Study of Mechanical Oscillations by Using Torsion Pendulum

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The torsion pendulum is a way for us to study S.H.M, damped oscillation and forced oscillation with external sinusoidal driving force.Meanwhile,we can do the analysis of the torsion constant κ from two different methods and calculate the values for damping coefficient “ λ ” and resonant frequency “ ω_R ” .

1. Introduction

For this experiment, the main target is to operate the torsion pendulum and to study many kinds of oscillations,such as Simple Harmonic Motion underdamped oscillation as well as the forced oscillation with external sinusoidal driving force. The “ Oscillation Phenomenon ” refers to the angular displacement θ of the torsion pendulum owing to rotating around an axis throughout the object, so we can write down this relation:

$$\boldsymbol{\tau} \equiv \frac{d\boldsymbol{L}}{dt} = -\kappa\boldsymbol{\theta} \qquad (1)$$

where $\boldsymbol{\tau}$,the derivative of angular momentum **L** with respect to time, is the torque due to rotation, and κ is the torsion constant.

Recall that the definition of angular momentum:

$$\boldsymbol{L} \equiv \boldsymbol{r} \times \boldsymbol{P} = \{I\}\boldsymbol{\omega} = \{I\}\frac{d\boldsymbol{\theta}}{dt} \qquad (2)$$

where **r** is the position vector,**P** is the linear momentum, $\boldsymbol{\omega}$ is the angular velocity and $\{I\}$ is the moment of inertia,which is a tensor [1].

Given that the rotation process is under the condition that the rotational axis is fixed and passes through the origin in the Cartesian coordinate we had selected,and then the tensor of the moment of inertia can be reduced into a simple scalar. Therefore we can obtain the differential equation by combining (1) and (2):

$$\frac{d^2\theta}{dt^2} + \frac{\kappa}{I}\theta = 0 \qquad (3)$$

The ODE is similar to the ODE of S.H.M , so we can easily gain its general solution and period T:

$$T = 2\pi\sqrt{\frac{\kappa}{I}} \qquad (4)$$

Besides,when we consider the damping force which is proportional to angular velocity ,the damping coefficient is λ as well as applying an external sinusoidal driving force $F_0 cos\omega_d$t, then the new differential equation will be this form:

$$\frac{d^2\theta}{dt^2} + \frac{\lambda}{I}\frac{d\theta}{dt} + \frac{\kappa}{I}\theta = \frac{F_0}{I}cos\omega_d t \qquad (5)$$

For inhomogeneous ODEs, the general solution can be viewed as a complementary solution(or called transient solution) pluses the particular solution(or called steady-state solution) which depends on the source term $\frac{F_0}{I}cos\omega_d t$.

For a long enough time, the transient solution would “die out”,hence we just need to interest the particular solution[2]:

$$\theta(t) = \theta_m \sin(\omega_d t - \varphi) \qquad (6)$$

where
$$\theta_m = \frac{F_0}{I\sqrt{\left(\omega_0^2 - \omega_d^2\right)^2 + (\lambda\omega_d)^2}} \qquad (7)$$

$$\varphi = tan^{-1}\left(\frac{\lambda\omega_d}{\omega_0^2 - \omega_d^2}\right) \qquad (8)$$

and ω_0 is the natural frequency($\omega_0^2 \equiv \frac{\kappa}{I}$).

By the First Derivative Test, we can find the specific angular frequency occurring when the maxima of θ_m is achieved (The first derivative of θ_m should be zero):

$$\omega_R = \sqrt{\omega_0^2 - \frac{\lambda^2}{2}} \qquad (9)$$

where ω_R is so called the “Resonat Frequency”.

Moreover, if we say that the specific two amplitudes which is 0.707($1/\sqrt{2}$) times of the maximum of θ_m are called ω_+ and ω_-, then We can define the width of frequency Δω

$$\Delta\omega \equiv \omega_+ - \omega_- \approx \lambda \qquad (10)$$

Note that the roughly approximation is under the circumstance that λ is quite small.Finally, we can define the quality factor Q as:

$$Q \equiv \frac{\omega_R}{\lambda} \approx \frac{\omega_0}{\Delta\omega} \qquad (11)$$

Q is often considered to be the sensity of the system reacting with the damping force. Oscillation is very important in Physics as well as common in nature.Mechanism, electronics, optical, biology,astrophysics and even quantum mechanism, all of them are related to either microscope or miniscope oscillaions. It's no exaggeration to say that the concept of oscillations is the most significant thing for us on the way to learn Physics.

