Electrodynamics (I): Final
7:00-9:30 PM, January 9, 2015

Total grade: 120

Useful information

(i) In the cylinderical coordinate (ρ, θ, z), if a vector field is given by $\vec{A} = A_\rho \hat{\rho} + A_\theta \hat{\theta} + A_z \hat{z}$, we have

$$\nabla \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} \hat{\rho} & \rho\hat{\rho} & \hat{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\theta & A_z \end{vmatrix}$$

(ii) In the spherical coordinate (r, θ, ϕ), if a vector field is given by $\vec{A} = A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$, we have

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

Problem 1 Answer the following questions briefly:

(a) 15% Explain the following terms briefly: continuity equation, bound current, displacement current, hysteresis loop, and motional emf.

(b) 3% Which is(are) the correct boundary condition in magnetostatics at the boundary between two different media?

(1) The component of $\vec{B}$ parallel to the surface must have the same value.

(2) The component of $\vec{H}$ parallel to the surface must have the same value.

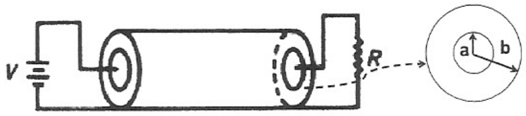
(3) The component of $\vec{B}$ normal to the surface must have the same value.

(4) The component of $\vec{H}$ normal to the surface must have the same value.

(c) 6% Two concentric metal spherical shells, of radius a and b, respectively, are separated by weakly conducting material of conductivity σ. Find the resistance of the system when two shells are maintained at a potential difference so that currents flow between two shells isotropically. What would be the inductance of the system?

(d) 5% An infinitely long circular cylinder carries a uniform magnetization M parallel to its axis. What are the magnetic fields (due to M) inside and outside the cylinder? What would be the magnetic fields if instead of being circular, the cross section is elliptical?

(e) 5% The following figure shows a circuit with a resistance R and a long coaxial cable. The cross section of the coaxial cable , shown in the right, shows that the radius of the inner cylinder is a, while that of outer cylinder is b. Let the length of the cable be L. What is the inductance in the circuit?



(f) 5% Following (d) (for circular cylinder), suppose that the magnetizatin $M = M_0 \cos(\omega t)$ is time-dependent but the quasi-static approximation is valid. If the radius of the cylinder is a, in the cylindrical coordinate (ρ, ϕ, z), assuming that the fields goes to zero as $\rho \to \infty$, find the electric field $\vec{E}(\rho, \phi, z, t)$ outside the cylinder $(\rho > a)$.

Problem 2

(a) 12% Consider a uniformly magentized sphere with the magnetization being along z-axis: $\vec{M} = M\hat{z}$. Let a be the radius of the sphere. In the spherical coordinate (r, θ, ϕ), show that for $r < a$ and $r > a$, the auxiliary field $\vec{H}$ can be expressed as the gradient of a scalar potential ϕ_H: $\vec{H} = -\nabla\phi_H$ with ϕ_H satisfying the Laplace equation. By using seperation of variables, find ϕ_H for $r < a$ and $r > a$. From here, find the magnetic field $\vec{B}$ inside the sphere.

(b) 6% Consider a charged spherical shell with radius a. Let σ be the surface charge density on the sphereical shell. If the spherical shell is spinning with angular velocity being $\vec{\omega} = \omega\hat{z}$, using results of (a), find the magnetic field $\vec{B}$ inside the spherical shell.

(c) 7% Following (b), find the magnetic force of the attraction between the northern and the southern hemispheres of the spinning charged spherical shell.

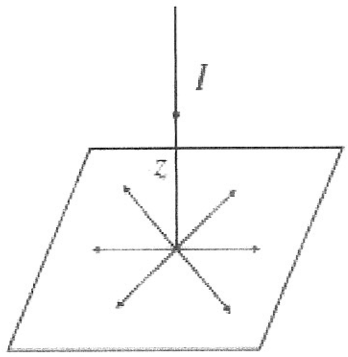
(d) 6% Using results of (a), find the magnetic field inside a sphere of linear magnetic material with magnetic susceptibility being χ_m in an otherwise uniform magnetic field $\vec{B}_0$.

Problem 3

As shown in the following figure, a current I flows down the z-axis from infinity and then spreads out radially and uniformly to infinity in the $z = 0$ plane.

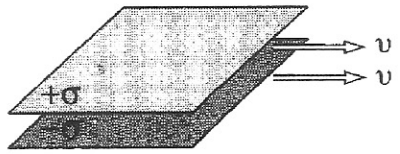
(a) 12% In the cylindrical coordinate (ρ, ϕ, z), find the mangetic field $\vec{B}(\rho, \phi, z)$ and the corresponding vector potential $\vec{A}(\rho, \phi, z)$ by setting $\vec{A}(\rho = 1) = 0$.

(b) 5% What is the boundary condition that $\vec{B}$ has to satisfy for the $z = 0$ plane? Check if the solution obtained in (a) satisfies the boundary condition.

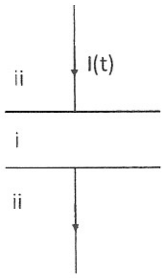


Problem 4

An large parallel-plate capacitor with uniform surface charge density σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v as shown in the following figure.



(a) 5% Find the magnetic field between the plates and also above and below them.
(b) 4% Find the speed v so that the magnetic force balance the electric force.
(c) 6% Suppose that the upper plate is at $z = d$ and the lower plate is at $z = 0$. v is along x-axis. Find the vector potential for all z by setting $A(z = 0) = 0$.
(d) As shown in the following figure, suppose that charge density σ is removed and the capacitor is fixed in space ($v = 0$). Suppose that plates are circular plates with radius being a and the capacitor is connected to other circuits by injecting or extracting current through conducting wires connecting perpendicular to the plates at the center. The voltage across the plate is $V_0 \cos \omega t$ at time t. By assuming that $d \ll a \ll c/\omega$, so that fringing of the electric field can be ignored and quasi-static appoximation is valid, answer the following questions:
(i) 6% In the cylinderical coordinate (ρ, ϕ, z) where ρ is measured from the center, find the electric fields $\vec{E}_1$ and the magnetic field $\vec{B}_1$ in the region labelled by (i) (for $\rho < a$).
(ii) 6% Find the current $I(t)$ in the wire as a function of



time t. From $I(t)$ and (a), find the electric field $\vec{E}_2$ and the magnetic field $\vec{B}_2$ in the region labelled by (ii) (for $\rho < a$).
(iii) 6% By using $\vec{B}_1$ and $\vec{B}_2$ found in (a) and (b), deduce the current density in the upper plate as a function of time t and ρ.