Test-2 EM Waves, Optics, and Fluids 05/11

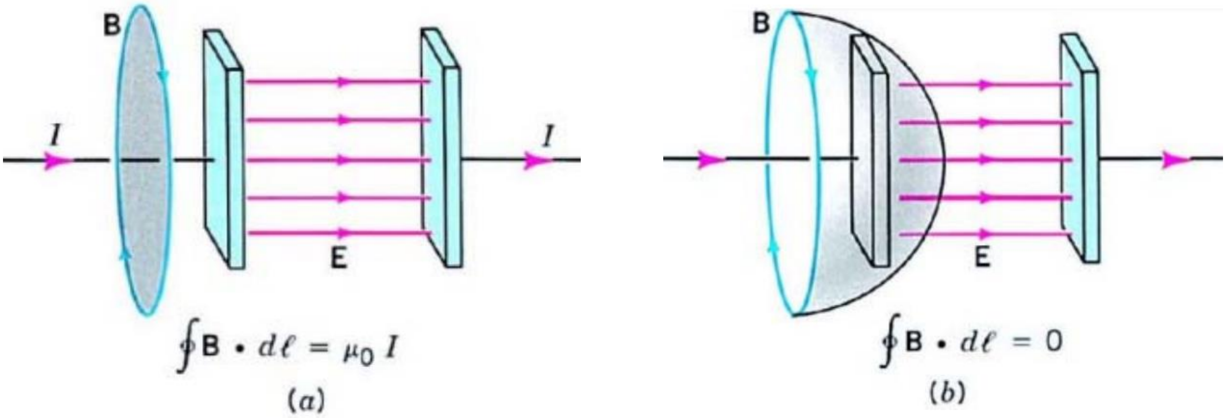
$c = 2.99792458\ \mathrm{m/s}$, $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$; $n = \sqrt{\varepsilon_r}$

1. (1) Why Ampère's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$ does not give consistent answers for the following two choices of surface in figures (a) and (b), respectively? [5%]

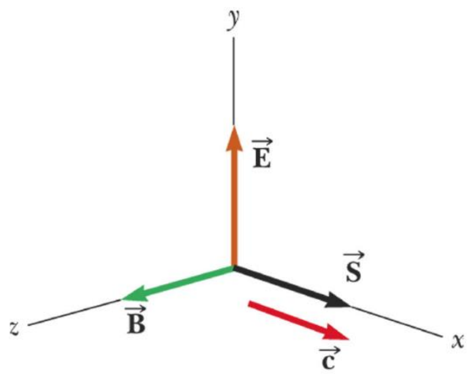
(2) Maxwell then proposed a new type of current to fix this inconsistency. What is its definition? [5%]

(3) Use the Ampère-Maxwell law to find the magnetic flux density B between the circular plates (of radius R) of a parallel-plate capacitor that is charging. Ignore the fringing field and consider the case of $r < R$. [10%]

Hint: $B = \frac{1}{2}\mu_0 \varepsilon_0 r \frac{dE}{dt} \quad (r < R)$



2. (1) The Poynting vector is defined by $\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}$ so that the three vectors **E**, **B**, and **S** are mutually orthogonal to each other. What is the physical meaning of the Poynting vector as well as its unit? [5%]



(2) In free space, the total energy density u of the EM wave is a sum of both that of **E** and of **B** fields:

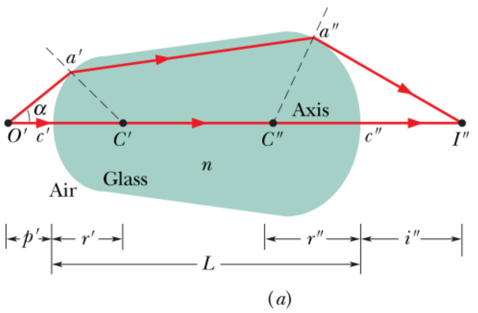
$$u_E = \frac{1}{2}\varepsilon_0 E_m^2; \quad u_B = \frac{1}{2\mu_0} B_m^2.$$

