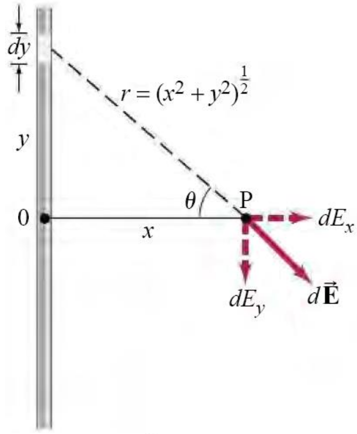
Test-1 Electrostatics, Magnetostatics, and EM Induction 03/30

$\varepsilon_0 = 8.854\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$; $e = 1.602\times10^{-19}\mathrm{C}$; $\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$

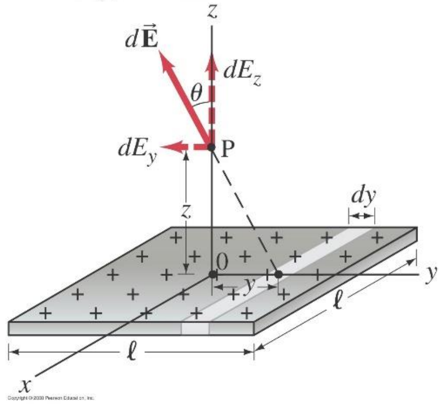
1. (1) The uniformly charged straight wire in the figure has the length ℓ, where point O (at $x=0$ and $y=0$) is at the midpoint. Show that the field at point P, a perpendicular distance x from O, is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{x\left(\ell^2+4x^2\right)^{1/2}},$$

where λ is the charge per unit length. [10%]

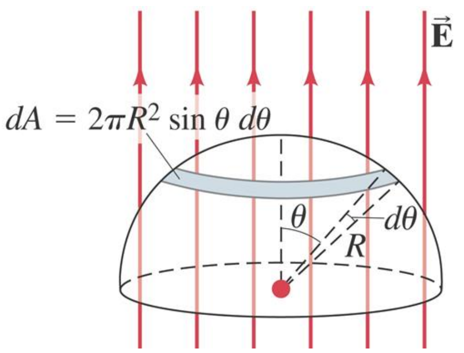


(2) Charge is distributed uniformly over a large square plane of side ℓ as shown. The charge per unit area is σ. In the limit $\ell\to\infty$, check the electric field at a point P a distance z above the center of the plane consistent with Gauss's law. [*Hint*: Divide the plane into long narrow strips of width dy: $\lambda = dq/\ell = \sigma dy$ and the result of (1); then sum the fields due to each strip to get the total field at P]. PS. $du/\left(1+u^2\right) = d\tan^{-1}u$. [10%]

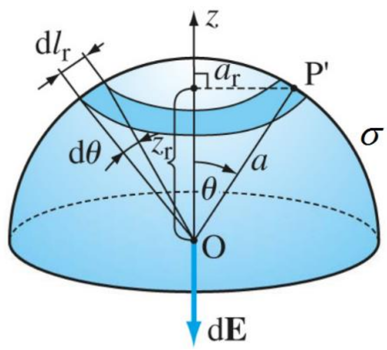


2. (1) A hemisphere of radius R is placed in a charge-free region of space where a uniform electric field exists of magnitude E directed perpendicular to the hemisphere's circular base, in the figure below on the left. (*a*) Using the definition of Φ_E through an "open" surface, calculate (via explicit integration) the electric flux through the hemi-sphere. [*Hint*: In the figure you can see that, on the surface of a sphere, the infinitesimal area located between the angles θ and $\theta+d\theta$ is $dA = \left(2\pi R\sin\theta\right)\left(Rd\theta\right) = 2\pi R^2\sin\theta d\theta$.] (*b*) Choose an appropriate gaussian surface and use Gauss's law to much more easily obtain the same result for the electric flux through the hemisphere. [10%]

(1)



(2)



(2) A hemispherical surface of radius a is uniformly charged with a charge density σ. The medium is air. Compute the electric field intensity vector and the electric potential at the center of the hemisphere (center of the corresponding full sphere). [10%]

