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$f(n) = (3 \cdot f(n-1) + 2 \cdot f(n-2) + f(n-3)) \bmod 7, \ n \geq 3$

$f(3) = (3 \cdot f(2) + 2 \cdot f(1) + f(0)) \bmod 7$

$= (9 + 4 + 1) \bmod 7$

$f(4) = (3 \cdot f(3) + 2 \cdot f(2) + f(1)) \bmod 7$

$= (0 + 6 + 2) \bmod 7$

$= 1$

$f(5) = (3 \cdot f(4) + 2 \cdot f(3) + f(2)) \bmod 7$

$= (3 + 0 + 3) \bmod 7$

$= 6$

$f(6) = (3 \cdot f(5) + 2 \cdot f(4) + f(3)) \bmod 7$

$= (18 + 2 + 0) \bmod 7$

$= 6$

$f(7) = (3 \cdot f(6) + 2 \cdot f(5) + f(4)) \bmod 7$

$= (18 + 12 + 1) \bmod 7$

$= 3$

$f(8) = (9 + 12 + 6) \bmod 7$

$= 6$

$f(9) = (18 + 6 + 6) \bmod 7$

$= 2$

$f(10) = (6 + 12 + 3) \bmod 7$

$= 0$

$f(11) = (0 + 4 + 6) \bmod 7$

$= 3$

$f(12) = (9 + 0 + 2) \bmod 7$

$= 4$ #

Problem 2 :

$3 \mid (5^{2n}-1)$ for every integer ≥ 0

1) Assume that $n=1$ to show the positive numbers

If $n=1$; $5^{2n}-1 = 5^2-1 = 24$ divisible by 3

$n=2$; $5^{2n}-1 = 5^4-1 = 624$ divisible by 3

2) Assume that 3 is a divisor of $5^{2n}-1$. show that 3 is a divisor of $5^{2(n+1)}-1$

$$5^{2(n+1)}-1 = 5^{2n} \cdot 5^2 - 1$$

$$= 25(5^{2n})-1$$

$$= (3 \cdot 8+1)(5^{2n})-1$$

$$= 3 \cdot 8(5^{2n})+5^{2n}-1$$

$$= 3 \cdot 8(5^{2n})+3k \text{ for some integer } k$$

$= 3[8(5^{2n})+k]$ since $8(5^{2n})+k$ is an integer, $5^{2(n+1)}-1$ is divisible by 3

Problem 3

$f(0)=1$

$f(n+1) = 2f(n)+n-1, n \geq 0$

Prove by mathematical induction that $f(n) = 2n-n$, for all integers $n \geq 0$

Assume that $n=0$ $\quad f(0+1) = 2 \times 1 + 0 - 1$

$= 2-1$

$= 1$

$f(1) = 1$ #

Suppose $n=1$

$f(n) = 2n-n$

$f(1) = 2(1)-1$

$f(1) = 1$

Both answers are 1, which is both the same

Hence these functions are true #

Problem 4

1) $n=1$; $1\cdot 1! = 1\cdot 1 = 1 = (n+1)! - 1 = 2! - 1$
$= 2-1$
$= 1$

2) Assume $f(k) = 1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! = (k+1)! - 1$

$$f(k+1) = 1\cdot 1! + 2\cdot 2! + \ldots + k\cdot k! + (k+1)\cdot(k+1)!$$
$$= (k+1)! - 1 + (k+1)\cdot(k+1)!$$
$$= (k+1)! \cdot ((k+1)-1) = (k+1)!\cdot k$$

Problem 5

Suppose $n=1$ $\left(\frac{d}{dx}\right)x = 1\cdot x^0$

$1 = 1$

LHS = RHS

So, this formula is true when $n=1$

$$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$

This formula is the product rule in calculus

Substitute u by x^k and v by x where k is any number

$$= \left(\frac{d(x^k)}{dx}\right)\cdot x + \left(\frac{d(x)}{dx}\right)\cdot x^k$$
$$= (k\cdot x^{k-1})\cdot x + x^k$$
$$= k\cdot x^{k-1+1} + x^k$$
$$= k\cdot x^k + x^k$$
$$= x^k(k+1)$$

LHS = RHS

Problem 6:

1) P(8) is the proposition that 8 baht of postage can be composed from 3 baht and 5 baht stamps, This is true, requiring 1 of each

2) $P(8) \wedge \ldots \wedge P(n) = P(n+1)$ for all natural numbers $n \geq 8$

1. The inductive hypothesis states that for all numbers m from 8 to n, m baht of postage can be composed from 3 and 5 baht stamps.
2. Prove: (n+1) baht of postage can be composed from 3 and 5 baht stamps
3. The cases where n+1 is 9 or 10 must be proved seperately, 9 baht can be composed from 3 3 baht stamps and 10 baht can be composed from 2 5 baht stamps.
4. For all natural numbers $n+1 > 10$, the inductive hypothesis entails the proposition $P(n-2)$. If (n-2) baht can be composed from 3 baht and 5 baht stamps then (n+1) baht can be composed from 3 and 5 baht stamps by simply adding one more 3 baht stamp.