109 學年下學期　　微積分　　期末考試題 110/06/18

請在空白紙上依序寫出各題答案，計算過程請另紙處理、不要寫在答案紙上。

每人限交 A4 大小答案紙 2 頁，第 1 頁首請附上學生證並寫上系級學號姓名。

1. (True-False；每小題 3 分，答錯倒扣(到本題 0 分為止)，棄答之小題不計分)

(a) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz^2+xz^2}{x^2+y^2+z^4}$ exists.　　(b) $\lim_{(x,y)\to(0,1)} \dfrac{x^2(y-1)}{x^2+(y-1)^2}$ exists.

(c) $\int_{-1}^{1}\int_{0}^{1} e^{-(x^2+y^2)}\cos y\,dx\,dy = 0$.　　(d) $\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx = \dfrac{\sqrt{\pi}}{2}$.

(e) $\oint_C (2x-y)dx+(x+3y)dy = 12\pi$, where C is the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$(counterclockwise).

(f) $\oint_C \dfrac{-y\,dx+x\,dy}{x^2+y^2}=0$, where C is the ellipse $\dfrac{(x-1)^2}{16}+\dfrac{(y-1)^2}{9}=1$.

(g) $\oint_C \dfrac{-y\,dx+x\,dy}{x^2+y^2}=0$, where C is the positively oriented boundary of

$1 \le \dfrac{(x-1)^2}{16}+\dfrac{(y-1)^2}{9} \le 4$.

2. (10 分) Let $g(x,y)=\begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{for } (x,y)\neq(0,0) \\ 0 & \text{for } (x,y)=(0,0) \end{cases}$ (此題請抄題)

(a) $\dfrac{\partial g}{\partial y}(x,0)=$______.　　(b) $\dfrac{\partial^2 g}{\partial x\partial y}(0,0)=$______.

(3-6 題，每題 7 分；請在(a)寫出可以真正算出的 iterated integral(含各個變數的上下限)，化簡為一個變數的積分寫在(b)，再將化簡整理後的答案寫在(c))

3. $\int_0^1\int_y^{\sqrt{y}} \dfrac{\sin x}{x}\,dx\,dy$=(3a)= (3b)= (3c).

4. Area A of the part of the surface $x=yz$ that lies inside the cylinder $y^2+z^2=9$ is

A=(4a)= (4b)= (4c).

5. Volume V of the solid bounded by the hyperboloid $\dfrac{x^2}{9}+\dfrac{y^2}{16}-\dfrac{z^2}{4}=1$ and the planes $z=-2$ and $z=2$ is V=(5a)= (5b)= (5c).

6. $\iint_\Omega xy\,dA$=(6a)= (6b)= (6c), Ω is the plane region in first quadrant bounded by the

curves $xy=1, xy=2, y=x^2, y=2x^2$.

(7-9 題，每題 8 分；請在答案紙上依題後說明寫出答案)

7. $\oint_C (z\vec{i}+2x\vec{j}+y\vec{k})\cdot d\vec{r} =$ (7), where C is the intersection of the cone $z=\sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=1$.(請寫(a)C 的參數式,(b)表為單變數的積分,(c)化簡後的答)
8. Solutions for the differential equation $y+(2xy-e^{-2y})y'=0$ are (8). (請寫出所有 solution curves)

9. $\iint_S z\,dS =$ (9), where S is the surface $z^2=1+x^2+y^2$ between planes $z=1$ and $z=\sqrt{10}$.

(請寫出(a)轉換為 multiple integral,(b)化簡為一個變數的積分,(c)化簡後的答)

10. (12 分)Find all critical points of function $f(x,y)=xy-x^2y-2xy^2$ and describe the geometric behavior (local max.? local min.? or saddle?) there. (此題請抄函數;請僅寫出答案,計算過程另紙處理)

11. (12 分)Points on the ellipse $\{(x,y,z)\,|\,y-2z=3, z^2=x^2+y^2\}$ closet to and farthest to the origin are(11a), (11b) respectively. (請依序寫出點的坐標)