Introduction to Logic
Assignment 3

King Mongkut's Institute of Technology Ladkrabang

August 28, 2021

Problem 1

Determine whether each sentence below can be translated into a formula in Propositional Logic using the given propositional letters and their specified meanings. If so, provide a formula that has the closest meaning to the sentence; otherwise, state that there is no translation.

Example. "If you have not paid your tuition fee, you will not be allowed to graduate."

p = You have paid your tuition fee.
g = You are allowed to graduate.

Ans. Yes. $\neg p \to \neg g$

$\wedge$ $\neg$

(a) "Sweden and Norway will both not adopt the Euro."

s = Sweden will adopt the Euro. $\neg s \wedge \neg n$
n = Norway will adopt the Euro.

$\neg$ ($\wedge$)

(b) "Sweden and Norway will not both adopt the Euro." [ambiguity] → shouldn't use in real life !!!

s = Sweden will adopt the Euro. $\neg(s \wedge n)$ It's not the case that Sweden & Norway will both adopt the Euro
n = Norway will adopt the Euro. $\neg s \vee \neg n$ Either Sweden or Norway (or both) will not adopt the Euro
$\neg s \wedge \neg n$ Both Sweden & Norway will not adopt the Euro

$\neg$ $\vee$

(c) "Our leader doesn't dye (his hair, use makeup, or wear a wig.)"

h = Our leader dyes his hairs. $\neg(h \vee m \vee w)$
m = Our leader uses makeup. De Morgan's law
w = Our leader wears a wig. $(\neg h \wedge \neg m \wedge \neg w)$

$\to$

(d) "By signing this document, you agree to the terms and conditions of this software."

s = You sign this document.
a = You agree to the terms and conditions of this software.

$s \to a$

(e) “Unless I see it with my own eyes, and hear it with my own ears, I never will believe it.” *(Charles Dicken)*

$s =$ I see it with my own eyes.
$h =$ I hear it with my own ears.
$b =$ I believe it.

$\neg(s \wedge h) \to \neg b$

(f) “The message was sent from an unknown system but it was not scanned for viruses.”

$u =$ The message was sent from an unknown system.
$s =$ The message was scanned for virus.

$u \wedge \neg s$

(g) “Access is granted whenever the user has paid the subscription fee and enters a valid password.”

$a =$ Access is granted.
$f =$ The user has paid the subscription fee.
$p =$ The user enters a valid password.

$(f \wedge p) \to a$

(h) “John has a belief that both Mary and Tom lied.”

$j =$ John has a belief.
$m =$ Mary lied.
$t =$ Tom lied.

No translation

(i) “Being affiliated with a major political party is not sufficient for you to become the President of the United States.”

$a =$ You are affiliated with a major political party.
$p =$ You are becoming the President of the United States.

no translation

(j) “High public debt and a sharp rise in consumer prices are necessary and sufficient conditions for an economic crises to happen in the country.”

$d =$ The country has high public debt.
$p =$ There is a sharp rise in consumer prices in the country.
$c =$ There is going to be an economic crisis in the country.

$(d \wedge p) \leftrightarrow c$

(k) John and Mary are friends.

$j =$ John is a friend. friends w/ each other : no translation
$m =$ Mary is a friend. John & Mary is my friend : $j \wedge m$

Problem 2

Suppose SE-Rocks is a popular rock band at KMITL, whose members are the following students in the Software Engineering program: Alex, Beth, and Carl. Let p_1, p_2, p_3, q_1, q_2 and q_3 be the following propositions:

p_1 : Alex is a lead singer.

p_2 : Beth is a lead singer.

p_3 : Carl is a lead singer.

q_1 : Alex plays guitar.

q_2 : Beth plays guitar.

q_3 : Carl plays guitar.

Write the following propositions about the band using p_1, p_2, p_3, q_1, q_2, q_3 and logical connectives.

(a) Beth does not play guitar and Carl is not a lead singer. $\neg q_2 \wedge \neg p_3$

(b) Neither Beth nor Carl is a lead singer. $\neg(p_2 \vee p_3) \equiv \neg p_2 \wedge \neg p_3$

(c) The band's lead singers also play guitar.
$((p_1 \wedge p_2) \vee (p_2 \wedge p_3) \vee (p_1 \wedge p_3)) \wedge (p_1 \rightarrow q_1) \wedge (p_2 \rightarrow q_2) \wedge (p_3 \rightarrow q_3)$
$(p_1 \rightarrow q_1) \wedge (p_2 \rightarrow q_2) \wedge (p_3 \rightarrow q_3)$

(d) There is one lead singer in the band. $(p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge \neg p_2 \wedge p_3)$

(e) At least two members of the band play guitar. $(q_1 \wedge q_2) \vee (q_2 \wedge q_3) \vee (q_1 \wedge q_3)$

Problem 3

Rewrite the following formulas by inserting all the omitted parentheses.

