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Final Examination 2
Applied Mathematics I(PHYS211000)
17 June 2014, 6.30 - 9.30 pm

Answer all questions. Each question carries 15 marks.
Simple calculator is allowed. No use of telephone.
You may answer in English or Chinese.

1. Find the eigenvalues and the corresponding eigenvectors for the matrix
$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}.$$
You should present the eigenvectors in orthonormal form.

2. (a) Change the order of integration in the following double integral
$$\int_0^{1/\sqrt{2}} dy \int_y^{\sqrt{1-y^2}} f(x,y)dx$$
(b) By performing a change of variables to the polar coordinates, calculate the integral
$$\iint_R \sin\sqrt{x^2+y^2}\,dxdy,$$
where R is the region $\pi^2 \le x^2+y^2 \le 4\pi^2$.

3. (i) State the divergence theorem
(ii) Use the divergence theorem to show that
$$\int_V (\varphi\nabla^2 G - G\nabla^2\varphi)dV = \int_S (\varphi\nabla G - G\nabla\varphi)\cdot d\mathbf{A},$$
where the surface S is the boundary of the volume V in $\mathbf{R}^3$.

4. By integrating both side with an arbitrary test-function, find the coefficients a, b, c, d, p and q in the following generalised function identities:
(a) $x^2\delta(x^2-x) = a\delta(x) + b\delta(x-c)$,
(b) $x\delta^{(3)}(x) = d\delta^{(2)}(x)$,
(c) $x\delta(3x-1) = p\delta(x-q)$.

5. (a) State the Green's theorem. Verify the Green's theorem for the following line integral
$$\int_C (y^3dx + x^2dy),$$
where C is the unit circle $x^2+y^2=1$.
(b) Calculate the line integral
$$\int_C e^{-(x^2-y^2)}(\cos 2xydx + \sin 2xydy),$$
where C is the unit circle $x^2+y^2=1$.

6. (a) State the Stoke theorem.

   (b) State the condition that the line integral

$$\int_C Pdx + Qdy + Rdz$$

   is independent of the path C.

   (c) Check if the line integral

$$I := \int_{(0,7,8)}^{(1,1,1)} (1 - \frac{1}{y} + \frac{y}{z})dx + (\frac{x}{z} + \frac{x}{y^2})dy - \frac{xy}{z^2}dz$$

   is path independent. If it is, calculate its value. If it is not, explain, why.

7. (i) Find the complex Fourier series for the periodic function of period 2π defined in the range $-\pi \leq x \leq \pi$ by $f(x) = \cosh x$.

   (ii) Using your result in part (i) to show that

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1} = \frac{1}{2}(\frac{\pi}{\sinh x} - 1)$$

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