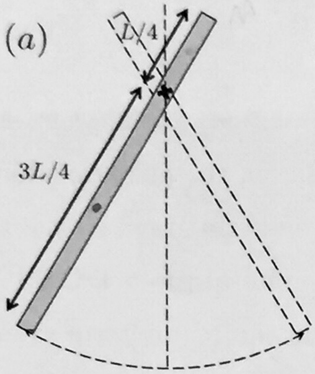
General Physics I – 2^{nd} Midterm Exam

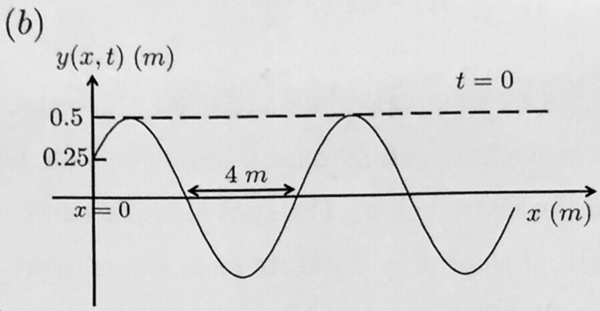
Exam Time: 10:10AM - 12:10PM

Q1. (60 pts) Fundamentals.

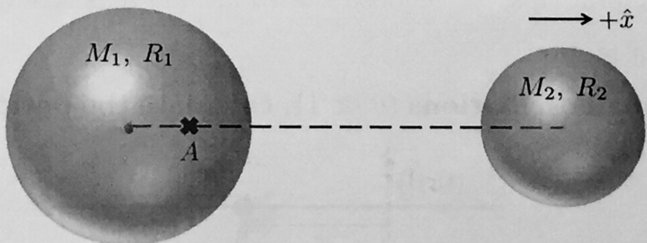
(a) (20 pts) A uniform rod of mass M and length L is rotated around an axis that is going through a point $L/4$ away from its end, see figure below. If the initial releasing angle is small, what would be the angular frequency of the swinging rod?

(b) (20 pts) A travelling harmonic wave is travelling in the $-x$ direction, and a snapshot of the wave taken at $t = 0$ is shown below. Given that the tension is 2 N and the linear mass density of the string is 0.5 kg/m, (1) write down the complete travelling wave expression $y(x,t)$. (2) What is the velocity of the particle of the string at $x = 0$ at this instant?





(c) (10 pts) Point A sits inside a shell of mass M_1 and radius R_1, while another massive shell of mass M_2 and radius R_2 is close by, see figure below. Point A is located on the joining line of centers of two shells. And it is at $R_1/2$ away from the center of shell 1, and $4R_2$ away from the center of shell 2. Find the gravitational field strength at point A (it is a vector).



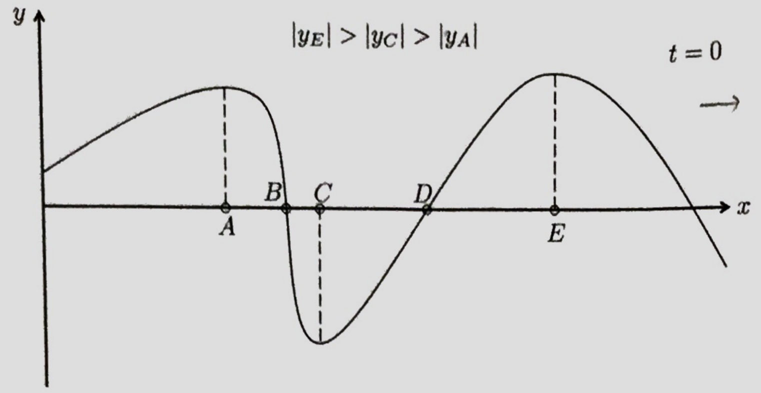
(d) (10 pts) **Find the steady state solution** of the following differential equation $\gamma\, dx/dt = -kx + F_0 \cos(\Omega t)$.

Q2. (20 pts) The string wave equation is $\partial^2 y/\partial t^2 = (F/\mu)\partial^2 y/\partial x^2$. Assume that **an arbitrary shape of wave** $y(x,t)$ travels in the $+x$-direction, answer the following questions,

(a) (5 pts) Use an arbitrary shape of wave to determine the string wave velocity.

(b) (10 pts) At a fixed position x, are the kinetic energy density (dK/dx) and the potential energy density (dU/dx) the same for an arbitrary shape of string wave? Show your proof.

(c) (5 pts) A snapshot of the string wave is taken at $t = 0$. Which segment of the string marked in the following figure possesses the largest energy density? **Explain.**



Q3. (40 pts) A simple pendulum of mass m_2, with a mass m_1 at the point of support which can only move horizontally on a frictionless track lying in the plane in which m_2 moves as shown in the figure below. The system is placed in a uniform gravitational field ($\vec{g} = -g\hat{y}$). The length of the rod is ℓ and its mass is negligible. **The angle θ DOES NOT have to to be small.** (And yes, you have seen this problem in the 1st midterm exam)

(a) (10 pts) Let's take an inertial frame whose origin is indicated in the figure below. Write down **the linear momentum of** m_1 **and** m_2 in terms of m_1, m_2, x, ℓ, θ, $\dot{x}$ and $\dot{\theta}$.

(b) (5 pts) We know that the total momentum in the x-direction is conserved. Use this conservation relation to **obtain a relation between $\dot{x}$ and $\dot{\theta}$.**

(c) (15 pts) **Write down the equation of the rotational motion (θ does not have to be small).** Helpful tips: (1) choose the rotational axis wisely and (2) use the correct form of equation of motion.

(d) (10 pts) **For small oscillations ($\theta \ll 1$), calculate the period of small oscillations.**