3. (1) In Fig. 25-59, two parallel-plate capacitors A and B are connected in parallel across a 600 V battery. Each plate has area 80.0 cm^2; the plate separations are 3.00 mm. Capacitor A is filled with air; capacitor B is filled with a dielectric of dielectric constant $\kappa = 2.60$. Find the magnitude of the electric field within (a) the dielectric of capacitor B and (b) the air of capacitor A. What are the free charge densities σ on the higher-potential plate of (c) capacitor A and (d) capacitor B? (e) What is the induced charge density σ' on the top surface of the dielectric? [10%]

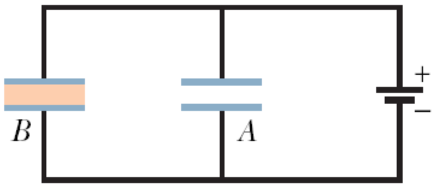


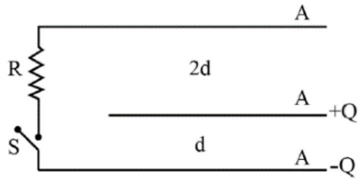
Fig. 25-59

Hint: (a) 200 kV/m; (b) 200 kV/m; (c) 1.77 μC/m2; (d) 4.60 μC/m2; (e) 2.83 μC/m2

(2) Three identical large metal plates of area A are small at distances d and 2d from each other. Top metal plate is uncharged, while the other metal plates have charges $+Q$ and $-Q$. Top and bottom metal plates are connected by switch S through a resistor of unknown resistance. What energy (in mJ) is dissipated in the resistor when switch is closed?

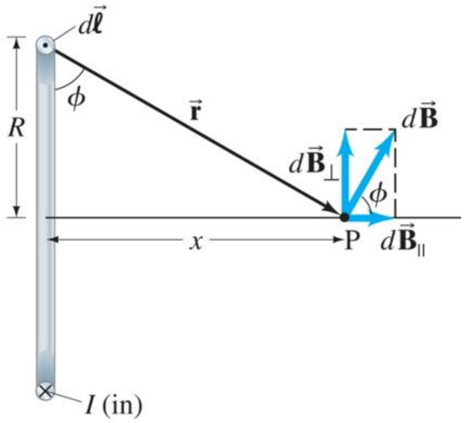
(given: $\frac{\epsilon_0 \mathbf{A}}{\mathbf{d}} = 6\,\mu\mathbf{F},\ \mathbf{Q} = 60\,\mu\mathbf{C}$) [10%]

Hint: $100 \times 10^{-12}\mathbf{J}$

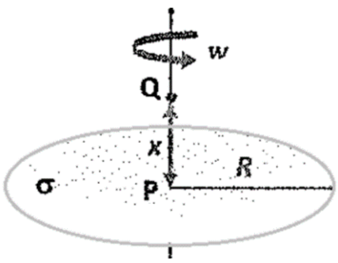


4. (1) Determine $\vec{\boldsymbol{B}}$ for points on the axis of a circular loop of wire of radius R carrying a current I. [10%]

Hint: $d\vec{\boldsymbol{B}} = \frac{\mu_o}{4\pi}\frac{Id\vec{\ell} \times \hat{\boldsymbol{r}}}{r^2} \xrightarrow{?} \vec{\boldsymbol{B}}(x) = \frac{\mu_o I R^2}{2\left(R^2 + x^2\right)^{3/2}}\hat{\boldsymbol{x}}$



(2) A disk of radius R has a total charge q uniformly distributed on it. The surface charge density of the disk is σ. The disk is rotating around a vertical axis through its center with angular frequency w.