E_m and B_m are the amplitudes of sinusoidal functions of **E** and **B** fields, respectively. Show that $u_E = u_B$. [5%]

(3) Examine that $I \equiv S_{\text{avg}} = \frac{1}{T}\int_{t_0}^{t_0+T} S(x,t)dt = \frac{E_m B_m}{2\mu_0} = c\{u_E + u_B\}_{avg}$. [5%]

(4) A radio station transmits a 10-kW signal at a frequency of 100 MHz. For simplicity, assume that it radiates as a point source. At a distance of 1.0 km from the antenna, find the energy incident normally on a square plate of side 10 cm in 5.0 min. [5%]

3. (1) In use of the figure below to show that $\dfrac{1}{p}+\dfrac{1}{i}=\dfrac{1}{f}=(n-1)\left(\dfrac{1}{r'}-\dfrac{1}{r''}\right)$ for a thin lens, *i.e.*, $L\to 0$. [5%]



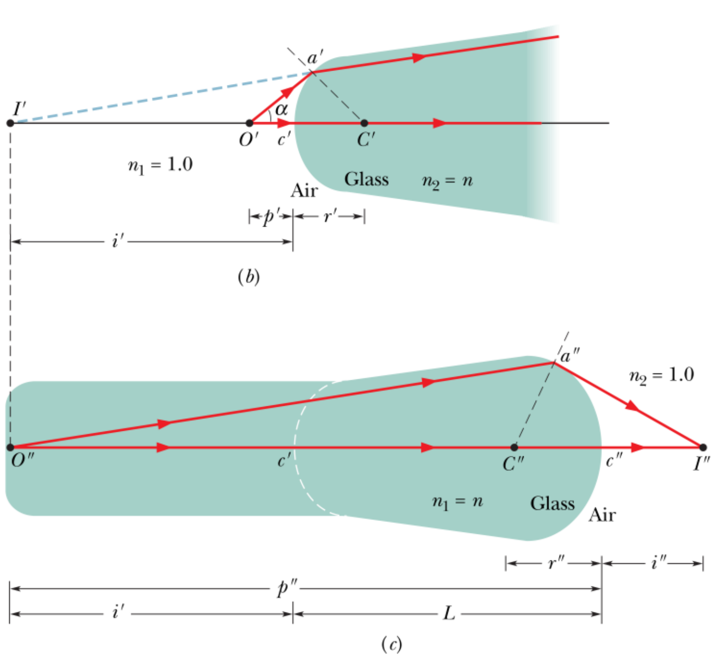


Fig. 34-24 (*a*) Two rays from point object O' form a real image I'' after refracting through two spherical surfaces of a lens. The object faces a convex surface at the left side of the lens and a concave surface at the right side. The ray traveling through points a' and a'' is actually close to the central axis through the lens. (*b*) The left side and (*c*) the right side of the lens in (*a*), shown separately.

Note HRW's textbook (the above, Walker-Halliday-Resnick) and that of Serway-Jewett both adopt the same convention of the lens-maker's equation; however, in Giancoli's textbook it is written in another form: $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)$.

(2) You grind the lenses shown in Fig.P3-a below from flat glass disks ($n=1.5$) using a machine that can grind a radius of curvature of either 40cm or 60cm. You select the side of radius 40cm radius toward the object and then hold the lens in sunshine to form an image of the Sun. What are the focal length f and image type (*real* or *virtual*) for this (meniscus concave) lens? [5%]

Fig.P3-a

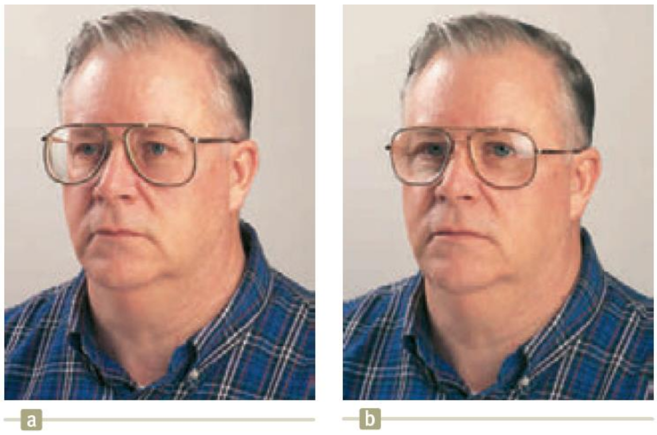
Fig.P3-B

(*a*) (*b*) (*c*) (*d*)

