2017 Spring PHYS2320 電磁學（Electromagnetism） Final 滿分 100

[Griffiths Chs. 10 and 12] 2018/06/12, 10:10am – 12:00am, 教師：張存續

1. Write the equations (if possible) and explain the following terms as clear as possible.
   (a) Lorentz gauge and Coulomb gauge. (4%)
   (b) Gauge transformations and gauge freedom. (4%)
   (c) Lienard-Wiechert potentials. (4%)
   (d) The two postulates of the special relativity (4%)
   (e) Conserved quantity and invariant quantity. (4%)

$\nabla\times(\nabla\times A) = \mu_0 J + \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial A}{\partial t}\right)$

$= \nabla(\nabla\cdot A) - \nabla^2 A.$

$-\left(\nabla^2 A - \mu_0\epsilon_0 \frac{\partial^2 A}{\partial t^2}\right) - \nabla\left(\nabla\cdot A + \mu_0\epsilon_0 \frac{\partial V}{\partial t}\right) = \mu_0 J.$

$\frac{1}{s}\int_0^\infty \frac{dk}{k\,2\sqrt{k-1}}$

2. (a) The transformations between two inertial systems S and $\bar{S}$ are $\bar{x} = \gamma(x - vt)$ and $\bar{t} = \gamma(t - vx/c^2)$. Show that when $\Delta t = 0$, $\Delta x = \Delta\bar{x}/\gamma$; but when $\Delta\bar{t} = 0$, $\Delta\bar{x} = \Delta x/\gamma$. Explain why the length relations depend on the simultaneity. (10%)
   (b) Show that $(E^2 - c^2B^2)$ is relativistically invariant. (10%)

$\Delta t = \frac{v}{c^2}\Delta x$ $\quad \Delta\bar{x} = \gamma(\Delta x - v\Delta t)$

$= \gamma\left(\Delta x - \frac{v^2}{c^2}\Delta x\right)$

$= \gamma\frac{1}{\gamma^2}(\Delta x)$

$\frac{1}{b^2+c^2t^2}$ $\quad \frac{1}{bc}\tan^{-1}(ct/b)$

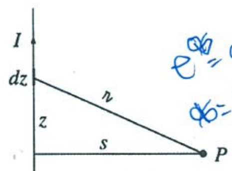
Let $t = \sqrt{k-1}$ $\quad dk = 2t\,dt.$

3. Show that the retarded potential satisfy the Lorentz gauge condition. (20%)
   [Hint: the retarded potentials $V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$ and $\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{\mathscr{r}}d\tau'$.]

$\frac{1}{s^2}\int\frac{dz}{(z/s)^2+1} = \frac{1}{s}\int\frac{dt}{t^2+1}$ $\quad \frac{Q}{4\pi s^2\epsilon_0}$ $\quad$ Let $t = z/s$, $s\,dt = dz.$

$\frac{dq}{dt} = I$ $\quad q = \int I = \frac{kt^2}{2}$

4. (20%) An infinite straight wire carriers a linearly increasing current: $I(t) = kt$ for $t > 0$.
   (a) Find the scalar and vector potentials. (10%)
   (b) Find the electric field generated. (10%)



$\nabla\times E = -\frac{\partial B}{\partial t}$ $\quad \oint E\cdot ds = -\frac{\mu_0 k}{2\pi s}$

$e^{\Phi} = \infty$ $\quad \Phi = \ln\infty$ $\quad \frac{}{2\pi s}$

$\frac{q}{4\pi\epsilon_0}\int\frac{dz}{z^2+s^2}$

$\frac{s}{\sin\theta}\Big|_{\frac{\pi}{2}}^{0}$ s $\frac{\cos\theta}{\sin^2\theta}\Big|_{\frac{\pi}{2}}^{0}$

$\nabla\cdot E = \frac{\rho}{\epsilon_0}$ $\quad \oint E\cdot ds = \frac{Q_{enc}}{\epsilon_0}$

$\frac{\mu_0 I}{2\pi R}$ $\quad \frac{\mu_0 kt}{2\pi s}$

5. The Maxwell equations can be written in terms of the field tensor.

$$F^{\mu\nu} = \begin{Bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{Bmatrix} \qquad \text{[Hint: } t^{\mu\nu} = \begin{Bmatrix} t^{00} & t^{01} & t^{02} & t^{03} \\ t^{10} & t^{11} & t^{12} & t^{13} \\ t^{20} & t^{21} & t^{22} & t^{23} \\ t^{30} & t^{31} & t^{32} & t^{33} \end{Bmatrix}\text{].}$$

(a) Find the corresponding Maxwell's equation for $\frac{\partial F_{\mu\nu}}{\partial x^\lambda} + \frac{\partial F_{\lambda\mu}}{\partial x^\nu} + \frac{\partial F_{\nu\lambda}}{\partial x^\mu} = 0$, when $(\lambda,\mu,\nu) = (1,2,3)$. (10%)

(b) Find the corresponding Maxwell's equation for $\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$, when μ =1, 2, and 3. (10%)