Quantum Physics (I): Final Jan. 7, 2003

Useful Integral:

$$\int_{-\infty}^{\infty} \exp(-ax^2)dx = \sqrt{\frac{\pi}{a}}$$

Problem 1 15% Find the hermitian conjugate of the following operators **(a)** $\frac{d}{dx} - ix$ **(b)** $\exp(i\hat{A})$, where $\hat{A}=\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}$. **(c)** the exchange operator. Which operator(s) is(are) unitary? Which operator(s) is(are) hermitian?
Problem 2 10% Briefly explain the following terms (a) nucleosynthesis (b) density of state (c) the exchange force (d) energy band
Problem 3 An electron is put in an oscillating electric field $E\cos\omega t$. It is therefore described by the Hamiltonian operator

$$H_0 = \frac{p^2}{2m} - (eE\cos\omega t)x.$$

(a)6% Find $\frac{d\langle x\rangle}{dt}$ and $\frac{d\langle p\rangle}{dt}$. **(b) 4%** If at $t = 0$, $\langle x\rangle = 0$, $\langle p\rangle = 0$, find $\frac{d\langle H_0\rangle}{dt}$ at $t = t_0$.
Problem 4 Consider N ($N \gg 1$) electrons (with mass being m) in a large three dimensional box of size $a \times b \times c$.
(a)7% Find number of states (including effects of spins) per energy level in terms of a, b, c, h and m.
(b)8% At zero temperature, find the degenerate pressure and the Fermi wavelength in terms of a, b, c, h, m and N.
(c)5% Now suppose $a = 2L$, $b = L$, and $c = 3L$. Find the total energy at zero temperature when $N = 8$.
Problem 4 Consider a two-particle wavefunction

$$\Psi(x_1, x_2) = Ne^{-(ax_1^2+2bx_1x_2+cx_2^2)/2},$$

where a and c are positive. N is real. **(a)7%**What is the condition that this wavefunction can be normalized and find the normalization factor N **(b) 8%** Calculating the correlation of the coordinates: $\langle(x_1 - \langle x_1\rangle)(x_2 - \langle x_2\rangle)\rangle$. What kind of the tendence does the correlation tell us? If x_1 and x_2 are independent, what value does you expect to obtain? **(c)5%**Now, suppose that these two particles are identical fermions. How should be the corrected wavefunction constructed from $\Psi(x_1, x_2)$? Find the condition that the correct wavefunction can be normalized and find the normalization wavefunction. What is the normalized wavefunction if these two particles are identical bosons?
Problem 5 10%Consider the white dwarf in the semi-relativistic treatment in which only the kinetic energy is treated ultra-relativistically. Show that there is critical mass M_c, beyond which no stable white dwarf exits. Find the approximated M_c in terms of the mass of sun ($M_S = 2 \times 10^{33}g$).
Problem 6 Consider a potential given by

$$\begin{aligned} V(x) &= \infty \qquad x < 0, \\ &= -V_0 \quad 0 < x < a, \\ &= 0 \qquad a < x. \end{aligned}$$

(a) 8 % Find the minimum V_0 so that at least, it can hold 6 electrons (including effects of spins) in the well ($0 < x < a$). **(b) 7%** Is there any relation of the bound states for this potential to the bound states of the following potential?

$$\begin{aligned}\tilde{V}(x) &= -V_0 \quad -a < x < a \\ &= 0 \qquad a < |x|\end{aligned}$$

Following (a), what is the minimum number of electrons that can be hold in the well of $\tilde{V}(x)$ when V_0 exceeds the minimum V_0? Explain your result briefly.

Problem 7 A particle of mass m in a symmetric infinite well ($-a < x < a$) is described by the following wavefunction at $t = 0$

$$\Psi(x,0) = \frac{N}{\sqrt{a}}\left[(3+2i)\cos(\frac{\pi x}{2a}) - 2\sin(\frac{\pi x}{a}) + 3i\cos(\frac{3\pi x}{2a})\right]$$

where N is a normalization constant. **(a) 5%** In the vector space analogy, we use a vector $\begin{pmatrix} a_1 \\ a_2 \\ . \\ . \\ . \end{pmatrix}$ to represent a wavefunction. If we use the normalized energy eigenfunctions of the particle as the basis, express the normalized $\Psi(x,t)$ as a vector in this basis. **(b) 5%** In the same basis, express the Hamiltonian as a matrix.