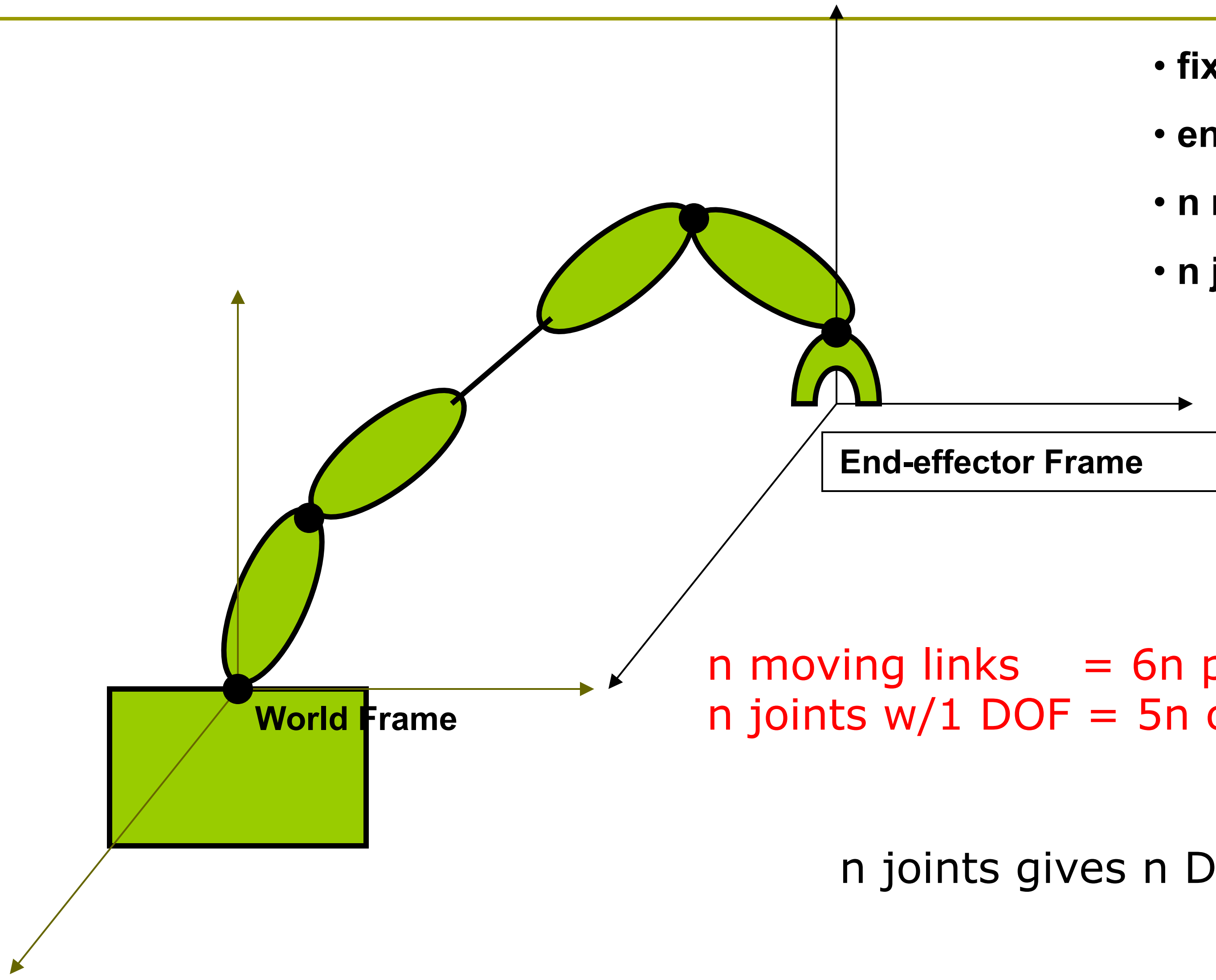
Robotics (ESE447)

