Optimal Decisions of Farmer and Purchasing Company Scheme in Agriculture Commodity Supply Chain Finance

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Abstract

Agriculture plays a critical role in the economies of many developing countries, with small farmers comprising a significant portion of the sector. However, these farmers often face challenges in accessing capital resources for inputs such as seeds, fertilizers, and machinery, leading to instability in the supply chain and financial risk for small farmers. The nature of agricultural products, which are often bulk commodities and subject to price fluctuations and climate disasters, also makes production and financing decisions difficult. Additionally, contract farming and insurance have been utilized to transfer risk to purchasing companies and insurance providers, respectively. Purchase order financing has also been suggested as a solution for small farmers to secure credit and expand production. In this paper, we propose a supply chain model framework to understand the decentralized decision-making process between small farmers and purchasing companies in one-to-one supply chains, using a Stackelberg game model. We analyze this process through the basic newsvendor model, the newsvendor model with financing, and the newsvendor model with insurance, and give the optimal decision of farmer and purchasing company.

Contents

1 Introduction

Agriculture occupies a pillar industry position in many developing countries, and the main body of this economy is more of a retail household, or a small farmer, which is different from the main body of large-scale mechanized farming. Due to underdeveloped logistics and information asymmetry, most of the agricultural products produced by these farming entities are circulated in the country and sold to relatively large purchasing companies.

It is worth noting that most of these farmers lack capital resources and cannot independently purchase capital inputs such as seeds, fertilizers, and agricultural machinery in the early stages of farming. This brings great instability to the entire supply chain and unignorable financial risk to small farmers.

Bulk agricultural commodity means any agricultural commodity that can be transported in bulk and can be temporarily stored in bulk quantities without undergoing processing or packaging. Such term also includes any commodity or product that is used by producers in the production of agricultural commodities and that can be stored or shipped in bulk, such as fertilizer and fuel.

At the same time, the nature of unstable output of agricultural products and their commodity attributes make production and financing decision of the supply chain particularly difficult. Price frictions and climate disaters often bankrupt smallholder farmers or cause huge losses to acquisition companies, which even cause the supply chain's breaking.

The World Bank emphasized that the importance of agricultural commodity financing needs to be brought to the table.[1](IFC 2014) In parts of Asia, North Africa, and Latin America, a variety of financing methods for farmers, supported by commercial banks, non-profit organizations, and governments, have been initiated. In China, the annual government work report and the No. 1 central administrative order continously focus on the issue of agriculture, rural areas and farmers, and the central bank also encourages and urges state-owned commercial banks to provide key financing support for agriculture with administrative orders to ensure the agricultural economy's healthy survival and development.

Farmers can adopt the format of agricultural insurance to transfer the risk of force majeure such as climate disasters to the insurance company. In order to lock in production, the purchasing company will also adopt the format of contract farming, signing purchase orders with farmers at a guaranteed bottom price before production[2](Van Bergen et al. 2019), so that farmers pass on the risk of uncertain demand.

For the initial capital investment, farmers will request loans from downstream purchasing companies or banks to ensure the smooth progress of production. But for vulnerable smallholder farmers, in order to avoid bad debts, banks are mostly unwilling to grant them a higher credit line outside of administrative instructions. Purchase order financing can better solve the credit problem in the financing process——guaranteed by the orders of large core enterprises can allow banks to increase the credit line for farmers and expand their feasible production set[3](Reindorp et al. 2018). Meanwhile, in order to achieve the expected profit, core company will lock the price through derivative financial instruments like future after signing the purchase order.