2. Method

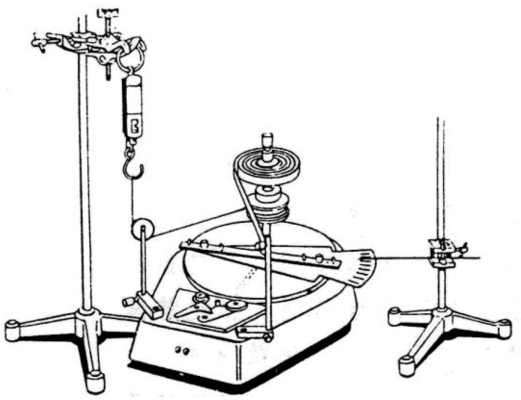

Fig.1:Set up for apparatus

In the experiment, we used the following equipment: a system of a disk upon other suspended by air, torsion pendulum apparatus, spiral spring, a spring balance, a computer, a motor, a rotation sensor, a magnet, a strand of thread and a DC supplier.

This experiment consisted of three main sections, for the first part the purpose was to measure the torsion constant κ.

Setting up the system in such a way that when we placed a mass on a thread attached to the central shaft which acts as the rotational axis for the disk. Then we used the spring balance to give five different constant forces which would pull the thread so that the disk start to rotate with an angle. Record the magnitude of the forces and measure the corresponding angles through DataStudio on computer.Finally, plotting torques against the angles displacement ($\tau - \theta$ graph). From the slope of the resulting straight line would be the value for the torsional constant κ.

For the second part of the experiment were using (4) to find the torsion constant κ and proceeding to compare this value with that obtained in the first part of the experiment. This can be viewed as the determination of (4) in turn.

During the process, we primarly need to measure the period for the oscillation of the disk without extra masses so that we can use (4) to obtain the moment of inertia of the disk. Next, we measure the periods for the oscillation of the disk with different combinations of additional masses.

Finally, plotting th square of periods against the additional moment of inertia ($T^2 - \Delta I$ graph). From the relation or the slope of the resulting straight line would be the value for the torsional constant κ.

In the third and finl part of the experiment are to study the damped and forced oscillations. We first aim to find the value of the damping coefficient λ by using the following relation:

$$\ln\left(\frac{\theta_A}{\theta_B}\right) = \frac{\lambda}{2}(t_B - t_A) \qquad (12)$$

By plotting the natural logarithm of the angle ratios against the time differences then we obtained the value for λ from the slope of the resulting straight line.

Next,we turned to seek the relation between the frequency and amplitude. Turn on the motor which is connected with the rotational axis through a thread, then the driving force would make the system start rotating.We found it by plotting the amplitude against the frequency and then proceeding to use Datastudio to do the curve-fitting to gain a function relation.

It turned out that giving Lorentzian distribution, which is quite similar to a Gaussian distribution but actually different,though.Meanwhile,we can get the quality factor Q from (11).

Finally, we would obtain another values of λ and Q by changing the position of the magnet which was used to supply the damping force and repeat the following.

3. Results

For the first part, we calculated the torsional constant κ from the slope of the straight line in Fig.1:

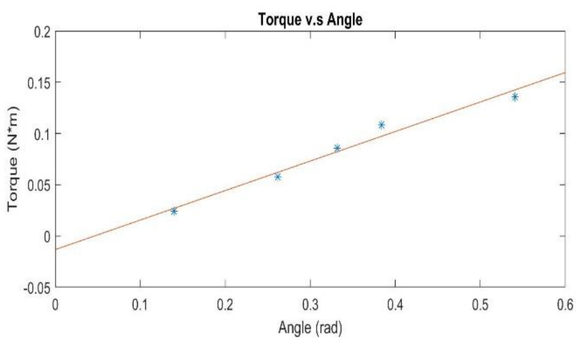

Fig.1: The torque acted on the disk against the angle rotated.

Curve-fitting:

f(x) = $mx + b$ = $0.288x - 0.0133$

The result κ was approximately 0.288 N·m / rad.

For the second part, we calculated the torsional constant κ the relation for the slope of the straight line in Fig.2:

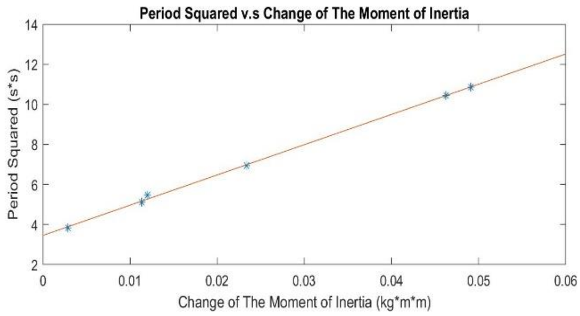

Fig.2: The torque acted on the system against the angle rotated.

