Electrodynamics-2, Final, June 14, 2016.

Single Choice (10 % each) *Just collect and summarize your answers(only) inside the boxes 1-5 on the cover page. No explanation is needed.*

$E = pc$

1. Which of the following is incorrect? (A) If a massless particle carries a momentum $10^2 kg.m/sec$, its energy is around $3 \times 10^{10} J$. (B) Lorenz gauge condition is Lorentz invariant. (C) ~~Tow~~ Two atomic clocks A and B are synchronized at a lab at the equator. You take B with you on a smooth 2-month cruise trip to the north pole and back. You will find that A is running behind B. (D) A light source with wavelength $3000\mathring{A}$ is moving toward you with a speed of $\frac{12}{13}c$. You cannot see it at all with your naked eyes.

C

2. Suppose in one frame, $c\phi = 10$ in SI unit where ϕ is the scalar potential, $\frac{\partial \phi}{\partial z} \neq 0$ and $\vec{A} = \vec{0}$. Which of the following (all in SI unit) is incorrect? (A) In this frame $\vec{B} = \vec{0}$. (B) There could be a frame where $\vec{E}' = \vec{0}$. (C)If in another frame $c\phi' = 13$, one could have $A'_x = -8$ and $A'_z = 2$. (D) It is impossible to find a frame where $c\phi' = 7$.

B.

$A_y = \sqrt{28}$ $\quad -100 = -196 + 64 + 4 + 28$

3. On a nylon filament 0.01 cm in diameter and 4 m long there are 5×10^{10} extra electrons distributed uniformly over the surface. Which is the following is incorrect? (A) In the rest frame of the filament, $\vec{E} \simeq -7.2 \times 10^{5}\hat{r}(V/m)$. (B) In the rest frame of the filament, $\vec{B} = \vec{0}$. (C) In a frame in which the filament is moving at a speed $\sqrt{0.99}c$ in a direction parallel to its length, $\vec{E} \simeq -7.2 \times 10^6\hat{r}(V/m)$. (D) In a frame in which the filament is moving at a speed $\sqrt{0.75}c$ in a direction transverse to its length, the electric field normal to both its length and the relative motion between the 2 frames is $E' \simeq 1.44 \times 10^6 (V/m)$.

A

4. You have a polarizer but you don't know its polarization direction. Which of following cannot be used to determine the polarization direction? (A) The light reflected from the lake in a clear day. (B) A ~~write~~ white cloud. (C) The crystal clean blue sky. (D) The transmitting light through a glass slab.

B

$V \simeq 0.$

5. A point charge q moves with constant velocity so that at time t' it is at point $(0, 0, vt')$. At time t, you locate at $(b, 0, 0)$ and use the Lorenz gauge. Which of the following is incorrect? (A) You see a scalar potential $q/(4\pi\epsilon_0)[\sqrt{b^2 + v^2t^2} + v^2t/c]^{-1}$. (B) You see a vector potential $qv\hat{z}/(4\pi\epsilon_0 c^2)[v^2t^2 + (1 - \beta^2)b^2]^{-1/2}$. (C) You see an electric filed whose x-component $E_x = qb(1 - \beta^2)/(4\pi\epsilon_0)[v^2t^2 + (1 - \beta^2)b^2]^{-3/2}$. (D) You see a magnetic field perpendicular to $\vec{E}$.

A

Long Questions *For the long questions, all answers must be supported by detailed calculation or reasoning. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to "guess" your results. No credit points will be granted if fail to do so.*

1. (20%) An EM plane wave is propagating in the vacuum. This wave is linearly polarized and it carries a momentum density of $3 \times 10^{-17} \frac{kg}{m^2 \cdot sec} \hat{x}$. Assuming that the speed of light

$(1-\beta^2)b^2 = b^2 - (\frac{V}{c})^2 b^2$

$b^2 + v^2t^2 + (\frac{V}{c})^2$

takes a value of $c = 3 \times 10^8 m/sec$ and $\mu_0 = 4\pi \times 10^{-7}$ (in SI unit). Construct and write down your own expressions for the corresponding E-field and B-field in numbers with 10% numerical accuracy. Specify clearly the wave length, frequency, and the units you used.

2. (15+10%) Charged particles in the universe can be accelerated up to enormous energies. These energetic particles rain down on the Earth as "cosmic rays". Particle physics determines that there is an ultimate limit to the energy of cosmic rays due to their collisions with the "cosmic microwave background radiation"(CMBR) that permeates the entire universe. Suppose the cosmic rays are protons(with rest mass $m_p \sim 1.67 \times 10^{-27}$kg.) If the CMBR consists of photons of energy 2.5×10^{-4} eV.[This is the energy corresponding to the temperature ($E = K_B T$) of $2.9°K$ that characterizes the CMBR.] The reaction that degrades the energy of the cosmic ray is the pion production

$$\gamma + p \rightarrow \pi^0 + p\,,$$

where γ is the CMBR photon and the π^0 is a "meson" with rest mass $m_{\pi^0} \simeq m_p/7$.
(a) What is the threshold energy of the proton (in eV, 10% numerical accuracy), i.e. what is the minimum proton energy necessary for the reaction to go in the frame where the photons have energy 2.5×10^{-4} eV?
(Hints (1): Use of Lorentz invariants will make this problem much simpler. (2): Consider the threshold condition in the center of mass of the final proton and pion. What is their configuration at threshold? (3): Get an approximation for $(1 - \beta)$ for $\gamma \gg 1$ in terms of γ.)
(b) Suppose the proton has just enough energy to make the reaction possible. What is its energy after the collision(in eV, 10% numerical accuracy)?

3. ((5+5)+10+5%) An insulating circular ring (radius a and mass m) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \cos\phi$, where λ_0 is constant and ϕ is the usual azimuthal angle. When the ring is at rest, we put a mark on it at $\phi = 0$. The ring is then set spinning by a machine at an angular velocity ω about the z axis. Assume that it can freely rotate without experiencing any friction, and $\omega a \ll c$.
(a) If at time t, the mark on the ring is pointing to an azimuthal angle $\phi(t)$ and the angular velocity is $\omega(t)$. Calculate the electric dipole moment and magnetic dipole moment of the ring at this instance.
(b) At some point, the power machine is detached and the ring spins on its own. Estimate how long does it take for the ring angular velocity slowing down to $\omega/2$? Check that your answer has the correct physical dimension.
(c) Where and how goes the mechanical angular momentum?