Midterm 1 for General Physics I

Date: Oct 20, 2013

(1) Please do not flip the sheet until instructed.
(2) Please try to be as neat as possible so that I can understand your answers without ambiguity.
(3) While it is certainly your rights to make wild guesses or memorize irrelevant details, I would truly appreciate if you try to make your answers logical.
(4) Good luck for all hard-working students!

Lecturer: Hsiu-Hau Lin

Midterm for General Physics I (Fall, 2013)

Lecturer: Hsiu-Hau Lin
(Dated: Oct 20, 2013)

1. Velocity addition (20%) Consider two inertia frames, Jack and Jill, with relative velocity v as discussed in class. In Jack view, a particle moving at constant velocity u along the x-axis traces out the trajectory $(x, t) = (ut, t)$. Utilize Lorentz transformation to find the trajectory (x', t') in Jill's view and the corresponding velocity u'. Compare your result with that obtained by the formula of velocity addition.

2. Circular motion (20%) A particle of mass m moves in circular motion of radius R at constant speed v in the $x - y$ plane. **(A)** Write down the equations of motion (EOM) in the two dimensional Cartesian coordinates. **(B)** Compare these EOMs with the simple harmonic motion and discuss their connection in details.

3. Shooting bullet (20%) A bullet of mass m is shooting down vertically with initial velocity $(0, 0, -v_0)$ and collides with a block of mass M at rest in the horizontal plane. The gravitational acceleration is g. After penetrating into the block with depth h, the mutual friction f stops the bullet inside the block and both come to rest. **(A)** Write down the equations of motion for the bullet and the block and find the solutions for their motions. **(B)** Does your answer obey the conservation of momentum? Explain your reasoning in glory details with clear logic flows.

4. Center of mass (20%) A horizontal rod of length L and mass M has a non-uniform linear density, $\lambda(x) = \lambda_0 x^2$, where $0 \leq x \leq L$ and λ_0 is some constant. **(A)** Express λ_0 in terms of the mass M and the length L. **(B)** Find the center of mass of the non-uniform rod. Compare your answer with that for a cylindrical cone taught in class and discussion their connection in details.

5. Conservation of momentum (20%) Write a short essay (less than two pages of your answer booklet) on *"The relation of Newton's third law and the conservation of momentum"*. Use the elastic collision of two blocks with a spring between them as a demonstrating example. You may emphasize how EOM gives rise to all collision details beside the conservation of momentum. You may also discuss the breakdown of Newton's third law if you really understand what you are talking about. Try to arrange all your personal understanding in logical order and cook up a readable essay on the subject.

6. Energy-momentum vector (Bonus 20%) In special relativity, time and space can be combined together into a four-vector (ct, x, y, z). Lorentz transformation thus takes the 4×4 matrix form,

$$\boldsymbol{L} = \begin{pmatrix} \gamma & -\gamma v/c & 0 & 0 \\ -\gamma v/c & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

relating the four-vectors (ct, x, y, z) and (ct', x', y', z'). It turns out that all four-vectors are related by the same Lorentz matrix $\boldsymbol{L}$. For instance, energy and momentum form another four-vector (E, cp_x, cp_y, cp_z) and transform according to the same matrix $\boldsymbol{L}$.

Let us consider the Jack-and-Jill inertia frames with relative velocity $\boldsymbol{v} = (v, 0, 0)$ again. Jack shoots out a periodic electromagnetic wave along the positive x-axis, which can be viewed as a steam of photons with energy E and momentum $\boldsymbol{p} = (p, 0, 0)$. According to Maxwell's equations, the energy and the momentum are related, $E = pc$. Jill will observe a different energy-momentum vector $(E', cp'_x, cp'_y, cp'_z)$ after Lorentz transformation. Using Einstein's relations for photons,

$$E = hf, \qquad p = \frac{h}{\lambda},$$

where h is the Planck constant and f, λ denote frequency and wavelength respectively, find the relation between f', f and also that between λ', λ. Explain the consequence of your results and how it may deviate from the classical wave theory.