電磁學 (Electromagnetism)　Final Exam　滿分 100

[Griffiths Chs. 10 and 12]　2018/06/11,　10:10am — 12:00am,　教師：張存績

1. Write the equations (if possible) and explain the following terms as clear as possible.
   (a) The Lorentz gauge and the Coulomb gauge. (4%)
   (b) Gauge transformations and gauge freedom. (4%)
   (c) Phase velocity and group velocity. (4%)
   (d) The two postulates of the special relativity. (4%)
   (e) Invariant quantity and conserved quantity. (4%)
   (f) Lienard-Wiechert potentials. (5%)
   (g) Hidden momentum. (5%)

2. (a) The transformations between two inertial system S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta\bar{x}/\gamma$; but when $\Delta\bar{t} = 0$, $\Delta\bar{x} = \Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (8%+2%)
   (b) Show that ($\mathbf{E}\cdot\mathbf{B}$) is relativistically invariant. (10%)
   [Hint: $\bar{E}_x = E_x$, $\bar{E}_y = \gamma(E_y - vB_z)$, $\bar{E}_z = \gamma(E_z + vB_y)$;
   $\bar{B}_x = B_x$, $\bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z)$, $\bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$]

3. (a) $\vec{\eta} = \boldsymbol{r} - \boldsymbol{r}'$, $\eta = |\boldsymbol{r} - \boldsymbol{r}'| = c(t - t_r)$, and $\mathbf{v} = \mathbf{v}(t_r)$. Find $\boldsymbol{\nabla}\eta$ and $(\vec{\boldsymbol{\eta}}\cdot\boldsymbol{\nabla})\mathbf{v}$. (10%)
   (Note: Express your answer in terms of $\boldsymbol{\nabla}t_r$.)
   (b) Show that the retarded potentials satisfy the Lorentz gauge condition $\boldsymbol{\nabla}\cdot\mathbf{A} + \mu_0\varepsilon_0\partial V/\partial t = 0$.
   (10%) [Hint: the retarded potentials $V(\mathbf{r},\mathrm{t}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r',t_r)}{\eta}d\tau'$ and $\mathbf{A}(\mathbf{r},\mathrm{t}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(r',t_r)}{\eta}d\tau'$]

4. Suppose V=0 and $\mathbf{A} = A_0\cos(kx - \omega t)\hat{\mathbf{z}}$, where A_0, ω, and k are constants.
(a) Find **E** and **B** (10%)
(b) Use the gauge function $\lambda = xt$ to transform the potential V' and $\mathbf{A}'$. (10%)
(c) Find the news $\mathbf{E}'$ and $\mathbf{B}'$. Comment on the gauge freedom. (8%+2%)
   [Hint: $\mathbf{A}' = \mathbf{A} + \boldsymbol{\nabla\lambda}$ and $V' = V - \partial\lambda/\partial t$]