – path trajectory:

Given the following timeline table:

Time (seconds)	**Theta-1 (degrees)**	**Theta-2 (degrees)**	**Theta-3 (degrees)**
0	0	0	90
2	30	-10	70
4	45	130	-85
6	150	10	70
8	180	0	-90

The following figure demonstrates the path trajectory calculating by MATLAB function spline fit with the given timeline table. Starting and ending slopes for each joints trajectory are set to zero.



There're two functions to deal with the path trajectory control. The first one "CreatTimeTble" fits the spline curve and creates more detailed version of "TimeLineTable", then passes the variable to the second function "path_trajectory". The second function, which has two input variable "T" (current time) and "TimeLineTable" (more continuous version of the given table), finds the closest time and return the corresponding joint position "q". The simulation makes it to the end stopping at (180,0,-90) degree, and the gains change the simulation. The higher gain makes the joints follow closer to the desired path trajectory, in other words, the actual path trajectory is not exact what we set in the "TimeLineTable".

Experiment with various K_p: $1, 10, 100, 200$. $K_v = 2\sqrt{K_p}$.

K_p	1	10	100	200
gif	4_1.gif	4_2.gif	4_3.gif	4_4.gif
Figure	4_1	4_2	4_3	4_4

Figure. 4_1

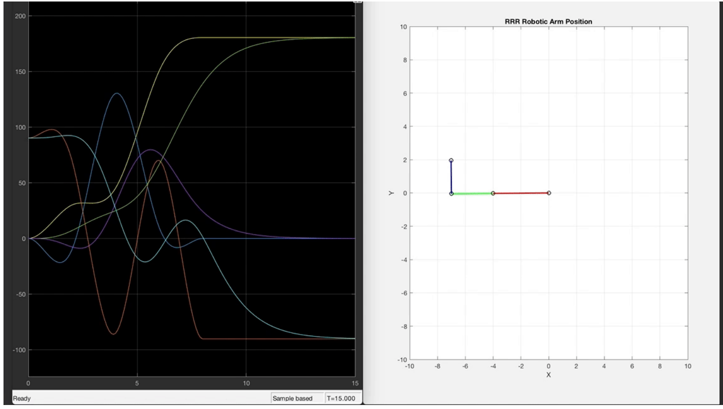


Figure. 4_2

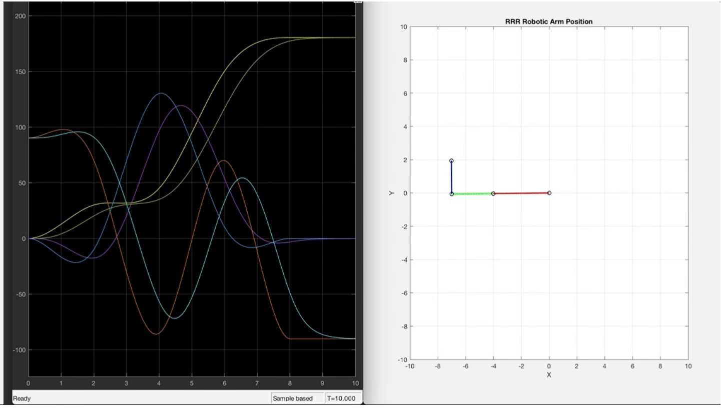


Figure. 4_3

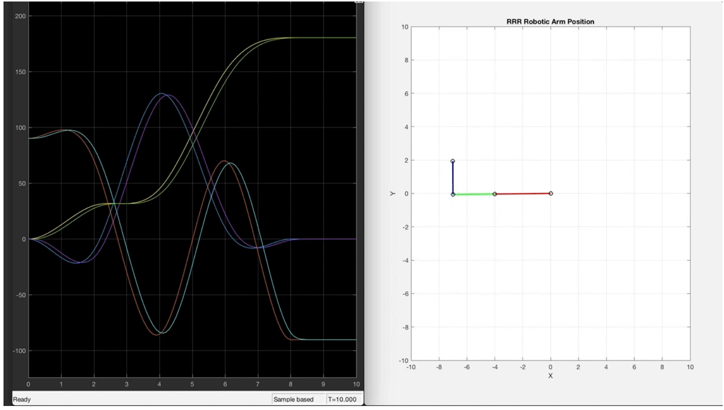
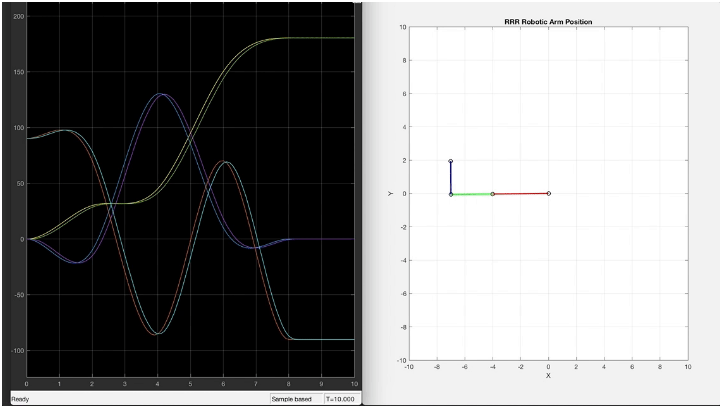
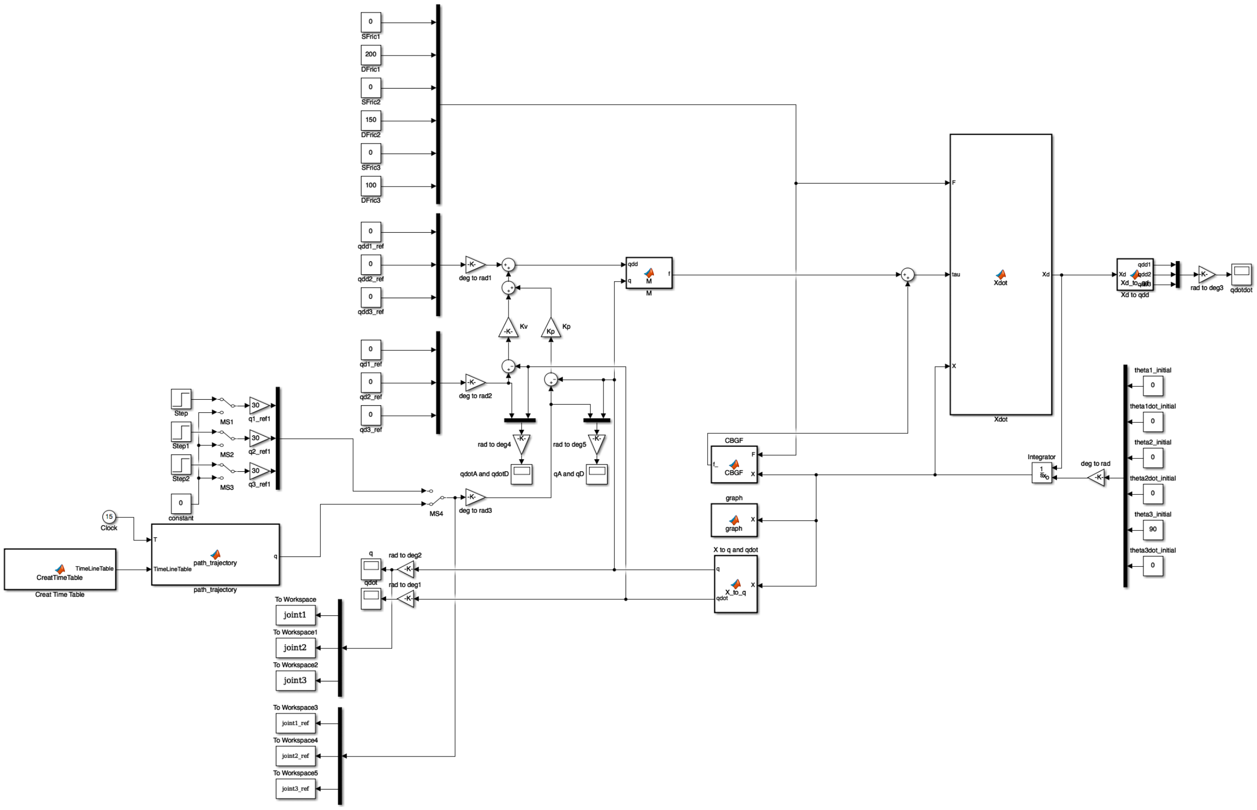


Figure. 4_4



Final Design:


0
SFric1
200
DFric1
0
SFric2
150
DFric2
0
SFric3
100
DFric3
0
qdd1_ref
0
qdd2_ref
0
qdd3_ref
-K-
deg to rad1
qdd
q
M
f
Kv
Kp
0
qd1_ref
0
qd2_ref
0
qd3_ref
deg to rad2
rad to deg4
qdotA and qdotD
rad to deg5
qA and qD
Step
Step1
Step2
constant
MS1
MS2
MS3
MS4
30
q1_ref1
q2_ref1
q3_ref1
deg to rad3
15
Clock
T
TimeLineTable
CreatTimeTable
Creat Time Table
path_trajectory
q
qdot
rad to deg2
rad to deg1
To Workspace
joint1
To Workspace1
joint2
To Workspace2
joint3
To Workspace3
joint1_ref
To Workspace4
joint2_ref
To Workspace5
joint3_ref
CBGF
f_
F
X
graph
X to q and qdot
X_to_q
F
tau
X
Xd
Xdot
Xd_to_qdd
qdd1
qdd2
qdd3
Xd to qdd
rad to deg3
qdotdot
Integrator
1/s
x0
deg to rad
theta1_initial
theta1dot_initial
theta2_initial
theta2dot_initial
theta3_initial
90
theta3dot_initial