The whole process can be summarized by the following process:

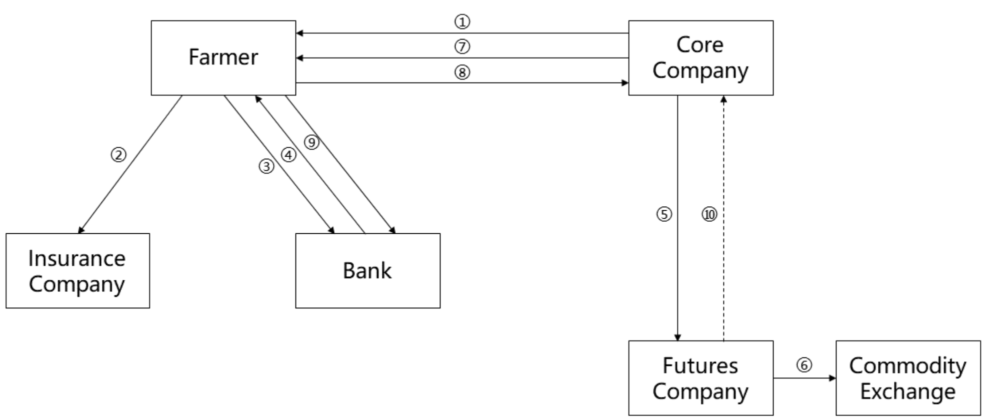

Figure 1: Contract Farming Purchasing Order Financing Process

1. Core company signs purchase orders with guaranteed lowest price with farmer
2. Farmer purchases agricultural insurance
3. Farmer applies for financing from bank with purchasing orders
4. Bank grants credit and advances after setting the advance ratio and interest rate
5. Core company puts forward requirements for value preservation to futures company

6. Futures company uses the ordered commodities to open a position on the exchange
7. Core company delivers payment to farmer
8. Farmer delivers commodities to core company
9. Farmers repay loans to bank
10. Futures company pays futures hedging profits to core company (possible)

We propose and focus on this decentralized decision-making problem for one-to-one supply chains, to facilitate our analysis, we develop a supply chain model framework to advance our understanding of how decisions of farmer interface with production uncertainty.

In order to simplify the game process, we describe the decision-making between the farmers and the purchasing company as a Stackelberg game in which the purchasing company plays the role of leader, and analyze the supply chain layer-by-layer for decision-making through the basic newsvendor model, the newsvendor model considering capital constraint and financing, and the model considering financing and insurance. The game process is as follows:

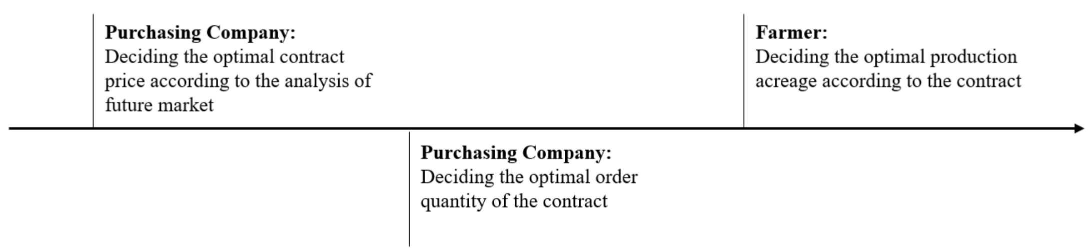

Figure 2: Game Process

Through the solution of different models (from basic to complex) in the game process, the purchasing company's price and order quantity decision and the farmer's production decision under equilibrium conditions and the sensitivity analysis conclusions are finally obtained.

The shortcomings of this essay is that, due to time and capacity constraints, we do not compare different financing methods, nor do we conduct numerical experiments to testify and visualize the results. The above deficiencies will be filled in the future.

2 Literature Review

2.1 Bulk Agricultural Commodity Supply Chain

In this section, we will review and summarize the problems in soybean supply chain management raised by the previous, and also explain the agricultural supply chain tools and methods from the perspective of supply chain finance, under the condition of which we discuss the optimal decision of farmers.

As Tsolakis et al.(2014)[4] covered all the aspects of agricultural supply chain in his paper and defined it as a set of processes in a ”farm-to-fork” sequence including farming (i.e. land cultivation and production of crops), processing/production, testing, packaging, warehousing transportation, distribution and marketing, for bulk products supply chain, which is different from the fresh food, people pay more attention to the link from the farm to the processing enterprise. For instance, only 6% of the world's total soy production is used in the form of whole beans, while the remaining 94% is crushed[5] (Oliveira and Schneider 2016).

