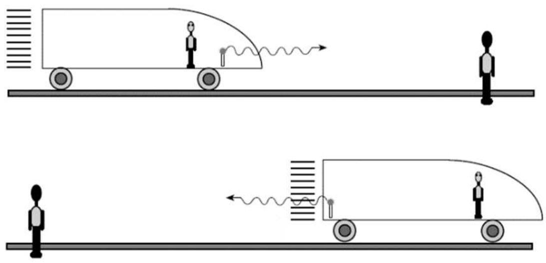
Test-3 Special Relativity and Quantum Physics, Set No.1 06/22

I. (1) Why is the momentum of a particle $\gamma m v$? [4%]

(2) Explain why massive particles cannot reach c, the speed of light. [2%]

(3) Why do massless ($m = 0$) particles all travel at c? [2%]

(4) It seems an experimental fact that there are no massless charged particles. Give your reasoning on it. [2%]



(5) Sofia (observer O') is moving with speed v toward Alex (observer O) in his positive direction shown in the figure below. She emits a beam of light, whose energy and momentum she measures to be E' and p' with $E'=p'c$. Alex receives this signal and measures its energy and momentum to be E and p. Relate E and p to E' and p' *via* Lorentz transformation. [4%]

(6) A photon moves with the same speed with respect to all observers, whether its momentum is or not the same? Resolve this issue. [2%]

(7) By using the Planck - Einstein relation: $E = \dfrac{hc}{\lambda}$, derive the relativistic Doppler effect. [4%]

II. Davisson-Germer Experiment The Bragg condition for constructive interference is $n\lambda = 2d \sin\theta = 2d \cos\alpha$. The spacing of Bragg planes d is related to the spacing of the atoms D by $d = D \sin\alpha$; thus

$$n\lambda = 2D \sin\alpha \cos\alpha = D \sin 2\alpha$$

or

$$n\lambda = D \sin\varphi \qquad \textbf{5-5}$$

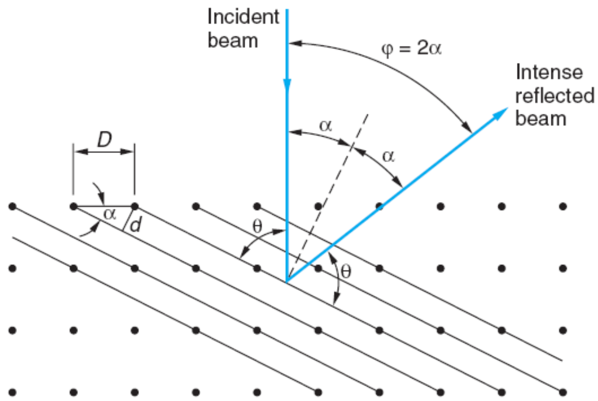
where $\varphi = 2\alpha$ is the scattering angle.

The spacing D for Ni is known from x-ray diffraction to be 0.215 nm. The wavelength calculated from Equation 5-5 for the peak observed at $\varphi = 50°$ by Davisson and Germer is, for $n = 1$,

$$\lambda = 0.215 \sin 50° = 0.165 \text{ nm}$$

The value calculated from the de Broglie relation for 54-eV electrons is

$$\lambda = \frac{1.226}{(54)^{1/2}} = 0.167 \text{ nm}$$



The above text is taken from some typical textbook of Modern Physics. Compared with the the following paragraph extracted from a popular webpage, the interpretaion of the diffraction data seems inconsistant. Is anything wrong in the content of either one? Or, are they both wrong? [20%]

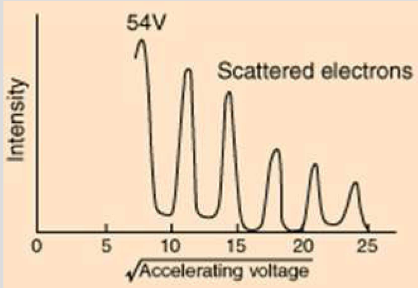
PS. Must reason any numbers you present.

*http://**hyperphysics.phy-astr.gsu.edu**/hbase/quantum/DavGer2.html*

In the historical data, an accelerating voltage of **54 volts** gave a definite peak at a scattering angle of **50**$^\circ$. The Bragg angle θ corresponding to that scattering angle is **65**$^\circ$, and for that angle the calculated lattice spacing is **0.092 nm**. For that lattice spacing and scattering angle, the relationship for wavelength as a function of voltage is empirically.

$$\lambda = \frac{h}{\sqrt{2m_e eV}} = 0.167\,\text{nm} \equiv \lambda_1 \quad \text{and} \quad \lambda_n = \frac{\lambda_1}{n}$$

$$\theta = \frac{180^\circ - \phi}{2} = 65^\circ \;\Rightarrow\; d = \frac{n\lambda}{2\sin\theta} = 0.092\,\text{nm}$$



Trying this relationship for ***n*=1,2,3** gives values for the square root of voltage **7.36**, **14.7** and **22.0**, which appear to agree with the first, third and fifth peaks above. Then what gives the 2nd, 4th and 6th peaks? *Perhaps they originate from a different set of planes in the crystal. Those peaks satisfy a sequence 2,3,4, suggesting that the first peak of that series would have been at 5.85.* That corresponds to an electron wavelength of 0.21 nm and a lattice spacing of 0.116 nm ?? I don't know if that makes sense. I need to look at the original article.

