Geomagnetic Survey through Faraday's Law of Induction

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Faraday's Law of Induction is a fundamental law of electromagnetism predicting how magnetic fields will interact with electric circuits to generate an electromotive force, which is called electromagnetic induction. It's been widely implemented on genarators,electrical motors and transformers.

1. Introduction

The geomagnetic field serves to deflect most of the solar wind, whose high-energy, charged particles would destroy the ozone layer that protects the Earth from harmful ultraviolet radiation in the way that giving sufficient energy to transform ozone into oxygen.

Moreover,humans have used it for navigation for a long time.For some kinds of species,their behaviors even depend on the direction and the intensity of geomagnetic field.

For this experiment,the main purpose is to measure the magnitude and the orientation of geomagnetic field. However, the magnitude of the geomagnetic field as we expected is too small to measure accurately, hence we do it in the way－Faraday's Law of Induction:

$$\varepsilon = -\frac{d\phi}{dt} \tag{1}$$

EMF is proportional to the rate of change of magnetic flux which was defined by Michael Faraday:

$$\phi \equiv \mathrm{N}(\mathbf{B}\cdot\mathbf{A}) \tag{2}$$

where B is the magnetic field, A is the vector area of the coil,and N is the number of turns of the coil.In Cartesian coordinate,magnetic field B can be represented as:

$$\mathbf{B} = B\mathrm{x}\boldsymbol{x} + B\mathrm{y}\boldsymbol{y} + Bz\boldsymbol{z} \tag{3}$$

Then we suppose the coil rotated around the z axis with the constant angular frequency, and then **A** and ϕ can be represented as this form:

$$\mathbf{A} = A\cos(\omega_z t+\theta)\mathbf{x} + A\sin(\omega_z t+\theta)\boldsymbol{y} \tag{4}$$

$$\phi = NA[B_x\cos(\omega_z t+\theta) + B_y\sin(\omega_z t+\theta)] \tag{5}$$

Thus,we can write EMF by Faraday's Law:

$$\varepsilon_z(t) = \omega_z NA[B_x\sin(\omega_z t+\theta) - B_y\cos(\omega_z t+\theta)]$$

$$\varepsilon_{z,p} = \omega_z NA\sqrt{{B_x}^2 + {B_y}^2} \tag{7}$$

By analogy, $\varepsilon_{x,p}$ and $\varepsilon_{y,p}$ can also be represented as this way :

$$\varepsilon_{x,p} = \omega_x NA\sqrt{{B_y}^2 + {B_z}^2} \tag{8}$$

$$\varepsilon_{y,p} = \omega_y NA\sqrt{{B_x}^2 + {B_z}^2} \tag{9}$$

Then we can compute the magnitude and the direction of geomagnetic field:

$$B_E = \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2} = \sqrt{\frac{\left(\frac{\varepsilon_{x,p}}{\omega_x}\right)^2 + \left(\frac{\varepsilon_{y,p}}{\omega_y}\right)^2 + \left(\frac{\varepsilon_{z,p}}{\omega_z}\right)^2}{2N^2(\pi R)^2}} \tag{10}$$

$$\psi = \sin^{-1}\frac{B_z}{B_E} = \sqrt{\frac{\left(\frac{\varepsilon_{x,p}}{\omega_x}\right)^2 + \left(\frac{\varepsilon_{y,p}}{\omega_y}\right)^2 - \left(\frac{\varepsilon_{z,p}}{\omega_z}\right)^2}{\left(\frac{\varepsilon_{x,p}}{\omega_x}\right)^2 + \left(\frac{\varepsilon_{y,p}}{\omega_y}\right)^2 + \left(\frac{\varepsilon_{z,p}}{\omega_z}\right)^2}} \tag{11}$$

where ψ is the magnetic dip ,which can stand for the direction of geomagnetic field.

2. Method

In this experiment,we used coils with radius 20 cm,and 10 cm ,which both of them are 10 turns. Set them on the DC motor , LH Microvoltmeter and digital oscilloscope.

First of all ,we shall standardize the LH Microvoltmeter to know the true amplification of the digital signal. Combined with the signal generator,we can directly obtain the data, that is, the real amplification.

Second, we set up the apparatus.Put the larger coil on the DC motor,and measure the angular velocity of the rotating coil ω along the three different axises(x,y,z) under the voltage 4V and specific frequency. Through digital oscilloscope, we would get the amplitude of the periodic signals which is in terms of EMF,respectively.

As we finished all of computation, we replaced the larger coil with the smaller one and repeated as the following.

3. Results and Disscusion

The result we calculate in the case that using the coil with radius 20 cm is :

$B_E = 5.14 \times 10^{-5}$ T ; $\psi = 22°14'24''$.

In another case that coil with radius 10 cm is :

$B_E = 5.74 \times 10^{-5}$ T ; $\psi = 35°15'36''$.

However, the standard value for the magnitude and the direction of geomagnetic field is :

$B_{E,0} = 3.64 \times 10^{-5}$ T ; $\psi_0 = 37°18'$

Coil Radius	Percentage of error for magnitude of geomagnetic	Percentage of error for magnetic dip
20 cm	41.3%	32.3%
10 cm	57.7%	1.9%

For both cases,the values of experiment have amount of error,especially using the small coil to measure the geomagnetic field. Due to it , We considered there were two possible reasons.

One reason was the magnitude of geomagnetic field is so weak. As we used our instrument to detect it, some inevitable noise interferences would cause the difference between the standard value and experimental value. In addition, we kept detecting an unknown DC signal during the experiment, hence the error is reasonable.

The other reason was that environmental interferences. When we did this experiment, other groups were operating the Helmholtz coil,which would generate the strong magnetic field .Moreover, the experiment where we did was not in a widely ground but in a building. Thus, we thought it was perhaps the primary and the most important reason.

In conclusion, we couldn't avoid the outer environmental interferencse , yet we were still able to get the roughly value of geomagnetic field and its direction.

Appendix

The real amplification of the LHMicrovoltmeter:

Input(mV)	Output(mV)	Real Amplification	Standard Amplification
2.030	4.80	236%	100%
0.216	4.96	2296%	1000%
0.108	24.0	21429%	10000%
0.012	24.9	207500%	100000%

For the large coil under input 4.5 V , the data from oscilloscope:

Axis	Amplitude (mV)	Angular frequency (1/s)	Frequency (1/s)	10times period (s)
z	520	2.09	0.33	30
x	520	2.51	0.40	25
y	520	2.62	0.42	24

Axis	EMF(V)	ε/ω	$(\varepsilon/\omega)^2$
z	1.25×10^{-4}	5.98×10^{-5}	3.58×10^{-9}
x	1.25×10^{-4}	4.98×10^{-5}	2.49×10^{-9}
y	1.25×10^{-4}	4.78×10^{-5}	2.29×10^{-9}

For the small coil under input 8.5 V, the data from oscilloscope:

Axis	Amplitude (mV)	Angular frequency (1/s)	Frequency (1/s)	10times period (s)
z	160	2.62	0.42	24
x	160	2.62	0.42	24
y	160	2.62	0.42	24

Axis	EMF(V)	ε/ω	$(\varepsilon/\omega)^2$
z	3.86×10^{-5}	1.47×10^{-5}	2.17×10^{-10}
x	3.86×10^{-5}	1.47×10^{-5}	2.17×10^{-10}
y	3.86×10^{-5}	1.47×10^{-5}	2.17×10^{-10}