Defined by Duan and Liu[6] (2012) as the relationship between the upstream businesses, ports and downstream enterprises, which formed in bulk commodities port logistics activities, the bulk commodity port supply chain contains three categories, energy commodities, basic raw materials and bulk agricultural commodities, including soybean commodity, which is also characterized as follows:

- Transacted in bulk leading to high demanding on storage, distribution and transportation;
- Transaction is strictly controlled by the country and vulnerable to macroeconomic;
- Costs effect transaction price.

However, there are still problems in the continuous development of the bulk agricultural commodity supply chain. Van Bergen (2019) raises up the instability of the output of most agricultural products, which will greatly hurt smallholder farmers with weak capital.[7] Agricultural suppliers struggles through seasonal harvesting which leads to large inventory build-ups, facing with flat demand, fitting for supply chain finance tools.

Another concern is the damage to the environment during cultivation. Ercin et al.[8](2012) and Ferreira et al.[9](2016) suggested excessive water use and deforestation during cultivation, which directly increased greenhouse gas emissions[10](Borzoni 2011). Meanwhile, excessive transport distances (soybeans cross borders) also lead to excessive fossil fuel use in the bulk agricultural products supply chain.[11](He et al. 2019)

2.2 Purchase Order Financing

Supply chain finance plays an important role in the intersection of operation and financing. For small and medium-sized enterprises, their credit limit and capital constraints are relatively obvious. Therefore, how to provide financing support for capital-constrained supply chains through financial instruments It has always been the focus of scholars' attention.

For the purchase order financing (POF) widely used in the soybean industry chain, as a form of pre-shipment financing (financing can be completed before the goods are shipped), M.Reindrop et al.[3] (2018) quantified capital friction and pointed out that this method can expand the production of suppliers feasible set. Tang et al.[12] (2018) discussed POF and the buyer direct financing (BDF) and developed a game model to capture the difference between the two innovative formats. And some trade-related literature also pointed out that this financing model can help small and medium-sized enterprises manage high inventories or meet large orders[13] (Fullen 2006), in fact, it is also realistic for the widely used contract agricultural production, especially the production of bulk agricultural products.[14] (Rüsch et al. 2013)

2.3 Stochastic Output Supply Chain

In Scarf's[15] (1957) research, the problem of output uncertainty was first involved. Based on the EOQ model, the retailer's optimal inventory problem was studied when the output was random. Yano and Lee[16] (1995) made a relatively extensive review on the situation of output randomness in the supply chain.

Ma et al.[17] (2013) consider the role of the spot market in the supply chain when the market demand is uncertain in the supplier managed inventory supply chain. Based on an additive stochastic output model, Wang and Tang[18] (2014) studied the production decision-making, allocation and inventory control problems when the output is uncertain.

Zhao and Wu[19] (2009) earlier studied the role of revenue-sharing contracts in the coordination of a supply chain consisting of an

agricultural product producer and an agricultural product retailer when the output and demand random variables are uniformly distributed. Ling et al.[20] (2011) studied the model of risk sharing under random output and demand and the supply chain coordination problem when both output and demand are evenly distributed under the background of the agricultural-supermarket docking model. Nong and Pang (2013) studied that in the supply chain of agricultural products, due to the randomness of the output of agricultural products, the price fluctuation of agricultural products is also relatively large.

2.4 Summary and Insights

To summarize, scholars have made many important achievements on the decision-making problems of agricultural supply chain management under the environment of uncertain output or demand, and the financial problems of capital-constrained agricultural supply chain. A solid theoretical foundation has been laid.

However, in the existing research, the research on the output and order decision-making problem of combining agricultural production insurance with supply chain financial instruments is relatively lacking, and there is room for more in-depth research.