Kinematics

Fixed base manipulators



- fixed base
- end-effector
- n rigid body links
- n joints w/1 DOF

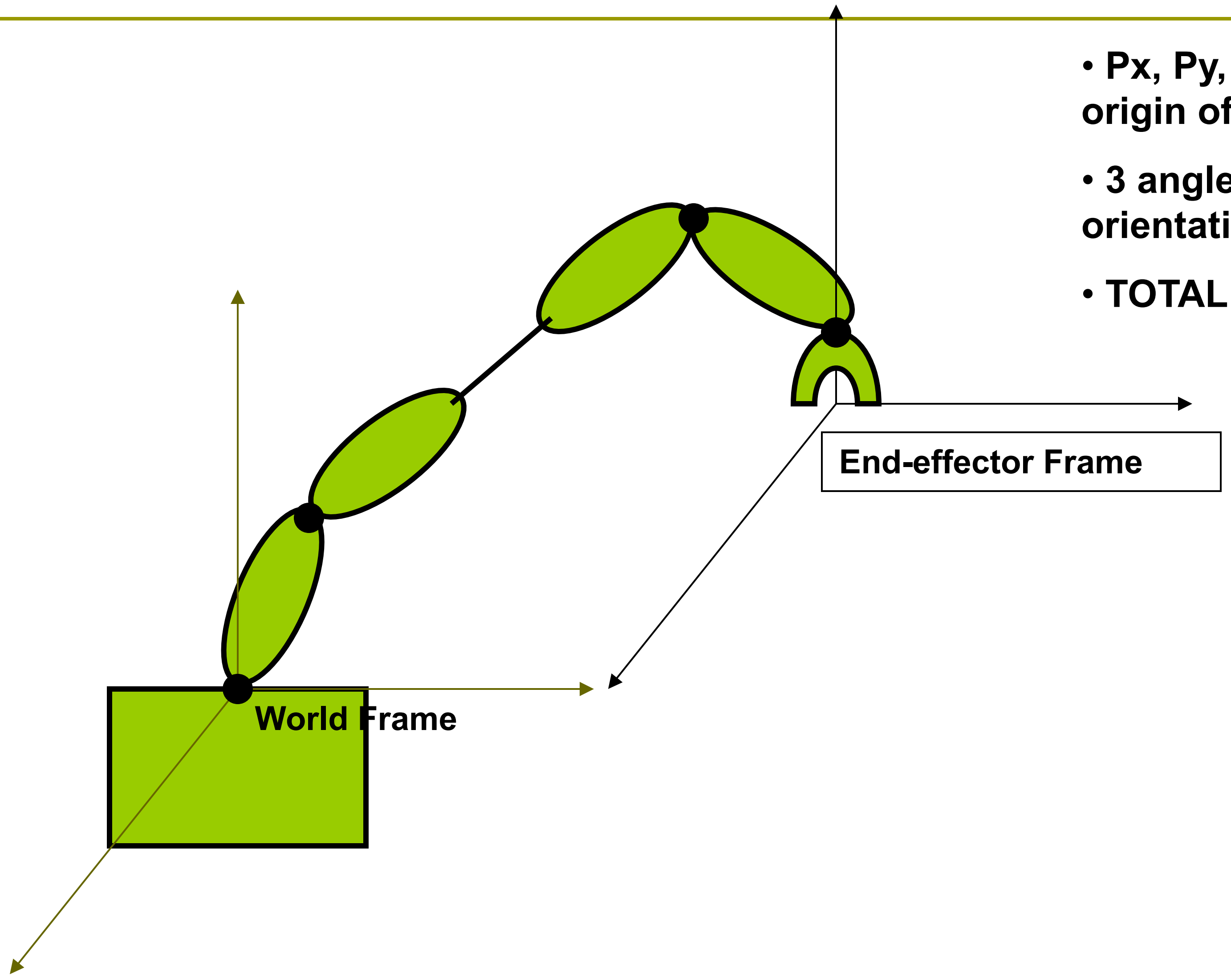
n moving links = 6n parameters
n joints w/1 DOF = 5n constraints