(3) Figure above (Fig.P3-b) shows four thin lenses, all of the same material, with sides that either are flat or have a radius of curvature of magnitude 20cm. Without written calculation, rank the lenses according to the magnitude of the focal length. [5%]

(4) How did Galileo compose lenses to make his telescope so as to discover mountains on the Moon in 1609? Or, any special considerations should be taken on the focal length of each lens? [5%]

4. (1) In the figures as shown, which glasses (the left: <u>a</u> or the right: <u>b</u>) correct nearsightedness and which correct farsightedness? Explain. [4%]



(2) A nearsighted eye has near and far points of 14 cm and 20 cm, respectively. Diverging lenses (of the focal length –18 cm) are needed for this person to see distant objects clearly. After wearing eyeglasses, what will be the near point? Assume that the lens is 2.0 cm from the eye (typical for eyeglasses). [4%]

Hint: ***Near point*** is the closest distance at which eye can focus clearly; normal is about 25 cm. ***Far point*** is the farthest distance at which object can be seen clearly; normal is at infinity.

(3) You are in an airplane at an altitude of 10,000 m. If you look down at the ground, estimate the minimum separation s between objects that you could distinguish. Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and λ = 500 nm. [4%]

Hint: For a circular aperture of diameter D, the central maximum has an angular width:

$$\theta_{\min} = 1.22\frac{\lambda}{D}$$ (or, written as the Rayleigh criterion, θ_R).

(4) Because of the diffraction, the best eye resolution is limited no better than 5x10^{-4}, a light has a maximum useful magnification of about _______ and the smallest objects it can resolve have a size of about _______. Explain the reasons of your choice. [4%]

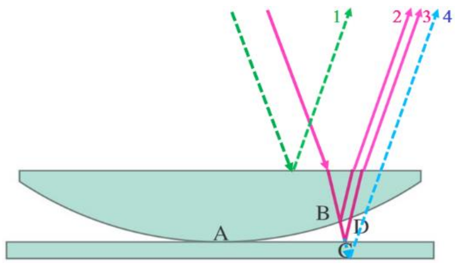
（A）50×; 2000 nm （B）100×; 1000 nm （C）500×; 200 nm
（D）2000×; 50 nm （E）5000×; 20 nm

(5) What is the difference between spherical and chromatic aberration? [4%]

5. (1) Interference is the most essential feature of waves. Give ***five*** examples in **different** types of wave interference (without lengthy descriptions). [5%]

(2) What is the minimum thickness of an ideal optical coating material whose index of refraction is n = 1.225 designed to eliminate reflected light at wavelengths (in air) around 550 nm when incident normally on glass for which n = 1.50? [10%]

(3) Why do we analyze Newton's rings, as shown in the figure, by taking into account only the interference between the reflected rays 2 and 3, but excluding others, say, reflected rays 1 and 4? [5%]



6. (1) Considering the intensity patterns from **6**-slits **diffraction grating**, what is the key factor making the features shown in Fig.P6-a different from that of Fig.P6-b? Explain in detail (in particular, ***analytically***). Notice that D is the width of each slit and d is the spacing of neighboring slits. [5%]

