Classical Dynamics (I) 10/29/2018

- 1st midterm (ten points each) -

1. (Example 1-8) Find the components of the acceleration vector $\vec{a}$ in cylindrical coordinates.
2. (Prob.1-26) A particle moves with v=const. along the curve $r = k(1 + \cos\theta)$. Find $\vec{a} \cdot \hat{e}_r$, $|\vec{a}|$, and $\dot{\theta}$ where $\vec{a}$ denotes the acceleration vector.
3. (Prob.1-37) Solve the integral $\int_S \vec{A} \cdot d\vec{a}$ where $\vec{A} = (x^2 + y^2 + z^2)(x, y, z)$ and the surface S is defined by the sphere $R^2 = x^2 + y^2 + z^2$.
4. (Prob.2-8) A projectile is fired with velocity v_0 so that it passes through two points both a distance h above the horizontal. Show that if the gun is adjusted for maximum range, the separation of the points is $d = (v_0/g)\sqrt{v_0^2 - 4gh}$.
5. (Prob.2-12) A particle is projected vertically upward in a constant gravitational field with an initial speed v_0. If it experiences a resisting force kmv, find the speed of the particle when it returns to the initial position.
6. (Prob.2-49) Two gravitationally bound stars with unequal masses m_1 and m_2, separated by a distance d, revolve about their center of mass in circular orbits. Find the period.
7. (Prob.3-9) A particle of mass m is at rest at the end of a spring of force constant k that hangs from a fixed support. At t=0, a constant downward force F is applied to the mass and acts for a time t_0. Find the displacement of the mass at $t > t_0$.
8. (Prob.3.29) Obtain the Fourier series representing the periodic function

$$F(t) = \begin{cases} 0, & -2\pi/\omega < t < 0 \\ \sin\omega t, & 0 < t < 2\pi/\omega \end{cases}$$

9. (Prob.3.31) A damped linear oscillator, originally at rest in its equilibrium position, is subjected to a forcing function $F(t)$. Find the response function. Is it necessary to distinguish among under-, critically, and over-damped cases? *Hint*: Fourier convolution, $F^{-1}[\tilde{f}(\omega)\tilde{g}(\omega)] = \int_{-\infty}^{\infty} f(t')\, g(t - t')dt'$, may come in handy.
10. Given that the motion of a physical pendulum of length ℓ and mass m obeys $\ell\ddot{\theta}/3 = -g\sin\theta$. If the amplitude A is small, but not that small, $\sin\theta$ need to be expanded to the order of θ^3. Please analyze its motion.

Classical Dynamics (I) 12/3/2018

\- 2nd midterm (ten points each, and ten bonus points) -

1. (Example 5.4) Consider a thin uniform disk of mass M and radius R. Find the force on a mass m located at height z along the axis of the disk.

2. (Prob.5-13) For a spherical planet (density ρ_1, radius R_1) and with thick spherical cloud of dust (ρ_2, R_2), find the potential at $r < R_1$, $R_1 < r < R_2$, and $r > R_2$?

3. (Prob.5-14) Find the gravitational self-energy of a sphere of mass M and radius R.

4. (Prob.6-4) Show that the geodesic, i.e., the shortest path between two points on a right circular cylinder is a segment of a helix.

5. (a) (Example 6.2) Find the fastest path, called a cycloid, that brings a particle under gravity from rest to a new position below, but not directly under, its starting point.
   (b) (Prob.6-6) Show that, if the final position is at the minimum point of the cycloid, the travel time will be independent of the starting point.
   (c) (5 bonus points) Tie a ribbon to the wire of a moving bicycle tire. Show that the trajectory sketched out by the ribbon obeys a cycloid.

6. (Example 6.3) It is known that, when the surface area generated by revolving a line connecting (x_1, y_1) and (x_2, y_2) about y-axis is minimized, the equation of line is a catenary. Show that the shape of a necklace or suspension bridge also obeys catenary.

7. (Prob.6-7) A light passes from a medium with refraction index n_1 to another with n_2. Use Lagrangian mechanics to (a) derive the Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and (b) (5 bonus points) prove that angles of reflection and incidence are equal.

8. (Example 6.6) (a) Prove that a circular curve (of length ℓ) bounded by the x-axis on the bottom that passes through points $(\pm a, 0)$ encloses the largest area. The value of endpoints a is determined by the problem.
   (b) Show that this curve is a semicircle.

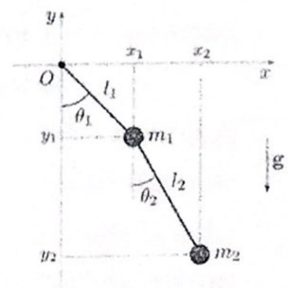
Classical Dynamics (I) 1/9/2019

\- Final (ten points each) -

1. (Prob. 7-15) A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k.

   (a) Please write down the equation of motion.

