Magnetic Moment Survey by Using Helmholtz Coil and Anti-Helmholtz Coil

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The Helmholtz coil, a device for producing a region of nearly uniform magnetic field, is named after the German physicist Hermann von Helmholtz. It consists of two identical circular magnetic coils placed symmetrically along the same axis, and separated by a distance h equal to the radius R of the coil. Each coil carries an equal current in the same direction. Not only building nearly uniform magnetic fields, but Helmholtz coils are also used in scientific apparatus to cancel external magnetic fields, just like geomagnetic field.Meanwhile, the Anti-Helmholtz coil is very similar to the Helmholtz coil, and the only difference between them is that the current directions carried by circular coils are in opposite directions.The Anti-Helmholtz coil can be used to set up the Magneto-Optical trap and the Quadrupole trap.

1. Introduction

The magnetic moment is a quantity that represents the magnetic field magnitude and orientation of any object that produces a magnetic field,such as magnets, electromagnets, molecules, and even electrons. For most cases, " magnetic moment " often refers to magnetic dipole moment which is "the second(the first-order)term in the multiple expansion of the magnetic moment."

The multipole expansion is very frequently used in the study of electromagnetic and gravitational fields where the fields at distant points are given in terms of sources in a small region for the quick,good approximation.It can be expressed as a sum of terms with progressively dependent of angle.[1] Such as the first term(the zeroth-order) is called monopole is independent of angle.
The following terms are dipole, quadrupole, hexadecapole,octuple moment and so on and so forth.That is,the higher order term it is, the more sensitively it varies with angles.

In analogy with the electric moment, but it's not complete.For experimental reason,we haven't been observed the evidence of the existence for magnetic monopole ,compared with the corresponding electric monopole－charges. That is the reason why the magnetic moment often refers to magnetic dipole moment(the zeroth-term doesn't exist).In addition, it also explained that why Maxwell's equations are not perfectly symmetric between electric and magnetic field:

Gauss' Law for Electricity:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{1}$$

Gauss' Law for Magnetism:

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

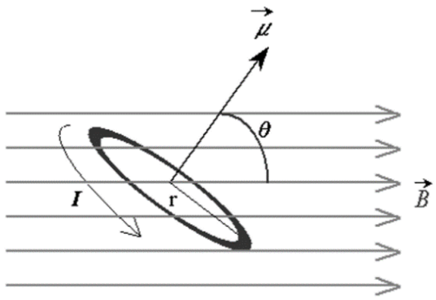

Fig.1: The torque is produced by magnetic moment and the external magnetic field.

According to Fig.1 description, the definition of magnet moment is :

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = \mu \mathrm{B} \sin\theta \hat{\boldsymbol{n}} \tag{3}$$

where $\boldsymbol{\tau}$ is the torque applying on the magnetic moment, $\boldsymbol{\mu}$ is the magnetic moment, and $\mathbf{B}$ is the external magnetic field.

Note that the magnetic moment is an unknown physical quantity in experiments,that is, we have to use the relation to indirectly measure the data.

Besides,the other form of magnetic moment is associated with the intrinct magnetism of elementary particles which is called the spin in quantum mechanics:

$$\boldsymbol{\mu_s} = g_s \frac{\mu_B}{\hbar} \mathbf{S} \tag{4}$$

In z component:

$$\boldsymbol{\mu_z} = -g_s \mu_B \boldsymbol{m_s} \tag{5}$$

where $\boldsymbol{\mu_s}$ is the magnetic moment due to the spin of a electron, $\boldsymbol{\mu_z}$ is the z component of $\boldsymbol{\mu_s}$, $\mathbf{S}$ is spin angular momentum, $\boldsymbol{m_s}$ is the spin quantum number, μ_B is the Bohr magneton, $g_s$is the electron spin g-factor and $\hbar$ is the

reduced planck constant.
The concept of the spin of elementary particles had been proposed by German physicist Ralph Kronig,and two of Dutch-American physicists George Uhlenbeck and Samuel Goudsmit in Copenhagen in 1925.

For this experiment, the primary target for us is to measure the magnetic force acting on the magnetic moment in the external magnetic field generated by the Helmholtz coil and the Anti-Helmholtz coil.Next,we measure the magnitude of the magnetic moment by using (3) and the result we obtained from the last step.

