

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{E} d\vec{l} = -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\vec{E} = -\nabla \times \vec{A}$$

$$\vec{J} = nev_d$$

普通物理期中考(5/16/2017) 時間:10:00-13:00

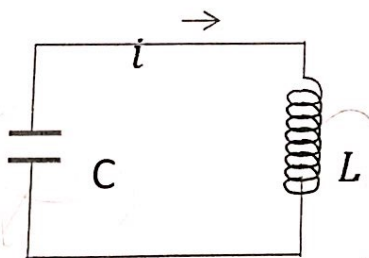
將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。 $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$

1. (a) Write out the names and mathematical expressions in integral form for Maxwell's equations in vacuum. [8%] (b) Derive the differential forms of Maxwell's equations from their integral forms. [8%] (c) Define vectors \vec{D} and \vec{H} and use them to derive two of the four Maxwell's equations in matters which use \vec{D} and \vec{H} . [4%]

2. (a) Draw a picture to describe how a Hall effect measurement is done. Explain why Hall effect measurements can be used to identify the type/sign of the carrier charge. [5%]

- (b) Show that the carrier concentration can be determined by Hall effect measurements? [5%]

3. Derive (a) the charge $q(t)$ of the capacitor and the current $i(t)$, as functions of time t , in an LC circuit, where the capacitor was initially charged with an electric charge Q and there is no current at $t=0$. [5%] (b) What are the energy stored in the capacitor and the energy stored in the inductor, both expressed in terms of C and Q , as functions of time? [5%]

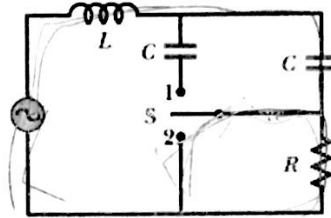
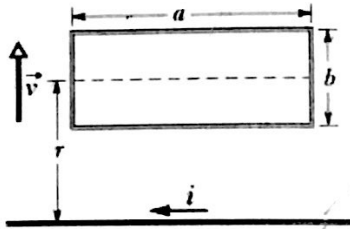


$$q(t) = Q(1 - e^{-t/\tau})$$

4. Use Biot-Savart law to calculate the magnetic field (a) at a distance R from a long straight wire carrying a current i [5%] and (b) at a distance z , along the axis, from the center of a circular loop of radius R and current i . [5%]

5. A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the (a) magnitude and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding? [10%]

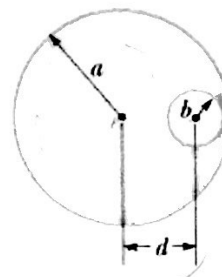
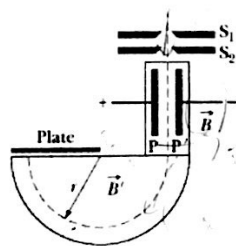
6. A rectangular loop of wire with length $a = 2.2$ cm, width $b = 0.80$ cm, and resistance $R = 0.40$ m Ω is placed near an infinitely long wire carrying current $i = 4.7$ A. The loop is then moved away from the wire at constant speed $v = 3.2$ mm/s. When the center of the loop is at distance $r = 1.5b$, what are (a) the magnitude of the magnetic flux through the loop and (b) the current induced in the loop? [10%]



Handwritten notes for problem 7: $\frac{\delta \Phi_B}{\delta t}$ and $1 + \frac{1}{2} = \frac{3}{2}$

7. The ac generator supplies 120 V at 60.0 Hz. With the switch open as in the diagram, the current leads the generator emf by 20.0° . With the switch in position 1, the current lags the generator emf by 10.0° . When the switch is in position 2, the current amplitude is 2.00 A. What are (a) R , (b) L , and (c) C ? [15%]

