Gravitational Constant Determination by the Principle of Optical Lever

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The principle of the optical lever provides a sensitive method of measurement. It is a convenient device to magnify a small displacement and thus to make possible an accurate measurement of it , that is , the optical lever remains a very useful approach in sensitive non-contacting measurements.

1. Introduction

For this experiment, the main target is to observe the moving reflected light spot and record the positions per 30 seconds for using the principle of optical lever to roughly calculate the value for gravitational constant.

Before hundreds of year British scientist Henry Cavendish used the torsion balance apparatus which was invented by English geologist John Michell to measure the gravity between masses.

The experiment, performed in 1797—1798, is the first success to yield the accurate value of the gravitational constant and gives a proof for the Newton's Law of Universal Gravitation which was given from *Mathematical Principles of Natural Philosophy* written by English physicist Isaac Newton in 1687[1]:

$$\mathbf{F} = \frac{GMm}{r^2} \qquad (1)$$

where G is the gravitational constant, and **F** and **r** are the gravity and the distance between masses M and m, respectively.

Hence the experiment has been named as the " Cavendish experiment ".

During the process, we should considered the net torque of the small ball with mass m and radius r due to attraction of the large ball pair with mass M and radius R:

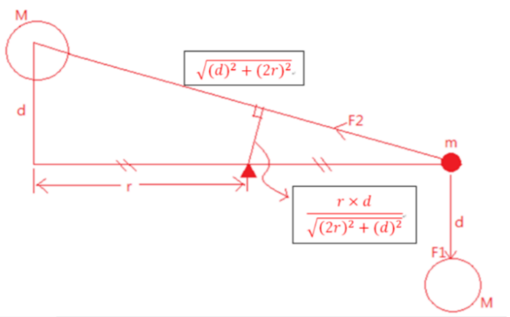

Fig.1:The torque acting on the small ball m Due to the pair of lage balls.

$$\boldsymbol{\tau}_1 = \sum_{i=1}^{2} \boldsymbol{r}_i^* \times \boldsymbol{F}_i = \alpha \frac{GMmr^*}{d^2} \qquad (2)$$

where $\boldsymbol{\tau}$ is the torque, $\boldsymbol{r}^*$is the position vector, d is the distance between the large ball and the small ball, and α is the correction term for the torque of the small ball attraction of large ball pair:

$$\alpha \equiv \left(1 - \frac{1}{\left(\sqrt{1+\left(\frac{2r^*}{d}\right)^2}\right)^3}\right) \qquad (3)$$

The small ball would start to move around "peroidicly", just like underdamped oscillation. Hence we set a flat mirror on the torsion balance apparatus,and let the laser beam reflected on it.According to the law of reflection and the principle of optical level,the following relation we would obtained:

$$4\theta = \frac{\Delta x}{L} \qquad (4)$$

where 4θ is the change of the angle for the refletion light which implied that the change of angular displacement for the small ball is 2θ , Δx is the difference of the equilibrium positions for the light spots on ruler due to large ball pair, and L is the distance between ruler and mirrow.

Moreover, the quartiz wire in the torsion balance apparatus would also rotate because of torques:

$$\tau = 2\tau_1 = \kappa\theta = I\left(\frac{2\pi}{T}\right)^2 \theta = 2mr^{*2}\left(\frac{2\pi}{T}\right)^2 \theta \quad (5)$$

where κ is the torsion constant of quartiz wire, I is the moment of inertia for the system, and T is the period of oscillation.

To summarize all of formulas and we can get the relation formula of G directly:

$$G = \frac{\pi^2 r^* d^2 \Delta x}{\alpha M L T^2} \qquad (6)$$

We can find the fact that gravitational constant G is proportional to the $\frac{\Delta x}{T}$, so we seek to measure the period T and the position difference Δx in order to determine the value of G.

2. Method

In the experiment, we used the following equipment: a pair of large lead balls with mass 1500 g and radius 32 mm, a pair of small lead balls with mass 15 g and radius 6.9 mm, a device of laser with 650 nm wave length, a ruler and the torsion balance apparatus.

The most important and the only procedure of the experiment is determination of G.

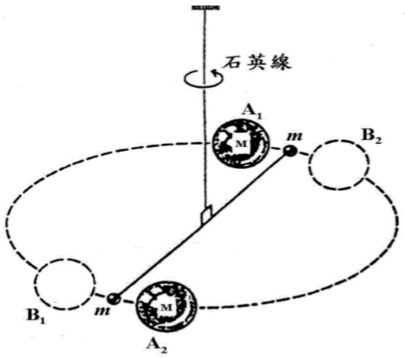

Fig.2: The setting for this experiment

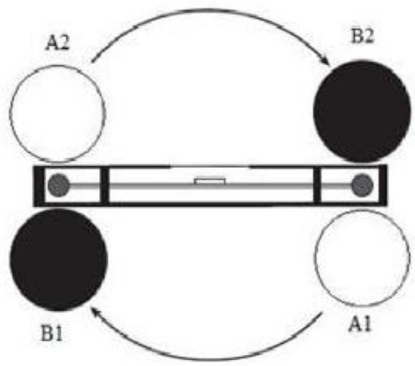

Fig.3: The position of pairs of large and small balls.

