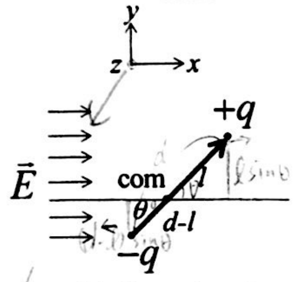
普通物理期中考(3/31/2016) 時間:10:00-13:00

將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。

$$Note: \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\ \mathrm{N\cdot m^2/C^2}$$

$\vec{P}\times\vec{E} = \tau$

1. Derive (a) the torque applied by a uniform electric field $\vec{E}$ on a initially stationary electric dipole $\vec{p} = q\vec{d}$. [5%] and (b) the its corresponding potential energy $U(\theta)$. [5%] [Let $U(90°)=0$.]



立体角 $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ #

2. (a) Show that Gauss' law can be derived from Coulomb's law. [10%] (b) Derive the differential form of Gauss's law from its integral form. [5%]

3. A total charge Q is uniformly distributed throughout a spherical volume of radius R. Calculate the electric potential energy stored in this charge distribution. [10%]

4. Derive the energy density as a function of electric field intensity E in the presence of dielectrics. [10%]

$\frac{dU}{d\tau} = \frac{1}{2}\varepsilon_0 \kappa E^2$

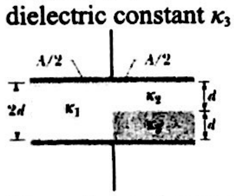
5. Consider a simple microscopic model for Ohm's law in which conduction electrons are free to move throughout the sample of a conducting material while colliding with atoms of the conducting material. Let τ be the average time between collisions and n is the concentration of conduction electrons. The electron mass and charge are m and e, respectively. Find an expression for the material's resistivity in terms of m, n, e, and τ. [5%]

$V_d = a\tau$

6. A current is set up through a truncated right circular cone of resistivity ρ, left radius a, right radius b, and length L. Assume that the current density is uniform across any cross section taken perpendicular to the length. What is the resistance of the cone? [5%]

$R = \int_0^L \frac{\rho\,dx}{\left(\frac{b-a}{l}\cdot x + a\right)^2 \pi} = \frac{\rho l}{\pi(b-a)}\left(\frac{1}{a} - \frac{1}{b}\right)$

7. A parallel-plate capacitor has a plate area A and a plate separation $2d$. The left half of the gap is filled with material of dielectric constant κ_1; the top of the right half is filled with material of dielectric constant κ_2; the bottom of the right half is filled with material of dielectric constant κ_3. What is the capacitance? [5%]



κ_2, κ_3 串

κ_1 和 $(\kappa_2\kappa_3)$ 並聯

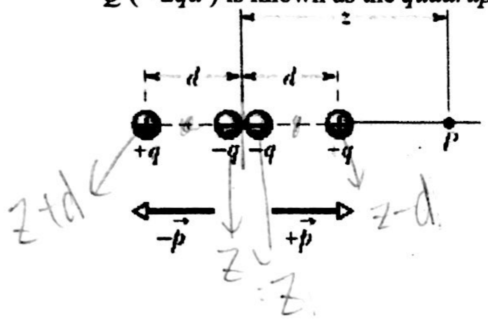
$q = \int_\tau \rho\, d\tau = \int_0^R \frac{\rho_s r}{R} \times 4\pi r^2 dr = \rho_s R^3 \pi$

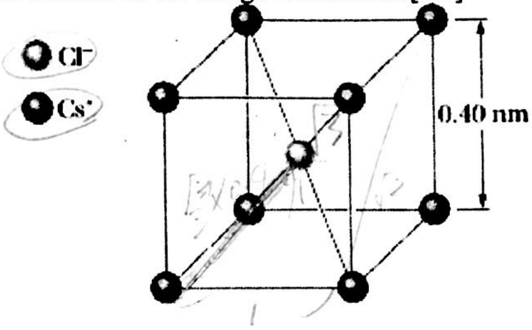
8. The volume charge density of a solid nonconducting sphere of radius $R = 5.60$ cm varies with radial distance r as given by $\rho = \rho_s r/R$. (a) What is the sphere's total charge?[5%] (b)What is the field magnitude E at a distance $r<R$ from the center of the sphere ? [5%]

(b) $\oint_s \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{\rho\pi r^4}{R} = E\, 4\pi r^2$

