

General Physics I – Final Exam

Exam Time: 10:10AM - 12:10PM

Useful formulas:

$$f' = \frac{v \pm v_o}{v \pm v_s} f_0, \quad dS = nC_V \frac{dT}{T} + nR \frac{dV}{V}, \quad \int_0^\infty e^{-ax^2} dx = \sqrt{\frac{\pi}{4a}}, \quad \int_0^\infty x e^{-ax^2} dx = \frac{1}{2a}.$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \sqrt{\frac{\pi}{16a^3}}, \quad \int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2}.$$

Q1. (50 pts) Fundamentals.

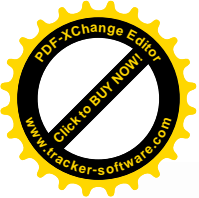
(a) (15 pts) Assume that the speed of sound to be 340 m/s (relative to the stationary air) in this question. A truck travels at 30 m/s to the east and a car travels at 60 m/s along the same straight road to the west. They approach each other. The car's horn has a natural frequency of 900 Hz . If the wind is blowing from west to east at 10 m/s , (1) what is the wavelength of the sound wave observed by the driver on the truck? (2) And what is the frequency observed by the driver on the truck?

(b) (15 pts) Two police cars have identical sirens that produce a frequency of 670 Hz . A stationary listener is standing between two cars. One car is parked and the other is approaching the listener and both have their sirens on. The listener notices 10 beats per second. Find the speed of the approaching police car. (Take the speed of sound to be 340 m/s .)

(c) (10 pts) An ideal 2D diatomic gas composed of N non-rigid rotators (the bond is NOT rigid and can be treated as a spring) reaches the thermal equilibrium at temperature T . Determine the total internal energy of the gas, and **Explain your answer**.

(d) (10 pts) Two different ideal gases A and B with the same temperature T are separated apart, and the volume ratio is $V_A : V_B = 3 : 7$, as shown in the figure below. Now, the partition suddenly vanishes, and two gases undergo adiabatic free expansions to fill the whole volume. Assume that there is no interaction between gas A and gas B . Determine the change in the entropy of the whole system.

A $0.3V_0$	B $0.7V_0$
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Q2. (40 pts) Let us consider an ideal 2D monatomic gas composed of N atoms (each has a mass m) confined in a square box of an area $A = L \times L$ at temperature T . Note that the ideal gas law may or may not be correct in 2D, so DO NOT use it until you prove it.

(a) (10 pts) Use the kinetic theory to derive a relation between the pressure P (defined as the force per unit length on the wall in 2D) and the mean kinetic energy of a gas molecule $\bar{K} = m\bar{v}^2/2$. That is to show that $P = \alpha\bar{K}$, and $\alpha = ?$.

(b) (10 pts) Obtain the Maxwell-Boltzmann distribution $f(v)$ defined by $dN = f(v)dv$. You need to calculate the normalization constant in order to receive full credits.

(c) (5 pts) Use the Maxwell-Boltzmann distribution $f(v)$ to calculate the mean kinetic energy of the gas molecule. Together with (a), you can see, for a 2D gas, whether $P \cdot A = Nk_B T$.

(d) (5 pts) Use the Maxwell-Boltzmann distribution $f(v)$ to calculate v_{av} .

(e) (10 pts) For an adiabatic process, obtain a relation $PV^\gamma = \text{Constant}$, and $\gamma = ?$

Q3. (30 pts) The Stirling engine cycle is composed of 4 segments as shown in the figure below. $A \rightarrow B$ and $C \rightarrow D$ are isothermal expansion and compression process, respectively. And $B \rightarrow C$ and $D \rightarrow A$ are isochoric (fixed volume) processes. There are n moles of ideal gas whose molar specific heat is $C_V = \frac{5}{2}R$. The volume ratio has a simple relation that $\ln(V_2/V_1) = 2$. The temperatures of heat reservoirs are $T_H = 400\text{ K}$ and $T_C = 200\text{ K}$, respectively.

(a) (10 pts) Evaluate the total heat absorbed in a cycle.

(b) (10 pts) Evaluate the total work output in a cycle.

(c) (5 pts) Find the thermal efficiency ϵ of this Stirling cycle.

(d) (5 pts) Find the change in the entropy of the gas from $D \rightarrow A$.

