Mathematical Physics 311: Mid-term Examination (4-28-2021)

2:20pm – 4:20pm

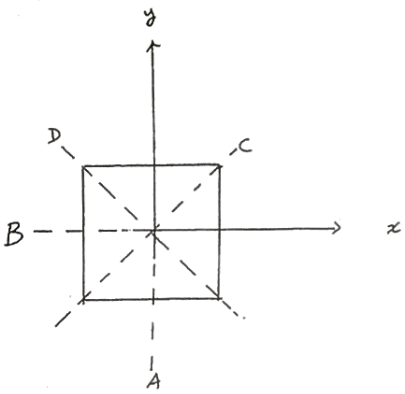
(1) 20 pts

The group D_4 has 8 group elements, which leaves a square invariant (see the figure).

(a) Construct the 3×3 matrix representation of the group D_4 using the basis vectors $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$, where the z-axis is the four-fold symmetry axis.

(b) Find the characters of the three dimensional representation obtained in part (a).

(c) Use the following character table to show that it can be reduced to a two-dimensional and a one-dimensional irreducible representation.



	$\{E\}$	$\{C_4^2\}$	$\{C_4, C_4^3\}$	$\{A, B\}$	$\{C, D\}$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
B_1	1	1	-1	1	-1
B_2	1	1	-1	-1	1
E	2	-2	0	0	0

(2) 30 pts

Consider the 6-dimensional function space V consisting of homogeneous polynomials of degree 2 in real variables (x, y, z):

$$f(x, y, z) = ax^2 + by^2 + cz^2 + dxy + eyz + gzx$$

where a, b, c, d, e, g are complext constants. If (x, y, z) transform under the dihedral group D_4 as the coordinates of a 3-vector, then we obtain a 6-dimension representation of D_4 on V.

(a) Find the characters of the representation for each class.

(b) Decompose the representation into irreducible representations (use the table in question (1)).

(c) Determine the basis of each irreducible representation that you found in part (b).