**General Physics II – Midterm Exam**

**Exam Time: 10:10AM - 12:00PM**

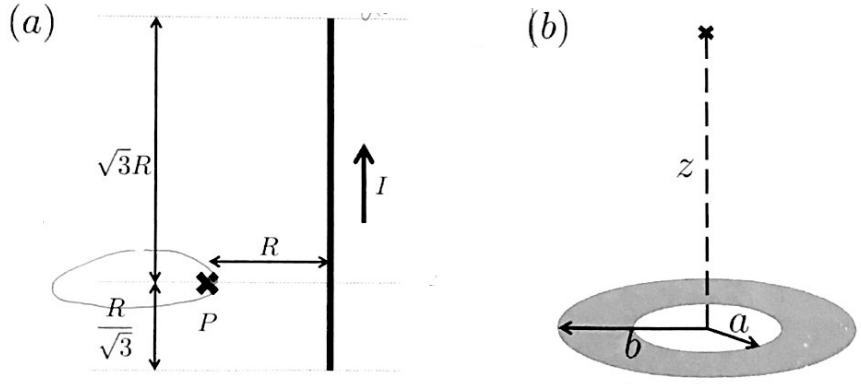
**Please explain all your answers and reasoning in a clear fashion.**

Coulomb's constant: $k = 1/4\pi\epsilon_0$. Coulomb's law: $\vec{F} = (1/4\pi\epsilon_0)(q_1 q_2/r_{12}^2)\,\hat{r}$.
Gauss's law: $\oiint \vec{E}\cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, $\nabla\cdot\vec{E} = \rho(\vec{r})/\epsilon_0$. Electric potential energy density: $\epsilon_0 E^2/2$.
Lorentz force $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$. Force acting on a segment of wire $d\vec{F} = Id\vec{\ell}\times\vec{B}$.
Biot-Savart law $d\vec{B} = (\mu_0/4\pi)(Id\vec{\ell}\times\hat{r}/r^2)$. Ampère's law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$.
Divergence theorem $\oiint \vec{E}\cdot d\vec{A} = \iiint(\nabla\cdot\vec{E})\,dV$.

**Q1.** (40 pts) Fundamentals.

**(a)** (10 pts) A finite straight wire carries a current $I$ flowing in the $y$-direction, see figure below. What is the magnitude of the magnetic field at point $P$ which is of a distance $R$ away from the wire?

**(b)** (20 pts) A disk of radius $b$ has a concentric hole of radius $a$, and it has a uniform surface charge density $\sigma$. Find the electric field above the center of the disk at a distance of $z$. **Hint**: It might be easier to obtain the solution by using the concept of electric potential.

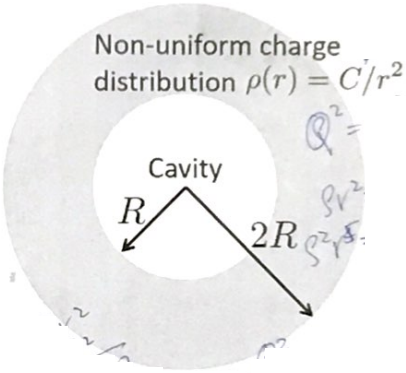



**(c)** (10 pts) The conservation of mass or charge (even the energy) can be well described by the continuity equation. Therefore, for a system of charges, the rate of change in the charge density $\rho$ is related to the divergence of the current density $\vec{J}$. Please derive in detail the continuity equation.

**Q2.** (42 pts) Charges are distributed **"non-uniformly"** throughout a **sphere** of a charge density $\rho(r) = C/r^2$ between two concentric spheres of radius $R$ and $2R$, as shown in the figure below.

**(i)** (30 points) Calculate and plot the magnitude of the electric field as a function of the radial distance $r$ (plot clearly for the following three regimes: $r < R$, $R < r < 2R$, and $r > 2R$; it is helpful to determine where the extreme value occurs).

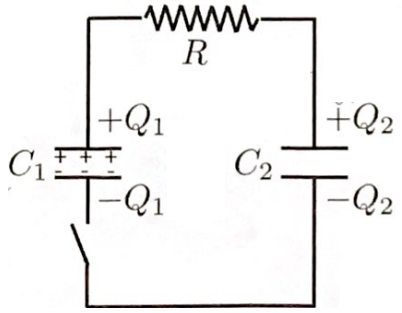
**(ii)** (12 points) Evaluate the electric potential energy of the system.




**Q3.** (32 pts) An RC circuit consists of a resistor $R$, and two capacitors $C_1$ and $C_2$, see figure below. At $t = 0$, the charges stored on $C_1$ and $C_2$ are $Q_0$ and 0, respectively. And the switch is also closed at $t = 0$. It is clear that the current $I$, and charges $Q_1$, $Q_2$ (on $C_1$ and $C_2$, respectively) are functions of time. Please use only $R$, $C_1$, $C_2$, $Q_0$, $I$, $Q_1$, and $Q_2$ in answering the following questions.

**(i)** (8 pts) Write down all Kirchhoff's rules for this circuit, and the relation between $I$ and $dQ_1/dt$, and the relation between $I$ and $dQ_2/dt$.

**(ii)** (24 pts) Obtain solutions for $Q_1(t)$ and $Q_2(t)$.




Initially,

$$Q_1(0) = Q_0$$

$$Q_2(0) = 0$$