Geometry Final Exam (January 8, 2015)

(3:30-6:00 PM, total score 130)

1. (15%) Find the mean curvature and Gaussian curvature of

$$\vec{X}(u,v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2).$$

2. (15%) Determine all the surfaces of revolution with mean curvature $H \equiv 0$ with reasons.

3. (15%) Consider the surface $\vec{X}(u,v) = (a \sinh u \cos v, a \sinh u \sin v, av)$ where a is some nonzero constant. Find its mean curvature. (Hint: you may check the Laplacian of the parametrization)

4. (15%) Prove that there are no compact (i.e., bounded and closed in R^3) minimal surfaces.

5. (15%) Justify why the surface below are not pairwise locally isometric:
   a. Sphere
   b. Cylinder $\{(x,y,z) \in \mathbb{R}^3 | x^2 + y^2 = 1\}$
   c. Saddle $z = x^2 - y^2$

6. (15%) Show that if $\vec{X}$ is an orthogonal parametrizations, that is, $F = 0$, then

$$K = \frac{-1}{2\sqrt{EG}}[(\frac{E_v}{\sqrt{EG}})_v + (\frac{G_u}{\sqrt{EG}})_u]$$

7. Consider the torus of revolution generated by rotating the circle $(x-a)^2 + z^2 = r^2, y = 0$ about the z-axis ($a > r > 0$). The parallels generated by the points $(a+r, 0), (a-r, 0), (a, r)$ are called the maximum parallel, the minimum parallel, and upper parallel respectively. Check which of these parallels is
   a.(5%) A geodesic
   b.(5%) A line of curvature
   and (10%) compute the geodesic curvature of upper parallel.

8. (20%) Consider the surface of revolution with the parametrization

$$x = f(v)\cos u, \quad y = f(v)\sin u, \quad z = g(v).$$

   Compute all Christoffel symbols and state the differential equations for geodesics.

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