$\frac{1Q}{4\pi\epsilon_0}\ \frac{\frac{1}{2}}{R}$

前 $V = \frac{1}{4\pi\epsilon_0}\ \frac{Q}{a}$

後 $V = \frac{1}{4\pi\epsilon_0}$ $\quad \frac{-Q}{4\pi\epsilon_0}\ \frac{\frac{1}{2}}{R}$ $\quad \frac{-Q}{4\pi\epsilon_0} \times \frac{\frac{1}{2}r^2}{R^3}$

Eletrodynamics-1, Midterm, Nov. 3, 2015.

Single Choice (10 % each) *Just collect and summarize your answers(only) inside the boxes 1-4 on the cover page. No explanation is needed.*

1. Consider a vector field $\vec{F} = (xy, 2, 0)$ and three line integral paths: P_I : from $(0,0,0)$ to $(1,0,0)$, P_{II}: from $(1,0,0)$ to $(0,1,0)$, and P_{III}: from $(0,1,0)$ to $(0,0,0)$. Which of the following is incorrect? (A)$\int_{P_{II}} \vec{F}\cdot d\vec{l} = 13/6$. (B)$\int_{P_{III}} \vec{F}\cdot d\vec{l} = -2$. (C)$\oint_{I,II,III} \vec{F}\cdot d\vec{l} = -1/6$. (D)$\int_{P_I} \vec{F}\cdot d\vec{l} = 0$. A

2. Consider a vector field $\vec{F} = (x, y, z)$, and the volume $\mathcal{V}$ which is the region $x \geq 0, y \geq 0, z \geq 0$, and $x^2 + y^2 + z^2 \leq 1$. The 4 boundary facets of $\mathcal{V}$ are denoted as $S_1 : (z = 0, x^2 + y^2 \leq 1, x \geq 0, y \geq 0)$, $S_2 : (x = 0, z^2 + y^2 \leq 1, z \geq 0, y \geq 0)$, $S_3 : (y = 0, x^2 + z^2 \leq 1, x \geq 0, z \geq 0)$, and $S_4 : (x^2 + y^2 + z^2 = 1, x \geq 0, y \geq 0, z \geq 0)$. Which of the following is incorrect? (A) $\int_{\mathcal{V}} \nabla \cdot \vec{F} d\tau = \pi/2$. (B) $\int_{S_1} \vec{F} \cdot d\vec{a} = 0$. (C) $\int_{S_4} \vec{F} \cdot d\vec{a} = \pi/2$. (D) $\vec{F}$ can be expressed as $-\vec{\nabla}(r^2/2 + 7)$. D

3. A static charge distribution produces a radial electric field, $\vec{E} = A\frac{e^{-br}}{r^2}\hat{r}$, where A, b are constants. Which of the following is incorrect? (A) The charge density is $A\epsilon_0[4\pi\delta^3(\vec{r}) - be^{-br}/r^2]$. (B) $\nabla \times \vec{E} = 0$. (C) The total charge is $-4\pi A\epsilon_0$.(D) The electric potential is continuous at the origin. C

$-\frac{Q}{4\pi\epsilon_0}\int_0^r \frac{r r}{R^3}\, dr \quad \frac{-Q}{4\pi\epsilon_0} \times \frac{1}{2}\frac{r'^2}{R^3}\Big|_0^r =$

$\frac{\frac{3}{8}Q}{4\pi\epsilon_0 R}$

4. A uniformly charged spherical volume of radius R which contains a total charge Q. Which of the following is incorrect? (A) The potential difference between $r = 0$ and $r = R/2$ is $3Q/(32\pi\epsilon_0 R)$. (B) If $r < R$, $\vec{E}(r) = \frac{Q\hat{r}}{4\pi\epsilon_0}\frac{r}{R^3}$. (C)If $r > R$, $V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. (D) The work required to move a test charge q' from the point $(a, 0, 0)$ to the point $(0, 0, b)$ is $\frac{Qq'}{4\pi\epsilon_0}[\frac{3}{2R} - \frac{b^2}{2R^3} - \frac{1}{a}]$, where $b < R$ and $a > R$. D

$\frac{1}{(\frac{R}{2})^3}$

$=$

Long Questions *For the long questions, all answers must be supported by detailed calculation or reasoning. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to "guess" your results. No credit points will be granted if fail to do so.*

1. (10+10%) (a) A ring of radius R has a total charge $+Q$ uniformly distributed on it. Calculate the electric field and potential at the center of the ring.
   (b) Consider a charge $-Q'$ constrained to slid along the axis of the ring. Show that the charge will execute simple harmonic motion for small displacements perpendicular to the plane of the ring.

2. (10+10%) A sphere of radius R has uniform charge density ρ within its volume, except for a small spherical hollow region of radius R_2 and its center is located at $(a, 0, 0)$ relative to sphere's center.
   (a) Find the electric field at the center of the hollow sphere.
   (b) What is the electric potential at the same point?
   (Infinity is set as the reference point for electric potential and $(a + R_2) < R$.)

3. (10+10%) (a) Two equal charges $+Q$ are separated by a distance $2d$. A grounded conducting sphere is placed midway between them. What is the approximate radius, R, of the sphere if the two charges are to experience zero total force?
(b) What is the force on each of the two charges if the same sphere, with the radius determined in (a), is now charged to a potential V?
Hint: just keep the Taylor expansion to the leading order of (R/d)

4. (10+10+10%) An air-filled capacitor is made from two concentric cylinders. The outer cylinder has a radius of 1 cm.
(a) What is the radius of the inner conductor which allows a maximum potential difference between the conductors before breakdown of the air dielectric?
(b) What is the radius of the inner conductor which allows a maximum energy to be stored in the capacitor before breakdown of the air dielectric?
(c) Calculate the maximum potentials for cases (a) and (b) for a breakdown field in air.
Note: (1)Use the numerical values that $e^{-1} = 0.37$ and $e^{-1/2} = 0.61$ to get your final answers. (2) The breakdown field in air is $\left|\frac{d\vec{E}}{dr}\right| = 3 \times 10^6 V/m$.