$\sec\theta = \frac{1}{\cos\theta}$ $\quad x^{\frac{1}{2}} \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{\sqrt{x}}$

$\sec'\theta = \frac{(\cos\theta \times 0) - (-\sin\theta)\times 1}{\cos^2\theta} = \frac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta = \frac{df(x)^5}{dy} = \frac{df(x)^5}{df(x)}\frac{df(x)}{dx}$

CALCULUS, MIDTERM EXAM, 8:00am-09:50am,11/29/2016
(TOTAL 100 PTS)

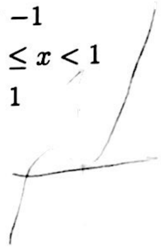
1.(50%) Find the following values:

$\frac{d(\sec x)^5}{d(\sec x)} \cdot \frac{d\sec x}{dy}$

(a) $\lim_{x\to -1} f(x)$, where

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$$

$= 5\sec^4 x \cdot \tan x \sec x$



(b) $\lim_{h\to\infty} \frac{\sqrt{9+h}-3}{h}$; (c) $\lim_{\theta\to 0^+} \theta\cot 3\theta$;

(d) $\frac{d}{dx}\sqrt{x+\sec^5 x}$; (e) $y''(\frac{1}{\sqrt{2}})$, where $x^2+y^2=1$.

$\frac{1}{2\sqrt{2}}$ $\quad -\frac{2}{3}+1>0$ $\quad \frac{-2}{x}+1$

2. (10%) A particle moves along a straight line and its position $s(t)$ at time t is given by the function $s(t) = t^3 - 9t^2 + 30t + 4$. What is the maximal velocity on the range $1 \le t \le 4$?

$x^{\frac{1}{3}}(4+x)$

$4\times(-2) + (-2)^{\frac{4}{3}} > 0$

3. (10%) Sketch the graph of $f(x) = 4x^{1/3} + x^{4/3}$. Indicate local extrema, inflection points, concave structure, asymptotes, and intercepts. $\rightarrow$ max, min

4. (10%) Find the linear approximation of $\sqrt{1.1}$ by using $f(x) = \sqrt{x}$.

5. (10%) The equation $x = \frac{1}{2}\cos x$ has a solution r near 0.5. Use Newton's method to find r accurate to two decimal places.

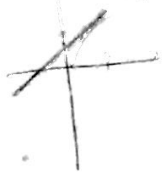
$216\times 2 = 432$ $\quad \frac{-2}{x}+1=0$ $\quad \frac{2}{x}=1$

6. (10%) As the sun sets behind a 120-foot building, the building's shadow grows. How fast is the shadow growing (in feet per second) when the sun's rays make an angle of 45^o (or $\pi/4$ radians).
(Note: the earth rotates once every 24 hours, or 86,400 seconds)

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad = 2f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{3}(x)(\frac{-2}{x}+1)$ $\quad x\neq 0$ $\quad \frac{1}{3}x^{-\frac{2}{3}}(-2x^{-1}+1)$ $\quad \frac{2}{x}+1=0$

$\Rightarrow \frac{-2}{x}+1 = \frac{-2+x}{x}$ $\quad = \frac{1}{3\sqrt[3]{x^2}}$ $\quad \frac{2}{x}=-1$ $\quad x=2$



$(x-2)'$ $\quad x<2 \Rightarrow f(x)<0$ $\quad x>2 \Rightarrow f(x)>0$

1

$\frac{4}{3}x^{-\frac{2}{3}}$ $\quad -\frac{8}{9}x^{-\frac{5}{3}} + \frac{4}{9}x^{-\frac{2}{3}}$

$\frac{d\sqrt{2x-x^2}}{dx} = 2-2x \frac{}{\sqrt{}}$ x^{-1}

CALCULUS, FINAL EXAM, 2017/01/13
(TOTAL 100 PTS)

$\frac{d\log_2\frac{x+1}{x}}{dx} = \frac{1}{(\ln 2)\frac{x}{x+1}} \cdot \frac{x-(x+1)}{x^2}$

