Quantum Physics (II): Midterm April 24, 2019

Useful Informations:

$$Y_{1\pm1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}}e^{\pm i\phi}\sin\theta, Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$\text{Bohr radius} = a_0 = \frac{\hbar}{m_e c\alpha}$$

$$\alpha = \frac{e^2}{\hbar c}(\text{Gauss}) \text{ or } \frac{e^2}{4\pi\varepsilon_0\hbar c}(\text{MKS}) \ (m_e\text{=electron mass, } c\text{=speed of light})$$

$$hc/e = 4.14\times10^{-7} Gauss - cm^2$$

Problem 1 24% Explain the following terms or answer the questions briefly.
(a) space quantization **(b)** Zeeman effect **(c)** Landau levels **(d)** The Aharanov-Bohm effect
(e) A particle of mass m is influenced by a central potential $V(r)$. If the wavefunction of the particle is known to be the form: $R(r)\sin(\theta)\cos(\phi)$ in the spherical coordinates (r,θ,ϕ), what is the equation that the radial wavefunction $R(r)/r$ obeys?
(f) Following (e), if the particle is placed in a magnetic field, $\vec{B}(\vec{r},t) = \nabla\times\vec{A}(\vec{r},t)$, what would be the probability density $\vec{j}(\vec{r},t)$ if the wavefunction is given by $\psi(\vec{r},t)$?

Problem 2 10% The relativistic analog of the Schrodinger equation for a spin 0 particle in a Coulomb potential is

$$\left(\frac{E}{\hbar c} + \frac{Z}{4\pi\varepsilon_0\hbar c}\frac{1}{r}\right)^2\Psi = -\nabla^2\Psi + \left(\frac{mc}{\hbar}\right)^2\Psi.$$

By comparing to the problem of the hydrogen atom, find the spectrum of E.

Problem 3 Consider a system described by

$$H = \frac{L_x^2+L_y^2}{2I_1} + \frac{L_z^2}{2I_2} + aL_x + bL_y + cL_z.$$

(a) 7% If $I_1 > I_2$ and $a=b=c=0$, find the energy spectrum of the system, and compare the spectrum with respect to the case when $I_1 = I_2$. **(b)**7% Suppose we want to describe the molecule CN as a dumbbell consisting of two masses M_1 and M_2 attached by a rigid rod of length d. If we denote the rod as z axis and describe the rotation of the dumbbell in a plane about axises going through the center of mass and perpendicular to the rod, show that the Hamiltonian for describing the motion of CN is also in the form of H. Find the corresponding parameters (I_1, I_2, a, b, and c) and the energy spectrum in terms of d, M_1 and M_2. **(c)**7% If $I_1 = I_2 = I$, $a=1$, $b=-3$, and $c=2$, find the energy spectrum.

Problem 4
(a) 10% A researcher prepares a hydrogen in the state: $n=2$, $l=1$,and $m=-1$. The quantization axis he uses is perpendicular to the ground. This hydrogen is later sent to his colleague in the same laboratory. His colleague, however, uses a quanzation axis parallel to the ground. Find the probablities for his colleague to find this hydrogen atom in all possible states specified by (n , l ,m) using his quantization axis.
(b) 8% Following (a), suppose that instead of being in the state: $n=2$, $l=1$,and $m=-1$, the angular wavefunction of the prepared hydrogen is given by

$$f(\theta,\phi) = N[1+\sin\theta\sin\phi].$$

Let the axis that points out from the ground be denoted by z axis and two axes being parallel to the ground being denoted by x axis and y axis. x, y, and z form a Catersian coordinate system. Find the expectation value of $\mathbf{L}^2$ and possible values one would get if one measures $L_x + L_z$.

Problem 5

(a) 8% Consider a hydrogen atom in a small but uniform magnetic field $\mathbf{B}$. If we neglect spins, plot a diagram showing how the energies of the atom change versus the magnitude of the magnetic field (for $n \leq 3$). Consider transitions from states in $(n = 3, l = 2)$ to $(n = 2, l = 1)$, draw a diagram indicating possible transitions.

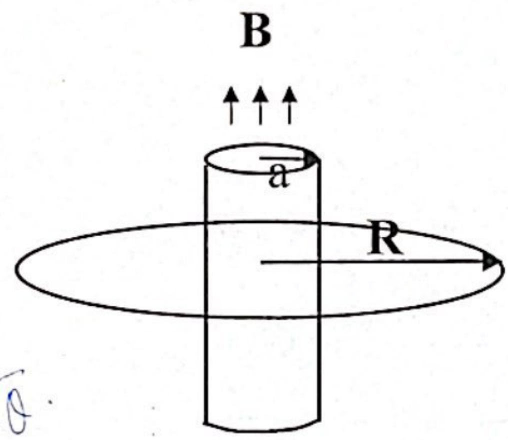
(b) 9% Consier a positronium atom that consists of an electron and a positron (charge $= +e$, $m = m_e$) in a hydrogen-like bound state. Write down the Hamiltonian for this system in the presence of a constant external magnetic field (ignoring the spins of the electron and positron) there is no Zeeman effect. Will the positronium atom exhibit the Zeeman effect? Why?

Problem 6 10% Imagine a particle confined to move in a circle of radius R (see Fig.1). Along the axis, we apply a magnetic field B inside a small solenoid with radius a. Let the flux of the magnetic field be $\phi = \pi a^2 B$. By choosing the vector potential appropriately, find the energy eigenvalues and the corresponding normalized energy eigenfunctions.

$A = \phi$

μB. $\frac{e\hbar}{2mc}$

$\nabla = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}$.



$\frac{1}{R}\frac{\partial}{\partial\theta}$.

FIG. 1.