$C = \epsilon_r C_{vac.}$

左

$\frac{1}{2}(\epsilon_A C_{vac}) \frac{1}{2}(\epsilon_B C_{vac})$

Eletrodynamics-1, Final, Jan. 12, 2016.

Single Choice (10 % each) *Just collect and summarize your answers(only) inside the boxes 1-7 on the cover page. No explanation is needed.*

1. Which of the following is incorrect? (A)$\mu_0 = 4\pi \times 10^{-7} N/A^2$ in SI unit, (B)$\mu_0\epsilon_0 = 1/c^2$ in both SI and Gaussian Unit, (C) Charge in Gausssian unit carries fractional dimensionality of length, time, and mass,(D) 1 Gauss $= 10^{-4}$ T.

B

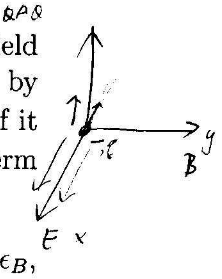
2. Which of the following is incorrect? (A) $\nabla \cdot \vec{A}$ always vanishes, (B)$\nabla \cdot \vec{B}$ always vanishes, (C) An $\vec{A} = \frac{B_0}{2}\hat{n}\times\vec{r}$ represents a constant $\vec{B} = B_0\hat{n}$, (D) An $\vec{A} = \frac{B_0}{2}\hat{n}\times\vec{r}+6r^2\hat{r}$ represents a constant $\vec{B} = B_0\hat{n}$.

AD

$r\sin\theta$

3. Which of the following is incorrect? (A)Since $\vec{B}$ does not do work so magnetic field carries no energy, (B) the force on a particle with charge q in EM fields is given by $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ in SI unit, (C) A negative charge moves toward $-\hat{z}$ in long term if it is placed in $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{y}$, (D) A positive charge moves toward $-\hat{y}$ in long term if it is placed in $\vec{E} = E_0\hat{x}$ and $\vec{B} = B_0\hat{z}$.

A



4. Two slabs A and B of identical shape and mass but different dielectric constant ϵ_A and ϵ_B, $\epsilon_B > \epsilon_A$, are halfway inserted between the plates of an isolated parallel-plate capacitor, Fig.1. Assume that they were held at rest until $t = 0$ and they can move freely in between the plates afterwards. Which of the following is correct? (A) Initially, both A and B move toward right, (B) At the end, the electrical energy stored in the capacitor will be smaller than that at the beginning. (C) A moves to left and B moves to right (D) the potential difference across the capacitor is a constant over time.

BA

$E = \frac{V}{d}$

E charge $\to \int E\,dl$ charge

5. Three identical electric dipoles with $\vec{P} = p\hat{z}$ are located at $\vec{r_1} = (0,0,0)$, $\vec{r_2} = (0,0,d)$, and $\vec{r_3} = (d,0,0)$ respectively. Which of the following is incorrect? (A)the force between p_1 and p_2 is attractive (B)the force between p_1 and p_3 is repulsive (C) the magnitude of the force between p_1 and p_2 is $3p^2/(4\pi\epsilon_0 d^4)$ (D)the magnitude of the force between p_1 and p_3 is $3p^2/(4\pi\epsilon_0 d^4)$.

A

6. A long non-magnetic cylindrical conductor with inner radius a and outer radius b carries a current I, Fig.2. Assume the current density in the conductor is uniform. Which of the following is incorrect? (A) $\vec{B} = 0$ inside the hollow space (B) $\vec{H} = I/(2\pi r)\hat{\phi}$ outside the conductor (C) $\vec{H} = \frac{I}{2\pi r}(r^2 - a^2)(b^2 - a^2)^{-1}\hat{\phi}$ within the conductor (D) If the magnetic permeability of the material is $2.1 \times 10^{-5} N/A^2$, the magnitude of $\vec{B}$ outside the conductor is greater than $\frac{\mu_0 I}{2\pi r}$.