1.(60%) Find the following values and integrals:

(a) $\lim_{x\to 0^+} x\left(\log_2(x+1) - \log_2 x\right)$; (b) $\int_0^{\pi/3} \cos x \ln(\sin x)dx$; $x \cdot \frac{\frac{x-(x+1)}{x^2}}{(\ln 2)\frac{x}{x+1}}$

$\checkmark(c)$ $\int x(1+\csc^2 x)dx$; $\checkmark(d)$ $\int_1^2 \frac{4x+9}{x(x+3)^2}dx$; $= \frac{-1}{x(x+1)\ln 2}$

(e) $\int \underbrace{x\sqrt{2x-\overset{u}{x^2}}dx}_{dv}$; $= uv - \int v\,du$ (f) $\int_1^\infty \frac{dx}{\sqrt{x}(x+1)}$.

$\frac{du}{dx} = x$ $\quad \frac{1}{2}x^2\sqrt{2x-x^2} - \int \frac{1}{2}x^2 d\sqrt{2x-x^2}$

$v = \frac{1}{2}x^2$ $\quad$ x

2. (10%) Set $f(x) = \int_0^x e^{-t^2}dt$, where $-\infty < x < \infty$

$\frac{dv}{dx} = \sqrt{2x-x^2}$

(a) On what intervals is f concave upward? $\frac{df(x)}{dx} = e^{-t^2}$

(b) Prove that $\int_0^1 e^{-t^2}dt > \frac{1}{4}\left(e^{-1/16} + e^{-1/4} + e^{-9/16} + e^{-1}\right)$.

$2-2x \frac{1}{2\sqrt{2x-x^2}}$

$\checkmark 3.$ (10%) Let $f(x) = \sin^{-1} x \quad (0 \le x \le 1)$. $\quad \frac{(1-x)}{\sqrt{2x-x^2}}$

~~(a) Sketch the graph of $y = f(x)$.~~

(b) Find the volume of the solid obtained by rotating the graph of $y = f(x)$ about the x-axis.

4. (10%) The growth of a cell population generally follows the law:

$$\frac{dN}{dt} = \alpha N - \beta N^2,$$

where $N(t)$ is the total population at time t and α, β are positive constants. If $N(0) < \alpha/\beta$, find the extrema of $N(t)$.

5. (10%) Evaluate $\quad -2x1$

$$\lim_{n\to\infty} \frac{\tan\frac{1}{2n} + \tan\frac{3}{2n} + \tan\frac{5}{2n} + \cdots + \tan\frac{2n-1}{2n}}{n}$$

$\ln x$

by first finding a function f such that the limit is equal to

$$\int_0^1 f(x)dx.$$

$\frac{d\left(\frac{1}{\sqrt{2x-x^2}}\right)}{dx}$

$\frac{d?}{dx} = \ln x$

$\left(\frac{1}{2x+3}\right)' = 2\cdot\frac{-1}{(2x+3)^2}$ $\quad d\,\frac{1}{f(x)} = \frac{1}{f(x)^2}$

$\frac{d\,\frac{1}{\ln x}}{dx} = -1(\ln x)^{-2}\cdot\frac{1}{x}\,dx$

$\int \frac{1}{t}\,\frac{1}{\ln t}\,dt$

CALCULUS, MIDTERM EXAM, 04/11/2017

(TOTAL 100 PTS)

1.(20%) Determine which of the following converges:

$$(a)\ \sum_{n=1}^{\infty}\frac{1}{(n+2)\ln(n+1)};\qquad (b)\ \ 1+\sum_{n=1}^{\infty}\frac{(-1)^n 10^{2n}}{2^{2n}(n!)^2}.$$

2. (20%) Let $f(x)=\dfrac{1}{1-x}+\left(\dfrac{x}{1!}-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+-\cdots\right)$.

(*a*) Write out the power series expansion of $f(x)$ with the base point 0.

