National Tsing Hua University
Department of Physics

PHYS3340 **Optics II** **2021 Spring**

Final Exam (Open Book)

注意：每個答案皆要有嚴謹的推導過程或詳細的推論理由。考題總分為 121 分。

常數： In *SI* units, $\varepsilon_0 = 8.85\times10^{-12}$, $\mu_0 = 4\pi\times10^{-7}$, c (真空光速) $= 3\times10^{8}$, h (Plank constant) $= 6.6\times10^{-34}$.

1. (16 points) Figure 1 shows a rectangular aperture on the yz plane and a screen on the YZ plane. Inside the aperture, the electric field of the wavelength λ is given by $E_0 e^{i[\omega t+\phi(y,z)]}$, where E_0 is a constant and $\phi(y,z)$ is a function of y and z. Outside the aperture, the electric field is zero. The distance between the center of the aperture (i.e., the origin of yz plane) and that of the screen (i.e., the origin of YZ plane) is D. (a) If the screen locates at the far field, what is the intensity on the screen as a function of Y and Z? (b) If the screen locates at the near field, what is the intensity on the screen as a function of Y and Z? [注意：寫下積分式即可，清楚表示積分的上下限與被積函數，任何未出現於題目或圖的數學符號要有定義，被積函數必須在遠場或近場的條件下作化簡。]

2. (12 points) Light from a laboratory sodium lamp has two strong yellow components at 589.5 nm and 590.1 nm, and is incident normally (零度入射角) on a grating having 8000 lines per centimeter. The first order spectrum is observed on a screen 2.50 m from the grating. (a) How far apart are the two sodium lines? (b) What is the width of each line? (c) What is the blaze angle of the grating to maximize the first order spectrum?

3. (15 points) Figure 2 顯示不同大小的圓孔之近場繞射結果，字母順序代表圓孔由小至大，光源為雷射光，光源至圓孔的距離及圓孔至觀測螢幕的距離保持固定。若最小的圓孔直徑為 1 mm，且正好涵蓋 1 個 Fresnel zone，(a) 那個圖形是圓孔直徑為 $\sqrt{2}$ mm 的繞射結果? (b) 那個圖形是圓孔直徑為 $\sqrt{3}$ mm 的繞射結果? (c) 那個圖形是圓孔直徑為 $\sqrt{5}$ mm 的繞射結果? 並請說明理由。

4. (12 points) 請以中文(專有名詞可用英文)詳細回答下列的問題：(a) 光的頻譜(frequency spectrum)與其 autocorrelation function 之關係是什麼? (b) 如何利用干涉儀(interferometer)測量一道光的 coherence time?

5. (30 points) 非相干單色(incoherent monochromatic)光源產生雙狹縫干涉圖樣如 Figure 3 所示，光源寬度用 A 表示，雙狹縫間距用 B 表示，每條狹縫的寬度用 C 表示，光源與雙狹縫之間的距離用 D 表示，雙狹縫與顯示干涉圖的屏幕之間的距離用 E 表示。請回答下列問題。(a) [5 points] Figure 3 左圖和右圖的 visibility 分別是多少？(b) [8 points] 當 $u = 0$ 時局部出現最大值(見左圖)的條件是什麼? 當 $u = 0$ 時局部出現最小值(見右圖)的條件是什麼？[注意：答案以 A、B、C、D、E 表示。] (c) [5 points] 虛線代表干涉圖案的包絡線。請問什麼是包絡線的成因? (d) [6 points] 假設 $C \ll \lambda$ (其中λ是光源的波長)。Δt 表示光分別從二個狹縫抵達螢幕某一位置 u 之時間差。τ 表示光源的相干時間(coherence time)。若 $u = 2$ 時，$\Delta t \approx \tau$，請問左圖會變成什麼樣子(請畫出來)?

(e) [6 points] 若 $u = 8$ 時，$\tau \gg \Delta t$，請問右圖會變成什麼樣子(請畫出來)？[注意：(d) 與 (e) 畫圖的範圍為 $-8 \le u \le 8$，標示最大值與最小值的 u 位置，以及最大值與最小值的相對大小。]

6. (20 points) As shown in Figure 4, there are 12 vertical, incoherent, infinitesimally narrow line sources of the wavelength λ. They are used to illuminate a pair of exceedingly narrow vertical slits in a aperture screen. The distance bewteen the light sources and the screen is A. The separation between the two slits is B. Two adjacent line sources separate by C, where $A \gg 12C$. (a) What is the condition that the fringe pattern of the two slits in the far field has the maximum visibilty? What is the value of this maximum visibility? (b) What is the condition that the fringe pattern of the two slits in the far field has the minimum visibilty? What is the value of this minimum visibility? Express your answers by A, B, C, and λ. [注意：答案不含積分式。若有積分式，必須計算出結果。]

7. (16 points) The function $h(x)$ arises from the convolution of the two functions $f(x)$ and $g(x)$, where $f(x)$ is shown by the red line in Figure 5 and $g(x) = -\delta(x+d) + \delta(x-d) - \delta(x-2d)$. [注意：必須清楚標示 $h(x)$ 的縱座標與橫座標。] (a) Please sketch $h(x)$. (b) Please calculate $H(k)$, where $H(k)$ is the Fourier transform of $h(x)$.

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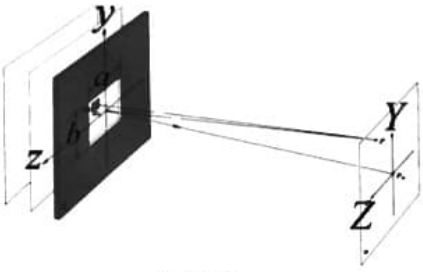

Figure 1

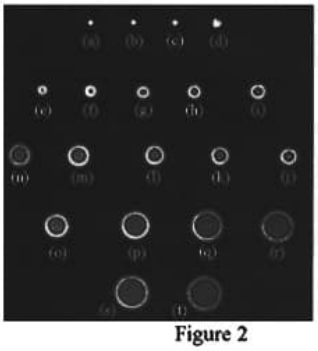
Figure 2

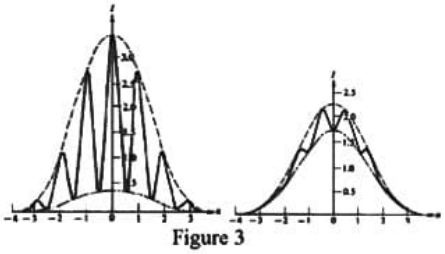
Figure 3

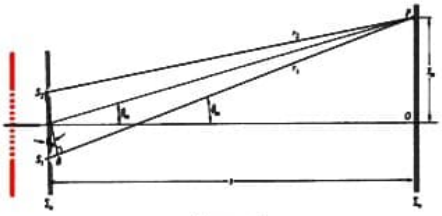
Figure 4

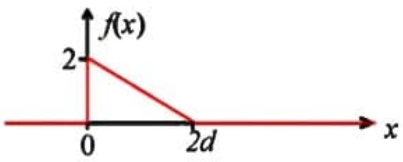

Figure 5