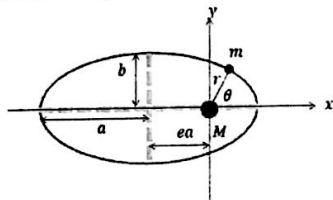普通物理期中考(12/06/2016) 時間:10:00-13:00

每大題含1-3小題,每小題5分,總分100分。將計算或推導過程清楚寫在答案本中,計算題最後答案畫長方格圈示。

1. Derive Kepler's (a) 2nd law and (b) 3rd law from Newton's law of gravitation.

   Hint: Kepler's 1st law: $\dfrac{1}{r} = \dfrac{GMm^2}{l^2}(1 + e\cos\theta)$; $b = a\sqrt{1-e^2}$

2. Consider a fluid of density ρ. (a) Derive Bernoulli's equation from Newton's second law . (b) Derive the equation of continuity using conservation of mass and divergence theorem.

3. A mass m is attached to a spring of spring constant $m\omega^2$ and is subject to a damping force $-2m\beta v$. The mass has an initial displacement $x(0) = 0$ and an initial velocity $v(0) = v_0$ at time $t=0$. Derive (a) the displacement of mass from its equilibrium point $x(t)$ and (b) the velocity $v(t)$ as a function of time.

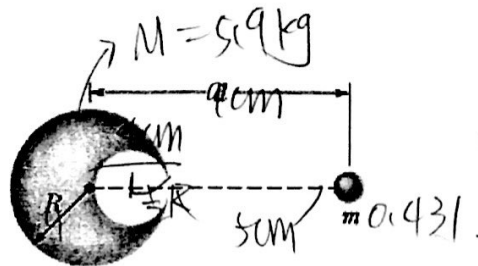4. (a) Derive the wave equation for waves on a taut string of tension τ and linear density μ.
   (b) A taut string of length L, tension τ and linear density μ is fixed at both ends. Derive the resonant frequencies for this string.

5. A stream of water flowing through a hole at depth $h = 12$ cm in a tank holding water to height $H = 40$ cm. (a) At what distance x does the stream strike the floor? (b) At what depth should a second hole be made to give the same value of x? (c) At what depth should a hole be made to maximize x?
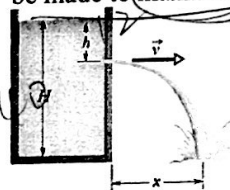
6. A uniform rope of mass m and length L hangs from a ceiling. (a) What is the speed of a transverse wave on the rope as a function of y, the distance from the lower end? (b) What is the time a transverse wave takes to travel the length of the rope ?

7. A spherical hollow inside a lead sphere of radius $R = 4.00$ cm; the surface of the hollow passes through the center of the sphere and "touches" the right side of the sphere. The mass of the sphere before hollowing was $M = 5.9$ kg. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass $m = 0.431$ kg that lies at a distance $d = 9.00$ cm from the center of the lead sphere, on the straight line connecting the centers of the spheres and of the hollow?

$M = 5.9 \, kg$

$A_1 V_1 \left(\dfrac{cm^3}{s} \right)$

$4 cm$

m

R

$5 cm$ $m \; 0.431$

P v_2 d_2 d_1 $v_1 \geqslant 0$ $1 \, atm$

$cm^2 \; cm$
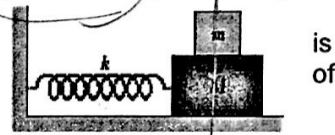
8. As shown in the figure below, water flows through a horizontal pipe and then out into the atmosphere at a speed $v_1 = 30$ m/s. The diameters of the left and right sections of the pipe are 5.0 cm and 3.0 cm. (a) What volume of water flows into the atmosphere during a 10 min period? In the left section of the pipe, what are (b) the speed v_2 and (c) the gauge pressure?

9. An aluminum wire, of length $L_1 = 60.0$ cm, cross-sectional area 1.25×10^{-2} cm², and density 2.60 g/cm³, is joined to a steel wire, of density 7.80 g/cm³ and the same cross-sectional area. The compound wire, loaded with a block of mass m = 10.0 kg, is arranged so that the distance L_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up on the wire by an external source of variable frequency; a node is located at the pulley. (a) Find the lowest frequency that generates a standing wave having the joint as one of the nodes. (b) How many nodes are observed at this frequency?

L_1 L_2

Aluminum Steel

m

$60 cm$ 86.6 $10 kg$
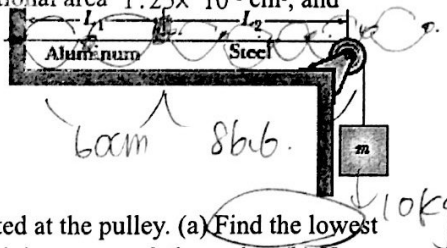
10. Two blocks (m =1.8 kg and M =10 kg) and a spring (k = 200 N/m) are arranged on a horizontal, frictionless surface. The coefficient of static friction between the two blocks is 0.40. What amplitude of simple harmonic motion of the spring–blocks system puts the smaller block on the verge of slipping over the larger block?

k m

对于（整体 或 其中之一）

对 M+m : $F = (m+M) \dfrac{d^2 X}{dt^2} = -kX$

对 m : $F = mg\mu = m\dfrac{d^2 X}{dt^2}$

对 M :

M
f