3 Model Description, Notations and Assumptions

Consider a small capital-constrained bulk agricultural commodity farmer, its sales channel is completely dependent on the local purchasing company, since the individual farmer has no say in the price setting of the purchase contract, it is a price taker in the contract.

The purchasing contract is designed with 2 key variables of order quantity q_0 and guaranteed floor price p_f, while the spot price of the soybean at the time of delivery is stochastic, marked as p_s. p_s follows a distribution with a probability density function (PDF) g, a cumulative distribution function (CDF) G and a complementary cumulative distribution function (CCDF) $\overline{G}$.

The decision variable is the acreage that the farmer will produce q. The characteristics of agricultural products determine that the yield per unit acreage of products is unstable, and it is described by a continuous random variable ξ. It follows a distribution with a PDF f, a CDF F and a CCDF $\overline{F}$.

We assume that the amount of capital owned by farmers before planting is 0, and the unit acreage production cost is c.

Due to the properties of bulk agricultural products, their storage and circulation restrictions can be ignored. The demand faced by purchasing enterprises is d, and d is a continuous random variable that follows a probability distribution, whose PDF is h, CDF is H, CCDF is $\overline{H}$. The uncertainty of the price is included in the equilibrium price decision of the purchasing company's contract design, which means the final delivery price of the farmer is $\max\{p_s, p_f\}$.

LEMMA 1. *The equilibrium condition of the purchasing company's pricing decision in the case of futures hedging is*

$$E[\max\{p_f, p_s\}] = E[p_s] \tag{1}$$

The purchase company's final contract guaranteed floor price is

$$p_f^* = \frac{\int_0^\infty p_s g(p_s)\, dp_s}{1 + \int_{p_s}^\infty \overline{G}(y)\, dy} = \frac{E[p_s]}{1 + n(p_s)} \tag{2}$$

Note. The $n(p_s)$ is defined as the loss function of p_s.[21]

Farmer submits loan requests to the bank with purchase orders. The bank evaluates the purchase contract and proposes interest rates r_b and advance ratios (λ). The loan amount can be described as $\lambda q_0 p_f^*$. Since the advance ratio and loan interest rate are limited by administrative orders, we assume they are strictly exogenously given. Also, considering a perfect capital market, we assume the tax and risk-free interest rate are both 0.

Meanwhile, the insurance fee is relative with product output on a fixed rate, marked as r_i, and the whole insurance fee can be described as $r_i q$. Once the output does not reach the bottom threshold (q_b) set by the insurance, the insurance company will compensate the part of the production that does not meet the standard per unit plot at a fixed unit price p_i.

Notation	*Meaning*
q	Farmer's production acreage decision variable
q_0	The quantity of the contract which purchasing company offers
ξ	Random variable of yield per unit area
μ	Average of ξ
p_s	Random variable of spot price at the time of delivery
b	Unit stock-out loss due to failure to meet demand
d	Random variable of market demand
p_f	Guaranteed floor price of the purchasing contract
v	Price for products that are not collectively sold
c	Cost per unit input
r_b	Bank loan interest rate
λ	Advance rate of bank loan
r_i	Insurance fee rate
p_i	Compensation price
q_b	Bottom yield per acreage of insurance compensation

Table 1: Notations

4 Optimal Decisions

4.1 Basic Model

In the basic model, only considering the uncertainty of price and output, ignoring the impact of capital constraints and agricultural industry insurance on farmer's profit, the farmer's profit function can be described by a newsvendor model as:

$$\pi_1 = \min\{\xi q, q_0\} \cdot \max\{p_s, p_f^*\} - cq + v(\xi q - q_0)^+ \tag{3}$$

The expectation of the profit function is:

$$E[\pi_1] = \int_0^{\frac{q_0}{q}} (E[p_s] - v)(\xi q - q_0) f(\xi)\, d\xi + E[p_s]q_0 - cq + v(\mu q - q_0) \tag{4}$$