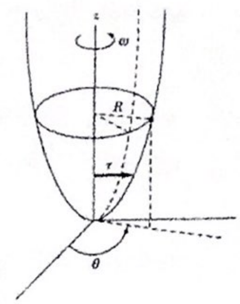
   (b) If the displacement is small, find the periods.



2. A double pendulum (see Fig.1) consists of two point masses, $m_{1,2}$, and weightless strings of length, $\ell_{1,2}$.

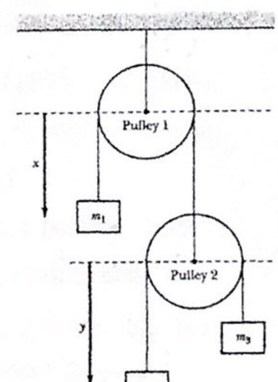
   5 (a) Derive the equations of motion.

   (b) Assuming $\theta_{1,2} \ll 1$, find the periods of this motion.



3. (Example 7.7) A bead slides in Fig.2 along a smooth wire bent in the shape of $z = cr^2$. The bead rotates in a circle of radius R when the wire is rotating about its vertical symmetry axis with angular velocity ω. $\boxed{\text{Find } c.}$

4. (Example 7.8) Determine the equations of motion by Lagrangian dynamics for a double pulley system in Fig.3.

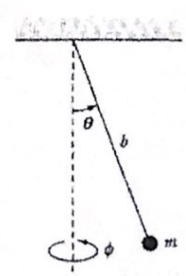


5. (Example 7.10) A particle of mass m starts at rest on top of a smooth hemisphere of radius a. Use Lagrangian dynamics.

   5 (a) If the particle slides, find the force of constraint and determine the angle at which it leaves the hemisphere.

   3 (b) (Prob.7-3) If we replace the particle by a ball of moment $I = 2mb^2/5$, find the force of constraint that keeps it rolling without slipping. Note the angle of rotation for the ball is $\frac{x}{b} + \frac{x}{a+b}$ when its center moves a distance x.

7 6. (Example 7.12) Use Hamiltonian dynamics to find equation of motion for a spherical pendulum of mass m and length b in Fig.4.



7. Use Lorentz transform to prove time dilation and length contraction.

3 8. Prove $E^2 = (m_0c^2)^2 + (pc)^2$ where E/p are relativistic energy/ momentum and m_0 the rest mass.

9. (5 points each) (a) In the rest frame, a fast moving ladder contracts and can fit into a barn shorter than its rest length. But to an observer moving with the ladder it is impossible. How to solve this discrepancy. (b) Two spaceships of proper length $L_{1,2}$ approach each other with speeds $v_{1,2}$. Find the time it takes for the ships to pass each other in the rest frame and observed by the two pilots.

Classical Dynamics (I) 1/9/2018

- Makeup exam (ten points each) -

1. (Prob. 1-26) A particle moves with v=const. along the curve $r = k(1 + \sin\theta)$. Find $\vec{a} \cdot \hat{e}_r$, $|\vec{a}|$, and $\dot{\theta}$ where $\vec{a}$ denotes the acceleration vector.
2. (Prob. 1-37) Solve the integral $\int_S \vec{A} \cdot d\vec{a}$ where $\vec{A} = (x^4 + y^4 + z^4)(x, y, z)$ and the surface S is defined by the sphere $R^2 = x^2 + y^2 + z^2$. Hint: Spherical integration of $\int_V x^4 dv$ and $\int_V y^4 dv$ are the same as $\int_V z^4 dv$ which is easier.
3. (Prob. 2-12) A particle is projected vertically upward in a constant gravitational field with an initial speed v_0. If it experiences a resisting force kmv^2, find the speed of the particle when it returns to the initial position.
4. (Prob. 2-49) Two gravitationally bound stars with unequal masses m_1 and m_2, revolve about their center of mass in circular orbits with period T. Find the separation between these two stars.
5. (Prob. 3-31) Find the response of a damped linear oscillator, originally at rest in equilibrium position, to a periodic force $F(t) = \begin{cases} 0, & -2\pi/\omega < t < 0 \\ \sin\omega t, & 0 < t < 2\pi/\omega \end{cases}$. *Hint:* You may need Fourier convolution, $F^{-1}[\tilde{f}(\omega)\tilde{g}(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t')\, g(t - t')dt'$.
6. (Prob. 5-13) For a hollow planet of mass density ρ and radius $R_1 < r < R_2$, find the gravitational potential at $r < R_1$, $R_1 < r < R_2$, and $r > R_2$?
7. (Example 6.2) Find the fastest path, called cycloid, that brings a particle under gravity from rest to a new position below, but not directly under, starting point.
8. (Example 6.3) (a) Find the form of catenary that connects (x_1, y_1) and (x_2, y_2) and minimizes the surface area generated by revolving this line about the y-axis.
   (b) Show that the shape of a necklace or suspension bridge also obeys catenary.
9. (Example 6.6) Prove that a semi-circular curve (of length ℓ) bounded by the x-axis on the bottom encloses the largest area.