Theoretical Mechanics II, Spring 2020

First Midterm Examination

Time: 10:10 – 12:00, April 8, 2020
Venue: 019 Physics, 501 Physics

> This is a closed book exam. No search on the web or related electronic books is allowed. Useful formulas and quantities are provided in the end of the exam papers.

Please answer the following questions. There are 4 questions in total.

1. [25%] **The center of mass frame.** A missile of mass M is launched with a velocity $\boldsymbol{u} = u_x\hat{\boldsymbol{x}} + u_y\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ denote the horizontal and vertical directions, respectively. At the highest point (i.e. apex) in its trajectory, the rocket explodes and breaks into two fragments of mass m_1 and m_2 that separate in a horizontal direction in the original plane of motion. Show that the fragments strike the ground at a distance apart by

$$D = \frac{u_y}{g}\sqrt{\frac{2MK_e}{m_1 m_2}}, \tag{1}$$

where K_e is the kinetic energy produced by the explosion, g the gravitational acceleration, and $M = m_1 + m_2$.

   (a) [5%] Describe the motion of the center of mass (CM).

   (b) [10%] Solve D in a reference frame that has the origin fixed at the launch point of the missile.

   [10%] Solve D in a reference frame moving with the CM.

2. [25%] **Newton's rule for inelastic collisions.** Let the coefficient of restitution be ε. Answer the following questions.

   (a) [10%] Consider oblique impact of a sphere of mass m on a smooth floor at an angle α from the normal with velocity $\boldsymbol{u}$ (Fig. 1). Find the velocity $\boldsymbol{v}$ and the rebound angle θ of the sphere after leaving the floor.

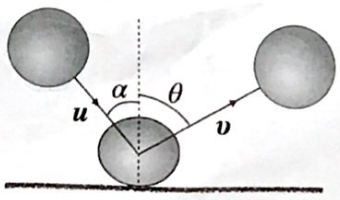

Figure 1: Inelastic rebound of a sphere on a smooth floor.

(b) [15%] A ball falls from a height H onto a horizontal floor. Let the height reached by the ball in the nth rebound is H_n. Find H_n as a function of H. Find the total bounce time T before the ball comes to rest. You may neglect air resistance and any variation in the gravitation acceleration, g.

[25%] **Differential cross section in a central force field.** Consider the elastic scattering of a particle from an impenetrable sphere with the potential

$$U(r) = \begin{cases} 0, & r > a \\ \infty, & r < a \end{cases} \tag{19}$$

(a) [15%] Calculate the differential cross section $\sigma(\theta)$ and the total cross section σ_t. *Hint:* In the CM frame, an elastic scattering of a particle of mass μ in a central force field follows

$$\theta = \pi - 2\Theta \tag{20}$$

$$\Theta = \int_{r_{\min}}^{\infty} \frac{\frac{b}{r^2}\mathrm{d}r}{\sqrt{1 - \frac{b^2}{r^2} - \frac{U}{K_0'}}}, \tag{21}$$

where $K_0' = \frac{1}{2}\mu u_1^2$ is the kinetic energy of incident particle m_1 in the CM frame.

(b) [10%] Explain the results in (a). How do these results make sense?

4. [25%] **Basic rocket motions.**

(a) [15%] A rocket with mass m_0 starts from a speed v_0 in free space by ejecting exhaust. At what fraction of the initial mass is the momentum a maximum.

(b) [10%] A rocket has an initial mass of m_0 and a fuel burn rate of $\dot{\alpha} = -\dot{m}$, where m is the mass of the rocket. What is the minimum exhaust speed, u, that will allow the rocket to lift off immediately after ignition?