(($\boldsymbol{i}$)) Show that the $\vec{\boldsymbol{B}}$-field at distance x from the center P of the disk (*i.e.*, point Q in the figure) is

$$\vec{\boldsymbol{B}}(x) = \frac{\mu_o \sigma w}{2}\left(\sqrt{R^2+x^2} - 2|x| + \frac{x^2}{\sqrt{R^2+x^2}}\right)\hat{\boldsymbol{x}} \qquad [5\%]$$

Hint: $\vec{\boldsymbol{J}}_S = \sigma\vec{\mathbf{v}} = \sigma\vec{\boldsymbol{w}}\times\vec{\boldsymbol{r}}$, where $0 < r \le R$

PS. $$\frac{r^3 dr}{\sqrt{r^2+x^2}^{\,3}} \underset{u\equiv\sqrt{r^2+x^2}}{=} \frac{(u^2-x^2)rdr}{u^3} = \left(u^2-x^2\right)\frac{udu}{u^3}$$

(($\boldsymbol{ii}$)) In the previous problem, by taking the limit: $|x| >> R$, derive the effective magnetic moment of this rotating disk. [5%]

5. (1) How does the ***magnetic*** Lorentz force turn a moving charged particle, but not affecting the particle's kinetic energy? [5%]

(2) Regarding the Hall effect, the Hall voltage $V_H = \dfrac{iB}{\ell ne}$ is derived according to the balance condition of forces: $q\vec{\boldsymbol{E}} + q\vec{\mathbf{v}}\times\vec{\boldsymbol{B}} = 0$. What does it mean? [5%]

(3) Why Hall effect and its applications are so important? Give an example. [5%]

(4) Answer the following question. [5%]

Suppose we use a Hall probe to measure the magnitude of a constant magnetic field. The Hall probe is a strip of copper with a height of 1 mm. We measure a voltage of 0.15 μV across the probe when we run a current 2 A through it. What is the magnitude of the magnetic field? (The density of copper is 8.96 g/cm3, and 1 mole of copper is 63.5 g and each copper atom has one conduction electron)

6. (1) A solenoid with N turns has a length l and a radius r. For what current will the energy density within the solenoid be u?

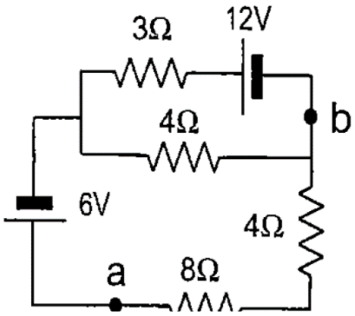
(a) $\sqrt{2u/\mu_0 N}$, (b) $\sqrt{2ul/\mu_0 N}$, (c) $\sqrt{2u^2 l/\mu_0 N^2}$, (d) $\sqrt{2ul^2/\mu_0 N^2}$, (e) none of the above.

Derive you result in use of Ampère's law first. [10%]

(2) If the current changes with time, $dI(t)/dt > 0$, show that the induced electric field is

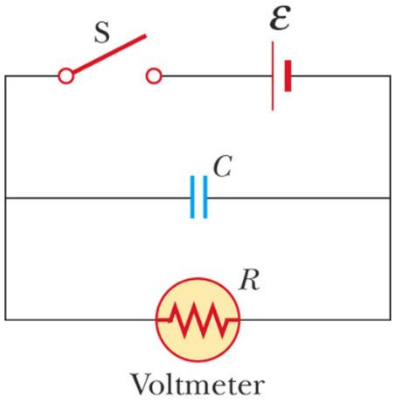
$$E = \frac{1}{2}a\frac{dB}{dt} = \frac{\mu_0 a N}{2\ell}\frac{dI}{dt} \qquad [10\%]$$

7. (1) For the circuit shown in the right figure, what is the potential difference between points **a** and **b**? (hint: you can add only one battery each time, and combine the two results together.)



Hint: $V_{ab} = 11.25\,\mathrm{V}$ [10%]

(2) Is anything wrong on the following circuit? If not, show $I_R(t)$ and $V_C(t)$ curves as the switch turns on (closed) at $t=0$. [10%]



8. (1) 1 eV = ________ J [5%]

(2) 1 T = ________ gauss [5%]

(3) The unit of magnetic flux: [Φ_m] = ________ [5%]

(4) The unit of LR: [LR] = ________ (in MKSA) [5%]