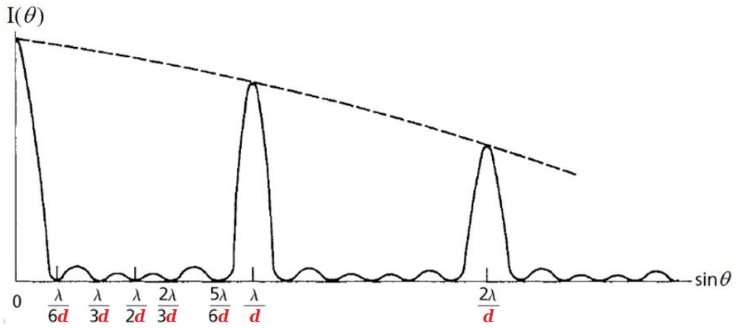


Fig.P6-a

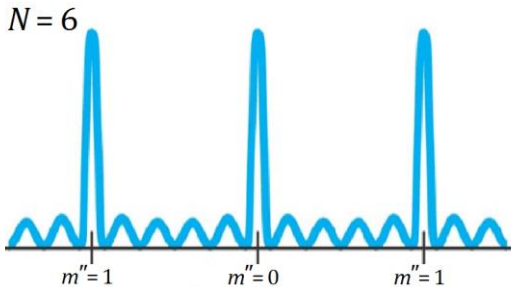


Fig.P6-b

(2) In Fig.P6-a, how to get the half width at $\sin\theta = 2\lambda/d$? Namely, $\Delta\theta_{hw}$ = ? [5%]

PS. $\Delta\theta_{hw}$ is also written as $\Delta\theta_{m''}$ somewhere and $m'' = 2$ in this problem.

(3) Suppose $d = D$ in a double-slit apparatus, so that the two slits merge into one slit of width $2D$. Show that

$$I_2(\theta) = I_0\left[\frac{\sin(2\beta)}{\sin\beta}\right]^2\left(\frac{\sin\alpha}{\alpha}\right)^2, \quad \text{where } \alpha = \frac{\pi}{\lambda}D\sin\theta \text{ and } \beta = \frac{\pi}{\lambda}d\sin\theta,$$

reduces to the correct equation for single-slit diffraction. [10%]

7. (1) The ***E***-field of time- harmonic electromagnetic waves can be expressed as

$$\vec{E}(\vec{r},t) = \vec{E}_0\sin\left(\vec{k}\cdot\vec{r} \pm \omega t + \phi_0\right) = \mathrm{Im}\left\{\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}\pm\omega t+\phi_0)}\right\},$$

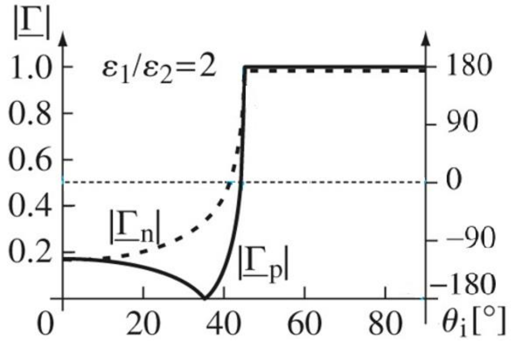
where $\mathrm{Im}\{\underline{C}\}$ means that picks up the imaginary part of the complex quantity $\underline{C}$ only. In this form, why we call the function of $\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}\pm\omega t+\phi_0)}$ wave a plane wave? [5%]

(2) In free space, why electromagnetic waves are transverse? Give a simple proof. [5%]

(3) Referring to the figure shown on the right, the reflection coefficient $\underline{\Gamma}$ is the ratio of the complex amplitude $E_{m,r}$ of the reflected wave to that $E_{m,i}$ of the incident wave. For the parallel-polarized plane wave,

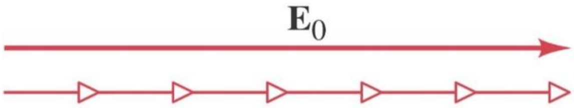
$$\underline{\Gamma}_p = \frac{\underline{E}^{(p)}_{m,r}}{\underline{E}^{(p)}_{m,i}} = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i},$$

where θ_t is the refracted angle of transmission. Show that is $\theta_{\text{Brewster}} = \tan^{-1}(n_2/n_1)$. [10%].