(a) $(((p \vee (\neg q)) \vee (r \wedge p)) \vee (q \wedge (\neg r)))$

(b) $((p \wedge (\neg q)) \rightarrow (p \vee q))$

Precedence of logical operations
1) $\neg$ negation
2) $\wedge$ conjunction
3) $\vee$ disjunction
4) $\rightarrow$ implication
5) $\leftrightarrow$ biconditional

Problem 4

Suppose $\phi = ((p \wedge q) \rightarrow r) \leftrightarrow ((p \rightarrow r) \wedge (q \rightarrow r))$

(a) Describe a truth assignment which makes ϕ **true**.

(b) Describe a truth assignment which makes ϕ **false**.

$((p \wedge q) \rightarrow r) \leftrightarrow ((p \rightarrow r) \wedge (q \rightarrow r))$

p : True
q : True
r : True

p : True
q : False
r : False

TRUTH TABLE

p a proposition	$\sim p$ not p (negation)
T	F
F	T

T = true
F = false

p a proposition	q a proposition	$p \& q$ p and q (conjunction)	p or q p or q, inclusive (inclusive disjunction)	p or q p or q, exclusive (exclusive disjunction)	$p \rightarrow q$ if p then q (implication)	$p \leftrightarrow q$ p if and only if q (biconditional)
T	T	T	T	F	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T

Problem 5

Show by means of a truth table that the formulas $p \leftrightarrow q$ and $(p \vee \neg q) \wedge (\neg p \vee q)$ are logically equivalent.

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg p \vee q$	$(p \vee \neg q) \wedge (\neg p \vee q)$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	T	T

equivalent

Problem 6

Table 1 lists some well-known logical equivalences in propositional logic.

Theorem 1 (Replacement Theorem) *Suppose ϕ is a formula and ψ is a subformula of ϕ. And suppose ψ' is a formula such that $\psi \equiv \psi'$. If ϕ' denotes the formula resulted from replacing an occurrence of ψ in ϕ by ψ', then $\phi \equiv \phi'$.*

The Replacement Theorem allows us to convert a formula into an equivalent one by replacing some subformula ψ in the original formula by any formula equivalent to ψ. The following example shows that the formulas $\neg(\neg p \wedge \neg q)$ and $p \vee q$ are logically equivalent by using repeated applications of the Replacement Theorem and the logical equivalences in Table 1.

Example 1

$$\begin{aligned} \neg(\neg p \wedge \neg q) &\equiv \neg(\neg p) \vee \neg(\neg q) && \textit{by E16} \\ &\equiv p \vee \neg(\neg q) && \textit{by E9} \\ &\equiv p \vee q && \textit{by E9} \end{aligned}$$

By applying the Replacement Theorem and the logical equivalences listed in Table 1, show (as in the previous example) that each pair of formulas below are logically equivalent.

(a) $\neg(p \rightarrow q)$ and $p \wedge \neg q$

(b) $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $(\neg p \vee q) \wedge (p \vee \neg q)$

(c) $p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \rightarrow r)$

(d) $(q \vee \neg p) \rightarrow (q \wedge p)$ and $p \wedge (r \rightarrow p)$

a $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$ E20
$\equiv \neg\neg p \wedge \neg q$ E17
$\equiv p \wedge \neg q$ E9

b $(p \wedge q) \vee (\neg p \wedge \neg q) \equiv ((p \wedge q) \vee \neg p) \wedge ((p \wedge q) \vee \neg q)$ E15
$\equiv (\neg p \vee (p \wedge q)) \wedge ((p \wedge q) \vee \neg q)$ E11
$\equiv (\neg p \vee (p \wedge q)) \wedge (\neg q \vee (p \wedge q))$ E11
$\equiv ((\neg p \vee p) \wedge (\neg p \vee q)) \wedge (\neg q \vee (p \wedge q))$ E15
$\equiv ((\neg p \vee p) \wedge (\neg p \vee q)) \wedge ((\neg q \vee p) \wedge (\neg q \vee q))$ E15
$\equiv (T \wedge (\neg p \vee q)) \wedge ((\neg q \vee p) \wedge (\neg q \vee q))$ E6
$\equiv (T \wedge (\neg p \vee q)) \wedge ((\neg q \vee p) \wedge T)$ E6
$\equiv (T \wedge (\neg p \vee q)) \wedge (T \wedge (\neg q \vee p))$ E10
$\equiv (\neg p \vee q) \wedge ((\neg q \vee p) \wedge T)$ E1
$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ E1
$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ E11

