Calculus Midterm Exam 109/04/28

1. (20 points) Find each of the following：

   (a) Area of the region common to the interiors of $r=3\cos\theta$ and $r=1+\cos\theta$.

   (b) Center and radius of the intersection circle of the plane $x+2y-2z=1$ and sphere $x^2+y^2+z^2-4x+2z=1$.

   (c) Area of the surface generated by revolving the curve $6xy=x^4+3\,(1\le x\le 3)$ around x-axis.

   (d) All vectors $\vec{v}$ satisfying $(-\vec{i}+2\vec{j}+2\vec{k})\times\vec{v}=4\vec{i}+5\vec{j}-3\vec{k}$.

2. (30 points) Find the interval of convergence for each of the following series, also find the sums of (c)、(d) : (6 points each for (a)、(b)，9 points each for (c)、(d))

   (a) $\sum_{k=1}^{\infty}\frac{(-1)^k}{\sqrt[3]{k}}(x+1)^k$

   (b) $\sum_{n=1}^{\infty}n!(2x+3)^n$

   (c) $\sum_{n=0}^{\infty}(n+2)x^n$

   (d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{2n+1}$

3. (20 points) Find each of the following if it exists, explain why if it doesn't：

   (a) $\lim_{n\to\infty}(7^n+3^n)^{1/n}$

   (b) $\lim_{x\to 0}\frac{(x-\sin x)(\cos 2x-1)}{x^2(e^{2x^3}-1)}$

   (c) $f^{(100)}(0)$ for $f(x)=\begin{cases}\frac{\sin x}{x} & (x\neq 0)\\ 1 & (x=0)\end{cases}$

4. (16 points) (a) Use binomial series to find the Taylor series expansion of the function $f(x)=\sqrt{1+x^2}$ (with explicit general terms).

   (b) Evaluate $\lim_{x\to 0}\frac{\sqrt{1+x^2}-1-\frac{1}{2}x^2}{e^{x^2}-1-x^2}$ if it exists.

5. (24 points) (a) Find the Fourier series for the function $f(x)=\pi-|x|$

   (b) Use the fact $x\sim 2\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sin kx}{k}$ on $(-\pi,\pi)$ to find the Fourier series of x^2 on $[-\pi,\pi]$.

   (c) Find the sum $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}$, $\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$ and $\sum_{k=1}^{\infty}\frac{1}{k^4}$