Curve-fitting:

f(x) = $mx + b$ = $150.9x + 3.457$

Using the relation below:

$$T^2 = \frac{4\pi^2}{\kappa}(\Delta I + I_0) \qquad (13)$$

where ΔI is the extra moment of inertia due to the combination of mass evaluated by Parall Axis Theorem, I_0 is the moment of inertia for the disk only.

The result κ was approximately 0.262 N·m / rad.

For the third part, we first figured out the damping coefficient and then plotted amplitude of the forced oscillation against the angular frequency to observe what the specific angular frequencies were the resonant frequencies achieved:

Case I:

$\lambda_1 = 0.25\text{kg/s}$; $\omega_R = 2.936\ rad/s$

$\omega_+ = 3.133\ rad/s$; $\omega_- = 2.806\ rad/s$

$\Delta\omega \equiv \omega_+ - \omega_- = 0.326\ rad/s$; $Q \equiv \frac{\omega_R}{\lambda} = 9.006$

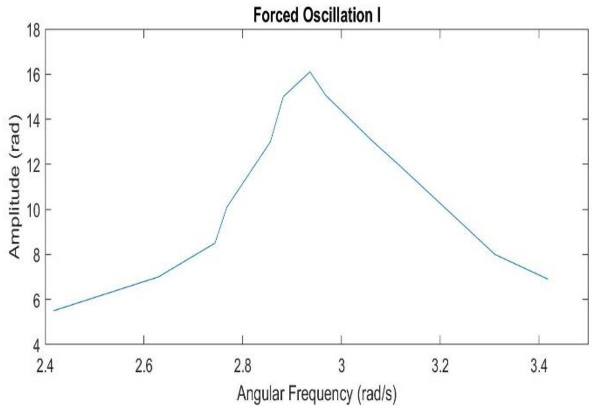

Fig.3:The amplitude against the angular frequency for Case I

Curve-fitting(Origin Pro):

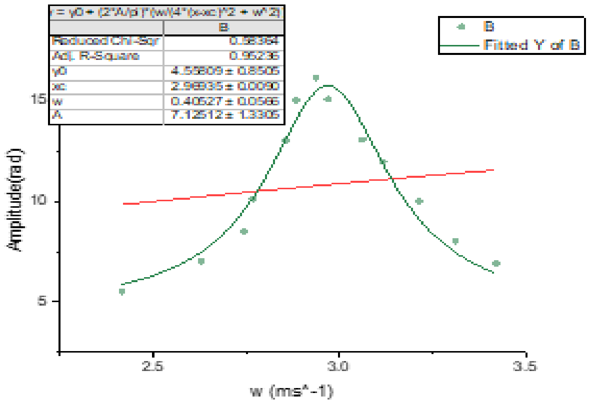

Fig.4:The amplitude against the angular frequency for Case I with curve-fitting.

Case II:

$\lambda_2 = 0.322\text{ kg/s}$; $\omega_R = 2.843\ rad/s$

$\omega_+ = 3.038\ rad/s$; $\omega_- = 2.658\ rad/s$

$\Delta\omega \equiv \omega_+ - \omega_- = 0.38\ rad/s$; $Q \equiv \frac{\omega_R}{\lambda} = 8.829$

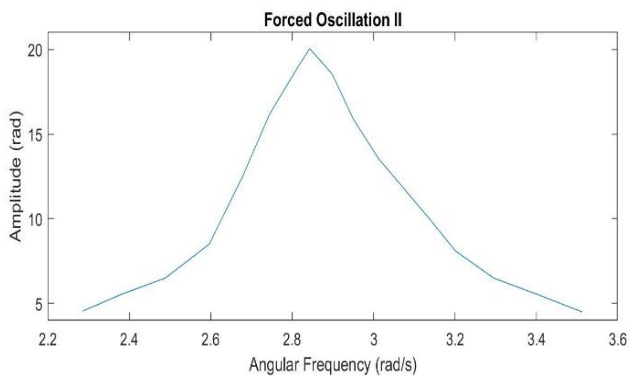

Fig.5:The amplitude against the angular frequency for Case II

Curve-fitting(Origin Pro):

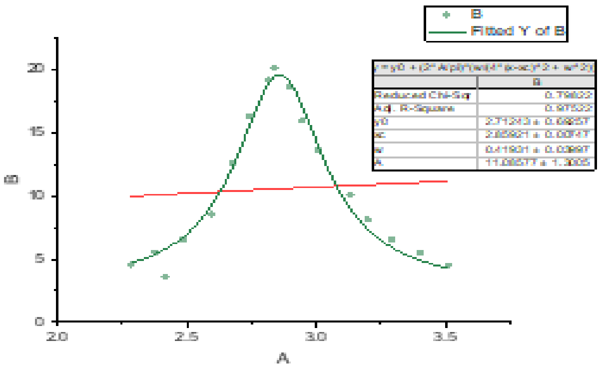

Fig.6:The amplitude against the angular frequency for Case II with curve-fitting.

