Theoretical Mechanics I, Fall 2020

First Midterm Examination

Time: 10:10 – 12:00, October 28, 2020

Venue: 019 Physics, 313 Physics, 203 General Physics Lab

This is a closed book exam. No search on the web or related electronic books is allowed. Useful formulas and quantities are provided in the end of the exam papers.

Please answer the following questions. There are 4 questions in total.

1. [25%] A boat of mass m gliding with initial velocity v_0 in water is slowed by a viscous retarding force of magnitude $F = -ke^{\gamma v}$.

   (a) [10%] Find an expression for the speed $v(t)$.

   (b) [15%] Find the travel distance for the boat to stop.

2. [25%] Motions of a spaceship.

   (a) [10%] A spaceship with mass m_0 starts from a speed v_0 in free space by ejecting exhaust. At what fraction of the initial mass is the momentum a maximum?

   (b) [5%] Before entering its orbit, the spaceship must escape from the Earth gravity by using a rocket. Let the rocket has an initial mass of m_0 and a fuel burn rate of $\alpha = -\mathrm{d}m/\mathrm{d}t$, where m is the mass of the rocket. What is the minimum exhaust speed, u, that will allow the rocket to lift off immediately after ignition?

   (c) [10%] After launch, consider the motion merely vertical. Let the gravitational acceleration, g, be a constant and neglect air resistance. Further assume a constant fuel burn rate, i.e. $\mathrm{d}m/\mathrm{d}t = -\alpha$, where $\alpha > 0$. Determine the terminal velocity v_f of the spaceship under the Earth's gravity after a flight time t_f.

3. [25%] An object of mass m is subject to a one-dimensional restoring force $F_r = -kx$ and a frictional force of constant magnitude $F_f = \mu N$, where μ is the coefficient of kinetic friction between the mass and the horizontal surface on which it slides, and $N = mg$ is the normal force pushing upward by the surface. Assume the initial conditions of $x_0 = A$ $(A > 0)$ and $v_0 = 0$. Denote the natural frequency as $\omega \equiv \sqrt{k/m}$.

   (a) [15%] Determine the displacement $x(t)$ and $v(t)$ of the object during the first cycle of its motion. Be aware that F_f is always in the opposite direction of the instantaneous $v(t)$.

   (b) [10%] Determine the fractional loss of total energy after the first cycle of its motion.

4. [25%] A particle of mass m is subject to a one-dimensional restoring force $F_r = -kx$ and a harmonic driving force $F_d = F_0 \cos\omega t$, where k, F_0 and ω are positive constant.

(a) [8%] Let the natural frequency of the oscillation be $\omega_0 \equiv \sqrt{k/m}$. Determine the position $x(t)$ for the initial conditions $x = 0$ and $v = 0$ at $t = 0$.

(b) [10%] If the driving frequency ω is very close to the natural frequency ω_0, find the two frequencies that can describe $x(t)$ as a combination (product) of two oscillating motions. Plot $x(t)$ over two periods of the lower frequency and label the two corresponding time scales.

(c) [7%] Sketch an equivalent electrical circuit for this mechanical oscillating system.