We suppose that the magnetic force is conservative,and then it would be satisfied with the following relations[2] :

$$\begin{cases} \nabla \times \boldsymbol{F} = \boldsymbol{0} \\ \mathbf{F} = -\nabla \mathrm{U} \\ W \equiv \oint \boldsymbol{F} \cdot d\boldsymbol{r} = 0 \end{cases} \tag{6}$$

where U is the magnetic potential energy of the system,and it can be expressed as:

$$\mathrm{U} = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu \mathrm{B} \cos\theta \tag{7}$$

Therefore,we can derive the relation formula:

$$\mathbf{F} = -\nabla \mathrm{U} = \nabla(\boldsymbol{\mu} \cdot \mathbf{B}) \tag{8}$$

Or we can rewrite the form of the magnetic force when $\boldsymbol{\mu}$ is aligned with $\mathbf{B}$,and moving in the direction that $\boldsymbol{\mu}$ is paralled to $\mathbf{B}$:

$$\mathbf{F} = \frac{d(\boldsymbol{\mu} \cdot \mathbf{B})}{dz} = \mu \frac{dB}{dz} \tag{9}$$

2. Method

In the experiment, we used the following equipment: the magnetic moment measuring instrument, steel balls with mass 1g and the electronic power supplier.

This experiment consisted of four main sections, for the first part the purpose was to measure the spring constant k.

After weighing the steel balls and the magnetic moment, we put the spring slightly in to the glass tube.When the end of the spring achieved the height scale 15, then we finished the initial resetting. Using Hooke's Law:

$$F = -k\Delta x \tag{10}$$

We hanged the steel balls and magnetic moment on the spring to stretch as well as measured the distance from the equilibrium position Δx and the restoring force F. Plot the force F versus the distance Δx and then we obtained the value of k from the slope of the straight line.

The second part was setting up the Helmholtz coil and adjusted the height of the magnet. Observe the phenomena whether the magnet rotated or the length of the spring changed by alternating the direction of one or both current on coils.

The third part was setting up the Anti-Helmholtz coil and put the magnetic moment with spring into glass tube.Turn on the power,place the spring and magnetic moment at height scale 0. Then turn off the power,and record the last position that magnetic moment in order to calculate the change for length of the spring.
In addition,we used the result from first part to calculate the magnetic force.Repeat the step by changing magnitude of current(such as 1 , 2A). Finally, we used (9) and did curve-fitting on computer to gain the value of magnetic moment.

The final part was setting up the Helmholtz coil. Turn on the power and put the magnetic moment at height scale -5.After holding still,we recorded the scale a .Then turn off the power and wait for still,and we recorded the scale b. The distance Δx would equal to b – a. Using (10) and the result from the first part, we could obtain the magnetic force. Repeat the step by changing the positions of the magnetic moment and replace the Helmholtz coil into Anti-Helmholtz coil as well as repeat the following steps to gain the value of magnetic moment with curve-fitting.

3.Result

For the first part, we calculated force constant k from the slope of the straight line in Fig.2:

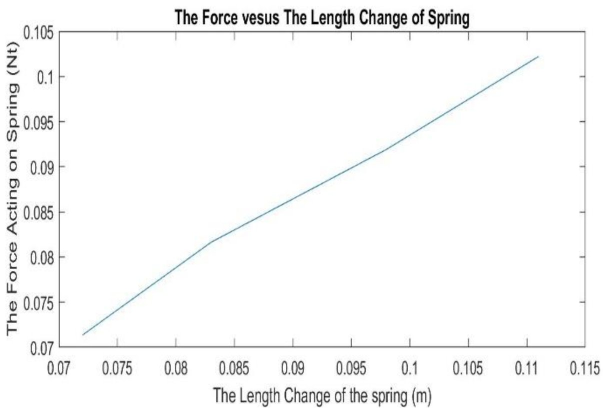

Fig.2: The force applied on the spring as

a function of the length change of the spring For the third part, we calculated the magnetic moment μ from the slope of the straight line in Fig.3:

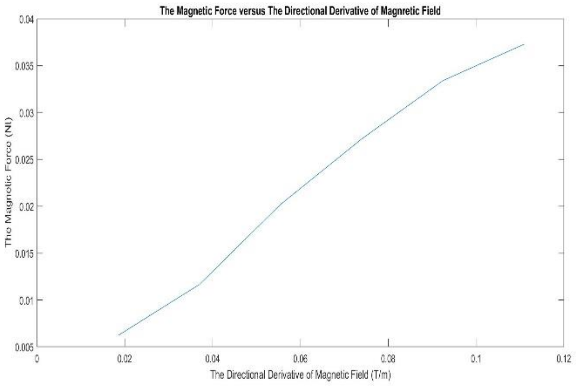

Fig.3: The magnetic force applied on the spring as a function of the derivative of magnetic field with respect to z.

