普通物理期中考(12/08/2015) 時間:10:00-13:00

每大題含2-3小題，每小題5分，總分100分。將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。

彈性

1. The center of mass of an extended object of mass M occupying a volume V is located at $\vec{r}_{COM} = (x_{COM}, y_{COM}, z_{COM})$ with respected to the origin near the surface of the Earth.

The object is subject to a uniform gravitational acceleration $-g\hat{k}$. Show (a) the torque by gravity on the object about the origin is $\vec{r}_{COM} \times (-Mg\hat{k})$, and (b) the gravitational potential for the object is Mgz_{COM} if the reference point is on the x-y plane.

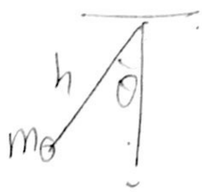
面積

2. (a) Derive Kepler's 2nd law from Newton's law of gravitation. (b) Derive the mechanical energy of a satellite of mass m moving around the Earth of mass M in an elliptical orbit of semi major axis *a*. E 橢圓

3. Consider a fluid of density ρ. (a) Derive Bernoulli's equation from Newton's second law . (b) Derive the equation of continuity using conservation of mass and divergence theorem.

4. (a) Derive the period of a physical pendulum of mass *m* and rotational inertia *I* about the pivot point which is at a distance *h* from the center of mass.

(b) The forced oscillations can be expressed as



$$x(t)=x_m'' e^{-\beta t}\cos(\omega' t+\phi'') + \frac{A}{\sqrt{(\omega^2-\omega_d^2)^2+4\omega_d^2\beta^2}}\cos(\omega_d t-\delta)$$

resonance frequency.

What is the resonant frequency? $\sqrt{\omega^2-2\beta^2}$

5. (a) Derive the wave equation for waves on a taut string of tension τ and linear density μ.

(b) A taut string of length L, tension τ and linear density μ is fixed at both ends. Derive the resonant frequencies for this string.

共振 $f = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$

6. A stream of water flowing through a hole at depth *h* = 12 cm in a tank holding water to height *H* = 40 cm. (a) At what distance *x* does the stream strike the floor? (b) At what depth should a second hole be made to give the same value of *x*? (c) At what depth should a hole be made to maximize *x*?

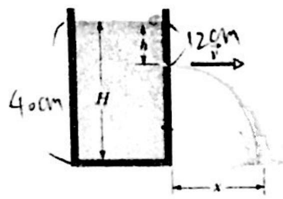
(b) $\sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = x$ 求 h

(c) 微分

By Bernoulli's equation. $= P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$ is constant.

$\rho g h = \frac{1}{2}\rho v^2 \quad v = \sqrt{2gh}$

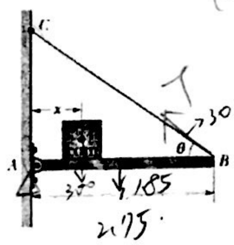
鉛直落下時間 $t = \sqrt{\frac{2(H-h)}{g}}$



7. Suppose the length L of the uniform bar is 2.75 m and its weight is 185 N. Also, let the block's weight W = 300 N and the angle θ = 30.0°. The wire can withstand a maximum tension of 500 N. (a) What is the maximum possible distance x before the wire breaks? With the block placed at this maximum x, what are the (b) horizontal and (c) vertical components of the force on the bar from the hinge at A?

(a) $300x + 185 \times \frac{2.75}{2} = 500 \times \sin 30^\circ \times 2.75 \Rightarrow x =$

(b) 靜力平衡 (對 bar 而言, 受4力).



單擺

扭擺

8. A 2.50 kg disk of diameter D = 42.0 cm is supported by a rod of length L = 76.0 cm and negligible mass that is pivoted at its end. (a) With the massless torsion spring unconnected, what is the period of oscillation? (b) With the torsion spring connected, the rod is vertical at equilibrium. What is the torsion constant of the spring if the period of oscillation has been decreased by 0.500 s? [Note: $I = \frac{1}{2}MR^2$ for a disk about the axis through its center. $\tau = -\kappa\theta$ for a torsion spring.]

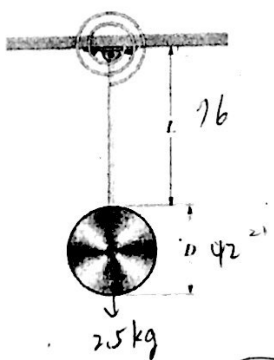
(b) $\vec{\tau} = -K\theta = I\frac{d^2\theta}{dt^2}$

$\frac{d^2\theta}{dt^2} = -\frac{K}{I}\theta \qquad T = 2\pi\sqrt{\frac{I}{K}}$

$I = \frac{1}{2}MR^2 + M(L+\frac{D}{2})^2$ (平行軸定理).

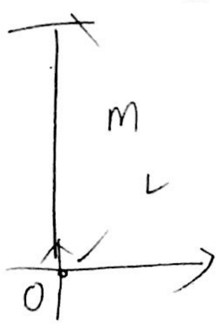
(a) $\vec{\tau} = mg(L+\frac{D}{2})\theta = I\frac{d^2\theta}{dt^2}$

$T = 2\pi\sqrt{\frac{I}{mg(L+\frac{D}{2})}}$ #



$\frac{1}{2}MR^2$

9. A uniform rope of mass m and length L hangs from a ceiling. (a) What is the speed of a transverse wave on the rope as a function of y, the distance from the lower end? (b) What is the time a transverse wave takes to travel the length of the rope ?



(a) $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{y \cdot \cancel{\mu} \times g}{\cancel{\mu}}} = \sqrt{yg}$

(b) $\int t = \int_0^L \frac{dy}{v} = \int_0^L \frac{dy}{\sqrt{yg}} = 2\sqrt{y}/\sqrt{g}\Big|_0^L = 2\sqrt{L/g}$ #