Mathematical Physics 311: Final (June 16, 2021 :: 2:20 pm)

Due June 18, 2021 :: 12:00 noon

(1) 30 pts

One description of spin 1 particles uses the following matrices

$$M_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \qquad M_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} , \qquad M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Show that $[M_x, M_y] = iM_z$

(b) Define

$$M_+ = M_x + iM_y , \qquad M_- = M_x - iM_y .$$

Show that

$$\begin{aligned} [M_z, M_+] &= M_+ \\ [M_z, M_-] &= -M_- \\ [M_+, M_-] &= 2M_z \end{aligned}$$

(c) Find the normalized eigenstates of M_z. Label them by $|1\rangle$, $|0\rangle$, and $|-1\rangle$.

(d) Evaluate

$$\begin{aligned} M_-|1\rangle &\quad , \quad M_+|1\rangle \\ M_-|0\rangle &\quad , \quad M_+|0\rangle \\ M_-|-1\rangle &\quad , \quad M_+|-1\rangle \end{aligned}$$

(2) 30 pts

A quantum mechanical system with **two** particles having angular meomenta $j_1 = 3/2$, $j_2 = 1$, respectively.

(a) Write down all the states in the basis of product states.

(b) What are the possible values for the total angular momentum j?

(c) Write down all the states of the total angular momentum j that you found in part (b) in the form of $|j_1 j_2;\ j\, m_j\rangle$.

(d) Express each of the eigenstates that you wrote down in part(c) in terms of the product states.

(3) 40 pts

Consider the Fredholm equation

$$\phi(x) = \lambda \int_{-\pi}^{\pi} (1 + \sin(x + t))\ \phi(t)\, dt\,.$$

(a) Using the method of separable kernel, determine the values for λ which allow solution for $\phi(x)$.

(b) Determine all the nonzero solutions for $\phi(x)$.

Hint: $\sin(x + t) = \sin x \cos t + \cos x \sin t$.