Quantum Physics (I): Midterm (2) Nov. 29, 2002

Problem 1 10% Which of the followings are linear operators? (a) the parity operator (b) $\hat{O}_1$: $\hat{O}_1\psi(x) = \frac{\hbar}{i}\frac{d\Psi(x)}{dx} - \frac{\beta x}{\Psi(x)}$, where β is some constant. (c) $\hat{O}_2$: $\hat{O}_2 = \hat{L}_1\hat{L}_2$ where $\hat{L}_1\hat{L}_2$ means $\hat{L}_2$ operates first and $\hat{L}_1$ operates later, and $\hat{L}_1$ and $\hat{L}_2$ are any two linear operators. (d) $\hat{O}_3$: $\hat{O}_3\Psi(x) = 4\Psi(x)$, $x \geq 0$; $\hat{O}_3\Psi(x) = 0$, $x < 0$ (e) $\hat{O}_4 : \hat{O}_4\Psi(x) = \int_{-\infty}^{\Psi(x)} f(x')dx'$, where the integral converges.

Problem 2 A particle of mass m in a symmetric infinite well ($-a < x < a$) is described by the following wavefunction at $t = 0$

$$\Psi(x,0) = \frac{N}{\sqrt{a}}\left[(3+2i)\cos(\frac{\pi x}{2a}) - 2\sin(\frac{\pi x}{a}) + 3i\cos(\frac{3\pi x}{2a})\right]$$

where N is a normalization constant. **(a) 5%** What's normalized $\Psi(x,t)$? **(b) 10%** Suppose one prepares 10^5 identical copies of the above particle-well system and performs the precise measurement of energy, what values would he get? Evaluate the average value and the uncertainty of energy he would obtain at time t. **(c) 5%** What is the probability for energy falling into this range $\frac{\hbar^2\pi^2}{ma^2} \leq E \leq 3\frac{\hbar^2\pi^2}{ma^2}$?

Problem 3 10% Consider the potential shown below

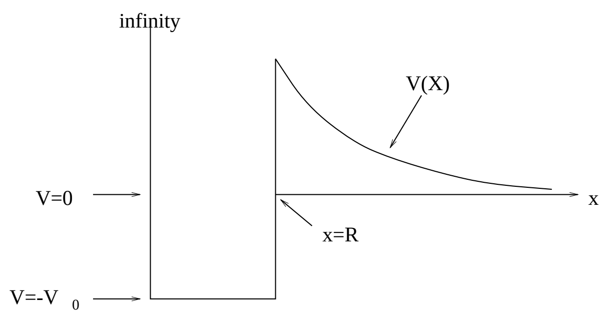

FIG. 1:

$$\text{with } V(x) = \frac{\hbar^2 a}{2mx^3} \text{ and } V_0 = \frac{\hbar^2}{2ma^2},$$ where a is some constant whose dimension is length.

Estimate the lifetime of a particle of energy $E \approx 0$ in this potential.

Problem 4 Consider a potential given by

$$\begin{aligned} V(x) &= \infty \qquad x < 0, \\ &= -V_0 \quad 0 < x < a, \\ &= 0 \qquad a < x. \end{aligned}$$

(a) 10% A plane wave e^{-ikx} is sent in from $+\infty$. Determine the form of the reflected amplitude R in the reflected wavefunction Re^{ikx}, and obtain the equation that allows one to determine R. **(b) 10 %** Find the minimum V_0 so that there are at least two bound states. **(c) 5%** Is there any relation of the bound states for this potential to the bound states of the following potential

$$\begin{aligned} \tilde{V}(x) &= -V_0 \quad -a < x < a \\ &= 0 \qquad a < |x| \end{aligned}$$

Problem 5 10%If the ground state of a particle with mass m in some unknown potential $V(x)$ is given by

$$\Psi(x) = N \exp[-(x/a)^2], \; a = \text{some length and } N \text{ is a normalization constant.}$$

Find the energy of this particle in the first excited state relative to the ground state energy in the same potential $V(x)$.

Problem 6 A particle of mass m is in the following potential

$$\begin{aligned} V(x) &= \beta\delta(x) \;\; \text{for } |x| < a, \\ &= \infty \;\; \text{for } |x| \geq a. \end{aligned}$$

(a) 10 % Show that there is a series of solutions that are not affected by the delta function. In addition to this series of solutions, derive the equation that other energy eigenvalues satisfy and show how the energy eigenvalues can be found. **(b) 7 %** Find energy eigenvalues in the limit of $\beta \to 0$ and $\beta \to \infty$, indicate their relations to those for infinite potential well. **(c) 8 %** Now suppose that the walls at $x = \pm a$ are removed so that $V(x) = 0$ for $|x| \geq a$. The particle is incident from the left with energy E_0, find the transimission coefficient.