Theoretical Mechanics II, Spring 2021

First Midterm Examination

Time: 10:10 – 12:00, April 7, 2021

Venue: 124 Physics

> This is a closed book exam. No search on the web or related electronic books is allowed. Useful formulas and quantities are provided in the end of the exam papers.

Please answer the following questions. There are 4 questions in total.

1. $\boxed{25\%}$ **The center of mass frame.** A missile of mass M is launched with a velocity $\boldsymbol{u} = u_x\hat{\boldsymbol{x}} + u_y\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ denote the horizontal and vertical directions, respectively. At the highest point (i.e. apex) in its trajectory, the rocket explodes and breaks into two fragments of mass m_1 and m_2 with an additional kinetic energy K_e produced by the explosion. Let the gravitational acceleration to be g.

   (a) $\boxed{10\%}$ Assume that the two segments separate in a horizontal direction in the original plane of motion after the explosion. Show that the fragments strike the ground at the same time with a distance separated by D. Solve D in a reference frame moving with the center of mass (CM). How does the separation D vary with other parameters?

   (b) $\boxed{15\%}$ Alternatively, if the explosion breaks the rocket into two fragments that separate in a vertical direction in the original plane of motion with m_1 traveling downward with a speed v_1 and m_2 upward. Describe the motion of the CM. Specify the flight time t_1 and t_2 after the explosion for the respective segment m_1 and m_2. What is the separation D of the two segments in this case?

2. $\boxed{25\%}$ **Newton's rule for inelastic collisions.** Let the coefficient of restitution be ε. Answer the following questions.

   (a) $\boxed{10\%}$ Consider oblique impact of a sphere of mass m on a smooth floor at an angle α from the normal with velocity $\boldsymbol{u}$ (Fig. 1). Find the velocity $\boldsymbol{v}$ and the rebound angle θ of the sphere after leaving the floor.

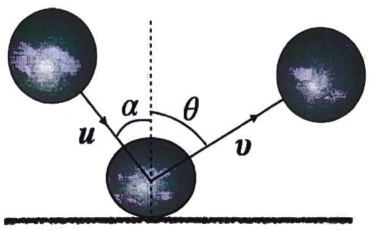

Figure 1: Inelastic rebound of a sphere on a smooth floor.

(b) $\boxed{15\%}$ A ball falls from a height H onto a horizontal floor. Let the height reached by the ball in the nth rebound is H_n. Find H_n as a function of H. Find the total bounce time T before the ball comes to rest. You may neglect air resistance and any variation in the gravitation acceleration, g.

3. $\boxed{25\%}$ **Differential cross section in a central force field.** Consider the elastic scattering of a particle from an impenetrable sphere with the potential

$$U(r) = \begin{cases} 0, & r > a \\ \infty, & r < a \end{cases} \tag{20}$$

(a) $\boxed{15\%}$ Calculate the differential cross section $\sigma(\theta)$ and the total cross section σ_t. *Hint:* In the CM frame, an elastic scattering of a particle of mass μ in a central force field follows

$$\theta = \pi - 2\Theta \tag{21}$$

$$\Theta = \int_{r_{\min}}^{\infty} \frac{\frac{b}{r^2}dr}{\sqrt{1 - \frac{b^2}{r^2} - \frac{U}{K_0'}}}, \tag{22}$$

where $K_0' = \frac{1}{2}\mu u_1^2$ is the kinetic energy of incident particle m_1 in the CM frame.

(b) $\boxed{10\%}$ Explain the results in (a). How do these results make sense?

4. $\boxed{25\%}$ **Effective force in a non-inertia frame.** In class, we show that the force on a particle with mass m can be expressed as

$$\boldsymbol{F} = m\boldsymbol{a}_f = m\ddot{\boldsymbol{R}}_f + m\boldsymbol{a}_r + m\dot{\boldsymbol{\omega}} \times \boldsymbol{r} + m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2m\boldsymbol{\omega} \times \dot{\boldsymbol{r}} \tag{31}$$

To an observer in the rotating coordinate system, the effective force on the particle is given by

$$\boldsymbol{F}_{\text{eff}} \equiv m\boldsymbol{a}_r. \tag{32}$$

(a) $\boxed{5\%}$ Explain each term in Eq. (31) and identify the non-inertial terms known as the centrifugal and Coriolis "forces."

(b) $\boxed{12\%}$ A charged particle moves under the combined effects of an electrostatic field $\boldsymbol{E}$ and a weak, uniform magnetostatic field $\boldsymbol{B}$. Show that the effect of $\boldsymbol{B}$ can be removed by transforming to a suitable rotating frame. Determine the conditions for such a rotating frame together with a feasible magnetostatic field strength B.

(c) $\boxed{8\%}$ Apply the result in part (b) to bounded motion of a particle with a constant charge q perturbed by the field $\boldsymbol{B}$. Describe the trajectory of the charged particle in an inertial frame. *Hint:* The general solutions for the bounded motion of a charged particle in an electrostatic field are elliptical orbits.