$\frac{dU}{dt} = \frac{dU}{dX} \cdot \frac{dX}{dt}$ $\sum_{i=1}^{n} m_i \frac{d^2 r_{com}}{dt^2} = M \cdot a_{com}$

普通物理期中考(10/27/2016) 時間:10:00-13:00
每大題含1-3小題，每小題5分，總分100分。將計算或推導過程清楚寫在答案本中，計算題最後答案畫長方格圈示。

3D

1. Consider a particle of mass m in the three-dimensional space. (a) Derive the work-kinetic energy theorem. (b) Show that the work done by a conservative force is equal to the decrease of potential energy associated with that force. (c) If the net force is conservative, show that the mechanical energy is conserved.

2. For a system of n particles, show (a) $\sum_{i=1}^{n} \vec{F}_{ext,i} = M\vec{a}_{COM}$, (b) $\sum_{i=1}^{n} \vec{p}_i = M\vec{v}_{COM}$, and (c) $\frac{d\vec{L}}{dt} = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_{ext,i}$.

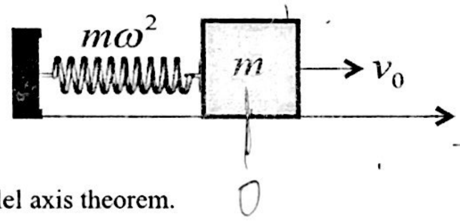
$m\frac{d^2X}{dt^2}$

$K = m\omega^2$

3. A mass m is attached to a spring of spring constant $m\omega^2$. The mass has an initial displacement $x(0) = 0$ and an initial velocity $v(0) = v_0$ at time $t=0$. Derive (a) the displacement of mass from its equilibrium point $x(t)$ as a function of time, and (b) the velocity $v(t)$ as a function of time. (c) Find the potential energy $U(x)$. Let $U(0) = 0$

?

$U(x) = \int m\omega^2 x(t)\,dX$



$U(x) =$

$\frac{1}{2}K(x(t))^2$

4. Prove the parallel axis theorem.

$U(x) = \frac{1}{2}m\omega^2 x^2$

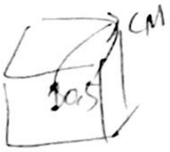
5. The center of mass of an extended object of mass M occupying a volume V is located at $\vec{r}_{COM} = (x_{COM}, y_{COM}, z_{COM})$ with respected to the origin near the surface of the Earth. The object is subject to a uniform gravitational acceleration $-g\hat{k}$. Show (a) the torque by gravity on the object about the origin is $\vec{r}_{COM} \times (-Mg\hat{k})$, and (b) the gravitational potential for the object is Mgz_{COM} if the reference point is on the x-y plane.

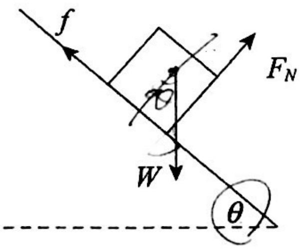
$\vec{\tau}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_x = (r_x, r_y, r_z) \times (0, 0, -Mg)$.

6. A crate, in the form of a cube with edge lengths of 2.0 m, contains a piece of machinery; the center of mass of the crate and its contents is located 0.50 m above the crate's geometrical center. The crate rests on a ramp that makes an angle θ with the horizontal. As θ is increased from zero, an angle will be reached at which the crate will either tip over or start to slide down the ramp. If the coefficient of static friction μ_s between ramp and crate is 0.60, (a) does the crate tip or slide and (b) at what angle θ does this occur? If $\mu_s = 0.70$, (c) does the crate tip or slide and at what angle θ does this occur?





7. A 6100 kg rocket is set for vertical firing from the ground. If the exhaust speed is 1200 m/s, how much gas must be ejected each second if the thrust (a) is to equal the magnitude of the gravitational force on the rocket and (b) is to give the rocket an initial upward acceleration of 21 m/s²?

$V_{rel} = 1200$

$e^{-i\omega t} - e^{i\omega t}$

$= i\sin(-\omega t)$

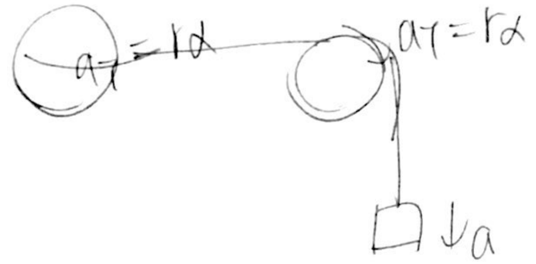
$\vec{\tau} = \sum r_i \times F_i$

$= \sum r_i \times [F_{ex}(i,t) + \sum_{j \neq i} F_{ji}]$ $-Mg\hat{k}$

$= (\sum r_i) \times F_{ex}(i,t) =$

8. A uniform spherical shell of mass $M = 4.5$ kg and radius $R = 8.5$ cm can rotate about a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}$ kg m² and radius $r = 5.0$ cm, and is attached to a small object of mass $m = 0.60$ kg. There is no friction on the pulley's axle; the cord does not slip on the pulley. (a) What is the acceleration of the object? (b) What is the tension of the horizontal section of the cord? (c) What is the speed of the object when it has fallen 328 cm after being released from rest? [Hint: the rotational inertia of the spherical shell is $2MR^2/3$]

3.28m (小心!)



$0.6 \times 9.8 - T = 0.6a$

$r \times T = \vec{\tau} = I\alpha$

$0.085\, T_2 = \frac{2}{3} \times 4.5 \times 0.085 \times a$

$0.05(T_1 - T_2) = 0.003 \times \frac{a}{0.05}$

$0.05\, T_1 = 0.21a = (5.88 - 0.6a) \times 0.05$

$0.24a$