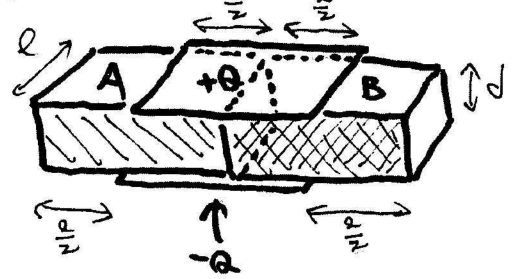
D

7. An oscillating magnetic field is arranged so that $\vec{B} = B_0 \sin\omega t\,\hat{z}$ for $r < a$, and $\vec{B} = 0$ for $r > a$. Which of the following is incorrect? (A) $\nabla \cdot \vec{B} = 0$ (B) $\vec{E} = +\partial\vec{A}/\partial t$ (C) $\vec{E} = -\frac{rB_0\omega}{2}\cos\omega t\,\hat{\phi}$ for $r < a$ (D) $\vec{E} = -\frac{a^2B_0\omega}{2r}\cos\omega t\,\hat{\phi}$ for $r > a$

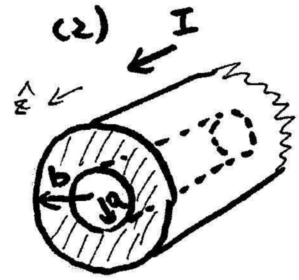
B

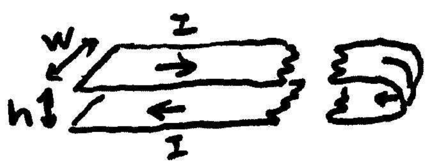
$E\,2\pi r = \{$

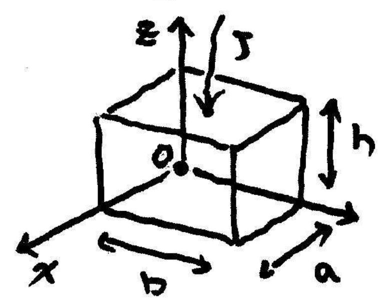
(1)



(2)







(3)

(4)

Long Questions *For the long questions, all answers must be supported by detailed calculation or reasoning. It is your responsibility to clearly state the logic of your answers. I will not make any attempt to "guess" your results. No credit points will be granted if fail to do so.*

1. (5+5+5+5%) A transmission line is constructed from two thin metal "ribbons", of width w, a very small distance $h \ll w$ apart, Fig.3. The current travels down one trip and back along the other. In each case, it spreads out uniformly over the surface of the ribbons.
   (a) Find the capacitance per unit length, $\mathcal{C}$.
   (b) Find the inductance per unit length, $\mathcal{L}$.
   (c) What is the product $\mathcal{LC}$, numerically? What does it mean?(Also specify the unit carefully.)
   (d) If the strips are insulated from one another by a nonconducting material of permittivity ϵ and permeability μ, what is the product $\mathcal{LC}$? What is the propagation speed?

2. (5+5+5+5%) A cylinder of radius R and infinity length is made of permanently polarized dielectric. The polarization vector $\vec{P}$ is everywhere proportional to the radial vector $\vec{r}$, $\vec{P} = a\vec{r}$, where a is a positive constant. The cylinder rotates around its axis with an angular velocity ω. This is a non-relativistic problem, $\omega R \ll c$.
   (a) Find the electric field $\vec{E}$ at a radius r both inside and outside the cylinder.
   (b)Find the magnetic field $\vec{B}$ at a radius r both inside and outside the cylinder.
   (c) What is the total electromagnetic energy stored per unit length of the cylinder before the cylinder started spinning?

(d) What is the total electromagnetic energy stored per unit length of the cylinder while it's spinning?

3. (10+10%) In Fig.4 a block of semiconductor(conductivity $=\sigma$) has its bottom face ($z=0$) attached to a metal plate (its conductivity $\sigma \to \infty$) which is held at potential $\phi = 0$. A wire with infinitesimal size carrying current J is attached to the center of the top face ($z = h$). The sides ($x = 0, x = a, y = 0, y = b$) are insulated and the top is insulated except for the wire. Assume that the charge density is $\rho = 0$ and $\epsilon_r = 1$ inside the block.
(a) Write down the equations satisfied by the potential inside the box and the general solution for the potential.
(b) Write down the boundary conditions for all faces and express the arbitrary constants in the solution from (a) in terms of the given quantities.
(Hint: (1)use Ohm's law to determine the B.C. for the vertical and the top surfaces.(2)you will need to implement the Dirac delta function.)