Final for Thermal and Statistical Physics

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1. Fermions in a box (20%) Two fermions are confined in an infinite potential well of unit length $L = 1$, occupying the following single-particle orbitals, 机率密度分部

$2\sin^2(\pi x)$

$$\psi_1(x) = \sqrt{2}\sin(\pi x), \qquad \psi_2(x) = \sqrt{2}\sin(2\pi x).$$

Compute the probability distribution $P(x_1, x_2)$ for the two-fermion systems. As shown in the figures below, which one should be the correct probability distribution $P(x_1, x_2)$ for the fermions?

$\sqrt{2}$ 1.414

(A) x_2 **(B)** x_2

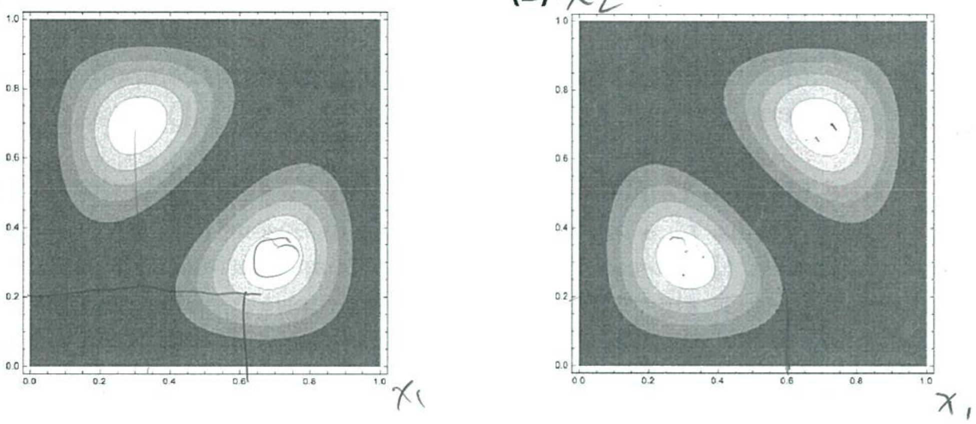

Figure 1: Probability distribution $P(x_1, x_2)$ presented as contour plot. Lighter color indicates larger probability density while darker color means smaller probability density.

2. Scattered bosons (20%) Two incident bosons hit the splitter from the left and get scattered into different states. The scattering matrix of the splitter acts as

$$R(\cdot) \longrightarrow -\frac{1}{2}R(\cdot) + \frac{\sqrt{3}}{2}L(\cdot), \qquad L(\cdot) \longrightarrow \frac{\sqrt{3}}{2}R(\cdot) + \frac{1}{2}L(\cdot),$$

where the dot stands for either particle 1 or 2. Compute the probabilities for all possible outcomes after the bosons passing through the splitter. How is your answer different from the classical expectation? Explain your answer.

解釋與 classical 差別

$\Big[\ -\frac{1}{2}$

用量子干涉估 BEC
畫出 BEC 相

3. Condensation temperature (20%) Two length scales are important when estimating the onset of Bose-Einstein condensation (BEC): de Broglie wave length and the average intermolecular distance. The thermal energy of a free particle is $\varepsilon = \frac{3}{2}\tau$. One can thus estimate the thermal de Broglie wave length λ_T accordingly. The average intermolecular distance a can be estimated from the particle density $n = N/V$. When the thermal de Broglie wave length λ_T is comparable to the intermolecular distance a, quantum effect shall be important, providing a simple estimate of the BEC transition temperature. Sketch the BEC phase diagram obtained by the above argument in the $\tau - n$ plane.

$a \to$ 密度

$\lambda \to T'$

4. Fermi energy (20%) Electrons with spin $s = \frac{1}{2}$ confined in two dimensions can be described by the following Hamiltonian,

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m}.$$

$\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$ 2維 $n_x^2 + n_y^2$

For a non-interacting Fermi gas of particle density n, compute the corresponding Fermi energy ε_F at low temperatures.

$\tau = 0$

5. Bose-Einstein condensation (20%) Upon cooling below the condensation temperature τ_c, a macroscopic fraction of bosons falls into the ground state, forming the so-called Bose-Einstein condensation. To describe the phase transition, one can construct the appropriate order parameter and study how it evolves with temperatures. Organize your understanding about BEC phase transition in clear logic and write down a short essay on this important and interesting phenomenon.

小論文