(*b*) What is its radius of convergence? What is its interval of convergence?

3. (20%) (*a*) Sketch the direction field of $y'=x+y$. $\quad y+x=u$

(*b*) Solve $y'=x+y$ by making the change of variable $u=x+y$. $\quad \frac{dy}{dx}+1=\frac{du}{dx} \to du = dy+dx$

4. (20%) A particle moves in the plane. Its position at time t is described by

$$x(t)=1+3t^2,\ y(t)=4+2t^3\qquad (0\le t\le 1).$$

(*a*) Find dy/dx at $t=1/2$.

(*b*) Find the arc length of the curve which this particle travels.

5. (20%) (*a*) Sketch the curve $r^2=\sin 2\theta$. $\quad \left(\frac{1}{\sqrt{2}}\right)^2\times 4 = 2$

(*b*) Find all points of intersection of the curves $r^2=\sin 2\theta$ and $r=2\cos\theta$. $\quad \frac{2}{3}\pi = \frac{\sqrt{3}}{2}$

(*c*) Find the area of the region inside both of the curves given in (*b*).

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$4\cos^2\theta$	4	2	1
$\sin(2\theta)$	0	1	$\frac{\sqrt{3}}{2}$

$\frac{x}{r}\ \frac{y}{r} = \overline{x^2+y^2}$ $\qquad \frac{3}{8}\quad \frac{1}{2}$

$r=\sin\theta$

CALCULUS, FINAL EXAM, 06/16/2017 08:00am-10:00am
(TOTAL 100 PTS)

1. (10%) Let $f(x) = x\cos x$.
   - (a) Find the power series expansion of f at $x = 0$.
   - (b) Find $f^{(99)}(0)$.

2. (25%) The motion of a particle is described by $\gamma(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$.
   - (a) Find the vector v for which $\|v\| = 2$ and v has the same direction with the unit tangent vector T at $t = 1$.
   - (b) Find the tangential component a_T of the acceleration $a(1)$.
   - (c) Find $\gamma'(t) \times \gamma''(t)$.
   - (d) Find the curvature of the curve at $t = 1$.
   - (e) Find the arc length function $s(t)$ and show that $s(t) \geq t$.

3. (15%) Let $f(x, y) = \sin(x + 2\pi y + xy)$, $u = (1, \sqrt{2})$, and $P = (0, 1)$.
   - (a) Find $D_u f(0, 1)$;
   - (b) Find the linearization of f at P.
   - (c) Find the equation of the tangent plane to $z = f(x, y)$ at $(0, 1, 0)$.

4. (10%) Suppose that f is differentiable and

$$\frac{\partial f}{\partial x}(1,0) = 1, \quad \frac{\partial f}{\partial y}(1,0) = 0,$$
$$\frac{\partial f}{\partial x}(-1,0) = -1, \quad \frac{\partial f}{\partial y}(-1,0) = -1.$$

$$\frac{\partial f(x,y)}{\partial x}$$

Find $z'(\pi)$, where $z(\theta) = f(\cos 2\theta, \sin 2\theta)$.

5. (15%) Let $f(x, y) = xy - x + 4$.
   - (a) Find all critical points of f.
   - (b) Minimize f subject to the constraint $x^2 + y^2 = 3$.
   - (c) Find the minimum value of f on $x^2 + y^2 \leq 3$.

6. (15%) Let $V(\Omega)$ denote the volume of the solid Ω bounded by the planes $x + 2y + z = 2$, $x = 0$, $y = 0$, and $z = 0$.
   - (a) Set up a double integral for $V(\Omega)$.
   - (b) Find the area of each x-cross section.
   - (c) Evaluate $V(\Omega)$.

7. (10%) Evaluate the double integral $\iint_D \ln(\sqrt{x^2 + y^2})\,dA$, where

$$D = \{(x, y) | 1 \leq x^2 + y^2 \leq 4\}.$$

$\int_1 \frac{1}{x}\,dx = \ln x + C$

$\frac{1}{2}\ln 2 - 0$