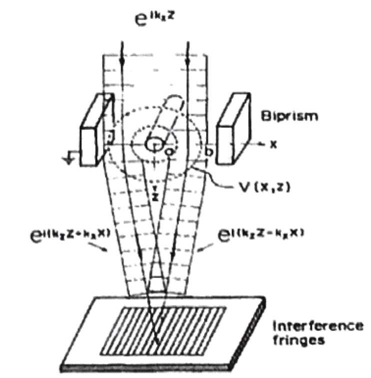
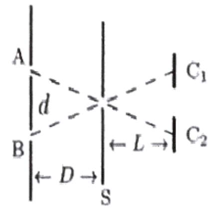
Quantum Physics (I): Midterm, Nov. 13, 2020

Total grade: 100

Problem 1 Explain, evaluate the following terms or answer the following quesitons briefly:
(a) 3% collapse of state
(b) 3% particle-wave duality
(c) 3% energy-time uncertainty relation
(d) 4% Compute the commutator $[\hat{x}, 1/(1+\hat{p}^2)]$, where $\hat{x}$ and $\hat{p}$ are position and momentum operators.
(e) 4% Indicate which of the following phenomena or conditions are related to the wave nature of the electron? (1) blackbody radiation (2) the Bohr quantization condition (3) the x-ray radiation from de-accelerated electrons (4) the photoelectric effect (5) the experiment on electron scattering from the Ni crystal done by Davisson and Germer.
(f) 6% Consider the double-slit experiment for the electron shown in the following figure. Suppose that the speed of the incident electron is half of the light speed, i.e., $c/2$, the electric potential for the central metal rod (labelled by a) is 10V, the radius of central metal rod is 0.5 μm, and the radius at grounded-plate is $b = 5mm$, estimate the spacing of the finge in the inteference pattern.



(g) 4% In the Compton experiment, if the observed x-ray is scattered from the original direction to $\theta = 60°$, find the recoil momentum of the electron that scatters by the observed x-ray.
(h) 4% In class physics, the quantity xp of a particle can be measured with x and p being the position and momentum of the particle. What would be the corresponding expression of the same quantity in quantum physics?
(i) 5% Consider the double-slit interference experiment that is carried out by sending one photon at a time. A method is devised to detect which slit the photon goes through by making a hole whose size covers the range of the central interference fringe exactly on the screen S (see the the following figure) so that photons coming from slit A trigger counter C_2 , while those coming from slit B trigger C_1. Here the setup assumes $D \gg d$ and $L \gg d$. It is claimed that in this way, one can detect which slit that the photon goes through without destroying the interference pattern. The claim is in conflict with the argument based on the uncertainty principle that we discussed in class. Which one is correct? If the above argument is wrong, where did the above arguement went wrong? Justify your answer.



Problem 2 Suppose that we prepare a cubic cavity (dimension is $L\times\ L\times L$) in equilibrium at temperature T. Inside the cavity, there will be radiations in various wavelengths. Let the energy density per unit wavelength for the radiation with wavelength λ be $u(\lambda, T)$.
(a) 4% If we drill a small hole on the wall with area a, find the radiation energy of wavelength λ that flows out from this hole per unit time.
(b) 5% Find the number of the electromagnetic waves (each allowed waves is called a mode) allowed in this cavity with wavelength in the range $(\lambda, \lambda + d\lambda)$.
(c) 5% What would be the Planck's expression for the average energy of each mode with wavelength λ ? What is the main assumption for Planck to get this expression?
(d) 10% Suppose that the radiation can be contained in a two dimensional cavity with dimension $L\times\ L$. Repeat your calculation for problem (a) and (b) by assuming that the only property of the radiation that changes is the dimension it lives, all the other properties remain the same. (Note that in this case, a hole becomes a segment. Therefore, the meaning of a is the length of the segment). Find the maximum wavelength of the radiation emitted per wavelength, i.e., derive the Wien's displacement law for the wavelength.

Problem 3 6% Use the uncertainty relation to estimate the range of motion for the ground state of a particle with mass m in the potential: $V(r) = V_0(\frac{r}{a})^k$ with k being a positive integer. Here r is the radius of the electron in spherical coordinates. Find the energy eigenvalues of this potential using the Bohr quanziation rule. What are the energy eigenvalues in the limit $k \to \infty$?

Problem 4 Consider a particle that moves freely in one dimension. Its initial ($t = 0$) wave function is given by

$$\Psi(x, 0) = Ae^{-|x|/L}$$

where A and L are positive constants.
(a) 8% Find the uncertainties $(\Delta x)_0$ and $(\Delta p)_0$ at $t = 0$.
(b) 3% Find the probability for finding this particle exactly at $x = L$.
(c) 7% Determine the uncertainties $(\Delta x)_t$ and $(\Delta p)_t$ at time t and verify the validity of the uncertainty relation

$$(\Delta x)_t(\Delta p)_t \geq \frac{\hbar}{2}.$$

(d) 2% Find the probability per unit time that the particle goes across the point x at $t = 0$.

Problem 5 A particle of mass m is in a symmetric infinite well ($-a < x < a$) with the following wavefunction at $t = 0$

$$\Psi(x, 0) = \frac{N}{\sqrt{a}}\left[(3 + 2i)\cos(\frac{\pi x}{2a}) - 2\sin(\frac{\pi x}{a}) + 3i\cos(\frac{3\pi x}{2a})\right]$$

where N is a constant. Suppose that we prepare 2×10^6 such particles in the same state. We now perform perform precise measurements of energy on each particle ~~at~~ tim3 t and obtain $\epsilon_1, \epsilon_2, \epsilon_3, ..., \epsilon_{10^6}$. Answer the following quesions:
(a) 7% What are possible values of ϵ_i one would get? Estimate how many times one would get for each of these values in the measurements.
(b) 7% Estimate $(\epsilon_1+\epsilon_2+\epsilon_3+\epsilon_4+\cdots+\epsilon_{10^6})/10^6 \equiv \bar{\epsilon}$ and $[(\epsilon_1-\bar{\epsilon})^2+(\epsilon_2-\bar{\epsilon})^2+(\epsilon_3-\bar{\epsilon})^2+(\epsilon_4-\bar{\epsilon})^2+\cdots+(\epsilon_{10^6}-\bar{\epsilon})^2]/10^6$