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Mid Examination 1
Applied Mathematics I(PHYS211000)
31 March 2014, 9.00 - 10.00 am

Answer all questions. Each question carries 20 marks.
Simple calculator is allowed. No use of telephone.
You may answer in English or Chinese.

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1. **(Determinant)**
   Using the Laplace expansion, evaluate the determinant of the matrx

$$A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & 1 \\ 3 & -3 & 4 & -2 \\ -2 & 1 & -2 & -1 \end{pmatrix} \tag{1}$$

2. **(Inverse)**
   Evaluate the determinant of the matrix

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 0 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix} \tag{2}$$

   and find its inverse A^{-1}.

3. **(Rank)**
   What is the rank of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}? \tag{3}$$

4. **(Eigenvalues and Eigenvectors)** Find the eigenvalues and the normalized eigenvectors of the matrx

$$A = \begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix}. \tag{4}$$

5. Let A be an $N \times N$ Hermitian matrix and consider the matrix

$$M = e^{iA}. \tag{5}$$

   (a) Show the M is unitary.
   (b) Given that A has eigenvalues λ_i and eigenvectors $v_i, i = 1, \ldots, N$. Determine the eigenvalues and eigenvectors of M. Show that the eigenvectors belonging to different eigenvalues are orthogonal to each other.

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