Calculus II Quiz 2 109/04/13

1. (5 points each) Find each of the following if it exists, give reason if it doesn't:

答題示例：(i) $\lim_{x\to 0}\frac{\sin^2 x}{x}=(\sin 0)\cdot 1=0$ 。(ii) $\lim_{x\to 0}\frac{\sin x}{x^2}=1/0$ 不存在

(a) $\lim_{x\to 0+}\frac{(1+x)^{1/x}-e}{x}$ (b) $\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos 11x}$

(c) $\lim_{x\to\infty}(x^2+1)^{1/\ln x}$ (d) $\lim_{x\to\infty} xe^{-x^2}\int_0^x e^{t^2}dt$

2. (6 points each) For each of the following series, determine whether it converges (absolutely or conditionally) or not, please indicate which theorem you use.

答題示例：(i) 由 integral test 得知 $\sum_{k=1}^{\infty}\frac{1}{k}$ 發散。

(ii) 由 alternating series test 知 $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}$ 收斂。

(a) $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{\sqrt{2k-1}}$ (b) $\sum_{k=0}^{\infty}(-1)^k(\frac{k}{k+102})^k$

(c) $\sum_{k=1}^{\infty}\frac{\sin(\pi k/4)}{k^2}$ (d) Find values of p for which $\sum_{k=2}^{\infty}\frac{\ln k}{k^p}$ converges.

(e) An example to explain that if $\sum_{k=1}^{\infty}a_k$ converges conditionally, $\sum_{k=1}^{\infty}a_k{}^2$ may diverge (please give the general term a_k in your answer).