Quantum Physics (II): Final June 15, 2021

Useful Information:
total grade = 100

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\begin{aligned}
Lande \text{ g } factor &= 1+\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} \\
\alpha &= \frac{e^2}{\hbar c}=\frac{1}{137}, \; e=4.8\times 10^{-10}(e.s.u)
\end{aligned}
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Problem 1
(a)4% Explain briefly what intrinsic parity is.
By definition, what are intrinsic pairities for proton and neutron ?
(b)7% Consider the reaction $\pi^- + d \to n+n$. It is known that the spin of π^- is zero and the angular momentum of the deuterium d is one with the orbital angular momentum between proton and neutron being one. If π^- is captured in a $L=1$ orbital state in the reaction, what are possible two-neutron states? Which states are allowed if the π^- had negative parity?
Problem 2 A non-relativistic, charged particle (charge $= q$) of mass m is placed in a magnetic field $\mathbf{B}$. Assume that the particle has spin 1/2, its gyromagnetic ratio is g_S and both q and g_S are positive, answer the following questions:
(a) 4% Suppose one can write $\mathbf{B}=\nabla\times\mathbf{A}$, what is the Hamiltonian H that describes this particle?
(b) 5% Let the spin part of H be H_S. Suppose that $\mathbf{B}$ is uniform and $B_x = B\sin\theta, B_y=0, B_z=B\cos\theta$. If one uses z-axis as the quantization axis, find the matrix form of H_S.
(c) 5% Suppose that instead of using z-axis as the quantization axis, one uses y-axis as the quantization axis. Find the matrix form of H_S.
(d) 5% Now suppose that a beam formed by such particles coming out from an oven is sent to perform the Stern-Gerlach (SG) experiment with 3 SG magnets in series. The three magnets in series are oriented with the angle between z-axis and the directions from north pole to south pole (the south pole is the pole that is sharpened in shape) being $\theta_1=0$, $\theta_2=\alpha$ and $\theta_3=\beta$. The azimuthal angles ϕ for three magnets are all zero. If the beam is blocked at the exit near the north pole of the first SG magnet, and is blocked at the exit near the south pole of the second SG magnet, find the percentage of the beam that would come out from the exit near the north pole of the 3rd SG magnet.
Problem 3 The hydrogen atom, in the simplest description, is characterized by the Hamiltonian $H=p^2/2m_e - e^2/r$, where m_e is the reduced mass of the proton and the electron. It is known that this Hamiltonian is accurate to $O(\alpha^2)$ with α be the fine structure constant. However, the energy spectrum of real hydrogen atoms has more structures. Answer the following question.
(a) 3% Write down two corrections of the order $O(\alpha^4)$ to the Hamiltonian that causes the fine structure of the spectrum in real hydrigen atoms.
(b) 4% Explain briefly what the 21 centimeter line is and explain its origin. Indicate the order of magnitude for the levels it originated from.
(c) 8% Sketch the energy levels that include the fine structure for $n\le 3$. Indicate each level by the spectroscopic notations and indicate number of degenerate states in each energy level.
(d) 5% Consider the energy levels that include the fine structure for $n=2$ and $l=1$. If we place the atom in a uniform magnetic field $B\hat{z}$, sketch how the spectrum changes when B increases from 0 to 5 Tesla.
Problem 4 Consider the Hamiltonian that describes the tunneling of N atom through the plane form three H atoms in the molecule NH_3 of the form $\hat{H}=\hat{H}_0+\lambda\hat{V}$ with

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\hat{H}_0=\begin{pmatrix} E_0^1 & 0 \\ 0 & E_0^2 \end{pmatrix}, \qquad \hat{V}=\begin{pmatrix} V_{11} & V_{12} \\ V_{12}^* & V_{22} \end{pmatrix}.
$$

(a) 8% Using the perturbation theory to write down the energy eigenvalues to first and second order in λ. Compute the exact eigenvalues, expand the results to first and second order in λ, and compare them with those obtained by the perturbation theory.
(b) 4% Find the corresponding eigenstates to first order in λ in the form given by the perturbation theory.
Problem 5
(a) 6% Use the Hund's rules to find the spectroscopic description $^{2S+1}L_J$ of the ground states of O ($Z=8$) and Fe ($Z=26$, $[Ar](4s)^2(3d)^6$).
(b) 5% Consider the carbon atom ($Z=6$) in the electronic configuration $(1s)^2(2s)^2(2p)^2$. Find all possible states in the spectroscopic description are consistent with the above electronic configuration. Which one has lowest energy according to the Hund's rules?

(c) 7% Consider the excited electronic configuration of the carbon atom $(1s)^2(2s)^2(2p)(3p)$. In terms of spectroscopic notations, $^{2S+1}L_J$, find all possible states that are consistent with the above electronic configuration. By considering the separation of electrons in spatial coordinates, indicate which state has the lowest energy.
(d) 6% Now consider the diatomic molecule C_2 in which each carbon atom is in the electronic configuration, $(1s)^2(2s)^2(2p)^2$ as the distance R between two carbon atoms goes to infinity. What is the electronic configuration of C_2 in the ground state? (in terms spectroscopic notations σ_g, σ_u, ...)
The energy levels of the diatomic molecule is a summation of electronic energy levels $E_n(R)$, vibrational levels E_v and rotational levels E_v. Sketch $E_n(R)$ versus R for C_2 and indicate E_v and E_v on the plot.

Problem 6

Consider the interaction of an electromagnetic field, $\vec{E} = \vec{A}\cos(\vec{k}\cdot\vec{r} - \omega t)$, with the energy level m and n of a hydrogen atom. Answer the following questions:
(a) 4% Assuming that the energies and wavefunctions corresponding to level m and n are E_m, E_n ($E_m > E_n$) and $\phi_n(\vec{r})$, $\phi_m(\vec{r})$ respectively, if one writes $\cos(\vec{k}\cdot\vec{r} - \omega t) = \frac{1}{2}\left[e^{i(\vec{k}\cdot\vec{r}-\omega t)} + e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\right]$ and makes the approximation, $e^{i\vec{k}\cdot\vec{r}} \approx 1$, i.e., the wavelength is much larger than the size of the atom, what is the transition rate from level m to level n to the leading order of A? Express your answer in terms of A, ω, E_m, E_n, $\phi_m(\vec{r})$, and $\phi_n(\vec{r})$. Is the transition from $2S$ to $1S$ allowed in this approximation?
(b) 10% The result of (a) is the transition rate for emitting a photon in a particular direction $\vec{k}$ and a particular electric field polarization. The spontaneous transition rate from m to n is the total transition rate in which one needs to integrating over all directions and all polarization directions. Find the spontaneous emission rate from m to n by expressing the result in terms of $d = |\int d^3r\phi_m^*(\mathbf{r})e\mathbf{r}\phi_n(\mathbf{r})|$ and ω. From the result, identify the Einstein's coefficients A_{21} and B_{21} by setting m as the index 2 and n as the index 1.