# Introduction to Electrodynamics-1, Midterm, Nov. 10, 2020.

$kz^2 - 0$

Problems with (*) are from your suggestions. Thanks!

1. (5%) Two point charges $Q_1$ and $Q_2$ are located at $\vec{r}_1$ and $\vec{r}_2$, respectively. Write down the corresponding charge distribution.
2. (5%) Explain why do sometimes $\frac{1}{2}\int \rho V d\tau$ and $\frac{\epsilon_0}{2}\int (\vec{E})^2 d\tau$ give different answers?
3. (5%) Can the vector field, $\vec{A}(x, y, z) = k[xy^2\hat{x} + 3x^3\hat{y} + yz^2\hat{z}]$ where $k$ is a constant, be a legitimate electric field? If not, why? If yes, what is the corresponding charge density?
4. (*5%) A conductor with a cavity in arbitrary shape is placed in an external electric field. Explain to your grandmom why there is no induced charge on the inner surface of the hole.

$q' = Q$

$b = r$

## Long Questions:

*All answers must be supported by detailed calculation or reasoning. It is your responsibility to state the logic of your answers clearly. I will not make any attempt to "guess" your results. No credit points will be granted if you fail to do so.*

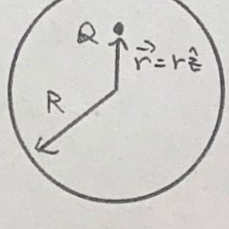
1. (* 10%) Two spheres, each of radius $R$ and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap with each other. Let's denote the vector from the positive center to the negative center as $\vec{a}$. What is the electric field in the region of overlap?
2. (* 5+5+5+5%) A point charge, Q, is at rest inside a thin metallic spherical shell but is not at its center; see figure below.
(A) What is the electric field outside the shell?
(B) What is the surface charge density on the outer surface?
(C) What is the force acting on the charge?
( Hint: Use the method of images.)
(D) What is the surface charge density on the inner surface?

$\frac{1}{4\pi\epsilon_0} \times \frac{-RrQ^2}{(R^2-r^2)^2}$

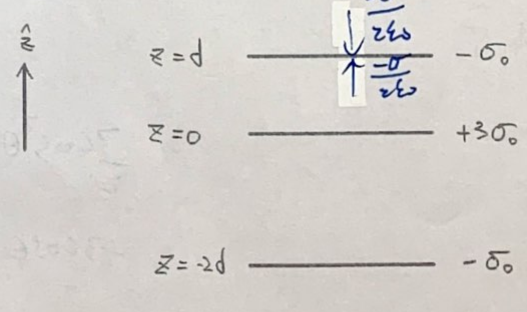
$|\vec{r} - \vec{a}| < R$

$|\vec{r}| < R$

3. (5+5%) Three infinite plane sheets, $z = -2d, z = 0$ and $z = d$, carry surface charge densities $-\sigma_0, +3\sigma_0$, and $-\sigma_0$, respectively.
(A) What are the electric field at point-A, $(0, 0, 3d)$, and point-B, $(d, -3d, -5d/3)$?
(B) What is the electric potential difference between point-A and point-B?




Prob. 2




Prob. 3

$-5A_2 R\, R_2 = \frac{2B}{3} R_2$ $\quad -3A_1 P_1 = A$

$\frac{Q}{V} = C \qquad V = \frac{Q}{C}$

$\frac{-2B}{15R} \qquad -\frac{A_0}{R} P_0 = \frac{B}{3} \qquad -\frac{BR}{3}$

$-(l+1) A_l R^{l-1} - l A_l R^{l-1}$

$A_l R^{l-1}(-l-1-l) \qquad A_l R^l = \frac{B_l}{R^{l+1}}$

$A_l R^{2l+1} = B_l$

4. (5+8+7%) A surface charge density $\sigma(\theta) = A\cos\theta + B\cos^2\theta$ is glued on the shell of radius $R$.
(A) Decompose the $\sigma(\theta)$ into a sum of Legendre polynomials.
(B) Solve and determine the electric potential for both $r > R$ and $r < R$.
(C) What is the electric field everywhere?

$A_l = B_l R^{-2l-1}$

$A_l R^{l-1} = B_l R^{-l-2}$

5. (10+10+10%) An air-filled capacitor is made from two concentric cylinders. The outer cylinder has a radius of 1 cm.
(A) What is the radius of the inner conductor, which allows maximum potential difference between the conductors before the breakdown of the air dielectric?
(B) What is the radius of the inner conductor, which allows a maximum energy to be stored in the capacitor before the air dielectric breakdown?
(C) Calculate the potential difference for cases (A) and (B).
Note: (1)Use the numerical values that $e^{-1} = 0.37$ and $e^{-1/2} = 0.61$ to get your final answers ( numbers, in SI units). (2) The breakdown field in the air is $|\vec{E}| = 3 \times 10^6 V/m$.

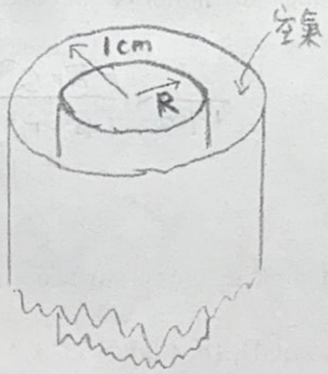
$\frac{2\pi R L \sigma = Q}{C}$

6. (* 10%) For the configuration shown in the figure, what is the total electric flux, namely $\int \vec{E} \cdot d\vec{a}$, passing the face $ABCD$?

$\frac{Q}{R}$

7. (* 10%) A very tiny hole is drilled on the spherical metal shell of radius $R$, and the object carries a total charge of $Q$. What is the electric field right in the tiny hole on the surface?

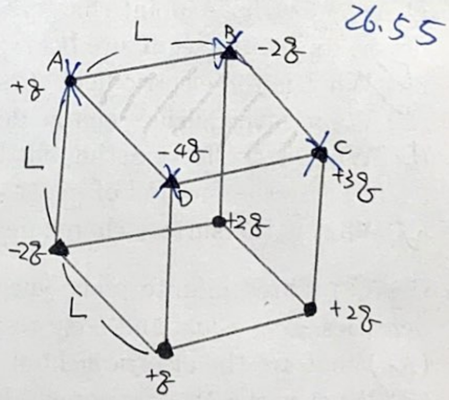
$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ $\qquad$ 26.55




Prob. 5

$\frac{\sigma}{2\varepsilon_0}$

$\frac{\sigma}{\varepsilon_0}$




Prob.6

$\frac{3\cos^2\theta}{2} - \frac{1}{2}$

$-3\cos\theta\sin\theta$