PROPOSITION 1. *The purchasing company is the leader of the Stackelberg game, as the price taker and follower, the optimal production input decision $q^*(q_0)$ of the farmer satisfies the following conditions:*

$$\int_0^{\frac{q_0}{q^*(q_0)}} \xi f(\xi)\, d\xi = \frac{c - v\mu}{E[p_s] - v} \tag{5}$$

and $q^*(q_0)$ is the reaction function of q_0

Proof:

Differentiate the expectation profit function with respect to q,

$$\frac{\partial E[\pi_1]}{\partial q} = \int_0^{\frac{q_0}{q}} (E[p_S] - v)\xi f(\xi)\, d\xi + v\mu - c \tag{6}$$

The second order partial derivative of the expected profit function with respect to q is,

$$\frac{\partial^2 E[\pi_1]}{\partial q^2} = -\frac{q_0^2}{q^3}(E[p_s] - v]f(\frac{q_0}{q}) < 0 \tag{7}$$

It can be seen that $E[\pi_1]$ is a concave function of q, and there is a global maximum value in the feasible interval. From the first-order condition, it can be seen that q should satisfy the condition in equation (5) when reaching the optimum, and it changes with the purchase amount q_0, so it is the reaction function of the purchase amount q_0.

COROLLARY 1. *There is a linear increasing relationship between the optimal production input of farmer and the purchase orders of purchasing company.*

Proof:

According to the implicit function theorem, the first-order partial derivative of $q^*(q_0)$ to q_0 is

$$\frac{\partial q^*(q_0)}{\partial q_0} = \frac{q^*(q_0)}{q_0} > 0 \tag{8}$$

Also, the second order partial derivative of the optimal input with respect to q_0 is

$$\frac{\partial^2 q^*(q_0)}{\partial q_0^2} = \frac{1}{q_0}\frac{\partial q^*(q_0)}{\partial q_0} - \frac{1}{q_0^2}q^*(q_0) = 0 \tag{9}$$

Obviously, equation (8) is a constant, q^* is a reaction function that follows a linear increasing relationship, which we define as k.

In the second stage of the game, as the leader, the purchasing company needs to make the decision on the optimal purchase amount q_0 according to the response function of the farmer's optimal input output $q^*(q_0)$ to its purchase amount q_0, and its profit function can be described as:

$$\begin{aligned}\pi_{r1} &= p_s \min\{d, q_0, \xi q_1^*(q_0)\} - p_f^* \min\{q_0, \xi q_1^*(q_0)\} \\ &\quad - b[d - \min\{q_0, \xi q_1^*(q_0)\}]^+ + v[\min\{q_0, \xi q_1^*(q_0)\} - d]^+\end{aligned} \tag{10}$$

The expectation profit function can be described as:

$$\begin{aligned}E[\pi_{r1}] = &\int_0^{\frac{q_0}{q_1^*(q_0)}} \int_0^{\xi q_1^*(q_0)} (E[p_s] + b - v)(d - \xi q_1^*(q_0))h(d)f(\xi)\, dd d\xi \\ &- \int_0^{\frac{q_0}{q_1^*(q_0)}} \int_0^{q_0} (E[p_s] + b - v)(d - q_0)h(d)f(\xi)\, dd d\xi \\ &+ \int_0^{q_0} (E[p_S] + b - v)(d - q_0)h(d)\, dd \\ &+ \int_0^{\frac{q_0}{q_1^*(q_0)}} (E[p_s] + b - p_f^*)(\xi q_1^*(q_0) - q_0)f(\xi)\, d\xi \\ &+ (E[p_s] + b - p_f^*)q_0 - bE[d]\end{aligned} \tag{11}$$

PROPOSITION 2. *The optimal purchase quantity of the purchasing company q_0^* satisfies:*

$$
\begin{aligned}
&(E[p_s]+b-v)F(q_0^*)-\int_0^{\frac{q_0^*}{q_1^*(q_0)}}(E[p_s]+b-p_f^*)[\xi\frac{\partial q_1^*(q_0)}{\partial q_0}|_{q_0=q_0^*}-1]f(\xi)\,d\xi \\
&+\int_0^{\frac{q_0^*}{q_1^*(q_0)}}(E[p_s]+b-v)[\xi F(\xi q_1^*(q_0^*))\frac{\partial q_1^*(q_0)}{\partial q_0}|_{q_0=q_0^*}-F(q_0^*)]f(\xi)\,d\xi \\
&=E[p_s]+b-p_f^*
\end{aligned}
\tag{12}
$$

