Quantum Physics (II): Final June 19 8AM-10AM, 2020

Useful Information: Total grade = 120

$$Lande\ g\ factor = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137},\ e = 4.8 \times 10^{-10}(e.s.u), \hbar = 1.05 \times 10^{-27} erg - s$$

Problem 1 16% Explain the following terms briefly:
(a) gyromagnetic ratio (b) Fermi's golden rule (c) 21 cm line (d) stimulated emission.

Problem 2 Consider two electrons in a spin singlet state.
(a) 4% If a measurement of the spin of one of the electrons shows that it is in a state with $s_y = 1/2$, what is the probability that a measurment of the x-component of the spin of the other electron yields $s_y = 1/2$?
(b) 6% If a measurement of the spin of one of the electrons shows that it is in a state with $s_y = 1/2$, what is the probability that a measurment of the x-component of the spin of the other electron yields $s_x = 1/2$?

Problem 3 In the simplest description of the hydrogen atom, the Hamiltonian for the electron (or the reduced mass) is $H = p^2/2m_e - e^2/r$. It is known that this Hamiltonian is not accurate enough. The energy spectrum of the hydrogen atom has more structures. The lowest order corrections are called finestructure of the spectrum.
(a) 5% Write down two possible corrections to the Hamiltonian that causes the finestructure of the spectrum. Estimate the order of magnitude of the energy splitting (in comparison to $m_e c^2$) due to these corrections.
(b) 7% Sketch the energy levels that include the finestructure for $n \leq 3$. Indicate each level by the spectroscopic notations and indicate number of degenerate states in each energy level.
(c) 6% Using the above correction of spectrum, now consider the transition of the hydrogen atom in the strong magnetic field case (the Paschen-Bark effect) from $n = 3, l = 2$ to $n = 2, l = 1$. Plot a diagram showing possible transitions and thus find number of lines in spectrum.
(d) 8% Consider the weak field case, plot a diagram showing possible transitions from $n = 3, l = 1$ to $n = 2, l = 0$. (Labelling states by the spectroscopic notations.)

Problem 4 8% A particle of mass m is in a 1D infinite potential well with width d and the zero of potential bottem being 0. We have solved this problem in the non-relativisitic limit in which the kinetic energy is $p^2/2m$. Find the relatistic correction of the energy eigenvalues to the order $1/c^2$.

Problem 5 (a)6% Write out the electronic configurations for the following atoms in the ground state N $(Z = 7)$, $Co(Z = 27)$, and $K(Z = 19)$ treating electrons as independent particles.
(b)6% Following (a), use the Hund's rules to find the spectroscopic description of the ground states of N $(Z = 7)$ and Ni $(Z = 28)$.
(c) 10% Consider an atom with Z electrons. Suppose that we want to model this atom without considering the Pauli exclusion principle. It is then reasonable to assume that all electrons are in the same single particle state. In this single particle state, the average distance between the neucleus and the electron is R. Furthermore, by using the uncertain principle, $p = \Delta p = \hbar/R$, one can express the kinetic energy in terms of R. In addition to the above interaction, there are Coulomb repulsions between electrons so that we can assume the average distance between any two electrons is fR, where f is an electron-electron avoidance parameter. (i) By minimizing the total energy with respect to R, express the ground state energy of this atom in terms of Z and the ground state energy (ε_0) of the hydrogen atom. (ii) It is found that the ground state energy of the Helium atom is $5.8\varepsilon_0$. From this, find f and calculate the ionization energy (the energy for removing one electron from the atom) in terms of Z and ε_0.
(d) 10% Fig. 1 shows the ground state and first four excited states of the helium atom. Draw a diagram to indicate the complete spectroscopic notation of each level and show the allowed dipole transition by drawing arrows on the diagram.

Problem 6

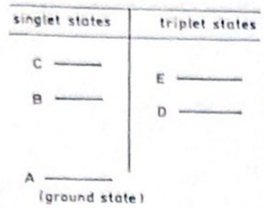

FIG. 1.

(a) 7% It is known that the ionization potential of sodium is 5.14eV and the electron affinity of fluorine is 3.40eV. Furthermore, the equilibrium separation of NaF (sodium fluoride) is 0.193nm and the dissociation energy of NaF is 4.99eV. Assuming the Pauli exclusion principle forces Na and F to repel each other with the form of the potential: $U_{ex}(r) = A/r^n$. Find A and n.
(b)4% The potential energies of nuclei in a diatomic molecule for two different electronic configurations are shown in Fig.2. Explain briefly which one has larger seperations between rotational energy levels, and which one has larger separations between low lying vibrational states.

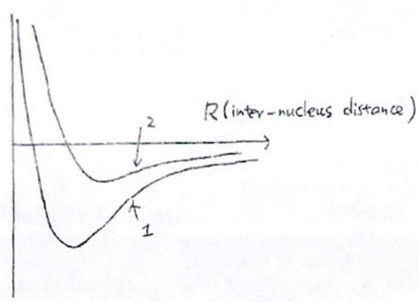

FIG. 2.

(c)4%In the above figure, mark vibrational and rotational levels schematically (Plot the diagram in your answering book). Indicate the order of magnitude of level spacing in terms of eV.
(d)6% Transitions from rotational levels of one vibrational state to the rotational levels of another vibrational state with no change in the eletronic state produces the so-called vibration-rotation spectrum. Consider the vibration-rotation spectrum from $n = 1$ to $n = 0$. Neglecting the mixing of the rotational and the vibrational motions. Plot a diagram showing allowed emission spectrum pattern, indicating the distance between the spectral lines and show that there is actually no transition line corresponding to the vbirational energy $\hbar\omega$, where ω is the vibrational frequency.
(e)7% Consider the effect due to the coupling between the rotational and the vibrational motion for nuclei, in the approximation for the potential

$$V(R) = \frac{1}{2}m\omega^2(R - R_0)^2 + \frac{l(l+1)\hbar^2}{2mR^2},$$

where R is the distance between two nuclei and l is the orbital angular momentum quantum number. By finding the minimum of $V(R)$, show that after taking the coupling into consideration, the rotational energy now takes the following form: $E_l = Al(l+1) + B[l(l+1)]^2 + \cdots$. Find A and B.