For the final part, we gained the value of magnetic moment with curve-fitting for the Helmholtz and Anti-Helmholtz coils:

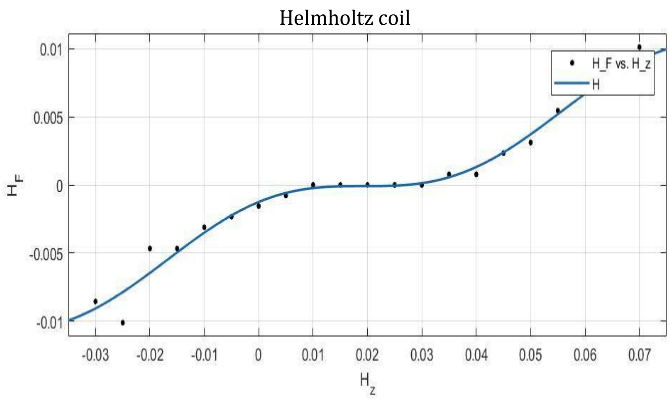

Fig.4: The magnetic force of Helmholtz coil as a function of the position changing in z direction.

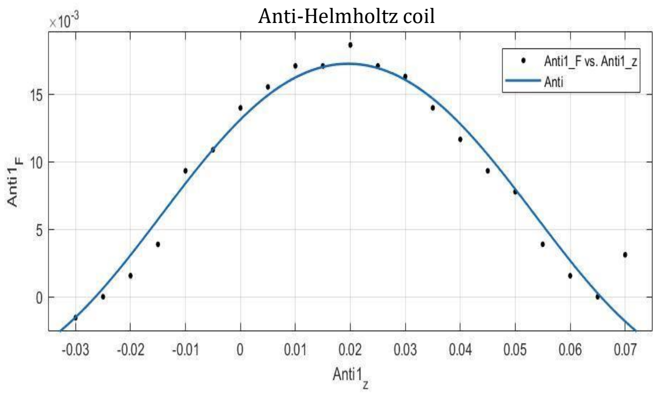

Fig.5: The magnetic force of Anti-Helmholtz coil as a function of the position changing in z direction.

4.Discussion

In the experiment, we first determined the value of force constant of the spring,which is roughly 0.777 $N{\cdot}m^{-1}$.Yet,we thought some scales on glass tube was hard to be observed, so we weren't able to record the absolute-precisely data .

Next, we observed the phenomena whether the magnet rotated or the length of the spring changed in the Helmholtz coil and the Anti-Helmholtz coil. We found that the spring kept slightly shaking but not strengthening in the Helmholtz coil.The reason of the phenomenon we thought was that the uniform magnetic field generated by the Helmholtz coil supplied the steady magnetic potential region.

In addition, we calculated the magnetic moment μ by using (9) and curve-fitting,there was a little difference between the experimental value we obtained and the standard value.We thought it might be due to the error from observing the scale to determine force constant in the first part.

As we did the curve-fitting on Matlab for the magnetic force varying with the position in the Helmholtz and Anti-Helmholtz coils, we found that the spring woudn't even strengthen in the steady magnetic potential region of Helmholtz coil ; however, the phenomenon didn't happen in Anti-Helmholtz coil. If we just considered the graph about the magnetic field varying with the position for it, we could know it better, through the graphs,and we could clearly knew that the phenomenon as above was quite reasonable.:

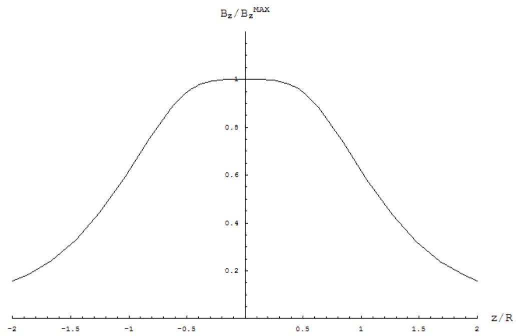

Fig.6: The magnetic field varying with the position for the Helmholtz coil

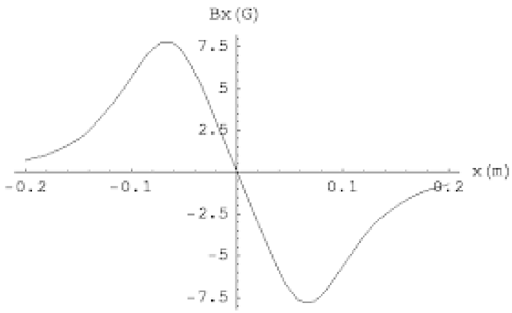

Fig.7: The magnetic field varying with the

position for the Anti-Helmholtz coil [3]

Reference

[1] David.J.Griffiths,*Introduction to Electrodynamics*,4th Ed,Pearson Education (2013)

[2] James Stewart, *Calculus:Early Transcendentals*,8th Ed,Cengage Learning (2015)

[3] Marcius H. T. Extavour, *Design and construction of magnetic elements for trapping and transport of cold neutral atoms*, August 2004.