# Thermal and Statistical Physics Spring 2021
# Midterm Exam

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**You must show your work. No credits will be given if you don't show how you get your answers.**

**You may use the following formula:**

• The first law the thermal dynamics (for a system with fixed number of particles) can be written as $TdS = dU + PdV$.

• The Helmholtz free energy is $F = U - TS$.

• The mean free path $l$ for a system with particles of diameter $d$ and number density $n$ is $l = \dfrac{1}{n\pi d^2}$.

• The flux $\vec{J}_A$ is defined as the amount of the quantity $A$ passing through a cross section that is normal to the direction of the flow of $A$ per unit time per unit area. The continuity equation is

$$\frac{\partial \rho_A}{\partial t} + \nabla \cdot \vec{J}_A = 0,$$

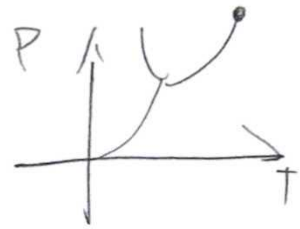
where $\rho_A$ is the density of $A$.

Ficks 2nd law ?

• Fick's law of diffusion: $\vec{J}_n = -D\nabla n$, where $n(\vec{r}, t)$ is the number density of particles and $\vec{J}_n$ is the flux of $n$. The diffusion coefficient $D = \dfrac{1}{3}\bar{v}l$, where $\bar{v}$ is the average speed.

• The viscosity coefficient $\eta$ is defined as $P_{ij} = \dfrac{F_i}{A_j} = -\eta \dfrac{dv_i}{dx_j}$, where $P_{ij}$ is the $i$-component of the shear force ($F_i$) per unit area normal to the $j$-direction ($A_j$), and $v_i$ is the velocity in the $i$-direction.

• The thermal conductivity $\kappa$ is defined as $\vec{J}_u = -\kappa \nabla T$, where $\vec{J}_u$ is the heat flux and $T$ is the temperature.



10 1. Consider a system with $N$ particles and volume $V$. It obeys the van der Waals equation

$$(P + a\frac{N^2}{V^2})(V - Nb) = Nk_BT,$$

where $P$ is the pressure and $T$ is the temperature. $a$ and $b$ are positive constants. (2 points each)

30

(a) Briefly explain why this system has two phases below certain temperature $T_c$?

(b) Briefly describe what "Maxwell equal-area construction" is and why it is needed to modify the van der Waals equation.

(c) Prove that the internal energy $U$ is a function of $V$ and $T$. 6 15

(d) Compared with the ideal gas equation $PV = Nk_BT$, the constant $b$ is a correction for the finite volume of a particle, and $a$ models the interactions among the particles. Is the interaction attractive or repulsive? Use your result from part (c) to explain. $\frac{1}{V^2}$

(e) Consider another system which obeys

$$(P - a\frac{N^2}{V^2})(V - Nb) = Nk_BT,$$

$V^{-2}$

does the system have a critical point? Explain why. $-2V^{-3}$

[*Note:* Partial credits may be given if you can explain in *physics* but don't know how to show it explicitly by math.]

12 2. **Ferroelectric** materials are ionic solids which can have non-zero electric polarization below some temperature without external electric fields. When there is no external electric field, the Landau free energy $F_L$ of a ferroelectric system can be written as

$$F_L = \frac{a}{2}m^2 + \frac{b}{4}m^4 + \frac{c}{6}m^6 + O(m^8),$$

where $m$ is the electric polarization (order parameter), $b < 0$, $c > 0$ and that's why we need to include the $m^6$ term.

(a) Explain why (in *physics*, not math) there are only terms in even powers of $m$? (2 points)

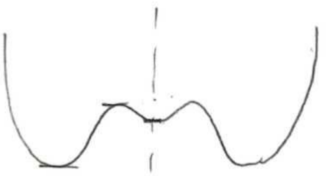
(b) Let $a = a_0(T - T_c)$ for some $a_0 > 0$. Show that when $T < T_c$ the system has a non-zero polarization $m$ at equilibrium. (3 points)
[*Hint:* Recall the formula for the roots of a quadratic equation $ax^2 + bx + c = 0$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.]

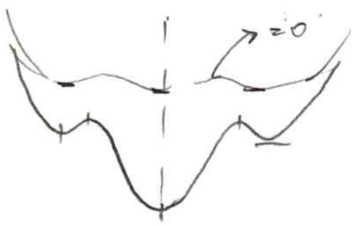
(b) $\begin{cases} m = 0 \\ m \neq 0 \rightarrow a + bm^2 + cm^4 = 0 \end{cases}$, $m^2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2c}$

$\boxed{\text{代入}}$ or $\boxed{\frac{\partial F}{\partial m}}$

$b^2 - 4ac > 0$

$m = 0$ or $m = \pm m_1$. $m_1^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$ $m_+ > m_- > 0$






(c) When $T > T_c$, the situation is a bit more complicated. Consider all possibilities of the size of $b$ ($|b|$), and show that $m \neq 0$ for $T_c < T < T_{tr}$ and $m = 0$ when $T > T_{tr}$ for some transition temperature $T_{tr}$. (5 points)

[*Note:* Here we are referring to the value of $m$ at the (true) stable state of the system.]

(d) [Bonus] Is the transition at $T = T_{tr}$ first order or second order? Explain your reasoning. (2 points)

[*Hint:* You might find it easier to make a few sketches of $F_L$ to explain your answers.]

5 3. From the Fick's law of diffusion, one may generalize this definition to $\vec{J}_A = -D\nabla\rho_A$ (correct up to an overall factor), with $A$ representing a physical quantity and $\rho_A$ is the density of $A$. Therefore one can deduce the relation of diffusion to other transport phenomena.

(a) From the definition of the viscosity coefficient $\eta$, explain why $\eta = D\rho$, where $\rho$ is the mass density. (2 points)

(b) Use the definition of the thermal conductivity and the continuity equation, show that

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T,$$ 2

where $D_T = \kappa / C_V$, with $\kappa$ as the thermal conductivity and $C_V$ is the heat capacity per unit volume. (2 points)

(c) In 1660 Robert Boyle performed an experiment to see how the swing of a pendulum slows down (damped) by the surrounding air. He put the pendulum in a box and used a pump to remove the air inside. He was surprised to find out that no observable change in the rate of damping of the swings when the pump was working. Can you explain why? (1 point)

$>1$ $7.5$

$13 + 8 + 6.5 + 1$

$= 28.5$

31.5