$\cancel{2}\pi \int_0^R \frac{ar^3}{\cancel{2}\epsilon_0}\,dr = \frac{\pi a}{\epsilon_0}\,\frac{R^4}{4}$

Electrodynamics-1 (2020F), Final exam, Jan. 12, 2021.

All answers must be supported by detailed calculation or reasoning. It is your responsibility to clearly state the logic of your answers. No credit points will be granted if you fail to do so.

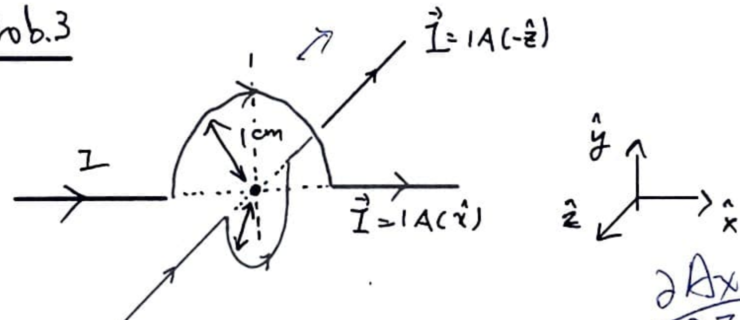
1. (6%) Name one paramagnetic material. Similarly, name one each for the diamagnetic and ferromagnetic material.

2. (8%) What do the electric/magnetic susceptibilities limits $\chi_e \to 0, +\infty$ and $\chi_m \to 0, -\infty$ correspond?

3. (10+5+5%) Two infinitely long wires both carry a current $I = 1$A. As shown below, they are bent to have a semi-circular detour around the origin, with radius 1cm.
   (a) Calculate the magnetic field at the origin. Give your answer both in T and Gauss. Also, note it is a vector.
   (b) A tiny magnet with a magnetic dipole moment $(10^{-9}/\pi)C \cdot m^2/sec$ is sitting at the origin and pointing to $\hat{y}$. How much energy is needed to rotate it to the $\hat{z}$ direction?
   (c) A particle with charge Q and mass m is put close to the origin with an initially tiny velocity in the y-direction. Describe its later motion.

$\mu_0 = ?$

Prob.3



$\frac{\partial A_x}{\partial z} = 0$

$\frac{\partial A_z}{\partial y}\hat{x} - \frac{\partial A_y}{\partial z}\hat{x} = B_0$

4. (5+5%) (a) What is the Coulomb gauge?
   (b) Construct a vector potential in the Coulomb gauge for a constant magnetic field $\vec{B} = B_0\hat{x}$, .

5. (10+10%) An infinitely long circular cylinder of radius R carries a constant magnetization $\vec{M} = M\hat{z}$ parallel to its axis.
   (a) Calculate the bound current densities $\vec{J}_b$ and $\vec{K}_b$.
   (b) Find the magnetic field inside and outside the cylinder.

$B \cdot 2\pi r = \mu_0 \int \vec{K}\cdot$

6. (10+10+10+10%) A cylinder of radius R and infinite length is made of a permanently polarized dielectric with $\chi_m = 0$. The polarization vector is given by $\vec{P} = a\vec{r}$, where a is a constant, and $\vec{r}$ is the radial vector in the cylindrical coordinates. The cylinder rotates around its axis with angular velocity ω, and $\omega R \ll c$.
(a) Find the bound charges σ_b and ρ_b.
(b) Find the electric field $\vec{E}$ inside and outside the cylinder.
(c) What is the total electromagnetic energy stored per unit length of the cylinder before the cylinder started spinning? (Note, there are two different kinds of energy. How do you understand the results?)
(d) Find the magnetic field $\vec{B}$ inside and outside the cylinder.

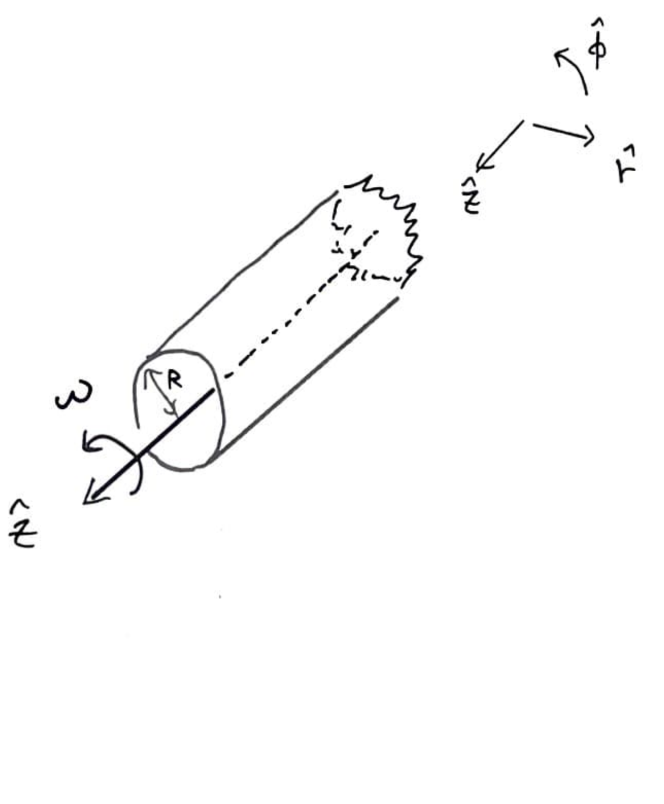
7. (10+10) An infinitely long cylindrical wire of permeability μ and radius R carries a steady current I along the z-axis. It is known that the current density is linearly proportional to r, the distance to the axis.
(a) Find $\vec{H}$ inside and outside the wire.
(b) Same as in (b), find $\vec{B}$ everywhere.

8. (10%) What is the total magnetic dipole moment if the current flows around the cubic edges in the way shown in the figure?

ar^3

$3ar^2$ $3ar^2$

Prob. 6



Prob. 8