Proof:

The first order partial derivative of expectation profit function with respect to q_0 is:

$$
\begin{aligned}
\frac{\partial E[\pi_{r1}]}{\partial q_0}&=\int_0^{\frac{q_0}{q_1^*(q_0)}}\int_0^{q_0}(E[p_s]+b-v)h(d)f(\xi)\,dd d\xi \\
&-\int_0^{\frac{q_0}{q_1^*(q_0)}}\int_0^{\xi q_1^*(q_0)}(E[p_s]+b-v)\xi\frac{\partial q_1^*(q_0)}{\partial q_0}h(d)f(\xi)\,dd d\xi \\
&-\int_0^{q_0}(E[p_S]+b-v)h(d)\,dd \\
&+\int_0^{\frac{q_0}{q_1^*(q_0)}}(E[p_s]+b-p_f^*)[\xi\frac{\partial q_1^*(q_0)}{\partial q_0}-1]f(\xi)\,d\xi \\
&+(E[p_s]+b-p_f^*)
\end{aligned}
\tag{13}
$$

The second order partial derivative with respect to q_0 is:

$$
\begin{aligned}
\frac{\partial^2 E[\pi_{r1}]}{\partial q_0^2}&=(E[p_S]+b-v)h(q_0)[F(\frac{q_0}{q_1^*(q_0)}-1] \\
&-\int_0^{\frac{q_0}{q_1^*(q_0)}}(E[p_S]+b-v)(\frac{\partial q_1^*(q_0)}{\partial q_0})^2\xi^2 h(\xi q_1^*(q_0))f(\xi)\,d\xi<0
\end{aligned}
\tag{14}
$$

The purchasing company's expected profit function is concave about q_0, which has a global maximum at the stagnation point.

4.2 Model Considering Capital Constraint and Financing

Considering capital constraint of the farmer and the approach of purchase order financing, the loan amount is $\lambda q_0 p_f^*$, and the profit function can be described as:

$$\begin{aligned} &\pi_2 = \min\{\xi q, q_0\} \cdot \max\{p_s, p_f^*\} - cq + v(\xi q - q_0)^+ - r_b \lambda q_0 p_f^* \\ &s.t.\ \lambda q_0 p_f^* \geq cq \end{aligned} \tag{15}$$

The constraint means that the loan amount must be larger than the total production cost, as the initial capital of farmer is assumed to 0. It can be transformed to $q/q_0 \leq \lambda p_f^*/c$, meaning that the first order derivative of farmer's optimal input decision q_2^* with respect to q_0 is constrained.

PROPOSITION 3. *According to Proposition 1 and Corollary 1, the optimal production decision of farmer q_2^* under the scenario of capital constraint and purchase order financing satisfies:*

$$\begin{cases} q_2^* = \frac{\lambda q_0 p_f^*}{c} & \text{if } \frac{\lambda q_0 p_f^*}{c} < q^* \\ \int_0^{\frac{q_0}{q_2^*(q_0)}} \xi f(\xi)\, d\xi = \frac{c - v\mu}{E[p_s] - v} & \text{if } \frac{\lambda q_0 p_f^*}{c} \geq q^* \end{cases} \tag{16}$$

Proof:

Since the expected profit function does not change its increase-decrease and concave-convexity after adding the interest loss item, considering the feasible domain limit of the decision variable q, if its value is limited to the left side of the stagnation point, then the profit function at this time is For monotonically increasing, the global maximum value can be obtained only by making the decision variable equal to the upper limit of the constraint. If the feasible region includes the stagnation point, the function will obtain the global maximum value still at the stagnation point.