8. (1) What does the "phasor" mean? How we benefit from it to simplify calculations? [4%]

(2) Referring to Fig.P6-a, the phasor relationship at the central maximum is denoted by

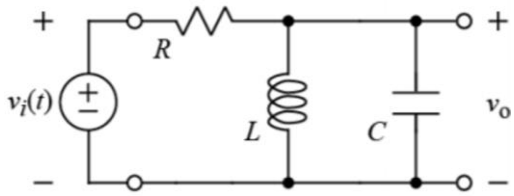


(Central maximum: $\theta = 0,\ \delta = 0$)

what the phasor relationship at the second minimum ($\sin\theta = \dfrac{\lambda}{3a}$) would be? [4%]

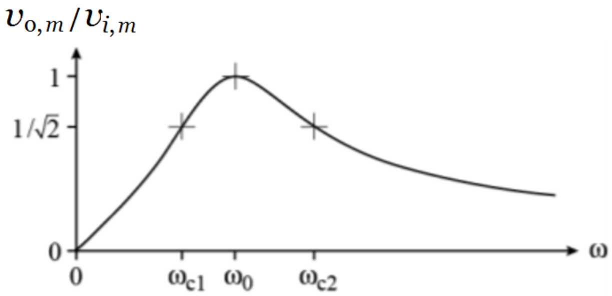
(3-i) Applying the method of phasor to the AC circuit as shown, where $v_i(t) = v_{i,\mathrm{m}} \sin \omega t$ and $V_R(t) = V_{R,\mathrm{m}} \sin(\omega t + \phi)$, explain that

$$\frac{V_{R,\mathrm{m}}}{v_{i,\mathrm{m}}} = \frac{R}{\left| R + \left(\dfrac{1}{jX_L} - \dfrac{1}{jX_C} \right)^{-1} \right|}.$$ [4%]



(3-ii) At what ω will this band-pass filter transmit maximal power to the output port? [4%]

(3-iii) Referring to the output response as shown below, what is the fractional change of energy dissipated by the resistor *versus* that supplied from the input source during one cycle? [4%]



9. (1) From an operational point of view, pressure is the force per unit perpendicular area over which the force is applied. However, pressure is a scalar, *not* the vector. Why is this? Give reasons precise and concise. [4%]

(2) How to convert 1 atm to about 14.7 psi? [4%]

(3) Bernoulli's principle is based upon the conservation of energy and its equation (in a simplest form) states that, $P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$. In what kind of flows of the fluid could apply this equation:

(i) compressible *vs* incompressible? [1%]

(ii) laminar *vs* turbulent? [1%]

(iii) viscous *vs* inviscid/non-viscous? [1%]

(iv) isobaric *vs* isentropic [1%]

(4) Why applying Bernoulli's principle is not able to explain the flight of nowadays heavy-duty aircrafts? Or, what another effect should be taken into account? [2%]

(5) In Fig.P9-a, how does a sailboat move upwind? [2%]

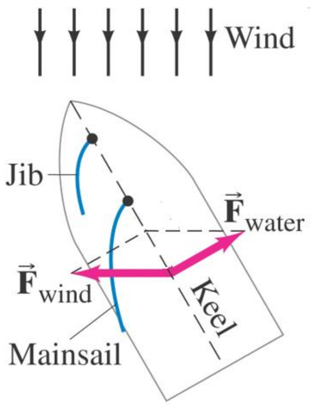


Fig.P9-a

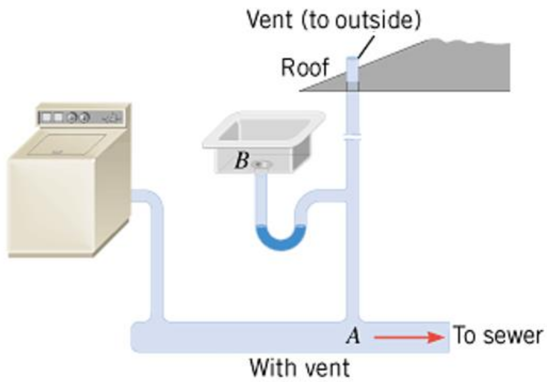


Fig.P9-b

(6) In a household plumbing system, a vent is necessary to equalize the pressures at points A and B, thus preventing the trap from being emptied. Explain. [2%]

(7) How Bernoulli's effect helps prairie dogs with air circulation? [2%]