$E = \frac{\rho_s r^2}{4\varepsilon_0 R}$ #

9. An electric quadrupole consists of two dipoles with dipole moments that are equal in magnitude but opposite in direction. Show that the value of E on the axis of the quadrupole for a point P a distance z from its center (assume $z \gg d$) is given by $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$, in which Q ($= 2qd^2$) is known as the *quadrupole moment* of the charge distribution. [5%]





10. In crystals of the salt cesium chloride, cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cube's center. The edge length of the cube is 0.40 nm. The Cs^+ ions are each deficient by one electron (and thus each has a charge of $+e$), and the Cl^- ion has one excess electron (and thus has a charge of $-e$). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs^+ ions at the corners of the cube? [5%] (b) If one of the Cs^+ ions is missing, the crystal is said to have a *defect*; what is the magnitude of the net electrostatic force exerted on the Cl^- ion by the seven remaining Cs^+ ions? [5%]

靜電力

$\boxed{\text{分段算}}$ 必考

11. A thick spherical shell of charge Q and uniform volume charge density ρ is bounded by radii r_1 and $r_2 > r_1$. With $V = 0$ at infinity, find the electric potential V as a function of distance r from the center of the distribution, considering regions (a) $r > r_2$, [5%] (b) $r_2 > r > r_1$, [5%] and (c) $r < r_1$. [5%]

$-\int_\infty^r \vec{E}\,dr$

積分2次

$\rho = \dfrac{Q}{\frac{4}{3}\pi(r_2^3 - r_1^3)}$

$\vec{E}=0$ $+Q, \rho$

$V = -\int \vec{E}\,d\vec{r}$

分2次算.

(a) $-\int_\infty^r \dfrac{\frac{4}{3}\pi(r_2^3-r_1^3)\rho}{4\pi\varepsilon_0 r'^2}\,dr' = \dfrac{(r_2^3-r_1^3)\rho}{3\varepsilon_0}\dfrac{1}{r}$

(b) $-\int_\infty^{r_2} \dfrac{\frac{4}{3}\pi(r_2^3-r_1^3)}{4\pi\varepsilon_0 r'^2}\rho\,dr' - \int_{r_2}^{r} \dfrac{\frac{4}{3}\pi(r'^3-r_1^3)\rho}{4\pi\varepsilon_0 r'^2}\,dr' = \dfrac{\rho}{3\varepsilon_0}\left(\dfrac{3}{2}r_2^2 - \dfrac{1}{2}r^2 - \dfrac{r_1^3}{r}\right)$

(c) 同 r_1 电位. ⇒ $\dfrac{\rho}{3\varepsilon_0}\left(\dfrac{3}{2}r_2^2 - \dfrac{3}{2}r_1^2\right) = \dfrac{(r_2^2-r_1^2)}{2\varepsilon_0}\rho$ #

∵内部 $\vec{E}=0$

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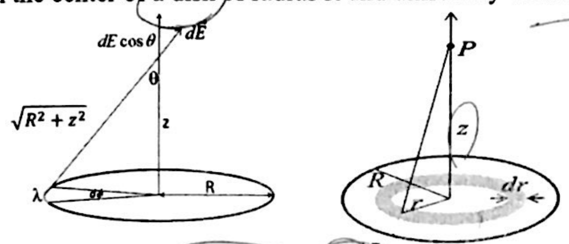
$$Note: \frac{1}{4\pi\varepsilon_0} = 8.99\times10^9\ \mathrm{N\cdot m^2/C^2}$$

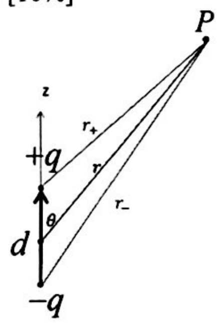
1. Derive (a) the electric field $\vec{E}$ at a distance z from the center of a circular ring of radius R and uniformly distributed charge q [5%] and (b) the electric field E at a distance z from the center of a disk of radius R and uniformly distributed charge q [5%]

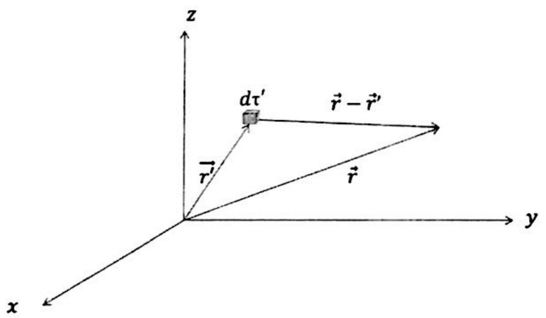


2. (a) Show that the electric potential $V(\vec{r})$ due to an electric dipole $\vec{P}$ at the origin is approximately equal to $\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}\cdot\vec{r}$. [10%] (b) Show that a polarized dielectric with electric polarization $\vec{P}$ carries a bound charge distribution with surface charge density $\sigma_b = \vec{P}\cdot\hat{n}$ and volume charge density $\rho_b = -\nabla\cdot\vec{P}$