n joints gives n DOF's

Definitions

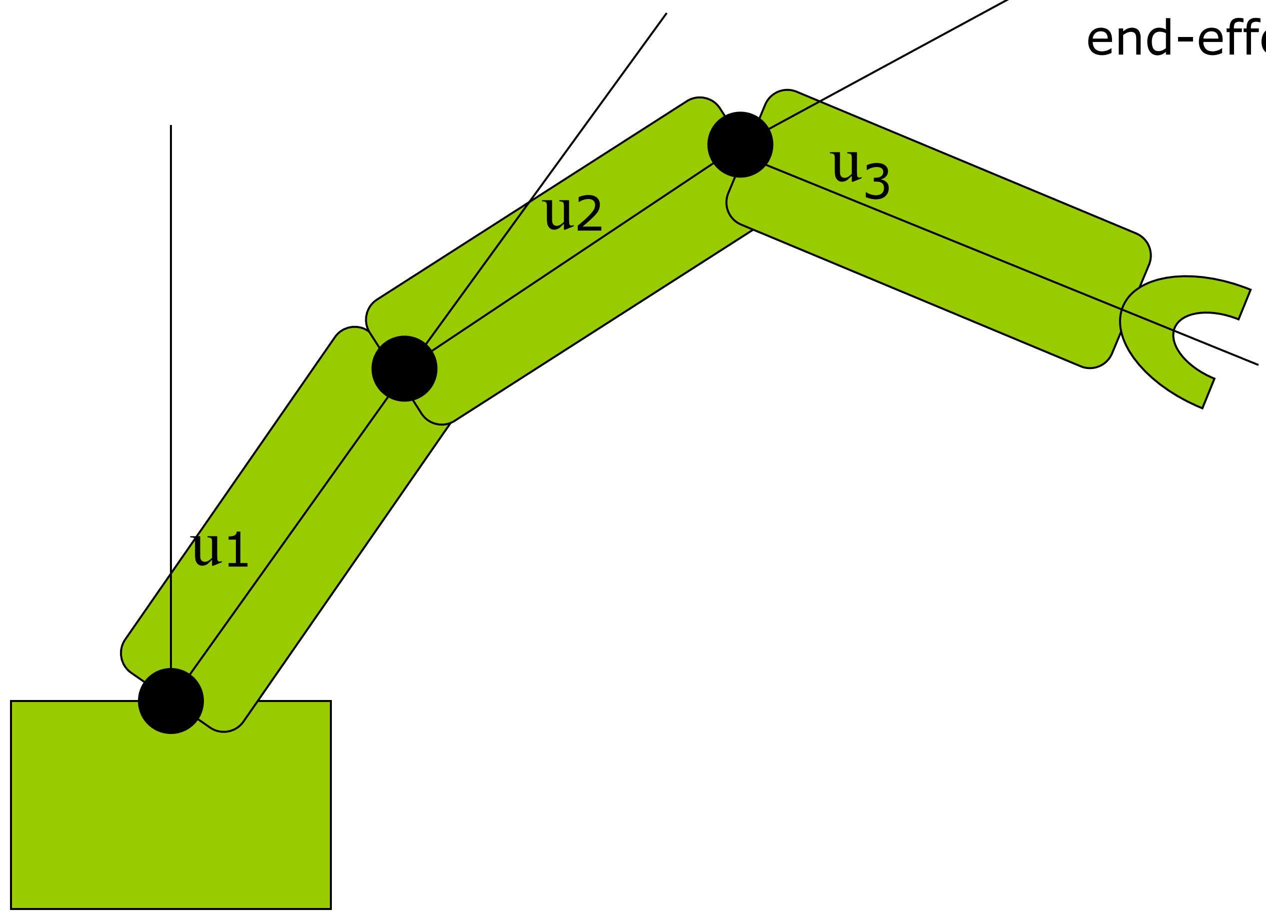
- “n” joints give maximum of “n” DOFs
- Each joint only having 1 DOF can only contribute 1 DOF to end-effector