4.Discussion

In the experiment we first determined the value of the torsion constant κ with two different methods . We found that a little difference between the values obtained from the first part and the second part. We thought the result from the first part was more accurate than that from the second part .

One reason is that we can directly measure the κ by pulling the spring balance and recording the change of angle rotated via DataStudio,which would be more accurate than that we calculated by measuring periods for the oscillation with various combination for the moment of inertia.

The other reason is that the thread we used in the first part could just endure under 5 Nt force, so we could neglect the restoring force of thread, which would also made the result more simply.

Then we measured the damping coefficients by changing the relative position of a magnet which was used to supply the magnetic damping force. We gained the value of damping coefficient ——0.246 kg/s and 0.322 kg/s, respectively.

Finally, we sought to find out the specific resonant frequency and the quality factor Q of the system. When we plotted amplitude of the oscillation against the angular frequency and did the curve-fitting to get the distribution function.

We could found that the curve fitting for the function obeyed the Lorenzian Distribution.

However,the data was not enough for us to construct the "smooth curve"with curve-fitting in both cases.Hence the Fig.3 and Fig.5 looked so creepy for the discrete data points.

References

[1] J. B. Marion, *Classical Dynamics of Particles & Systems*, 5^{th} Ed, Brooks/Cole Pub Co(2008)

[2] Erwin Kreyszig, *Advanced Engineering Mathematics*, 10^{th} Ed, John Wiley & Sons(2011)

Appendix

For the third part of this experiment,we did the curve-fitting by using the Origin Pro.

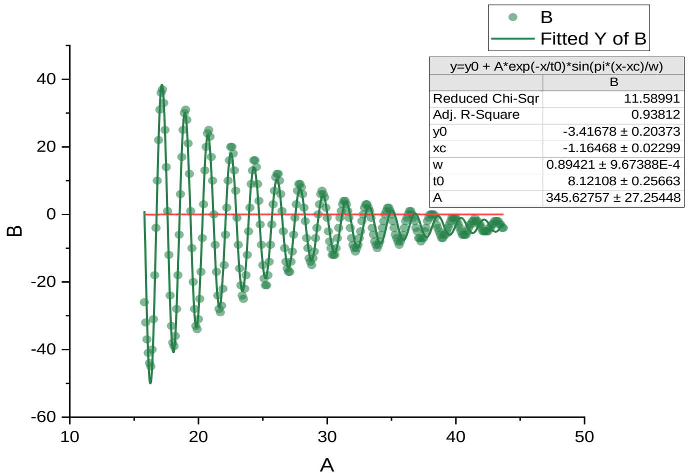

Fig 7:Damped Oscillation

with λ=0.246

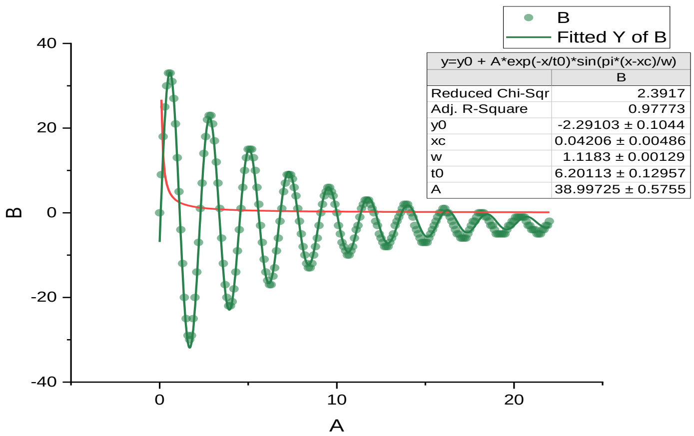

Fig.8:Damped Oscillation

with λ=0.32

For λ=0.246

ω(1/s)	Amplitude
2.42	5.50
2.63	7.00
2.74	8.50
2.77	10.10
2.86	13.00
2.88	15.00
2.94	16.10
2.97	15.10
3.06	13.10
3.12	11.95
3.21	10.00
3.31	8.00
3.42	6.90

For λ=0.322

ω(1/s)	Amplitude
2.42	3.60
2.29	4.55
2.38	5.55
2.49	6.50
2.60	8.50
2.68	12.50
2.74	16.20
2.82	19.10
2.84	20.05
2.90	18.55
2.95	15.90
3.01	13.55
3.14	10.05
3.20	8.10
3.30	6.50
3.41	5.50

3.51	4.50