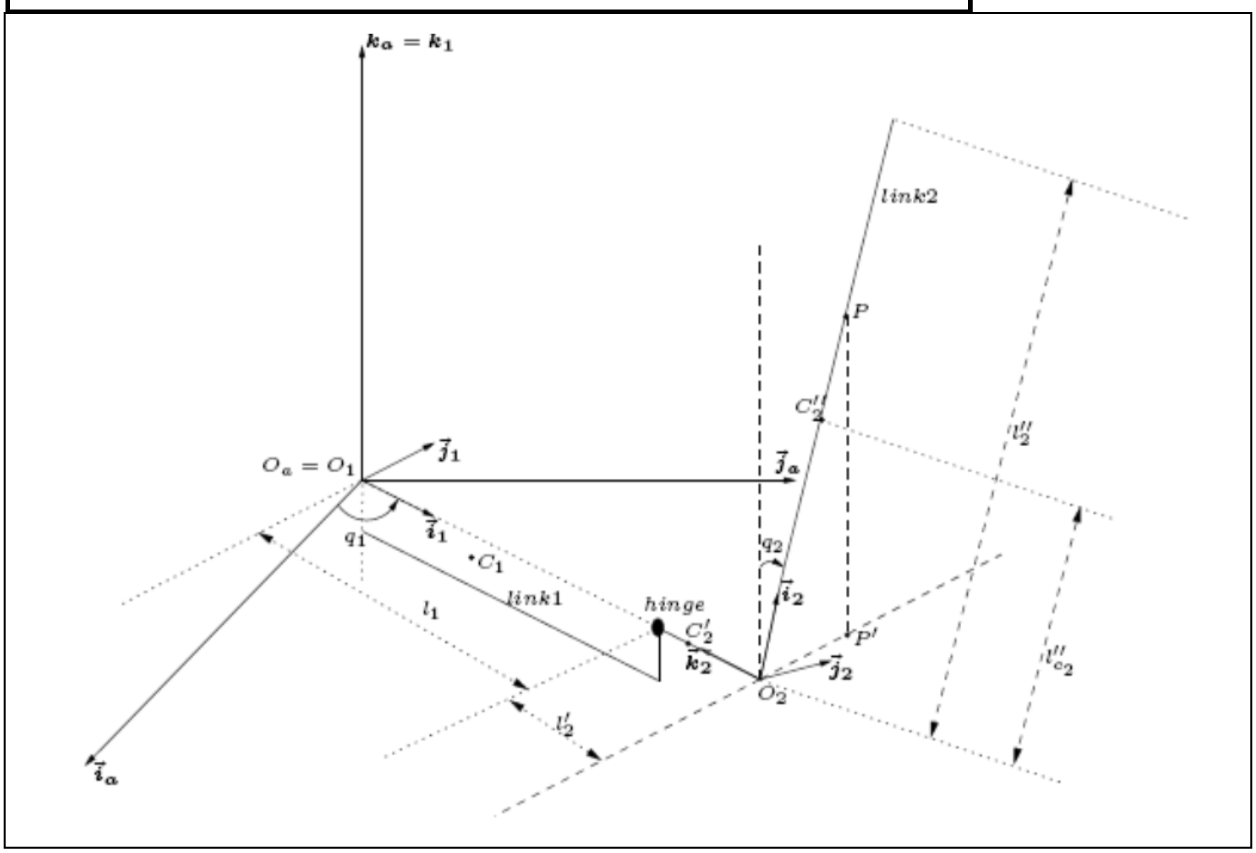
1. Items done this session:

TASK-1: Using the information provided in the "Dynamics" document, derive the dynamic equations for our "Robot" (single inverted pendulum as shown in the above figure). This should result in one equation for each joint (two equations).



In the first part of the Assignment_4, we started with derivation of mathematical expression for the RR robot arm model. Starting with formula of Lagrangian Dynamics Equation.

