108 學年下學期　微積分　期末考試題 109/06/19

1. (15 points) Evaluate each of the following if it exists, explain why if it doesn't.

   (a) $\lim_{(x,y)\to(0,1)} \dfrac{x^2(y-1)^2}{x^2+(y-1)^2}$　　(b) $\int_0^\infty \dfrac{\tan^{-1}(\pi x)-\tan^{-1}x}{x}\,dx$

   (c) $\oint_C \dfrac{-y\,dx+x\,dy}{x^2+y^2}$, where C is the polygon with vertices at $(1,0),(0,1),(-1,0),$ $(0,-2),(2,0),(2,2),(-2,2),(-2,-2),(1,-2),(1,0)$ in that order.

2. (15 points) Find points on the ellipse $\begin{cases} z^2=x^2+y^2 \\ x+y-z+1=0 \end{cases}$ that are closest and farthest from the origin.

10 points each for 3~10)

3. Find directional derivative of f at $(0,0)$ in direction $(\cos\theta,\sin\theta)$, where

$$f(x,y)=\begin{cases} \dfrac{2xy^2}{x^2+y^4} & \text{for } (x,y)\neq(0,0) \\ 0 & \text{for } (x,y)=(0,0) \end{cases}.$$

4. Find and classify all critical points of $f(x,y,z)=xy+x^2z-x^2-y-z^2$.

5. Find volume of the solid that lies outside $z=\sqrt{x^2+y^2}$ and inside $z=\sqrt{1-x^2-y^2}$

6. Find the centroid of the solid bounded above by the plane $z=y$ and below by the paraboloid $z=x^2+y^2$.

7. Evaluate $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dfrac{e^{-(x-y)^2}}{1+(x+y)^2}\,dx\,dy$ if it exists.

8. Find all solutions for $e^x\sec y+\tan y+y'=0$.

9. Find $\iiint_T \dfrac{1}{x^2+y^2+z^2}\,dV$, T be the solid $0\le x\le 3, 0\le y\le\sqrt{9-x^2}, 0\le z\le\sqrt{9-x^2-y^2}$.

10. Find work done by force field $\vec{F}=(y^2\cos x+z^3)\,\vec{i}+(2y\sin x-4)\,\vec{j}+(3xz^2+2)\,\vec{k}$ when moving a particle along curve $\gamma(t)=\sin^{-1}t\,\vec{i}+(1-2t)\,\vec{j}+(3t-1)\,\vec{k}\ (0\le t\le 1)$.