$\sqrt{\frac{2ax}{m(x^2+a^2)}} = v^2$

Classical Mechanics (I): Midterm 2017

November 3, 2017

Useful integrals:

$x = \frac{-a^2 \pm a\sqrt{a^2+4}}{2}$

$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

$$\int \frac{dx}{x(a^2+x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2+x^2}\right)$$

$$\int e^{ax}dx = \frac{e^{ax}}{a} \qquad (1)$$

$a(-x^2+a^2-a^2x)$

$(x^2+a^2)-2x^2-a^2x=0$

Problem 1 20%

(a) Explain briefly the following terms:

12. (i) quality factor of resonance (ii) limit cycle (iii) overdamped motion (iv) Lyapunov exponent

(b) Write down one example of non-linear oscillation and a distinct feature for non-linear oscillations. (No reasoning needs to be given for this problem.)

15 **Problem 2** Consider a particle of mass m that moves on a helix with the trajectory at the moment t being given by $\vec{r} = (3\cos\phi(t), 3\sin\phi(t), 4\phi(t))$, where $\phi(t)$ is some function of time t.

(a) 7% At $t=0$, $\phi(0)=\phi_0$, if the speed of the particle is fixed to be v, find $\phi(t)$ (assuming $\phi(t)>0$ for $t>0$) and the acceleration $\vec{a}(t)$ of the particle at $t>0$.

(b) 8% Now suppose that the helix is frictionless and the net force that acts on the particle is $(0,0,-F)$ with F being a positive constant. If at $t=0$, $\phi(0)=\phi_0$ and $\frac{d\phi(0)}{dt}=0$ find $\phi(t)$ and the speed $v(t)$ of the particle at a later time $t>0$.

$\vec{F}$; $\vec{r}$

Problem 3 A particle of mass m is subject to a force with the potential energy $V(x) = \frac{ax}{x^2+a^2}$, where a is a positive constant.

$(\vec{r}-\vec{g})\cdot\vec{\phi} = 0$

11. **(a) 6%** Find the position of stable equilibirum and the period of small oscillation about it.

(b) 9% If the particle starts to move from the stable equilibirum point with velocity v, find the range of v for which the particle (i) oscillates, (ii) escapes to $x=-\infty$, (iii) escapes to $x=+\infty$.

(c) 5% Sketch the phase diagram of this system.

$U(x) = 0$

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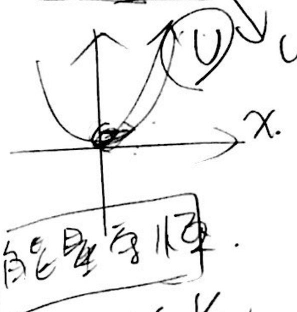
$\frac{1}{2}m\dot{x}^2 > V(x)$



$\rightarrow v$. 能量守恒.

$E = V + K$.

Problem 4 Consider a particle that is under the influence of the force $\vec{F} = 3x^2\hat{x} + (2xz - y)\hat{y} + z\hat{z}$.

15 **(a) 10%** Find the work that needs to be done to move the particle from $(0, 0, 0)$ to $(2, 1, 3)$ by two paths: (i) the straight line (ii) along the curve: $x(t) = 2t^2$, $y = t$, $z = 4t^2 - t$.

(b) 5% Is $\vec{F}$ a conservative force? Explain your result. If it is conservative force, find the potential.

Problem 5 10% A particle moves in a medium under the influemce of retarded force equal to $mk(v^3 + a^2v)$. Here v is the speed of the particle. k and a are constants. By considering all possible initial velocities, find the maximum distance that the particle can travel.

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Problem 6

10 **(a) 5%** Consider an underdamped oscillator of mass m. The natural frequency of the oscillator is ω_0 and the damping parameter is β. If the oscillator is driven by a force $F = \cos\omega t + 2\cos 2\omega t$, find the position of the oscillator $x(t)$ in the limit of $t \to \infty$.

(b) 10% Following (a), now the oscillator is driven by a force that starts at $t = 0$ as follows

$$F(t) = \begin{cases} 0, & t < 0 \\ A\sin\omega t, & t > 0 \end{cases}$$

If initially, the oscillator is at rest with $x(0) = 0$, use the Green's method to find $x(t)$.

(c) 5% Following (b), what would be $x(t)$ if initially the oscillator is at $x(0) = 0$ with a velocity v_0?

$m\ddot{x} + b\dot{x} + kx = H(t)\,A\sin\omega t$