2017 Fall PHYS2310 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 1-3] 2017/11/14, 10:10am – 12:00am, 教師：張存續

$\phi = 0 \sim 2\pi$

$\theta = 0 \sim \pi$

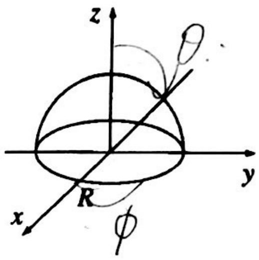
◇ Useful formulas $\nabla V = \dfrac{\partial V}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}v_\phi$$

1. (20%)

(a) Compute the divergence of the function $\mathbf{v} = r^2\cos\theta\hat{\mathbf{r}} + r^2\cos\phi\hat{\boldsymbol{\theta}} - r^2\cos\theta\sin\phi\hat{\boldsymbol{\phi}}$. (10%)

(b) Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (10%).



$\int (\nabla \cdot \vec{E})\, d\tau = \int \vec{E} \cdot d\vec{a}$

2. (20%)

(a) Prove that the normal component of **E** is discontinuous at any boundary, using Divergence theorem. (6%)

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(b) Prove that the tangential component of **E** is always continuous, using Stokes' theorem. (6%)

(c) Write down the normal and tangential component of electric fields immediately outside a metal surface with the surface charge density σ. (8%)

$\vec{E} = \dfrac{\sigma}{\epsilon_0}$

3. (20%) The potential of some configuration is given by the expression $V(\mathbf{r}) = Ae^{-\lambda r}/r$, where A and λ are constants.

(a) Find the energy density. (6%)

(b) Find the charge density $\rho(\mathbf{r})$. (6%)

$\int \vec{E} \cdot d\vec{a}$

(c) Find the total charge Q (do it two different ways) and verify the divergence theorem. (8%)

4 OK

$1+1=2$ $\vec{E}$ $r\,d\theta$

σ

4. (20%) Consider two concentric spherical shells, of radii a and b ($b>a$). The inner shell is connected to a potential $V(a,\theta)$ (to be given), while the outer shell is grounded $V(b,\theta)=0$.

(a) If $V(a,\theta)=V_0$ (constant), find the potential at $r<a$, $a<r<b$, and $r>b$. (10%)

(b) If $V(a,\theta)=V_0(1-\cos\theta-\cos^2\theta)$, find the potential everywhere between the shells ($a<r<b$). (10%) [Hint: use Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)=(3x^2-1)/2$].

$aA_0 + B_0 = aV_0$

$bA_0 + B_0 = 0$

$(a-b)A_0 = aV_0$

$A_\ell a^\ell = A_\ell a^\ell + B_\ell a^{-(\ell+1)}$

$B_\ell b^{-(\ell+1)} = A_\ell b^\ell + B_\ell b^{-(\ell+1)}$

$bB_0 + C_0 = 0$

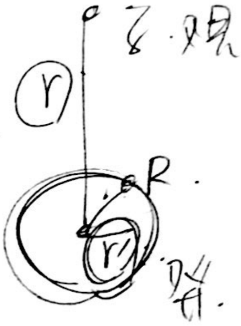
$aB_0 + C_0 = aV_0$

$(a-b)B_0 =$

5. (20%) A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta)=k\frac{R}{r^2}(R-2r)\sin\theta,$$

$\dfrac{1}{|r-r'|}$



where k is a constant, and r, θ are the usual spherical coordinates.

(a) Find the monopole, dipole, and quadrupole terms. (10%)

(d) Find the approximate potential for points on the z axis, far from the sphere. (10%)

[Hint: $V(\mathbf{r})=\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{1}{r}\int\rho(\mathbf{r}')d\tau'+\dfrac{1}{r^2}\int r'\cos\theta'\rho(\mathbf{r}')d\tau'+\dfrac{1}{r^3}\int(r')^2\left(\dfrac{3}{2}\cos^2\theta'-\dfrac{1}{2}\right)\rho(\mathbf{r}')d\tau'+\ldots\right)$].

$\rho(r')\,d\tau'$

(a)

$-\sin^2$

$\int \cos^2\sin^2 d\theta$

$=\left(\dfrac{\sin 2\theta}{2}\right)^2 d\theta$ $\quad \dfrac{1-\cos 4\theta}{4}$ $\quad \left(\dfrac{Rr'^3}{3}-2\dfrac{r^4}{4}\right)\Big|_0^R$

$=\dfrac{-1}{6}R^4$

$\int \cos\theta\sin^2\theta\, d\theta$

$=\sin^2\theta\, d\sin\theta$

$\int \sin^2\theta\, d\theta = \dfrac{1-\cos 2\theta}{2}$

$\dfrac{1}{2}-\dfrac{1}{3}$

$d r\, d\theta$

$RR-R$

$\dfrac{R^2r^2}{2}-\dfrac{2r^3R}{3}=\left(\dfrac{1}{2}-\dfrac{2}{3}\right)R^4$