

# Quantum Physics I Midterm Exam

2023/11/10 8:00~9:50

1. The wave function of a particle is given by

$$\psi(x) = \begin{cases} Ae^{\alpha x}, & x < 0 \\ Be^{-\alpha x}, & x > 0 \end{cases}$$

where  $A$ ,  $B$ , and  $\alpha$  are positive real constants.

- (a) Calculate  $A$  and  $B$  in terms of  $\alpha$  so that  $\psi(x)$  is normalized. (5 pts.)
- (b) Where is the particle most likely to be found? (5 pts.)
- (c) What is the probability of finding the particle between  $x = 0$  and  $1/\alpha$ ? (5 pts.)
- (d) What is the expectation value of the particle's position  $x$ ? (5 pts.)
- (e) What is the expectation value of the particle's momentum  $p$ ? (5 pts.)
- (f) Calculate the particle's wave function in the momentum space  $\phi(p)$ . (5 pts.)

2. A particle in the harmonic potential  $\frac{1}{2}m\omega^2x^2$  starts out in the state

$$\Psi(x, 0) = A[\psi_1(x) + 2\psi_2(x) + 2\psi_3(x)],$$

where  $\psi_n(x)$  is the  $n$ th excited state of the quantum oscillator.

- (a) Calculate  $A$  so that  $\Psi(x, 0)$  is normalized. (5 pts.)
- (b) If you measure the particle's energy, what values might you get and with what probabilities? (5 pts.)
- (c) Suppose the energy is measured (at time  $t_0$ ) to be the highest value in (b). What is the expectation value of  $x$  after the energy measurement (for time  $t > t_0$ )? (5 pts.)
- (d) What is the probability density of finding the particle at position  $x$  and time  $t > t_0$  after the energy measurement in (c)? (5 pts.)

3. Consider the potential

$$V(x) = \begin{cases} 0, & x \leq 0 \\ V_0, & x > 0 \end{cases}$$

where  $V_0$  and  $a$  are positive real numbers.

- (a) Calculate the transmission coefficient of a particle with energy  $E < V_0$ . (10 pts.)
- (b) Repeat (a) for  $E > V_0$ . (10 pts.)

4. The Hamiltonian for a certain three-level system is represented by the matrix

$$\hat{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix},$$

where  $a$ ,  $b$ , and  $c$  are real numbers.

(a) If you measure the energy, what are the possible values? (10 pts.)

(b) If the system starts out in the state

$$|S(0)\rangle = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix},$$

what is  $|S(t)\rangle$ ? What are the probabilities of measuring the energy at time  $t$  to be the values in (a)? (15 pts.)

5. Consider an experiment in which the position of an electron is measured by viewing the electron with a microscope while illuminating it. Use such a thought experiment to estimate the ultimate limit (minimal value) of  $\Delta p_x \Delta x$  set by the uncertainty principle. (10 pts.)