Focusing on the constraint in (15), and considering that the bank advance/loan ratio λ is a decision made by the bank after considering the comprehensive asset status of the farmer and the purchase order q_0, we have the following relationship:

COROLLARY 2. *The maximum profit π_2^* of the farmer increases with the two variables of bank advance ratio λ and order quantity q_0 until the following condition is* *satisfied:*

$$\int_0^{\frac{c}{\lambda p_f^*}} \xi f(\xi)\, d\xi = \frac{c - v\mu}{E[p_s] - v} \tag{17}$$

Moreover, the advance ratio and order quantity are related to the farmer's own conditions (including performance rate, credit, land quality and etc.)

4.3 Model Considering Financing and Insurance

In this section, we consider a model that is closer to the actual business model, that is, farmers need to purchase insurance to hedge against the risk that the actual yield ξ per area is too low. According to the assumption, it can be known that the insurance cost that farmers need to pay is $r_i q$, and the insurance compensation that farmers may obtain is $p_i q(q_b - \xi)^+$, and capital constraints need to be able to include planting costs and insurance costs in financing.

Under the above conditions, the farmer's profit function can be described as:

$$\begin{aligned} \pi_3 = &\underbrace{\min\{\xi q, q_0\} \cdot \max\{p_s, p_f^*\}}_{Revenue} - \underbrace{cq}_{Cost} + \underbrace{v(\xi q - q_0)^+}_{Unsold} - \underbrace{r_b \lambda q_0 p_f^*}_{Interest} - \underbrace{r_i q}_{Fee} + \underbrace{p_i q(q_b - \xi)^+}_{Claim} \\ s.t.\ \ &\lambda q_0 p_f^* \geq cq + r_i q \end{aligned} \tag{18}$$

The expectation of the profit function is:

$$E[\pi_3] = \int_0^{\frac{q_0}{q}} (E[p_s] - v)(\xi q - q_0) f(\xi)\, d\xi + E[p_s]q_0 - cq + v(\mu q - q_0) - r_i q + \int_0^{q_b} F(\xi)\, d\xi \cdot p_i q \tag{19}$$

PROPOSITION 4. *The optimal production input decision of farmer q_3^* satisfies:*

$$\begin{cases} q_3^* = \frac{\lambda q_0 p_f^*}{c+r_i} & \text{if } \int_0^{\frac{c+r_i}{\lambda p_f^*}} \xi f(\xi)\, d\xi < \frac{c+r_i-v\mu-\int_0^{q_b} F(\xi)\, d\xi \cdot p_i}{E[p_s]-v} \\ \int_0^{\frac{q_0}{q_3^*(q_0)}} \xi f(\xi)\, d\xi = \frac{c+r_i-v\mu-\int_0^{q_b} F(\xi)\, d\xi \cdot p_i}{E[p_s]-v} & \text{if } \frac{\lambda q_0 p_f^*}{c+r_i} \geq q_3^* \end{cases} \tag{20}$$

Proof:

The first order derivative of $E[pi_3]$ with respect to q is:

$$\frac{\partial E[\pi_3]}{\partial q} = \int_0^{\frac{q_0}{q}} (E[p_S] - v)\xi f(\xi)\, d\xi + v\mu - c - r_i + \int_0^{q_b} F(\xi)\, d\xi \cdot p_i \tag{21}$$

The second order derivative of $E[\pi_3]$ with respect to q is equal to equation (7) which is also positive. The expectation profit function is also concave.

According to the equation (20) and implicit function theorem, we can obtain sensitivity analysis conclusion as follows:

CONCLUSION 1. $\frac{\partial q_3^*}{\partial(c+r_i)} < 0$, $\frac{\partial q_3^*}{\partial v} > 0$, $\frac{\partial q_3^*}{\partial E[p_s]} < 0$, $\frac{\partial q_3^*}{\partial p_i} > 0$, *that is to say, in this case, the optimal input decision q_3^* of farmer decreases with the input cost per unit area (including planting and insurance costs) and the expectation of the spot price of bulk commodity at the delivery period; and increases with the processing price and insurance claim price.*

For purchasing company, The form of the profit function will not change with the financing and insurance behavior of farmers, only the reaction function will change with the capital constraints and new items, see conclusion 1.

