General Physics II – 1st Midterm Exam

Exam Time: 10:10AM - 12:00PM

Please explain all your answers and reasoning in a clear fashion.

Coulomb's constant: $k = 1/4\pi\epsilon_0$. Coulomb's law: $\vec{F} = (1/4\pi\epsilon_0)(q_1 q_2/r_{12}^2)\,\hat{r}$. Gauss's law: $\oint \vec{E}\cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, $\nabla\cdot\vec{E} = \rho(\vec{r})/\epsilon_0$. Electric potential energy density: $\epsilon_0 E^2/2$. For a simple RC circuit that obeys $\mathcal{E} - IR - Q/C = 0$, the solution is $Q(t) = \mathcal{E}C(1 - e^{-t/RC})$.

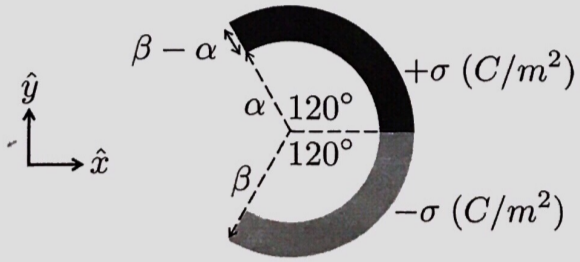
Q1. (60 pts) Fundamentals.

(a) (10 pts) Three conducting spheres of radius R, $2R$ and $3R$ initially carry charges $-Q$, Q and $3Q$, respectively. These spheres are separated far away and then are connected by wires. **(i)** Which sphere has the largest electric potential after the wires are connected? . **(ii)** Find the ratio of surface charge densities after the wires are connected $\sigma_1 : \sigma_2 : \sigma_3$.

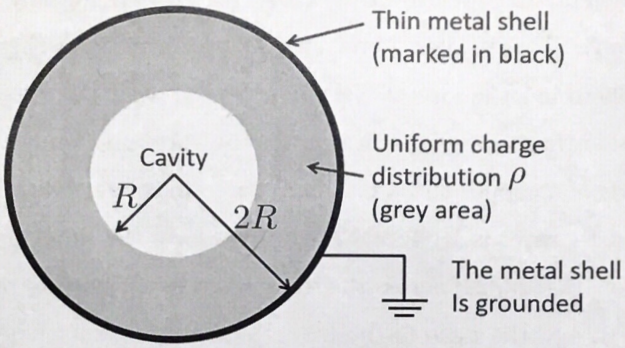
(b) (20 pts) The ends of a copper and a platinum wire are connected together. The ratio of the length of Cu and Pt wires is 1 : 4, and both wires have the same cross-section area. A potential difference of V is applied across the free ends. Given that the ratio of conductivity for Cu and Pt is 6 : 1, and the ratio of the free electron density for Cu and Pt is 4 : 3, **(i)** find the ratio of power dissipated in Cu and Pt wires. **(ii)** Which wire has a larger electric field within it? **(iii)** What is the ratio of (current density) through Cu and Pt wires? **(iv)** What is the ratio of drift speed of free electrons within Cu and Pt wires?

(c) (10 pts) Start with the integral form of the Gauss's law $\oint \vec{E}\cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$ to derive its differential form $\nabla\cdot\vec{E} = \rho(\vec{r})/\epsilon_0$.

(d) (20 pts) Given two concentric rings of surface charge density $+\sigma$ and $-\sigma$ respectively, see figure below. The concentric rings have the inner radius of α and outer radius β. Find the electric field at the center of the circle.



Q2. (30 pts) Charges are distributed uniformly throughout a **sphere** of a charge density ρ between two concentric spheres of radius R and $2R$. And it is enclosed by a thin metal spherical shell of radius $2R$ (negligible thickness), as shown in the figure below. The metal spherical shell is grounded. **(i)** Plot the magnitude of the electric field as a function of the radial distance r. **(ii)** Find the surface charge density on the inner surface of the spherical metal shell. **(iii)** By defining the ground potential to be 0, what is electric potential in the region $r < R$? **(iv)** How would you evaluate the electric potential energy of the system? Simply **describe** your method and **DO NOT calculate it.**



Q3. (30 pts) A charging RC circuit consists of a battery with emf $\mathcal{E}$, a resistor R, and two capacitors C_1 and C_2, see figure below. The two capacitors are identical when there are no dielectrics inserted (that is $C_1 = C_2 = C_0$ when placed in vacuum). In order to increase the capacitance, we place a dielectric material of the dielectric constant κ in C_2. At $t = 0$, there are no charges on capacitors, and we close the switch to charge these capacitors. Please only use $\mathcal{E}$, R, C_0, κ, Q_1, Q_2, I, I_1, and I_2 in answering the following questions. **(i)** Write down all Kirchhoff's rules for this circuit. **(ii)** Derive quantitative expressions for $Q_1(t)$ and $Q_2(t)$. **(iii)** What are the time constants for charging capacitors C_1 and C_2.