8. Bainbridge's mass spectrometer separates ions having the same velocity. The ions, after entering through slits, S_1 and S_2 , pass through a velocity selector composed of an electric field produced by the charged plates P and P' , and a magnetic field \mathbf{B} perpendicular to the electric field and the ion path (crossed field). The ions that then pass undeviated through the crossed \mathbf{E} and \mathbf{B} fields enter into a region where a second magnetic field \mathbf{B}' exists, where they are made to follow circular paths. A photographic plate (or a modern detector) registers their arrival. For given E , B , and B' , derive the charge-to-mass ratio q/m of the ions as a function of r , the radius of the circular orbit. [5%]



9. A long cylindrical conductor of radius a contains a long cylindrical hole of radius b . The central axes of the cylinder and hole are parallel and are distance d apart; current i is uniformly distributed over the tinted area. (a) What is the magnitude of the magnetic field at the center of the hole? [5%] (b) What is the magnitude of the magnetic field at the center of the conductor? [5%]



Handwritten notes for problem 9:

$$E = \frac{1}{2} \epsilon_0 E^2 + (v_L - v_C)^2$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I'$$

$$- \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I''$$

$$I' - I'' = I$$

$$\frac{I'}{I''} = \frac{a^2}{b^2}$$

5. To find minimum of $|B| \rightarrow \frac{d(\cos\theta + 0.6\sin\theta)}{d\theta} = 0$

$\Rightarrow -\sin\theta + 0.6\cos\theta = 0 \rightarrow \tan\theta = 0.6 \quad \theta = \tan^{-1} 0.6$

$\Rightarrow B = 0.1008 \text{ T}$

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將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。

$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$

1. (a) Write out the names and mathematical expressions in integral form for Maxwell's equations in vacuum. [8%] (b) Derive the differential forms of Maxwell's equations from their integral forms. [8%] (c) Define vectors \vec{D} and \vec{H} and use them to derive two of the four Maxwell's equations in matters which use \vec{D} and \vec{H} . [4%]

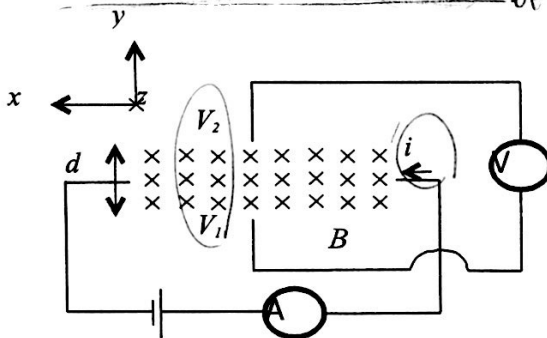
2. (a) A particle of mass m and charge q is moving in a uniform magnetic field $\vec{B} = B\hat{k}$. The initial velocity of the particle is $\vec{v}(0) = v\hat{j}$. (a) Derive the particle's velocity $\vec{v}(t)$ as

a function of time. [10%] (b) If the particle is initially located at $\vec{r}(0) = -\frac{mv}{qB}\hat{i}$, what is $\vec{r}(t)$? [5%]

3. (a) A loop of self-inductance L is carrying a current i . Derive the magnetic energy stored in this loop of current. [5%] (b) Derive the magnetic energy density from

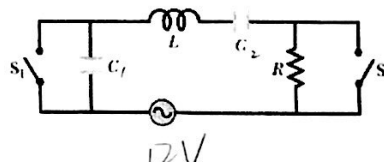
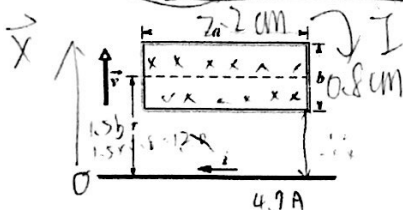
$U_B = \frac{1}{2} \int_V \vec{J} \cdot \vec{A} d\tau$ Hint: $(\nabla \times \vec{B}) \cdot \vec{A} = \nabla \cdot (\vec{B} \times \vec{A}) + \vec{B} \cdot (\nabla \times \vec{A})$

