# Quantum Physics (II): Midterm (1) Apr. 9, 2004

**Problem 1** Explain or evaluate the following terms briefly:

**(a) 3%** Gauge invariance

**(b) 3%** Landau levels

**(c) 3%** (integer) quantum Hall effect

**(d) 4%** $\langle l, m_1 | {L_x}^2 | l, m_2 \rangle$

**(e) 4%** $\langle \frac{1}{r} \rangle = \frac{Z}{n^2 a_0}$, compute $\langle \frac{1}{r^2} \rangle$

**Problem 2** An angular wavefunction is given by

$$f(\theta, \phi) = N[1 + \sin\theta \cos\theta \cos 2\phi]$$

**(a) 5%** Find *N* such that $f(\theta, \phi)$ is properly normalized.

**(b) 8%** Find the probability of finding $\mathrm{L_z} = -\hbar$? $\mathrm{L_z} = 2\hbar$?

**Problem 3** An electron in the Coulomb field of a proton is in a state described by the wave function

$$\frac{1}{6}\left[4\psi_{100}(\mathrm{r}) + 3\psi_{211}(\mathrm{r}) - \psi_{210}(\mathrm{r}) + \sqrt{10}\psi_{21-1}(\mathrm{r})\right]$$

**(a) 4%** What is the expectation value of the energy?

**(b) 4%** What is the expectation value of $\mathbf{L}^2$?

**(c) 4%** What is the expectation value if $L_z$?

**Problem 4** Consider a hydrogen atom in its ground state.

**(a) 5%** What is the expectation value of the kinetic energy?

**(b) 5%** What is the probability of finding the atom with a kinetic energy larger than the expectation value?

**(c) 10%** Calculate the uncertainty in the position variable. Do the same for the uncertainty in the momentum and check the validity of the Heisenberg inequality.

**Problem 5 10%** A particle of mass *m* in a three-dimensional harmonic oscillator of potential energy $m\omega^2 r^2/2$ has a spectrum given by

$$E = \hbar\omega(2n_r + l + 3/2)$$

where $n_r$ is a radial quantum number ($n_r = 0, 1, 2, 3, \ldots$) and *l* is the orbital angular momentum ($l = 0, 1, 2, 3, \ldots$). Suppose the particle has charge q and the harmonic oscillator is placed in a weak magnetic field B. Sketch the spectrum for the three lowest energy states

**Problem 6** A researcher prepares a hydrogen in the state: $n = 3$, $l = 2$, and $m = -1$. The quantization axis he uses is perpendicular to the ground. This hydrogen is later sent to his colleague in the same laboratory. His colleague, however, uses a quantization axis parallel to the ground.

**(a) 9%** Express the wavefunction $\psi_{32-1}$ the researcher prepared as a linear combination of the wavefunction which his colleague described.

**(b) 6%** Find the probabilities for his colleague to find this hydrogen atom in all possible states specified by ($n$, $l$, $m$) using his quantization axis.

**Problem 7** Consider a muonium atom that consists of an electron and an antimuon (charge = $+e$, $m = m_\mu$) in a hydrogen-like bound state. Due to the mass difference between the antimuon and the electron, muonium is similar to hydrogen atom.

**(a) 8%** Write down the Hamiltonian for this system in the presence of a constant external magnetic field.

**(b) 5%** Will Zeeman effect occur in the system? Explain your result briefly.