III. (1) Parity operator is defined as follows: $\begin{Bmatrix} 1-\text{dim.}, & \hat{\mathcal{P}}\psi(x) = \psi(-x) \\ 3-\text{dim.}, & \hat{\mathcal{P}}\psi(\vec{r}) = \psi(-\vec{r}) \end{Bmatrix}$. Show that, for the eigenstate $\psi(x)$ of a Hamiltonian $\hat{\mathcal{H}}$ with a symmetric potential $V(-x) = V(x)$, the corresponding eigenvalue of the parity operator $\hat{\mathcal{P}}$ are ± 1. [4%]

(2) Show that, in the previous case of a symmetric potential, $[\hat{\mathcal{H}}, \hat{\mathcal{P}}] = 0$. [2%]

(3) In 3 dimensions, for the Coulomb potential $V = V(r) = -\dfrac{k_e Z e^2}{r}$, the eigenfunction satisfies $\hat{\mathcal{P}}\psi(\vec{r}) = \psi(-\vec{r})$. Prove that $\hat{\mathcal{P}} Y_{\ell m}(\theta,\phi) = (-1)^{\ell} Y_{\ell m}(\theta,\phi)$. [4%]

(4) For the potential $V(x) = \begin{Bmatrix} -V_0 < 0, & \text{as } |x| < a \\ -V_0/2, & \text{as } a < |x| < 2a \\ 0, & \text{as } |x| > 2a \end{Bmatrix}$, why there exists at least one bound state which is an even function of x? (Give an answer without proof.) [5%]

(5) In a 1-dimensional potential, $V(x) = \begin{Bmatrix} 0, & \text{as } x < -b < 0 \\ -V_0 < 0, & \text{as } -b < x < 0 \\ \infty, & \text{as } x > 0 \end{Bmatrix}$ and V_0 as a function of b, derive the dependence of $V_0 = \dfrac{\pi^2\hbar^2}{8mb^2}$ such that there is just ***one*** bound state, of about ***zero*** binding energy, for a particle of mass m. [5%]

IV. (1) For the hydrogen atom, how is Schrödinger's model different from Bohr's? Please point out the most important FOUR characters as you know. [4%]

(2) In what kind of situation, Schrodinger's model would resemble Bohr's one? [2%]

(3) Why we say that, in general, the degeneracy of d-orbitals of an isolated atom is not 5x2 = 10 and, in fact, is separated into 6 and 4? [2%]

(4) Explain in detail why the intensity of $K\alpha_1$ x-ray fluorescence is two times of that of $K\alpha_2$ emission in almost all of atoms. [2%]

(5) Consider a particle of mass m attached to a rigid massless rod of fixed length R whose other end is fixed at the origin. The rod is free to rotate about the origin. The Hamiltonian of this system is given by

$$\hat{\mathcal{H}}_0 = \frac{\hat{\vec{L}}^2}{2I}, \quad \text{where} \quad I = mR^2 \quad \text{and} \quad \hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}.$$

The Schrödinger equation for the energy levels of the rigid rotator is given by $\hat{\mathcal{H}}_0 \psi(x) = E\psi(x)$. What are the possible energy eigenvalues of the system? [2%]

(6) In the presence of weak uniform B-field, the Hamiltonian becomes

$$\hat{\mathcal{H}} = \frac{\hat{\vec{L}}^2}{2I} + g_L \hat{\vec{L}} \cdot \vec{B}, \quad \text{where} \quad \vec{B} = B\frac{\vec{z}}{|\vec{z}|} \quad \text{and} \quad B = \text{constant}.$$

Compute the energy levels of the system as described. [2%]

(7) If there are radiative transitions between energy levels, draw a diagram to show transitions from $\ell = 1$ to $\ell = 0$ and denote the wavelength of each transition. [4%]

(8) The above model is a good one for diatomic molecules. Considering the nitric oxide molecule under 1 Tesla B-field, estimate what the energy splitting would be? [2%]

V. (1) How did Drude describe the electrical conductivity in metals? [2%]

(2) What are the key ingredients of quantum concepts applied onto Drude's model to make a better Interpretation of the Wiedemann-Franz Law? [4%]

(3) Show that, at $T = 0$ K, the average energy E_{avg} of the conduction electrons in a metal is equal to $\frac{3}{5}E_F$. (Hint: By definition of average, $E_{\text{avg}} = (1/n)\int_0^{E_F} EN_0(E)dE$, where n is the number density of charge carriers.) [4%]

(4) Referring to the plot as shown of each Density of States in different dimensions, what a feature you could derive from this plot? [2%]

(5) From the energy point of view, give the various aspects of E_F as you know. [4%]

(6) After proper doping, what advantages on the the electric properties of semiconductors would achieve? [4%]