End-effector in 3D-space



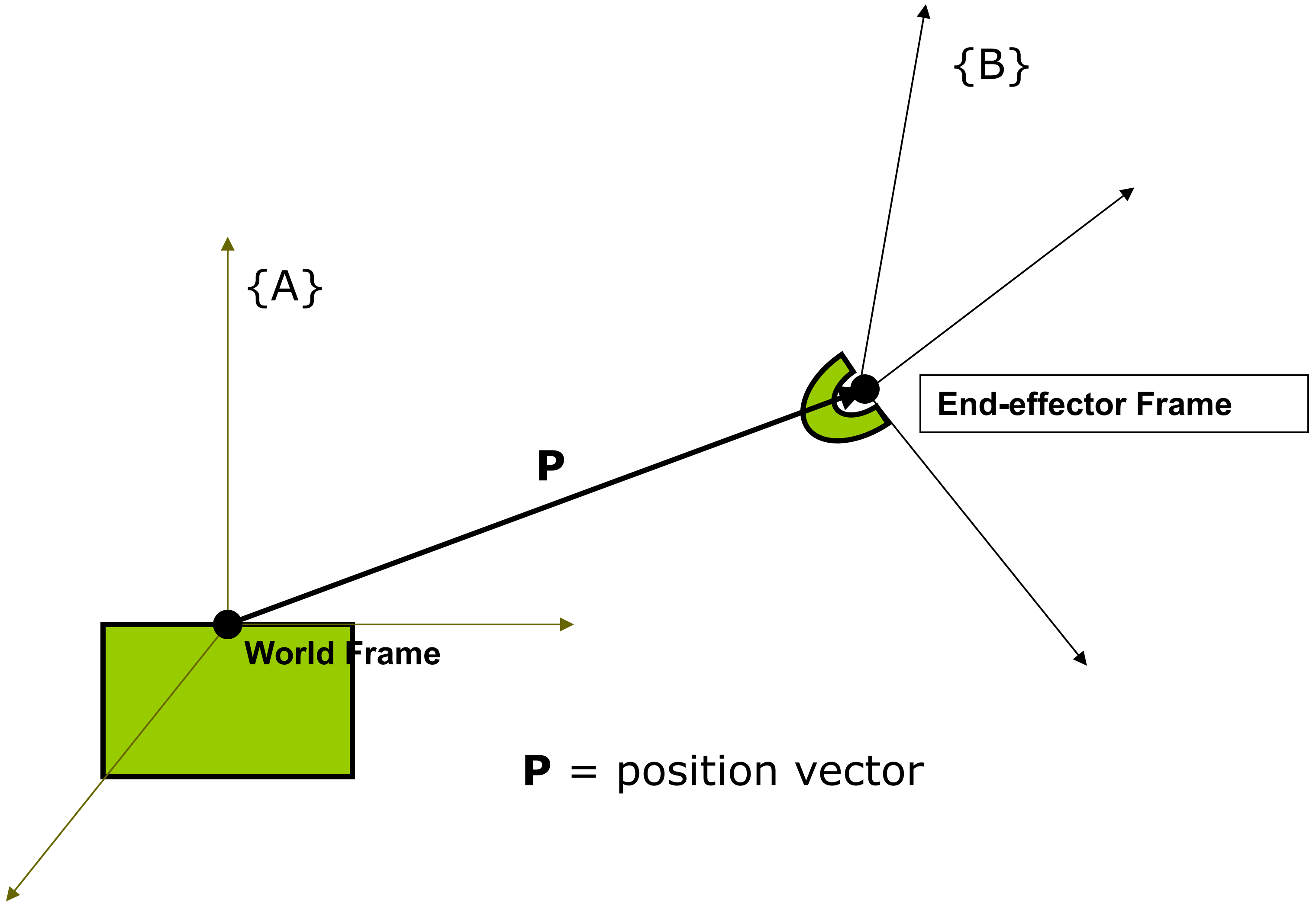
- Px, Py, Pz locates origin of frame
- 3 angles describes orientation of frame
- TOTAL – 6 parameters

Planar Manipulator



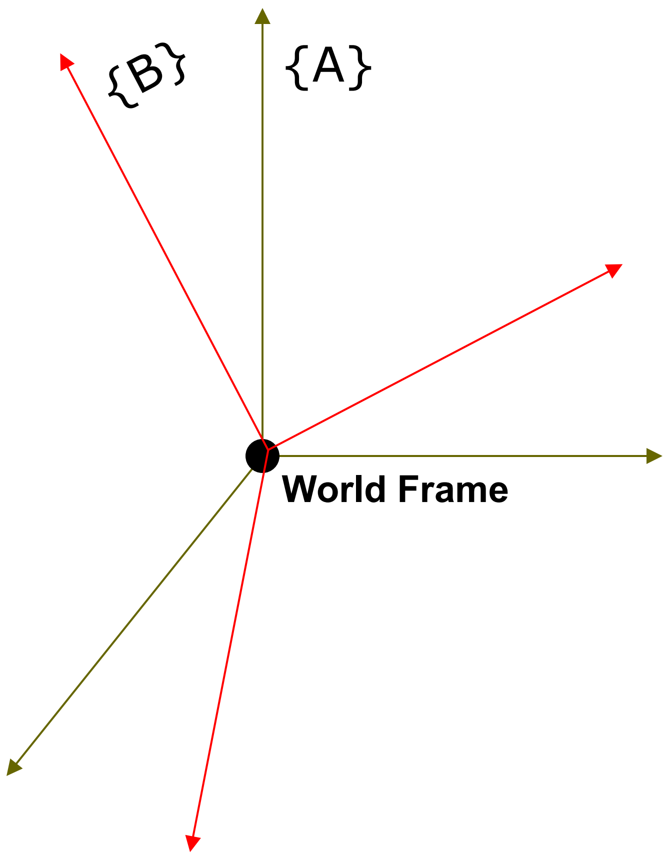
- RRR (3 revolute joints)
- How many DOF of end-effector?

