

Mid Examination 1  
Applied Mathematics II  
29 October 2014, 8.00 - 10.00 am

Answer all questions. Each question carries 25 marks.

Simple calculator is allowed. No use of telephone.

You may answer in English or Chinese.

1. (a) Find the general solution of

$$(1 + x^2)y' + 6xy = 2x \quad (1)$$

- (b) Find the general solution of

$$y'' + y' - 2y = 18xe^x. \quad (2)$$

2. Use the method of variation of parameters to find the solution of

$$y'' - y = \operatorname{sech} x \quad (3)$$

3. Determine the Green function for the following differential equations and write down a particular solution of the differential equation in terms of Green function.

- (a)

$$y'' = f(x) \quad \text{with } y(0) = 0 \text{ and } y'(1) = 0, \quad (4)$$

defined on the interval  $[0, 1]$ .

- (b)

$$y'' - y = f(x), \quad y(\pm\infty) = 0, \quad (5)$$

defined on  $(-\infty, \infty)$ .

4. Consider the equation

$$z^2 y'' + z y' + (z - 1)y = 0. \quad (6)$$

- (a) What kind of point is  $z = 0$ ? i.e. is it an ordinary point or a singular point?  
 (b) The differential equation has two solutions. Solve for one of the solutions  $y_1(z)$  near  $z = 0$  using the method of generalized power series.  
 (c) Using the method of Wronskian, determine the second solution in the form

$$y_2(z) = \alpha[y_1(z) \ln z + h(z)], \quad (7)$$

where  $\alpha$  is a constant and

$$h(z) = \sum_{n=-1}^{\infty} b_n z^n. \quad (8)$$

Determine the coefficients  $b_{-1}, b_0, b_1$ .