1. Items done this session:

TASK-1: Using the information provided in the “Dynamics” document, derive the dynamic equations for our “Robot” (single inverted pendulum as shown in the above figure). This should result in one equation for each joint (two equations).

In this session, we continue to work on mathematical derivation of Lagrangian Dynamics equation.

$$V = m_1 g z_{c1} + m_2 g l_{c2} \cos q_2 . \qquad \theta_4' = m_2 l_{c2} .$$

$$\mathcal{L}(q,\dot{q}) = K - V$$
$$= \tfrac{1}{2}[\theta_1' + \theta_3' \sin^2(q_2)]\dot{q}_1^2 + \theta_3' \cos(q_2)\dot{q}_1\dot{q}_2 + \tfrac{1}{2}\theta_2' \dot{q}_2^2 - m_1 g z_{c1} - \theta_4' g \cos q_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_1} - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q_1} = \tau_1$$

$$\cancel{[\theta_1' + \theta_2' \sin^2(q_2)]\ddot{q}_1 + \theta_3' \cos(q_2)\dot{q}_1\ddot{q}_2 +}$$

$$\frac{\partial \mathcal{L}(q_1,\dot{q}_1)}{\partial q_1} = 0$$

$$\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_1} = [\theta_1' + \theta_2' \sin(q_2)]\dot{q}_1 + \theta_3' \cos(q_2)\dot{q}_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_1} = 2\theta_2' \sin(q_2)\cos(q_2)\dot{q}_2\dot{q}_1 + [\theta_1' + \theta_3' \sin^2(q_2)]\ddot{q}_1 - \theta_3' \sin(q_2)\dot{q}_2^2$$
$$+ \theta_3' \cos(q_2)\ddot{q}_2 = \tau$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_2} - \frac{\partial \mathcal{L}(q,\dot{q})}{\partial q_2} = \tau_2 = 0$$

$$\frac{\partial \mathcal{L}(q,\dot{q})}{\partial q_2} = -\tfrac{1}{2}\cdot 2\theta_2' \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4' g \sin(q_2)$$
$$= -\theta_2' \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4' g \sin(q_2)$$

$$\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_2} = \theta_3' \cos(q_2)\dot{q}_1 + \theta_2' \dot{q}_2$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}(q,\dot{q})}{\partial \dot{q}_2} = \theta_3' \cos(q_2)\ddot{q}_1 + \theta_2' \ddot{q}_2$$

$$\theta_3' \cos(q_2)\ddot{q}_1 + \theta_2' \ddot{q}_2 - \theta_2' \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4' g \sin(q_2) = 0 .$$

$$V = [\theta_1 + \theta_2 \sin^2(q_2)]\ddot{q}_1 + \theta_3 \cos(q_2)\ddot{q}_2 + 2\theta \sin(q_2)\cos(q_2)\dot{q}_1\dot{q}_2 - \theta_3 \sin(q_2)\dot{q}_2^2 + \theta_5 \dot{q}_1$$

$$0 = \theta_3 \cos(q_2)\ddot{q}_1 + \theta_2 \ddot{q}_2 - \theta_2 \sin(q_2)\cos(q_2)\dot{q}_1^2 - \theta_4 g \sin(q_2) + \theta_6 \dot{q}_2$$

$$\theta_i = \theta_i' \frac{R_a}{k_r k_t} \quad i = 1,2,3,4 \qquad \theta_5 = \beta_1 \frac{R_a}{k_r k_t} + k_r k_t \qquad \theta_6 = \beta_2 \frac{R_a}{k_r k_t}$$

TASK-2: Arrange the equations in the matrix form shown below where *'v'* represents voltage and *'q'* represents the generalized coordinate system (joint variables).

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix} * \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11}(q,\dot{q}) & c_{12}(q,\dot{q}) \\ c_{21}(q,\dot{q}) & c_{22}(q,\dot{q}) \end{bmatrix} * \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_1(\dot{q}_1) \\ f_2(\dot{q}_2) \end{bmatrix} + \begin{bmatrix} g_1(q_1) \\ g_2(q_2) \end{bmatrix}$$

After finishing Task-1, where transfer energy with gear ratio to voltage, then we separated two equations of each robot joint into a matrix form with individual coefficient.

$$K_2 = \cancel{\frac{1}{2} J_1 \dot{q}_1^2} + \overbrace{\boxed{\frac{1}{2} m_2 (l_1 + l_2')^2 \dot{q}_1^2}}^{A} + \overbrace{\boxed{\frac{1}{6} m_2 l_2^2 \sin^2(q_2) \dot{q}_1^2}}^{C}$$
$$+ \underbrace{\boxed{\frac{1}{2} m_2 (l_1 + l_2) l_2 \cos(q_2) \dot{q}_1 \dot{q}_2}}_{B} + \underbrace{\boxed{\frac{1}{6} m_2 l_2^2 \dot{q}_2^2}}_{D}$$

$$K = K_1 + K_2$$

$$[\theta_1 + \theta_2 \sin^2(q_2)] \ddot{q}_1 + \theta_3 \cos(q_2) \ddot{q}_2 + 2\theta_2 \sin(q_2)\cos(q_2) \dot{q}_1 \dot{q}_2$$
$$- \theta_3 \sin(q_2) \dot{q}_2^2 + \theta_5 \dot{q}_1 = v$$

c_{11}

$m_{11}(q) = \theta_1 + \theta_2 \sin^2(q_2)$

$m_{12}(q) = \theta_3 \cos(q_2)$

$m_{21}(q) = \theta_3 \cos(q_2)$

$m_{22}(q) = \theta_2$

$c_{11}(q, \dot{q}) = 2\theta_2 \sin(q_2)\cos(q_2) + \theta_5$

$c_{12} = -\theta_3 \sin(q_2) \dot{q}_2$

$c_{21} = -\theta_2 \sin(q_2)\cos(q_2) \dot{q}_1$

$c_{22} = \theta_6$

$f_1(\dot{q}_1)$

$f_2(\dot{q}_2)$

$g_1(q_1) = 0$

$g_2(q_2) = -\theta_4 g \sin(q_2)$

$v_1 = m_{11} \ddot{q}_1 + m_{12} \ddot{q}_2$

$$V = M \ddot{q} + C \dot{q} + f + g$$

$$M \ddot{q} = [V - C\dot{q} - f - g]$$
$$\ddot{q} = M^{-1}[v - C\dot{q} - f - g]$$

2. Items for next session:

Implementing the equation into MATLAB function and build simulation.