Theoretical Mechanics II: Final Exam, June 22nd, 2015

Time: 6PM – 9PM

Write down your answers and explain your reasoning clearly. No references to any materials during exam.

Useful relations: For photons, $E = hf$. Displaced axis theorem, $I_{\alpha\beta} = I_{CM,\alpha\beta} + M(D^2\delta_{\alpha\beta} - D_\alpha D_\beta)$

$$\int \frac{x}{\sqrt{x^2+a^2}}\,dx = \sqrt{x^2+a^2} + Const$$

1. (a) (4 pts) Write down two postulates of special relativity.

   (b) (8 pts) **Derive the Lorentz transformations** between two inertial frames S and S' where these two frames coincide at t = 0, and S' frame is moving relative to S frame with velocity v in the +x direction.

   (c) (12 pts) Show that an isolated photon **cannot** be converted into an electron-positron pair, $\gamma \to e^- + e^+$.

2. (32 pts) One photon of energy E_1 moving along the *x*-axis is colliding with another photon of energy E_2 that is moving in the opposite direction to form a particle of mass *m*.

   (a) (10 pts) **Find the mass** of the particle in terms of E_1, E_2, and c.

   (b) (6 pts) What is the **frequency** of these two photons observed in the **center of momentum frame**? Express your answer in terms of E_1, E_2, and h (Planck's constant).

   (c) (16 pts) Right after the mass particle is formed, a constant force *f* is applied to the particle in the +*x*-direction. Find an expression for the position of the particle as a function of time. You can simply express the answer in terms of *f*, *m*, *t*, p_o (p_o is the initial momentum of the particle in x-direction right after the collision). The initial position of the particle (right after the collision) is set to be *x=0*.

3. (40 pts) Consider a **solid oblate ellipsoid** of constant density ρ,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$$

   (a) (8 pts) Find the **moment of inertia tensor** with respect to the **center of mass**.

   (b) (8 pts) Find the **moment of inertia tensor** with respect to **one end of the shorter semi-axis** (c-axis).

   (c) (8 pts) **Derive** Euler's equations for a torque-free rigid body.

   (d) (8 pts) Assume that there is no external torque acting on the ellipsoid. **In the body frame**, starting from Euler's equations to derive the **precession angular velocity about the z-axis** for the above solid oblate ellipsoid. Initially the oblate ellipsoid has the initial angular velocity $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$.

   (e) (8 pts) If the oblate ellipsoid is set to rotate about its x, y, and z axes respectively, which axes are the **stable rotational axes** (Please provide your analysis)?

4. (a) (6 pts) **Draw graphs** to define three Euler angles clearly.

   (b) (10 pts) From your plots, write down the **relations between angular velocities seen in body frame and $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$**.

5. (3 pts) Feel free to leave your comments for this course (leave it blank if nothing comes to your mind).