普通物理期末考(6/13/2017) 時間:10:00-13:00

將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。

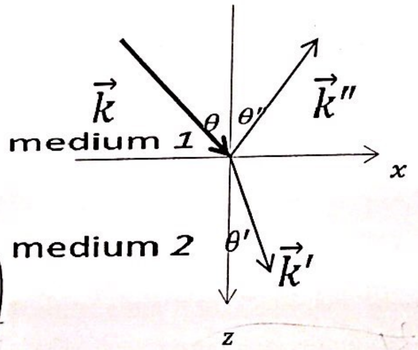
1. For a plane electromagnetic wave $\begin{cases} \vec{E}(\vec{r},t)=\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ \vec{B}(\vec{r},t)=\vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \end{cases}$ show that (a) $\vec{B}$ is perpendicular to $\vec{E}$ and $\vec{k}$, (b) $\vec{E}$ is perpendicular to $\vec{k}$, and (c) $\dfrac{|E|}{|B|}=c$. [15%]

2. (a) Show that Poynting vector is the energy current density of the electromagnetic wave. (b) Show that the intensity of a plane electromagnetic wave $\begin{cases} \vec{E}(\vec{r},t)=\vec{E}_0\cos(\vec{k}\cdot\vec{r}-\omega t) \\ \vec{B}(\vec{r},t)=\vec{B}_0\cos(\vec{k}\cdot\vec{r}-\omega t) \end{cases}$ is $\dfrac{E_0^{\,2}}{2c\mu_0}$ [10%]

3. An incident electromagnetic wave traveling in medium 1 (index of refraction $n_1$) towards medium (index of refraction $n_2$), is partially reflected by the interface between the two media (the x-y plane) and partially transmitted into medium 2.

$$\vec{E}=\begin{cases} \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}+\vec{E}_0'' e^{i(\vec{k}''\cdot\vec{r}-\omega'' t)} & z\le 0 \\ \vec{E}_0' e^{i(\vec{k}'\cdot\vec{r}-\omega' t)} & z>0 \end{cases} \quad \text{Note: } v=\frac{\omega}{k}=\frac{c}{n}$$

Show that (a) law of reflection and (b) law of refraction (Snell's law) can be derived from the requirement that boundary conditions must hold for all points on the interface for all time. [10%]




A|

$\boxed{\hat{A}\psi_i = a_i\psi_i}$

$\hat{A}=a_i$

4. Derive the eigenstates and eigenenergies of a particle of mass $m$ in an infinitely deep potential energy well. $V(x)=\begin{cases} \infty & x<0 \text{ or } x>L \\ 0 & 0\le x\le L \end{cases}$ [10%]

$a^*a.$

5. Show that a Hermitian operator has (a) real eigenvalues (b) orthogonal eigenfunctions. [10%]

$\boxed{\hat{A}=c_i|a_i\rangle}$

$a_i\langle\varphi_{a_i}|\varphi_{a_j}\rangle$ ok.

$(a_i-a_j)\langle\varphi_{a_i}|\varphi_{a_j}\rangle = 0$

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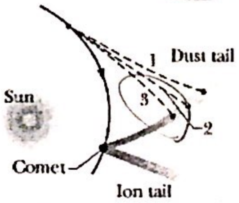
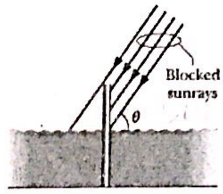
**6.** As a comet swings around the Sun which has a mass of $1.99\times10^{30}$ kg and emits light at an energy rate of $3.9\times10^{26}$ W, ice on the comet's surface vaporizes, releasing trapped dust particles and ions. The (electrically neutral) dust particles are pushed radially outward from the Sun by the radiation force on them from sunlight. Assume that the dust particles are spherical, have density $3.5\times10^{3}$ kg/m$^3$, and are totally absorbing. (a) What radius must a particle have in order to follow a straight path, like path 2 in the figure? (b) If its radius is larger, does its path curve away from the Sun (like path 1) or toward the Sun (like path 3)? Note: $G=6.67\times10^{-11}$ m$^3$/kg$\cdot$s$^2$ [10%]




**7.** A 2.00-m-long vertical pole extends from the bottom of a swimming pool to a point 50.0 cm above the water (index of refraction=1.33). Sunlight is incident at angle $\theta = 55.0°$. What is the length of the shadow of the pole on the level bottom of the pool? [5%]

8. The wave function for the hydrogen-atom quantum state which has $n = 2$ and $\ell = m_\ell = 0$, is

$$\psi_{200}(r) = \frac{4}{4\sqrt{2\pi}} a^{-3/2}\left(2 - \frac{r}{a}\right)e^{-r/2a},$$

in which $a$ is the Bohr radius and the subscript on $\psi(r)$ gives the values of the quantum numbers $n$, $\ell$, $m_\ell$. (a) Find the maximum of $\psi^2_{200}(r)$ for $0 < r < \infty$. (b) Find the radial probability density $P_{200}(r)$ for this state. (c) Show that the wave function $\psi_{200}(r)$ has been properly normalized? Note: $\int_0^\infty x^n e^{-x}dx = n!$. [15%]

$C^4_2 = 6.$

**9.** A hydrogen atom is excited by a UV light from its ground state (–13.6eV) to the state with $n=4$. (a) What is the wavelength of the UV light? [3%] (b) How many different wavelengths are there in the emitted light spectrum as the atom de-excites back to the ground state? [2%] (c) What is the shortest wavelength in the emitted spectrum? [2%] (d) What is the longest wavelength in the emitted spectrum? [3%]
Note: $hc=12400$ eV · Å

4
3
2
1

**10.** What is the kinetic energy of a relativistic electron with de Broglie wavelength 1.0 fm? Note that $hc=12400$ eV · Å and the electron mass energy is 0.511MeV. [5%]

$mc^2$

2
4
1