Electrodynamics (II): Midterm
6:30 PM-9:00 PM, April 17, 2015

Total grade 110

Useful information

The electric field and magnetic field along tranverse x and y directions inside a waveguide can be expressed in terms of the corresponding z component as follows

$$E_x = \frac{i}{(\omega/c)^2 - k^2)}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$
$$E_y = \frac{i}{(\omega/c)^2 - k^2)}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$
$$B_x = \frac{i}{(\omega/c)^2 - k^2)}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$
$$B_y = \frac{i}{(\omega/c)^2 - k^2)}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right),$$

where c is the speed of light, ω is the angular frequency, and k is the wavevector.

Problem 1

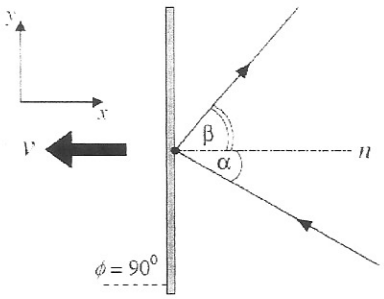
Answer the following questions briefly:

(i) 20% Explain the following terms briefly: Brewster angle, TM waves in wave guides, intensity of light, skin depth, time dilation.

(ii) 6% In a space Ω without any charges and electric currents, let the electric field and magnetic fields be $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$. What is the energy density one obtains if he tries to measure it at position $\vec{r}$ and time t? What is the differential equation that expresses the energy conservation for the electromagnetic fields in Ω?

(ii) 6% It is known that the wavenumber k and angular frequency ω of electromagnetic waves in a medium obeys the dispersion relation $k^2 = (\omega^2 - \omega_0^2)/c^2$, where ω_0 is a positive and real number. (1) Find the phase verlosity and group velocity for the propagation of electromagnetic wave in this medium. (2) What happens to the propagation of the electromagnetic waves when their angular frequence $\omega < \omega_0$?

(iii) 8% As shown in the following figure, a mirror moves with velocity v towards $-\hat{x}$ direction. A light is incident from right with α being the incident angle and β being the reflection angle. Express $\cos\beta$ in terms of $\cos\alpha$ and v.



Problem 2

In the circuit shown in below, the capacitor has circular plates of radius r_0 separated by a distance d. Suppose that $d \ll r_0$ and fringing fields can be neglected. Between the plates, it is vacuum. If at $t = 0$, there is a charge Q_0 on the capacitor and the switch is closed. In the quasi-static approximation, answer the following qeustions

(a) 10% Find the total electromagnetic energy E_{out} that flows out from the capacitor for $0 \le t < \infty$. Find the total energy dissipated E_d by the resistor for $0 \le t < \infty$

(b) 5% Let the total electomagnetic energy stored in the capacitor at $t = 0$ be E_{EM}. Find relations between E_d, E_{out}, and E_{EM} to express the energy conservation in this system.

Problem 3

Consider an infinite parallel-plate capcitor with the lower plate (at $z = -d/2$) carrying surface charge density $-\sigma$, and the upper plate (at $z = d/2$) carrying surface charge density $+\sigma$.

(a) 9 % Determine all nine elements of the stress tensor in the region between the plates. Display