$$p(x_2) = \begin{pmatrix} (l_1+l_2')\cos(q_1) - x_2\sin(q_2)\sin(q_1) \\ (l_1+l_2')\sin(q_1) + x_2\sin(q_2)\cos(q_1) \\ x_2\cos(q_2) \end{pmatrix}$$

$$V(x_2) = \frac{d}{dt}(p(x_2))$$

$$\vec{V}(x_2) = \begin{pmatrix} -(l_1+l_2')\sin(q_1)\cdot\dot{q}_1 - x_2\cos(q_2)\sin(q_1)\cdot\dot{q}_2 - x_2\sin(q_2)\cos(q_1)\dot{q}_1 \\ (l_1+l_2')\cos(q_1)\dot{q}_1 + x_2\cos(q_2)\cos(q_1)\dot{q}_2 + (-x_2\sin(q_2)\sin(q_1)\dot{q}_1) \\ -x_2\sin(q_2)\dot{q}_2 \end{pmatrix}$$

$$|V(x_2)|^2 = [(l_1+l_2')\sin(q_1)\dot{q}_1]^2 + [(l_1+l_2')\cos(q_1)\dot{q}_1]^2 + 2x_2(l_1+l_2')\cos(q_2)\cdot\dot{q}_2\dot{q}_1 + (x_2\sin(q_2)\dot{q}_2)^2$$
$$+ [x_2\cos(q_2)\sin(q_1)\cdot\dot{q}_2]^2 + [x_2\sin(q_2)\cos(q_1)\dot{q}_1]^2 + [x_2\cos(q_2)\cos(q_1)\dot{q}_2]^2$$
$$+ [x_2\sin(q_2)\sin(q_1)\dot{q}_1]^2$$

$$\cancel{= [(l_1+l_2')\dot{q}_1]^2 + 2x_2(l_1+l_2')\cos(q_2)\dot{q}_2\dot{q}_1 + x_2^2\dot{q}_2^2 - \cos^2(q_2)\cos^2(q_1)x_2^2\dot{q}_2^2}$$
$$\cancel{+ x_2^2[\cos^2(q_2)\sin^2(q_1)\dot{q}_2 + \sin^2(q_2)\cos^2(q_1)\dot{q}_1^2 + \sin^2(q_2)\sin^2(q_1)\dot{q}_1^2]}$$
$$\cancel{= [(l_1+l_2')\dot{q}_1]^2 + 2x_2(l_1+l_2')\cos(q_2)\dot{q}_2\dot{q}_1 + x_2^2\dot{q}_2^2}$$
$$\cancel{+ x_2^2\cos^2(q_2)\sin^2(q_1)\dot{q}_2 + \sin^2 q_2\,\dot{q}_1^2 x_2^2}$$
$$\cancel{= [x_2^2\sin^2(q_2) + (l_1+l_2')^2]\dot{q}_1^2}$$
$$= [(l_1+l_2')\dot{q}_1]^2 + 2x_2(l_1+l_2')\cos(q_2)\dot{q}_2\dot{q}_1 + x_2^2\dot{q}_2^2[\sin^2(q_2) + \cos^2(q_2)\sin^2(q_1) + \cos^2(q_2)\cos^2(q_1)]$$
$$+ x_2\dot{q}_1^2[\sin^2(q_2)\cos^2(q_1) + \sin^2(q_2)\sin^2(q_1)]$$

$$= [(l_1+l_2')^2 + x_2^2\sin^2(q_2)]\dot{q}_1^2 + 2x_2(l_1+l_2')\cos(q_2)\dot{q}_1\dot{q}_2 + x_2^2\dot{q}_2^2$$

$$K_2 = \frac{1}{2}\int_0^{l_2} \rho_2 A_2 |V(x_2)|^2\, dx_2$$
$$= \frac{1}{2}\rho_2 A_2\left(\dot{q}_1^2 l_2(l_1+l_2')^2 + \frac{1}{3}l_2^3\sin^2(q_2)\dot{q}_1^2 + l_2^2(l_1+l_2')\cos(q_2)\dot{q}_1\dot{q}_2 + \frac{1}{3}l_2^3\dot{q}_2^2\right]$$

$$= \frac{1}{2}m_2[\dot{q}_1^2(l_1+l_2')^2 + \frac{1}{3}l_2^2\sin^2(q_2)\dot{q}_1^2 + l_2(l_1+l_2')\cos(q_2)\dot{q}_1\dot{q}_2]$$

$$k_1 = \frac{1}{2}J_1\dot{q}_1^2$$

$$\theta_1' = J_1 + m_2(l_1+l_2')^2 \qquad \theta_2' = \frac{1}{3}m_2 l_2^2 \qquad \theta_3' = \frac{1}{2}m_2(l_1+l_2')l_2$$

$$K = \frac{1}{2}[\theta_1' + \theta_2'\sin^2(q_2)]\dot{q}_1^2 + \theta_3'\cos(q_2)\dot{q}_1\dot{q}_2 + \frac{1}{2}\theta_2'\dot{q}_2^2$$

2. Items for next session:

Continue to finish mathematical derivation of Task-1 for this problem.