Quantum Physics (I): Midterm (1) Oct. 25, 2002

Useful Integrals and notations:

$$\int_0^{\pi/2} x^2 \cos^2 x dx = \frac{\pi^3}{48} - \frac{\pi}{8}$$

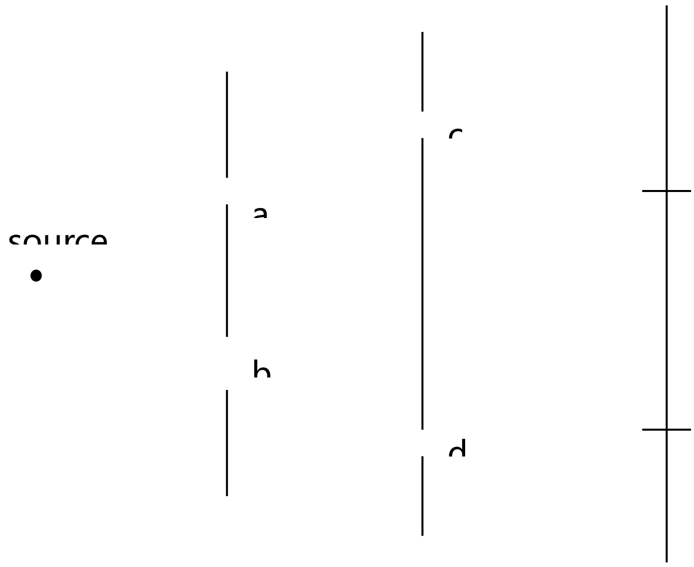
$$c = \text{speed of light}$$

$$\int_{-\infty}^{\infty} dx \exp(-\alpha x^2) = \sqrt{\frac{\pi}{\alpha}}$$

Problem 1 7% Indicate which of the following experiments are related to the wave nature of the electron? (a) blackbody radiation (b) the Bohr quantization condition (c) the Compton effect (d) the x-ray radiation from de-accelerated electrons (e) the quantum corral experiments done in the IBM laboratory (f) the photoelectric effect (g) the experiments of electron scattering from the Ni crystal done by Davisson and Germer.

Problem 2 Suppose that we prepare a cubic cavity (dimension is $L\times\ L\times L$) in equilibrium at temperature T. Inside the cavity, there will be radiations in various wavelengths. Let the energy density per unit frequency for the radiation with frequency ν be $f(\nu, T)$. **(a) 8%** If we drill a small hole on the wall with area a, find the radiation energy of frequency ν that flows out from this hole per unit time. **(b) 7%** Find the number of the electromagnetic waves (each allowed waves is called a mode) allowed in this cavity with frequency in the range $(\nu, \nu + d\nu)$. **(c) 5%** What is the Planck's expression for the average energy of each mode with frequency ν ? What is the main assumption for Planck to get this expression? **(d) 10%** Suppose that the radiation can be contained in a two dimensional cavity with dimension $L\times\ L$. Repeat your calculation for problem (a) and (b) by assuming that the only property of the radiation that changes is the dimension it lives, all the other properties remain the same. (Note that in this case, a hole becomes a segment. Therefore, the meaning of a is the length of the segment).

Problem 3 10% The following figure represents a generalized Young's experiments for the electron. We shall denote the "wavefunction" (more appropriate it should be termed "amplitude") for the electron to go from α to δ via β and γ as $\Psi(\alpha, \beta, \gamma, \delta)$. For example, the wavefunction for the electron to go from the source to arrive at the point A via b and c is $\Psi(s, b, c, A)$. Express the total probabilty for the electron to go from the source to the point B. What would the expression for the total probability if one uses the classical probability concept to calculate?



Problem 4 10% Use the uncertainty relation to estimate the ground state energy of a particle with mass m for the potential: $V(r) = \alpha r^4$, and find other energy eigenvalues of this potential using the Bohr quanziation rule. Here r is the radius of the electron in spherical coordinates.

Problem 5 **8%** Show that the following operator is a Hermitian operator:

$$\frac{x\frac{\hbar}{i}\frac{\partial}{\partial x}+\frac{\hbar}{i}\frac{\partial}{\partial x}x}{2}.$$

Therefore, it can be used to represent the classical quantity xp .

Problem 6 A particle is described by the following wavefunction

$$\begin{aligned}\Psi(x,t) &= A\cos\frac{\pi x}{a}e^{-iEt/\hbar} \quad \text{for } -a/2<x<a/2,\\ &= 0 \qquad\qquad\qquad\qquad \text{for } |x|\geq a/2,\end{aligned}$$

where E is the total energy and A is a normalization constant. Suppose that we prepare 2×10^6 such particles in the same state. We now perform precise measurements of position or momentum on each particle. Among these measurements, there are 10^6 measurements that measure the position and the other 10^6 for measuring the momentum. The results for positions are $\{x_1,x_2,x_3,x_4,...,x_{10^6}\}$ and those for momentum are $\{p_1,p_2,p_3,p_4,...,p_{10^6}\}$ **(a)** **8%** How many x_i's satisfy $-a/8\leq x\leq a/8$? Explain your estimation. **(b)** **15%** Estimate $(x_1^2+x_2^2+x_3^2+x_4^2+\cdots+x_{10^6}^2)/10^6$ and $(p_1^2+p_2^2+p_3^2+\cdots+p_{10^6}^2)/10^6$. Estimate the uncertainties for the above measurements of position and momentum.**(c)** **5%** Find the probabilty current at $x=0$.

Problem 7 **7%** Consider the problem of the spreading of a Gaussian wave packet for a free particle of mass m. If in the momentum space, the wavefunction $\phi(p)=\sqrt{\frac{a}{2\pi^{3/2}}}\exp[-a(p-p_0)^2]$, find the group velocity of this particle.What is the width of the wavepacket of this particle at time t?