End-effector FRAME description



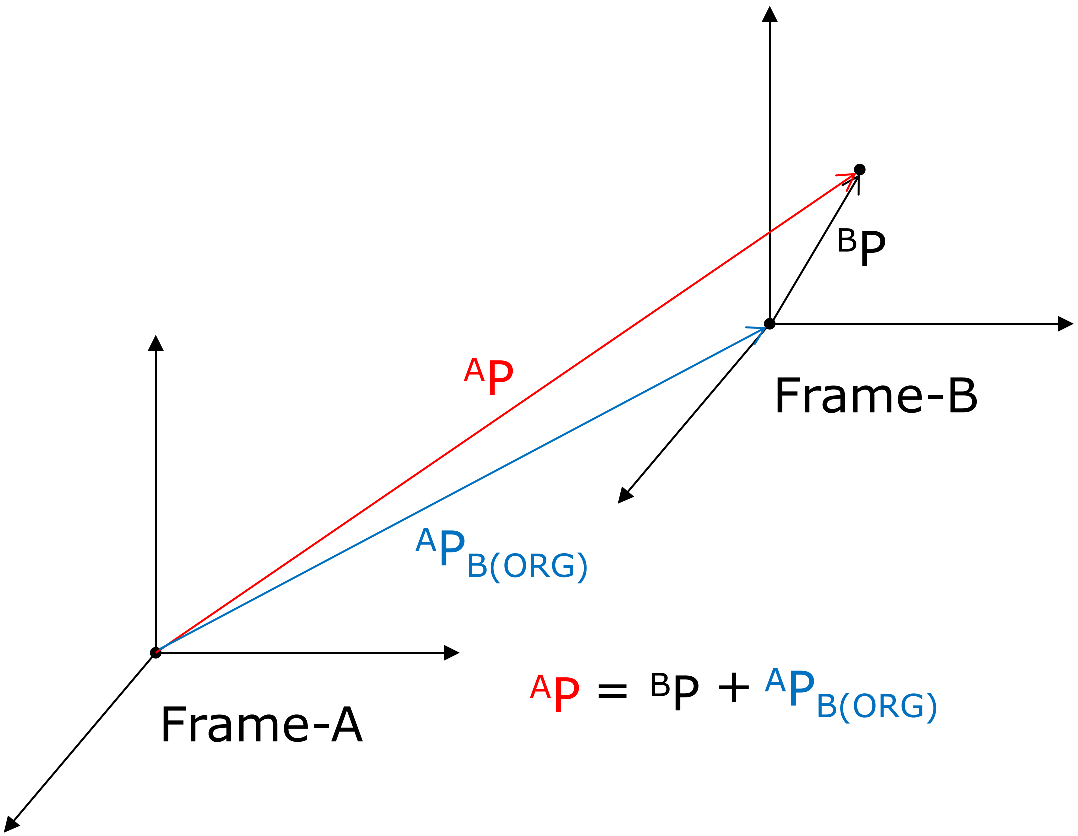
P = position vector

End-effector FRAME description



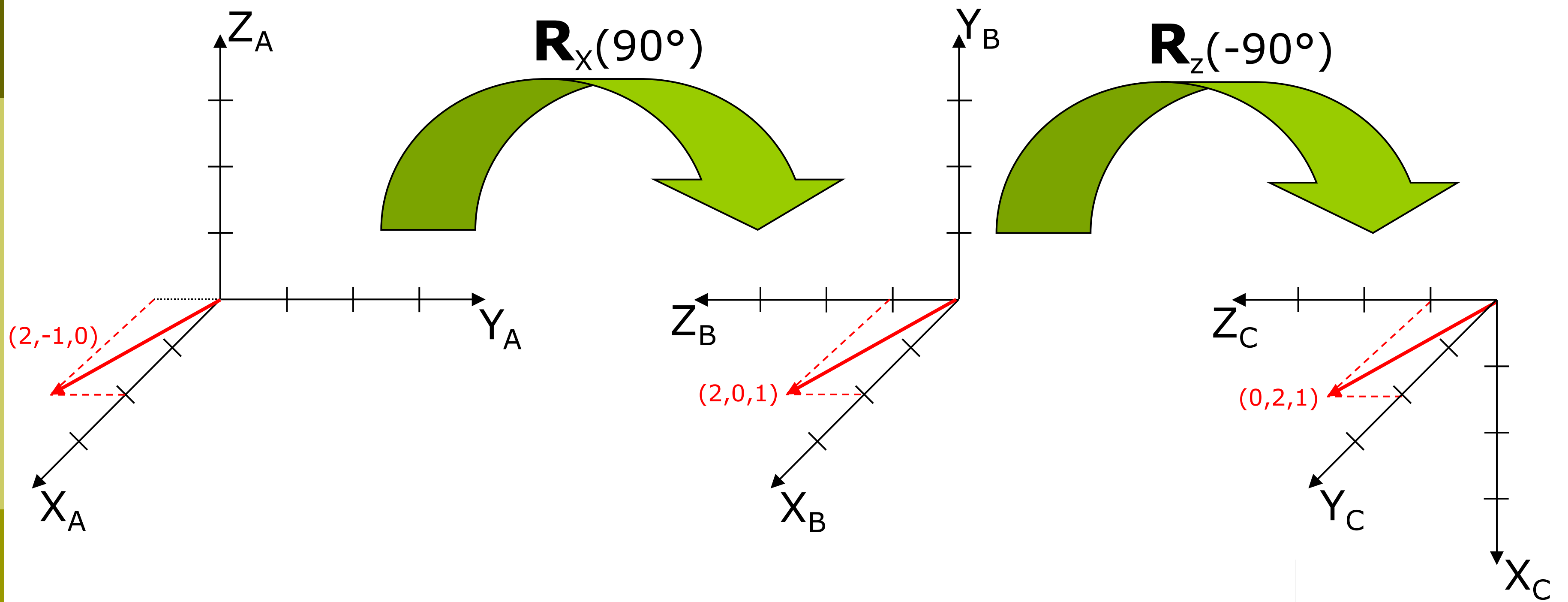
${}^{A}_{B}\mathbf{R}$ is the rotation matrix which describes the orientation of {B} with respect to {A}

Mapping - Translation



$$^{A}P = {}^{B}P + {}^{A}P_{B(ORG)}$$

Frame Rotation and Mapping



${}^{A}_{B}\mathbf{R} =$

${}^{B}_{C}\mathbf{R} =$

General Transformation

- Convert known vector ${}^{B}\mathbf{P}$ into base frame "A" (pure rotation)
- Translate vector ${}^{B}\mathbf{P}$ by distance between origins ("A" to "B")
- Write in homogeneous matrix form

$$ {}^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG} $$

$$ {}^{A}P = {}^{A}_{B}T\;{}^{B}P \quad \Longrightarrow \quad {}^{A}_{B}T = \left[\begin{array}{ccc|c} & {}^{A}_{B}R & & {}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

Transformation as an operator

PROBLEM: Given a vector in space ($\mathbf{P}_1$) --- translate and rotate that vector creating a new vector ($\mathbf{P}_2$)

- Translation is once again accomplished by adding a displacement vector to $\mathbf{P}_1$
- Rotation is accomplished via $\mathbf{P}_2 = \mathbf{R}\mathbf{P}_1$
- It also follows that : $\mathbf{P}_2 = \mathbf{T}\mathbf{P}_1$

Transformation RECAP

- ${}^{A}_{B}\mathbf{T}$ can be used as a description of frame "B" with respect to frame "A"
- ${}^{A}_{B}\mathbf{T}$ can be used to map a vector in frame "B" into frame "A"
- $\mathbf{T}$ can be used to operate on a vector thus creating a new vector

More properties

- Do Transform matrices multiply as did the Rotation matrices ??
- What is the Inverse of the Transform ??