PROPOSITION 5. *Under the conditions of purchase order financing and production insurance, considering the capital constraint, substituting the optimal conditions in Equation (20) into Equation (12), we can obtain the optimal purchase quantity decision of the purchasing company q_0^{**} satisfies:*

$$
\begin{cases}
(E[p_s]+b-v)F(q_0^{**}) \\
+\displaystyle\int_0^{\frac{c+r_i}{\lambda p_f^*}}(E[p_s]+b-p_f^*)f(\xi)\,d\xi & \text{if } \int_0^{\frac{c+r_i}{\lambda p_f^*}}\xi f(\xi)\,d\xi < \frac{c+r_i-v\mu-p_i}{E[p_s]-v} \\
-\displaystyle\int_0^{\frac{c+r_i}{\lambda p_f^*}}(E[p_s]+b-v)F(q_0^{**})f(\xi)\,d\xi \\
=E[p_s]+b-p_f^* \\
(E[p_s]+b-v)F(q_0^{**}) \\
-\displaystyle\int_0^{\frac{q_0^{**}}{q_3^*(q_0)}}(E[p_s]+b-p_f^*)[\xi\frac{\partial q_3^*(q_0)}{\partial q_0}|_{q_0=q_0^{**}}-1]f(\xi)\,d\xi & \text{if } \frac{\lambda q_0 p_f^*}{c+r_i}\geq q_3^* \\
+\displaystyle\int_0^{\frac{q_0^{**}}{q_3^*(q_0)}}(E[p_s]+b-v)[\xi F(\xi q_3^*(q_0^{**}))\frac{\partial q_3^*(q_0)}{\partial q_0}|_{q_0=q_0^{**}}-F(q_0^{**})]f(\xi)\,d\xi \\
=E[p_s]+b-p_f^*
\end{cases}
\tag{22}
$$

According to the equation (22) and implicit function theorem, we can obtain sensitivity analysis conclusion as follows:

CONCLUSION 2. $\frac{\partial q_0^{**}}{\partial E[p_s]}>0$, $\frac{\partial q_0^{**}}{\partial b}>0$, $\frac{\partial q_0^{**}}{\partial v}>0$, $\frac{\partial q_0^{**}}{\partial(c+r_i)}<0$, $\frac{\partial q_0^{**}}{\partial\lambda}>0$, *The optimal order quantity* q_0^{**} *of the purchasing company increases with the expectation of the spot price at the delivery period, unit shortage loss, product processing price, and bank advance ratio, and decreases with the increase of production cost.*

5 Conclusion

In the above discussion, we explored the simple newsvendor model in the case of uncertain demand and production, the extended model that incorporates purchase order financing and capital constraints, and the extended model that considers agricultural output insurance. Finally, the equilibrium conditions satisfied by the optimal production, quotation and order decisions of suppliers and retailers in a secondary supply chain are obtained (see Proposition 1-5), and a sensitivity analysis is carried out (see Conclusion 1, 2).

Obviously, for farmer, higher risk compensation (residual value and yield insurance compensation price) will give farmer more incentives to product more. Meanwhile, as the response function of the order quantity, under the scheme of purchase order financing, the optimal decision-making of farmer is still affected by the bank's advance ratio, and a higher advance ratio means that the bank has a more optimistic judgment on the farmer's asset status and production capacity, which gives farmer more incentives to product more as well.

For purchasing company, the judgment of the future price trend of bulk commodities will greatly affect the design of her purchase contracts, which is also in line with the actual business situation.

There are still many deficiencies in this essay, such as failing to carry out numerical experiments to testify and visualize calculation results, and failing to make empirical investigations. According to previous studies, we also consider the risk preference of farmer and uses the method of conditional value at risk (CVaR) to analyze the stochastic profit function[22](Sukono 2021). These works will be supplemented in the future.

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