Note: $\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{1}{|\vec{r}-\vec{r}'|^2}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$; $\nabla'\cdot\left(\frac{1}{|\vec{r}-\vec{r}'|}\vec{P}\right) = \nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\cdot\vec{P} + \left(\frac{1}{|\vec{r}-\vec{r}'|}\right)\nabla'\cdot\vec{P}$

[10%]

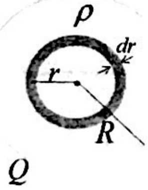


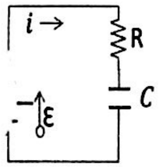


$-\frac{Q}{2\varepsilon_0 R^2 \pi}$

3. A total charge Q is uniformly distributed throughout a spherical volume of radius R. Calculate the electric potential energy stored in this charge distribution. [10%]

U



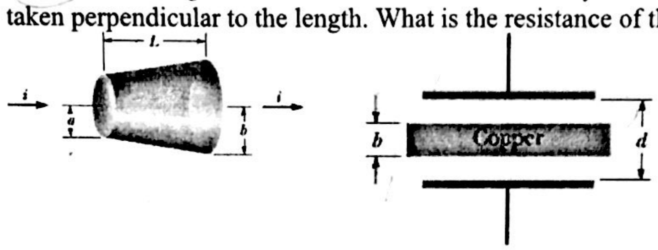


4. Derive the charge as a function of time, q(t), in a simple RC circuit for the following cases: (a) charging with an *emf* E and an initial current $q(0)=0$, and (b) discharging with E=0 and $q(0)=q_0$. [10%]

5. Consider a simple microscopic model for Ohm's law in which conduction electrons are free to move throughout the sample of a conducting material while colliding with atoms of the conducting material. Let τ be the average time between collisions and n is the concentration of conduction electrons. The electron mass and charge are m and e, respectively. Find an expression for the material's resistivity in terms of m, n, e, and τ. [5%]

6. A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude $E = Kr^4$, directed radially outward from the center of the sphere. Here r is the radial distance from that center, and K is a constant. What is the volume density ρ of the charge distribution? [5%]

7. radius b, and length L. Assume that the current density is uniform across any cross section taken perpendicular to the length. What is the resistance of the cone? [5%]



8. A slab of copper of thickness b is thrust into a parallel-plate capacitor of plate area A and plate separation d; the slab is exactly halfway between the plates. (a) What is the capacitance after the slab is introduced? [5%] (b) If a charge q is maintained on the plates, what is the ratio of the stored energy before to that after the slab is inserted? [5%] (c) How much work is done on the slab as it is inserted? [5%]

9. The volume charge density of a solid nonconducting sphere of radius $R = 5.60$ cm varies with radial distance r as given by $\rho = \rho_s r/R$. (a) What is the sphere's total charge?[5%] (b)What is the field magnitude E at a distance $r<R$ from the center of the sphere ? [5%]

10. A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with $V = 0$ at infinity). (a) What is the radius of the drop? [5%] (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop? [5%]

$\frac{A}{d}$ $\quad \frac{A\varepsilon_0}{b}K$ $\quad$ 500V $\quad \rho \to 0$ $\quad E=0$

$\frac{dU}{d\tau} = \frac{1}{2}\varepsilon_0 E^2 K$ $\quad E_0 = \frac{q}{A\varepsilon_0}$

$\frac{}{d}(K+1)$

$\frac{1}{\frac{A\varepsilon_0 \cancel{\times 2}}{(d-b)}}\cancel{\times 2} + \frac{b}{A\varepsilon_0 K} = \frac{(d-b)K+b}{A\varepsilon_0 K}$

$\frac{q}{\varepsilon_0 A}$ $\quad 500 = -\int_{\infty}^{R} \frac{30pC}{4\pi\varepsilon_0 r^2} \times dr$

$V = -\int_{-}^{+} \vec{E} \cdot d\vec{r}$

$\Rightarrow \frac{A\varepsilon_0 K}{(d-b)K+b} = \frac{A\varepsilon_0}{d} \times K'$

$K' = \frac{dK}{(d-b)K+b} = \frac{dK}{dK+(1-K)b}$