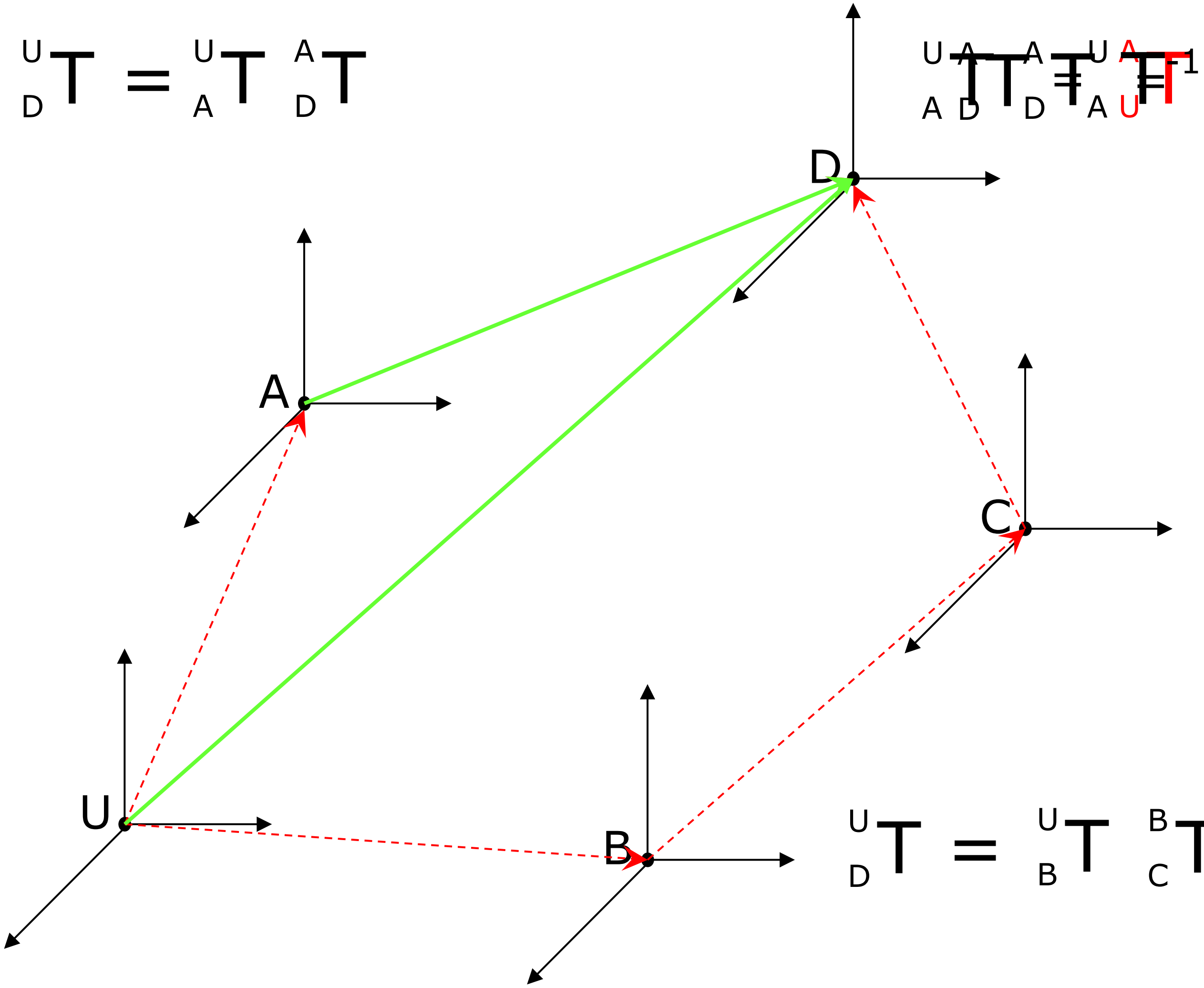
Transform Equation

$${}^{U}_{D}T = {}^{U}_{A}T\;{}^{A}_{D}T$$

$${}^{U}_{A}T\;{}^{A}_{D}T = {}^{U}_{B}T\;{}^{B}_{C}T\;{}^{C}_{D}T$$

$${}^{A}_{D}T = {}^{U}_{A}T^{-1}\;{}^{U}_{B}T\;{}^{B}_{C}T\;{}^{C}_{D}T$$

$${}^{A}_{D}T = {}^{A}_{U}T\;{}^{U}_{B}T\;{}^{B}_{C}T\;{}^{C}_{D}T$$



$${}^{U}_{D}T = {}^{U}_{B}T\;{}^{B}_{C}T\;{}^{C}_{D}T$$