As Fig.2, we placed pair of large balls at A_1A_2. After the pair of small balls moved slightly, we started to observe and record the positions of light spot on ruler per 30 seconds for 30 minutes.

Next, we changed the position of large ball pair into B_1B_2 and did the same way as the A_1A_2 step, keeping recording the positions of light spot on ruler per 30 seconds for 30 minutes.

Finally, plotting the position of light spot on the ruler against the time.Did the curve-fitting and we obtained the position function of time as well as the equilibrium position *x* and the period T. Using (6) to calculate the value of G, average of G,standard deviation of G and compare with the standard value in the world.

3. Results

The results we calculate for G:

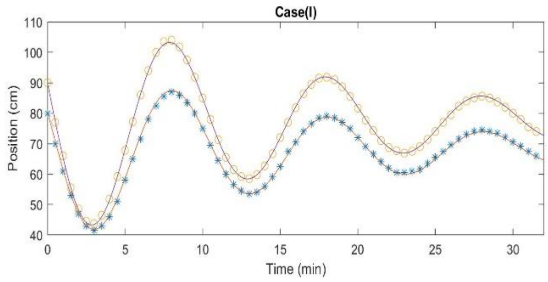

Fig.4 : The position varies with the time for case I, where the blue curve represented as A_1A_2 , and the orange one represented as B_1B_2

Case I :

T = 660 s , Δx =11.5 cm , L = 248.7 cm

$G_1 = 6.06\times 10^{-11}$ N-m^2/kg^2

Curve-Fitting:

$$A_1A_2 = y_0 + Ae^{\frac{-x}{t_0}}\sin(\pi\frac{x-x_c}{\omega})$$

$$= 67.98 - 31.38\exp(\frac{-x}{16.89})\sin(\pi\frac{x-0.6505}{5.012})$$

$$B_1B_2 = y_0 + Ae^{\frac{-x}{t_0}}\sin(\pi\frac{x-x_c}{\omega})$$

$$= 77.60 - 40.68\exp(\frac{-x}{17.25})\sin(\pi\frac{x-0.4928}{-5.017})$$

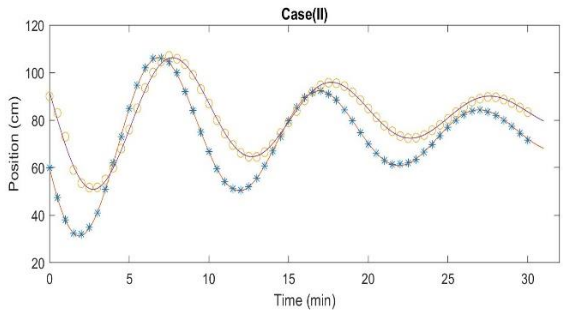

Fig.5 : The position varies with the time for case II, where the blue curve represented as A_1A_2 , and the orange one represented as B_1B_2

Case II :

T = 600 s , Δx = 10.8 cm , L = 268.7 cm

$G_2 = 6.38\times 10^{-11}$ N-m^2/kg^2

Curve-Fitting:

$$A_1A_2 = y_0 + Ae^{\frac{-x}{t_0}}\sin(\pi\frac{x-x_c}{\omega})$$

$$= 74.29 + 48.15\exp(\frac{-x}{17.2})\sin(\pi\frac{x-4.53}{5.012})$$

$$B_1B_2 = y_0 + Ae^{\frac{-x}{t_0}}\sin(\pi\frac{x-x_c}{\omega})$$

$$= 82.48 - 37.15\exp(\frac{-x}{17.45})\sin(\pi\frac{x-0.4029}{4.974})$$

Average of G	6.22×10^{-11} N-m^2/kg^2
Standard Derviation of G	1.11×10^{-12}

The standard value of G:

$G_0 = 6.67\times 10^{-11}$ N-m^2/kg^2

	Percentage of error for G
Case I	9.08%
Case II	4.37%

4.Discussion

During the process of this experiment , we found two main problems. First of all, we need to give the external energy to the torsion balance when changing the position of large ball pair from A_1A_2 to B_1B_2. Since the oscillation process was damped all the time, the system must processively run out of the mechanical energy.

Meanwhile,the speed of moving light spot would too slow for us to observe and record the positions of light spot. Hence,we had no choice but to reput the large ball pair to create the slightly shaking for the system, that is, supply the extra mechanical energy for the system.

Second, we couldn't measure the position of light spot until the oscillation of torsion balance had moved slightly. It implied that we couldn't accurately determine when the " right time " was for us to start observing the moving light spot. Hence,the total mechanical energy of the system that everytime we began to record was different. It would cause the higher and inevitable statistic error to some degree.

References

[1] J. B. Marion, *Classical Dynamics of Particles & Systems*, 5th Ed, Brooks/Cole Pub Co(2008)

Appendix

Grounded Data:

The distance between mirror and ruler:

L_1＝248.7 cm，L_2＝268.7 cm

Large ball：mass＝1500 g，radius＝32 mm

Small ball：mass＝ 15 g，radius＝6.9 mm

The distance between large and small ball:

d＝46.5 cm

The correction term of torque： α＝0.7969

Measurement Time Record:

Case I：

13:55～14:30 (A1A2)

15:32～16:04 (B1B2)

Case II：

16:19～16:51 (A1A2)

17:06～17:36 (B1B2)