109 學年上學期　微積分　期末考試題 2021/01/08

1. (30 points) Find each of the following if it exists, explain why if it doesn't：

   (a) $\lim\limits_{x\to\infty}\left(1-\sin\dfrac{3}{x}\right)^x$　(b) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}(1+x)}dx$　(c) $\displaystyle\int_0^{\pi/2}\frac{\sin^3\theta}{\sin^3\theta+\cos^3\theta}d\theta$

   $\displaystyle\int \frac{d\theta}{1+\tan^3\theta}$

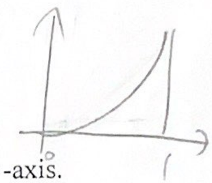
   (d) $(f^{-1})'(0)$ where $f(x)=\int_0^x 1+\cos(\sin t)dt$.

   (e) Volume of the solid obtained by rotating the triangle with vertices $(2,3),(2,5),(5,4)$ about line $x+y=1$.

2. (32 points) Find each of the following if it exists, explain why if it doesn't：

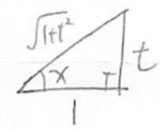
   (a) $\displaystyle\int_1^\infty \frac{\tan^{-1}x}{x^2}dx$　(b) $\displaystyle\int x^4(\ln x)^3\,dx$

   (c) A continuous function f satisfying $f(x)=1+\dfrac{1}{x}\displaystyle\int_1^x f(t)dt$ for all $x>0$.

   (d) Area of the surface obtained by rotating the curve $y=x^{3/2}\,(0\le x\le 1)$ about y-axis.

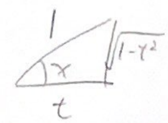
3. (10 points) Find a differentiable function $y=f(x)$ such that $f(0)=0$ and the arc length of the graph of f between $(0,0)$ and (x,y) is $\sin x+y$.

4. (24 points) Find all solutions for each of the following differential equations：
   (a) $xy'=y+x^2\sin x$ satisfying $y(\pi)=0$
   (b) $y''-2y'+5y=\sin x$ satisfying $y(0)=1$ and $y'(0)=1$
   (c) $y''+4y'+4y=\dfrac{e^{-2x}}{x^3}$.

   $\sin x\tan x$

   $\dfrac{\sin x}{t}\,dt$

5. (14 points) Let $f(x)=x\ln(1+x^{-1})$ for $x>0$. Find $\lim\limits_{x\to0+}f(x)$ and $\lim\limits_{x\to\infty}f(x)$; also show that $f(x)$ is strictly increasing (for $x>0$). (Hint: may have to consider $f''(x)$)

$\dfrac{2}{7}\times\dfrac{13^2}{468}\times\dfrac{\sqrt{13}}{2}-\dfrac{2}{3}\left(\dfrac{13}{4}\right)\dfrac{\sqrt{13}}{2}+\dfrac{10\cdot6}{15}$　$125-8=\dfrac{117}{3}$　$2C_1+\dfrac{4}{3}=1$

$=\dfrac{169\times3}{240}\sqrt{13}-\dfrac{130}{24\times10}\sqrt{13}+\dfrac{4}{15}$　$\left(\frac{1}{3}\times125-\frac{5}{2}\times25\right)-\left(\frac{1}{3}\times8-\frac{5}{2}\times4\right)$　$2C_1=-\dfrac{1}{3}$

$=39-\frac{5}{2}\times21=$　$C_1=-\dfrac{1}{6}$

$=\dfrac{507-130}{240}\sqrt{13}+\dfrac{4}{15}$　$125\left(\frac{1}{3}-\frac{2}{5}\right)$　$\dfrac{20}{3}-\dfrac{16}{9}=\dfrac{60-16}{9}=\dfrac{44}{9}$

$\dfrac{377}{240}\sqrt{13}$

$\begin{array}{r}125\\44\\\hline 81\end{array}$