Quantum Physics I Midterm Exam

2022/11/23 8:00–9:50

1. A room contains 16 people, whose ages are shown in the following table.

Ages	Number of people
25	2
26	1
28	3
30	2
32	5
34	3

(a) If you select one individual at random from this group, what are the probabilities of getting each of the 6 ages? (6 pts.)

(b) What are the most probable, median, and average ages? (6 pts.)

2. At time $t = 0$ a particle is represented by the wave function,

$$\Psi(x,0) = \begin{cases} Ax, & \text{if } 0 \le x \le a \\ B(b-x), & \text{if } a \le x \le b \\ 0, & \text{otherwise.} \end{cases}$$

(a) Normalize $\Psi(x,0)$ to find A and B. (6 pts.)
(b) Where is the particle most likely to be found at $t = 0$? (2 pts.)
(c) What is the probability of finding the particle to the **right** of a? (4 pts.)

3. A particle is in the following state at time $t = 0$.

$$\Psi(x,0) = \begin{cases} A(a^2 - x^2), & \text{if } -a \le x \le a \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the normalization constant A. (5 pts.)
(b) What is the expectation value $\langle x \rangle$ of its position? (5 pts.)
(c) What is the expectation value $\langle p \rangle$ of its momentum? (5 pts.)

$\frac{1}{2} 2x\,dx$ $\quad \frac{dx^2}{dx} = 2x$ $\quad \int e\, dx^2$ $\quad dx^2 = 2x\,dx$ $\quad \frac{dx^2}{dx} = 2x$

$\frac{de^{-\alpha x^2}}{dx} = -2\alpha x e^{-\alpha x^2}$

4. A particle is confined in the following potential.

$$V(x) = \begin{cases} 0, & \text{if} -a \le x \le a \\ \infty, & \text{otherwise.} \end{cases}$$

$\int x e^{-\alpha x^2} dx = \frac{1}{-2\alpha} \int -2\alpha x\, e^{-\alpha x^2} dx$

Find the allowed energies and the corresponding (normalized) wave functions. (20 pts.)

5. Find the expectation value of the potential energy in the nth excited state of the harmonic oscillator. (15 pts.)

6. A particle approaches the following step potential from $x < 0$,

$k = A \sin k$ $\quad \frac{n\pi}{a} x$

$$V(x) = \begin{cases} 0, & \text{if } x \le 0 \\ V_0, & \text{otherwise.} \end{cases}$$

$n = 1$ $\quad \frac{2mE}{\hbar^2} + \frac{2m(E - V_0)}{\hbar^2}$

(a) Find the reflection coefficient if the particle's total energy $E < V_0$? (10 pts.)

(b) Find the reflection coefficient if the particle's total energy $E > V_0$? (10 pts.)

7. A particle in the harmonic potential starts out in the state

$\psi_n = \frac{1}{\sqrt{n!}} (\hat{a}_+)^n \psi_0(x)$

$$\Psi(x, 0) = A[2\psi_0(x) - 5\psi_1(x)],$$

$c_0 = \alpha$

where $\psi_0(x) = (m\omega/\pi\hbar)^{1/4} e^{-m\omega x^2/2\hbar}$ and $\psi_1(x) = (2m\omega/\hbar)^{1/2} x\psi_0(x)$ are the wave functions of the ground and first excited states, respectively.

$\sqrt{\frac{m\omega}{\pi\hbar}} \cdot \sqrt{\pi} \cdot \sqrt{2}$

(a) Find A. (4 pts.)

$\frac{2m\omega}{\hbar} = \alpha$ $\quad 2\hbar$ $\quad E_0 = \frac{-iE_n}{\hbar}$

(b) Construct $\Psi(x, t)$ and $|\Psi(x, t)|^2$. (4 pts.)

(c) Find $\langle x \rangle$. (4 pts.)

(d) If you measure the energy of the particle, what values might you get and with what probabilities? (4 pts.)

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi, \quad -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = E\psi, \quad \int_{-\infty}^{\infty} x^{2n} e^{-\alpha x^2} dx = \sqrt{\pi}(-1)^n \frac{\partial^n}{\partial \alpha^n} \alpha^{-\frac{1}{2}}$$

$$\int \sin^n(x) dx = -\frac{1}{n} \sin^{n-1}(x) \cos(x) + \frac{n-1}{n} \int \sin^{n-2}(x) dx, \quad \hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x}$$