Eletrodynamics-2, Midterm, April 12, 2016.

Short questions

1. (10%) Radio station Philharmonic Radio Taipei Co. radiates a power of 2kW at about 90.7 MHz from its antenna in HuKou, approximately 10km from NTHU. Obtain a rough estimate of its electric field at NTHU in volts per meter.

2. (10%) How do you transform a linearly polarized plane EM wave into a right-handed circularly polarized EM wave?

3. (5% each) The electric filed of an EM wave in vacuum is given by

$$E_x = E_z = 0\,,\; E_y = 50\cos\left(4\pi \times 10^7 t - \frac{2\pi}{15}x\right)$$

where E is in volts/meter, t in seconds, and x in meters. Determine the following with their units specified clearly
(a) the frequency,
(b) the wavelength,
(c) the magnetic field,
(d) the direction of propagation of the wave,
(e) the Poynting vector,
(d) the field momentum density.

4. (5% each) For the given potentials

$$\phi(\vec{r},t) = 0\,,\; \vec{A}(r,t) = \frac{3}{4\pi\epsilon_0}\frac{Qt}{r^2}\hat{r}\,,$$

find the corresponding
(a) electric and magnetic fields,
(b) charge and current distributions.
(c) Find a gauge function $\lambda(\vec{r},t)$ to go to the Coulomb gauge,
(d) and verify that you get the same electric and magnetic fields.

Long Questions *All answers must be supported by detailed calculation or reasoning. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to "guess" your results. No credit points will be granted if fail to do so.*

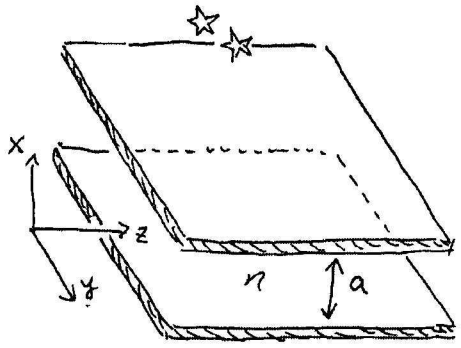

Fig. 1

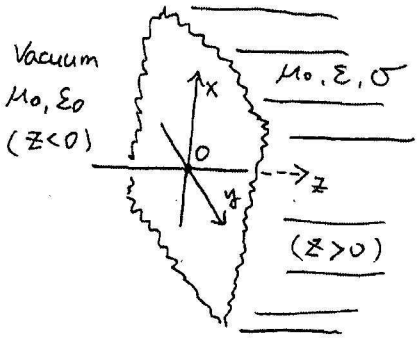

Fig. 2

1. (10+5+5%) A waveguide is formed by two infinite parallel perfectly conducting planes separated by a distance a, see Fig.1. The gap between the planes is filled with a gas whose index of refraction is n and n is independent of frequency. Consider the guided plane wave modes in which the field strengths are independent of the y variable.

   (a) For a given wavelength λ find the allowed frequency ω. What is the cut-off frequency?

   For each such propagating mode, find
   (b) the phase velocity v_p
   (c) and the group velocity v_g.

2. (10+10+10%) A polarized plane EM wave $\vec{E} = E_0\hat{y}\exp i(kz - \omega t)$ is incident normally on a semi-infinite material ($z > 0$) with permeability μ_0, dielectric constant ϵ, and a real conductivity σ, see Fig.2. Assume that $\sigma \gg \omega\epsilon$.

   (a) From Maxwell's equations derive an expression for the electric field in the material.
   (b) Find the magnetic field in the material.
   (c) For $z > 0$, calculate the Poynting vector averaged in time over a wave period.