1.

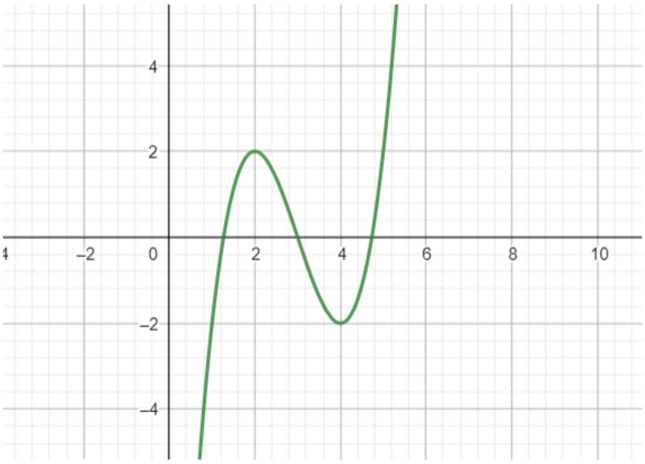
pf.

$f(x)=x^3-9x^2+24x-18$

$f'(x)=3x^2-18x+24=3(x-4)(x-2)$,x=4 or2

$f''(x)=6x-18=6(x-3)$, x=3

(表格省略)



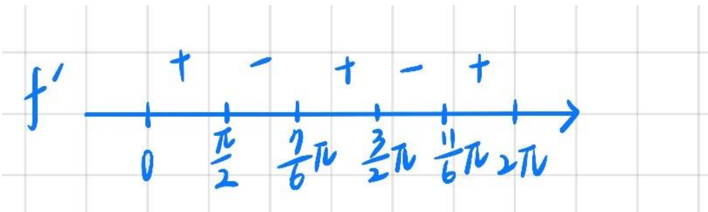
2.

pf.

$f(x)=2\sin x-\cos(2x)$

$f('x)=2\cos x+2\sin(2x)$

x = $\frac{\pi}{2}$、$\frac{7\pi}{6}$、$\frac{3\pi}{2}$ 、$\frac{11\pi}{6}$ are critical points



$f(0)=-1$

$f(\frac{\pi}{2})=2-(-1)=3$

$f(\frac{7}{6}\pi)=2\times(\frac{-1}{2})-(\frac{1}{2})=\frac{3}{2}$

$f(\frac{3}{2}\pi)=-2-(-1)=-1$

$f(\frac{11}{6}\pi)=2(\frac{-1}{2})-\frac{1}{2}=\frac{-3}{2}$

$f(2\pi)=0-1=-1$

$\because$ f is cont, on $[0, 2\pi]$
$\therefore$ By closed interval thm,
the maximum of f on $[0, 2\pi]$ is 3
the minimum of f on $[0, 2\pi]$ is $-\frac{3}{2}$

3.

3、 $f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$f(-1) = -a + b + c = 2 \quad \cdots ①$

$f'(-1) = 3a - 2b = 1 \quad \cdots ②$

$f''(-1) = -6a + 2b = 0 \quad \cdots ③$

$② + ③ \Rightarrow -3a = 1$

$a = \frac{-1}{3}$, $b = -1$, $c = \frac{8}{3}$

4.

4. let $f(x)=\sqrt{x}$, for $x>0$

$f'(x)=\frac{1}{2\sqrt{x}}$

$\because$ f is cont. on $[a,b]$ and

f is diff. on $(0,b)$

$\therefore$ By Mean Value thm, $\exists c \in (a,b)$, s.t.

$f'(c)=\frac{\sqrt{b}-\sqrt{a}}{b-a}$, $\exists d \in (0,a)$ s.t.

$f'(d)=\frac{\sqrt{a}-0}{a}=\frac{1}{\sqrt{a}}$, $f'(c)=\frac{1}{2\sqrt{c}}$

$f'(d)=\frac{1}{2\sqrt{d}}$, $c \in (a,b)$, $c \neq 0$

$d \in (0,a)$, $d \neq 0$

$\because \frac{1}{2\sqrt{c}} < \frac{1}{\sqrt{a}}$

$\therefore \sqrt{b}-\sqrt{a} < \frac{b-a}{\sqrt{a}}$

5.

5. $f(x)=\frac{x^2+1}{x^2-4}$ for $x \neq 2$, $x \neq -2$

$$f'(x)=\frac{2x(x^2-4)-(x^2+1)(2x)}{(x^2-4)^2}$$

$$=\frac{2x^3-8x-2x^3-2x}{(x^2-4)^2}$$

$$=\frac{-10x}{(x^2-4)^2}$$

$f''(x)=$ (省略) $=\frac{10(3x^2+4)}{(x+2)^3(x-2)^3}$

$\because f'' \begin{cases} >0, \text{ on } x>2,\ x<-2 \\ <0, \text{ on } -2<x<2 \end{cases}$

$\therefore$ By concave up thm

f $\begin{cases} \text{concave up on } & x>2,\ x<-2 \\ \text{down} & -2<x<2 \end{cases}$

6.抱歉各位，實在是年代太久遠，只能提供原考卷 QQ

6. Suppose $\exists\, a<b<c \in I$. $f(a)=f(b)=f(c)=0$.

$\because$ f is cont on I、f is diff. on I.

By Rolle's thm. $\exists\, d \in (a,b)$ s.t. $f'(d)=0$. +6

similarly $\exists\, e \in (b,c)$ s.t. $f'(e)=0$.

$\because$ f' is cont. on I. f' is diff. on I.

By Rolle's thm. $\exists\, g \in (d,e)$ s.t. $f'(g)=0$

$\Rightarrow$ f has at most two distinct real roots. on I.

7.

7. let $x_1, x_2 \in (a,b)$, $x_2 > x_1$

$\because$ f is cont. on $[a,b]$ and

f is diff. on (a,b)

$\therefore$ By Mean Value thm,

$f(x_2)-f(x_1)=f'(c)(x_2-x_1)$, $c\in(a,b)$

$\because$ $f'(c)<0$, $x_2-x_1>0$

$\therefore$ $f(x_2)-f(x_1)<0$

$\Rightarrow$ f is decreasing on $\mathbb{R}$

8.

8. let $x_1, x_2 \in I$, $x_2 > x_1$

$\because$ f is diff. on I, f is cont. on I

$\therefore$ By MVT, $f(x_2)-f(x_1)=f'(c)(x_2-x_1)$, $c\in I$

$\because$ f is an increasing function

9.抱歉各位，實在想不出來，又擔心是錯誤的，於是只放題目

9、題目：$f(x)=10$, $\forall x\in\mathbb{Q}\cap[a,b]$, f is cont. on $[a,b]$

prove $\int_a^b f(x)\,dx = 10(b-a)$