Theoretical Mechanics II, Spring 2020

# Final Examination

**Time:** 10:10 – 12:00, June 19, 2020
**Venue:** 019 Physics, 501 Physics

> This is a closed book exam. No search on the web or related electronic books is allowed. Useful formulas and quantities are provided in the end of the exam papers.

**Please answer the following questions. There are 3 questions in total.**

1. [25%] **Normal modes of a double pendulum.** A double pendulum consists of an inextensible string of negligible mass and length $2\ell$, with one end fixed and masses $m$ attached at the midpoint and the other end (Fig. 1). Assume small oscillations. Let the fixed end to be the origin of the coordinate system. Denote the position of $m_1$ as $(x_1, y_1)$ and the position of $m_2$ as $(x_2, y_2)$.

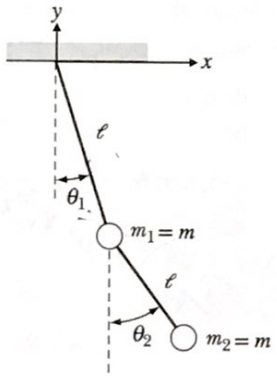


Figure 1: A double pendulum of two identical masses.

   (a) [5%] Write down the kinetic and potential energies.

   (b) [10%] Find the characteristic frequencies by solving the secular equation.

   (c) [10%] Find the normal coordinates and briefly describe the normal modes.

2. [20%] **Frequency splitting by coupling.** Three oscillators of equal mass $m$ are coupled such that the potential energy of the system is described by

$$U = \frac{1}{2}\left[\kappa_1(x_1^2 + x_2^2 + x_3^2) + 2\kappa_2(x_1x_2 + x_2x_3)\right] \tag{37}$$

   (a) [10%] Write down the kinetic and potential energies. Find the equations of motion by using the Lagrangian method.

(b) [10%] Find the characteristic frequencies of the system by solving the secular equation. Discuss the conditions to have three distinct oscillation frequencies.

3. [35%] **Triatomic linear molecules** are of fundamental importance in the space or laboratories. The structures of these simple molecules can be studied through their vibrational spectra. A good example is the triatomic linear molecular, hydrogen cyanide (HCN). Please answer the following questions for HCN.

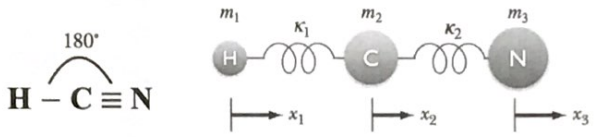


Figure 2: Structures of the triatomic linear molecule hydrogen cyanide (HCN).

(a) [5%] What is the degrees of freedom (DoF) for the longitudinal vibrational motion? Explain clearly how you calculate the numbers. How about the DoF for the transverse vibrational motion?

(b) [10%] Consider only the longitudinal vibrations and write down the kinetic and potential energies. ~~Find the equations of motion by using the Lagrangian method.~~

(c) [10%] Determine the characteristic frequencies of the system by solving the secular equation for the longitudinal vibrations.

(d) [10%] Using the results from part(c), find the normal coordinates and briefly describe the normal modes.