Electrodynamics-2, Final, June 22, 2021.

Important:

When you are done, scan your answer sheet into a PDF file and submit it to https://lms.nthu.edu.tw/course/51257 **by 12:00**.

(In the rare case that you have difficulties doing so, e-mail your PDF file to me by the deadline.)

1. (10%) What is the so-called *twin paradox*? How to resolve it?

2. (10%) What is the so-called *hidden momentum*? Name one example and briefly explain how the hidden momentum helps resolve the problem.

3. (15%) There are two electric dipoles oscillating in-phase at the same frequency ω. You can move and rotate them freely around the origin. If you want to generate a right-handed circular EM wave for an observer located at the x-axis in the distance much larger than the wavelength, what should you do?

4. (45%) Describe and elaborate, as much as you can, the three most exciting things you have learned in EM2.

5. (20%) Charged particles in the universe can be accelerated up to enormous energies. These energetic particles rain down on the Earth as "cosmic rays". Particle physics determines that there is an ultimate limit to the energy of cosmic rays due to their collisions with the "cosmic microwave background radiation"(CMBR) that permeates the entire universe. Suppose the cosmic rays are protons(with rest mass $m_p \sim 1.67 \times 10^{-27}$kg.) If the CMBR consists of photons of energy 2.5×10^{-4} eV.[This is the energy corresponding to the temperature ($E = K_B T$) of $2.9°K$ that characterizes the CMBR.] The reaction that degrades the energy of the cosmic ray is the pion production
$$\gamma + p \to \pi^0 + p\,,$$
where γ is the CMBR photon and the π^0 is a "meson" with rest mass $m_{\pi^0} \simeq m_p/7$.
(a) What is the threshold energy of the proton (in eV, 10% numerical accuracy), i.e. what is the minimum proton energy necessary for the reaction to go in the frame where the photons have energy 2.5×10^{-4} eV?
(Hints (1): Use of Lorentz invariants will make this problem much simpler. (2): Consider the threshold condition in the center of mass of the final proton and pion. What is their configuration at threshold? (3): Get an approximation for $(1 - \beta)$ for $\gamma \gg 1$ in terms of γ.)
(b) Suppose the proton has just enough energy to make the reaction possible. What is its energy after the collision(in eV, 10% numerical accuracy)?