4. In a Hall effect measurement, the sample is in a magnetic field of magnitude B . The thickness of the sample is l . The potential difference $V_2 - V_1$ is measured to be V . Derive the carrier concentration n . [5%]



5. A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What are the (a) magnitude [5%] and (b) angle (relative to the vertical) of the smallest magnetic field that puts the rod on the verge of sliding? [5%]

6. A rectangular loop of wire with length $a = 2.2 \text{ cm}$, width $b = 0.80 \text{ cm}$, and resistance $R = 0.40 \text{ m}\Omega$ is placed near an infinitely long wire carrying current $i = 4.7 \text{ A}$. The loop is then moved away from the wire at constant speed $v = 3.2 \text{ mm/s}$. When the center of the loop is at distance $r = 1.5b$, what are (a) the magnitude of the magnetic flux through the loop [5%] and (b) the current induced in the loop? [5%]



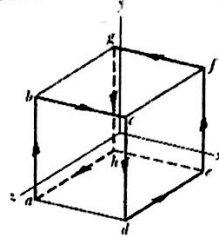
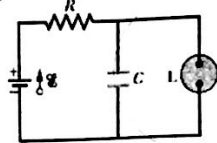
$\vec{F} = i\vec{L} \times \vec{B} = (mg + F \sin\theta)\mu$

①.

$\Rightarrow 50 \times B \cos\theta = (1 \times 9.8 + 50 B \sin\theta) \times 0.6 \Rightarrow 50 B (\cos\theta + 0.6 \sin\theta) = 9.8 \times 0.6$

7. A driven RLC circuit that contains two identical capacitors and two switches. The emf amplitude is set at 12.0 V, and the driving frequency is set at 60.0 Hz. With both switches open, the current leads the emf by 30.9° . With switch S_1 closed and switch S_2 still open, the emf leads the current by 15.0° . With both switches closed, the current amplitude is 447 mA. What are (a) R , (b) C , and (c) L ? [15%]

8. The figure shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L (of negligible capacitance) is connected in parallel across the capacitor C of an RC circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage V_L ; then the capacitor discharges completely through the lamp and the lamp flashes briefly. For a lamp with breakdown voltage $V_L = 72.0$ V, wired to a 95.0 V ideal battery and a 0.150 μF capacitor, what resistance R is needed for two flashes per second? [5%]



9. a conductor carries 6.0 A along the closed path $abcdefgha$ running along 8 of the 12 edges of a cube of edge-length 10 cm. (a) Taking the path to be a combination of three square current loops ($bcbgh$, $abgha$, and $cdefc$), find the net magnetic moment of the path in unit-vector notation. [5%] (b) What is the magnitude of the net magnetic field at the xyz coordinates of (0, 5.0 m, 0)? [5%]

$$\tan(-30.9^\circ) = (\omega L - \frac{1}{\omega C}) / R$$

$$\tan(15^\circ) = (\omega L - \frac{1}{\omega C}) / R$$

$$\text{② } LC \text{ 电路, 则 } V_R = 0 \Rightarrow V_L = V_C = I(X_L - X_C)$$

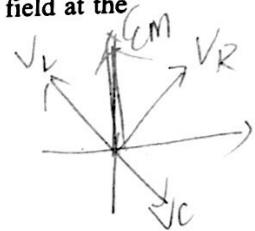
$$477 \text{ mA} = \frac{12}{\omega L - \frac{1}{\omega C}}$$

$$\text{由②③} \Rightarrow \frac{12}{477 \text{ m}} = \frac{\tan(15^\circ)}{R} \rightarrow R = 0.011 \Omega$$

$$0.06 R = 376.8 L - \frac{0.0053}{C}$$

$$0.27 R = 376.8 L - \frac{0.00265}{C}$$

$$0.81 R = \frac{0.00265}{C} \rightarrow C = 0.29 \text{ F} \quad L = 3.26 \times 10^{-5} \text{ H}$$



$$\frac{1}{\frac{1}{C} + \frac{1}{C}} = \frac{C}{2}$$