Geometry Midterm (April 30, 2015)

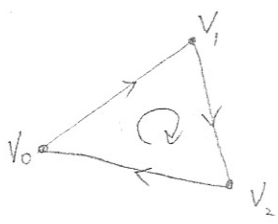
(3:30-6:00, Total score 110)

(1) (15%) Let $S \subset R^3$ be a regular compact connected orientable surface which is not homeomorphic to a sphere. Prove that there are points on S where the Gaussian curvature is positive, negative and zero.

(2) (10%) Prove Jacobi identity for vector fields

$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0$$

(3) (10%) Compute the Euler-Poincare characteristic of the surface $S = \{(x, y, z) \in R^3; x^2 + y^4 + z^6 = 1\}$.



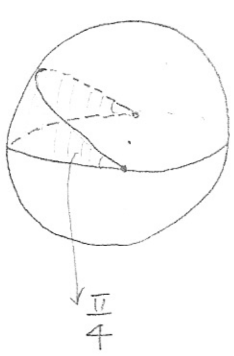
(4) (15%) Compute the homology group of the 2-simplex $K = [v_0, v_1, v_2]$

(5) (15%) Prove the following statement: Let S be a surface homeomorphic to a cylinder with Gaussian curvature $K < 0$. Show that S has at most one simple closed geodesic.

(6) (15%) Consider the surface $X(u, v) = (\cosh v \cos u, \cosh v \sin u, v)$. Compute the geodesic curvature of the curve of the intersection of the surface with the plane $z = 1$.

(7) (15%) Prove that, if there exist two simple closed geodesics on a compact connected surface of positive curvature, then the two geodesics must intersect.

(8) (15%) Show that Gauss-Bonnet Theorem is true in the following case: Consider the region on the unit sphere whose graph is as shown in the picture.



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