2017 Fall PHYS2310 電磁學（Electromagnetism) Final Exam. (double sides)

[Griffiths Chs. 4-7.1] 2018/01/11, 10:10am – 12:00am, 教師：張存續

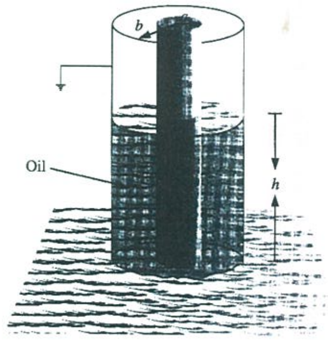
◇ Answer the questions as complete as possible.

1. (20%) Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at potential V, and the outer one is grounded (See the figure).

(a) Find the electric field **E** in the air part and the oil part? (8%)

(b) Find the capacitance? (6%)

(c) Find the height (h) that the oil rises in the space between the tubes? (6%)



2. (20%) Consider a uniformly polarized dielectric sphere of radius R. $\mathbf{P} = P_0\hat{\mathbf{z}}$

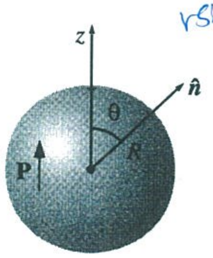
(a) Find the surface bound charge density σ_b and the volume bound charge density ρ_b. (10%)

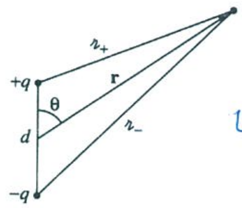
(b) Find the potential V of the dipole sphere for $r \geq R$. (10%)

[Hint: Use the dipole approximation, or $V = \dfrac{1}{4\pi\varepsilon_0}\oint_S \dfrac{\sigma_b}{\imath}da' + \dfrac{1}{4\pi\varepsilon_0}\int_v \dfrac{\rho_b}{\imath}d\tau'$].

$\sigma = P\cdot n$

$r\sin\theta\, dr\, d\theta$ $\rho_b = P\times n$ $\nabla\times P$





$-2\pi r^2(\cos\pi - \cos 0)$ $= 4\pi r^2$

$4\pi r^2$

$\int 2\pi r \cdot r\sin\theta$ $E_\perp = \text{Const}$

3. (20%) Boundary conditions and applications.

(a) $\nabla\cdot\mathbf{D} = \rho_f$. Find the boundary condition for the normal component of **D**, $D^\perp$. (6%)

(b) $\nabla\times\mathbf{H} = \mathbf{J}_f$. Find the boundary condition for the tangential component of **H**, $\mathbf{H}^{//}$. (6%)

(c) Consider the interface between two dielectric materials with ε_1 and ε_2 as shown in the figure. Find the relations between the normal and the tangential components of the electric fields. Assume that there is no surface charge, i.e., $\sigma_f = 0$. (8%)

$B_\parallel = \text{Cost}$ $B_\perp =$ $B_\parallel = \text{Cost}$ $H_\parallel = \text{Co}$

$2\pi r\ r\sin\theta\cdot d\theta$

$\sin 2\theta = 2SC$

$$\frac{\varepsilon_1,\ E_1^\perp,\ E_1^{//}}{\varepsilon_2,\ E_2^\perp,\ E_2^{//}}$$

nl.

4. (20%) An infinitely long solenoid with air core having a radius a and n closely wound turns per unit length, as shown in the figure. The windings are slanted (傾斜) at an angle θ and carry a current I.

(a) Find the z-component of the magnetic flux density (B_z) both inside and outside the solenoid. (10%) [Hint: Use Ampere's law.]

(b) Find the ϕ-component of the magnetic flux density (B_ϕ) both inside and outside the solenoid. (10%) [Hint: Use cylindrical coordinates, r, ϕ, z.]

$I \sin\theta$

$I \sin\theta.$

$\frac{\mu_0 I \sin\theta}{4\pi R}.$

$x = r\cos\phi$

$y = r\sin\phi.$

$z = z.$

$\oint H \cdot dl = I_{en}$

$\oint B \cdot dl = \mu_0 I.$

$\nabla \times H = I_{enc}.$

$\mu_0 n I$

$\frac{\mu_0 I}{2R}$

$\frac{\mu_0 I}{4\pi R}.$

5. (20%) A long cylinder of radius R carries a magnetization $\mathbf{M} = M_0 \hat{\mathbf{z}}$, where M_0 is a constant.

(a) Find $\mathbf{J}_b$ within the material and $\mathbf{K}_b$ on the surface of the material. (10%)

(b) Find the magnetic field $\mathbf{B}$ due to $\mathbf{M}$ for points inside ($r \le R$) and outside the cylinder ($r \ge R$). (10%)

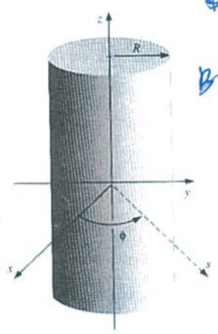
$K_b = M \times n$

$J_b = \nabla \times M.$

$\frac{B}{\mu_0} - M = H.$

$B = \frac{\mu_0}{4\pi} \frac{I \cdot dl \times \hat{r}}{r^2}$

$B = \mu_0 K_b$



10	10	10
2	3	5
2	5	10
2	5	8
2	3	6
2	3	10
2	4	8
2	5	8
0	3	6
4	6	8
3	5	6
3	5	6
34	67	87

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65