Theoretical Mechanics I, Fall 2020

SECOND MIDTERM EXAMINATION

Time: 10:10 – 12:00, December 9, 2020

Venue: 019 Physics, 313 Physics, 203 General Physics Lab

This is a closed book exam. No search on the web or related electronic books is allowed. Useful formulas and quantities are provided in the end of the exam papers.

Please answer the following questions. There are 4 questions in total.

1. $\boxed{25\%}$ Assume that the Earth has solid interior of uniform density $\rho = \frac{3}{4\pi}\frac{M}{R^3}$, where M and R are the mass and radius of the Earth, respectively. A particle is dropped into a hole drilled straight through the center of the Earth. Neglect the rotational effects and answer the following questions about the motion of the particle.

   (a) $\boxed{10\%}$ Show that the motion of the particle is simple harmonic.

   (b) $\boxed{15\%}$ Find the Lagrangian of the particle and determine the Lagrange's equation of motion. (*Hint:* In class, we have shown the gravitational potential for a point inside a spherical shell of uniform density ρ to be

$$\Phi(r) = -4\pi\rho G\left(\frac{b^2}{2} - \frac{a^3}{3r} - \frac{r^2}{6}\right), \tag{3}$$

   where a and b are the inner and outer radius of the shell.)

2. $\boxed{25\%}$ A particle of mass m is placed along the z-axis above a thin infinite sheet of surface density ρ (mass per unit area, e.g. $\mathrm{g\,cm^{-2}}$) at $z = 0$.

   (a) $\boxed{10\%}$ Calculate the gravitational force on the particle attracted by the sheet.

   (b) $\boxed{15\%}$ Alternatively, use the potential method to find the gravitational force. Since the sheet extends to infinity, you may want to first determine the force given by a sheet of finite size, such as radius R, by using the gravitational potential and then allow R to approach the infinity.

3. $\boxed{25\%}$ A particle of mass m slides on the surface of a fixed, smooth hemisphere of radius R. The particle starts at the top of the sphere ($\theta = 0$) with horizontal initial velocity v_0.

   (a) $\boxed{15\%}$ Find the force of constraint.

   (b) $\boxed{10\%}$ Determine the angle, θ_c, at which the particle leaves the hemisphere.

4. $\boxed{25\%}$ Consider two identical simple pendulums, each with mass m and length ℓ, connected by a spring of force constant k with negligible mass (Fig. 1). The motions of both pendulums stay in the x-y plane. Consider small oscillations, where the variation in y direction of the

177.02 $+35$

354

$\dot{z}mr^2\dot{\theta}$

pendulum motion may be neglected. The spring is not stretched when the pendulums are in equilibrium ($x_1 = x_2 = 0$).

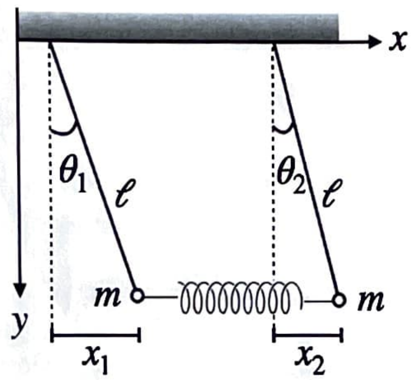

Figure 1: The double pendulum.

(a) $\boxed{15\%}$ Find the Lagrangian and write down the equations of motion. You may use the small angle approximation to drop higher order terms in (x_1/ℓ) or (x_2/ℓ).

(b) $\boxed{10\%}$ Solve the equations of motion to determine the proper frequencies of the system.