Table 1: Some Logical Equivalences

	Equivalences	Name
E1	$\phi \wedge \top \equiv \phi$	Identity Laws
E2	$\phi \vee \bot \equiv \phi$	
E3	$\phi \wedge \bot \equiv \bot$	Domination Laws
E4	$\phi \vee \top \equiv \top$	
E5	$\phi \wedge \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \vee \neg\phi \equiv \top$	
E7	$\phi \wedge \phi \equiv \phi$	Idempotent Laws
E8	$\phi \vee \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \wedge \psi \equiv \psi \wedge \phi$	Commutative Laws
E11	$\phi \vee \psi \equiv \psi \vee \phi$	
E12	$\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$	Associative Laws
E13	$\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$	
E14	$\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$	Distributive Laws
E15	$\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$	
E16	$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	
E18	$\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption Laws
E19	$\phi \vee (\phi \wedge \psi) \equiv \phi$	
E20	$\phi \rightarrow \psi \equiv \neg\phi \vee \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	

ⓒ $p \to (q \to r) \equiv p \to (\neg q \vee r)$ E20

$\equiv \neg p \vee (\neg q \vee r)$ E20

$\equiv (\neg p \vee \neg q) \vee r$ E13

$\equiv (\neg q \vee \neg p) \vee r$ E11

$\equiv \neg q \vee (\neg p \vee r)$ E13

$\equiv \neg q \vee (p \to r)$ E20

$\equiv q \to (p \to r)$ E20

ⓓ $(q \vee \neg p) \to (q \wedge p) \equiv \neg (q \vee \neg p) \vee (q \wedge p)$ E20

$\equiv (\neg q \wedge \neg\neg p) \vee (q \wedge p)$ E17

$\equiv (\neg q \wedge p) \vee (q \wedge p)$ E9

$\equiv (p \wedge \neg q) \vee (q \wedge p)$ E10

$\equiv (\overset{\phi}{p} \wedge \overset{\psi}{\neg q}) \vee (\overset{\phi}{p} \wedge \overset{\chi}{q})$ E10

$\equiv \overset{\phi}{p} \wedge (\overset{\psi}{\neg q} \vee \overset{\chi}{q})$ E14

$\equiv p \wedge (q \vee \neg q)$ E11

$\equiv p \wedge T$ E6

$\equiv p$ E1

$p \wedge (r \to p) \equiv p \wedge (\neg r \vee p)$ E20

$\equiv p \wedge (p \vee \neg r)$ E11

$\equiv p$ E18

$\therefore (q \vee \neg p) \to (q \wedge p) \equiv p \wedge (r \to p)$

	Table 1: Some Logical Equivalences	
	Equivalences	Name
E1	$\phi \wedge \top \equiv \phi$	Identity Laws
E2	$\phi \vee \bot \equiv \phi$	
E3	$\phi \wedge \bot \equiv \bot$	Domination Laws
E4	$\phi \vee \top \equiv \top$	
E5	$\phi \wedge \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \vee \neg\phi \equiv \top$	
E7	$\phi \wedge \phi \equiv \phi$	Idempotent Laws
E8	$\phi \vee \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \wedge \psi \equiv \psi \wedge \phi$	Commutative Laws
E11	$\phi \vee \psi \equiv \psi \vee \phi$	
E12	$\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$	Associative Laws
E13	$\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$	
E14	$\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$	Distributive Laws
E15	$\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$	
E16	$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	
E18	$\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption Laws
E19	$\phi \vee (\phi \wedge \psi) \equiv \phi$	
E20	$\phi \to \psi \equiv \neg\phi \vee \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \to \psi) \wedge (\psi \to \phi)$	

Table 1: Some Logical Equivalences		
	Equivalences	Name
E1	$\phi \wedge \top \equiv \phi$	Identity Laws
E2	$\phi \vee \bot \equiv \phi$	
E3	$\phi \wedge \bot \equiv \bot$	Domination Laws
E4	$\phi \vee \top \equiv \top$	
E5	$\phi \wedge \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \vee \neg\phi \equiv \top$	
E7	$\phi \wedge \phi \equiv \phi$	Idempotent Laws
E8	$\phi \vee \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \wedge \psi \equiv \psi \wedge \phi$	Commutative Laws
E11	$\phi \vee \psi \equiv \psi \vee \phi$	
E12	$\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$	Associative Laws
E13	$\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$	
E14	$\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$	Distributive Laws
E15	$\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$	
E16	$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	
E18	$\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption Laws
E19	$\phi \vee (\phi \wedge \psi) \equiv \phi$	
E20	$\phi \rightarrow \psi \equiv \neg\phi \vee \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$	