Pehuk

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If and only if

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Negation

p	$\neg p$
T	F
F	T

$$\sim p \wedge \sim q \equiv \sim(p \vee q)$$

Validity - if all intepretation make the formula true

Invalid - Some intepretation make the formula false

Satisfiability - it is possible to find an intepretation that make formula true

Unsatisfiability - None of the intepretation make formula true

Logical consequence - premises are true and conclusion also true

Logical equivalent - formula always have the same truth value

Conjunction (And)	Disjunction (Or)	If then
Although But Yet	Or	Sufficient
Though Even though	Either or	
Moreover Furthermore	Neither nor ($\sim p \vee q$)	
Whereas		

$\vdash$ derive using rules

$\vDash$ is a logical consequence of

logically implies

DD $\wedge$ $\vee$ $\leftrightarrow$ $\emptyset$ $\ulcorner$

ID $\dfrac{P}{\perp}$ Ass $\sim P$

CD $\rightarrow$ Ass ตัวหน้า

CNF - $\wedge$ เชื่อม formula
DNF - $\vee$ เชื่อม formula

literal - atomic formula or negetion of atomic formula

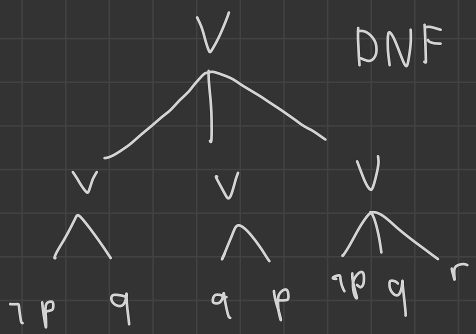
$$p \wedge q \equiv \neg(\neg p \vee \neg q)$$

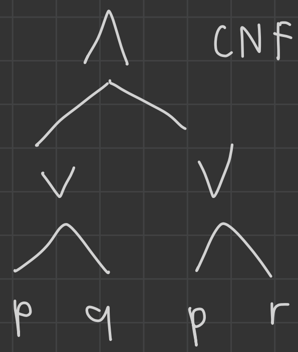
$$p \rightarrow q \equiv \neg p \vee q$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\equiv \neg(\neg(p \rightarrow q) \vee \neg(q \vee p))$$

$$\equiv \neg(\neg(\neg p \vee q) \vee \neg(\neg q \vee p))$$





ลำดับความสำคัญ

$\sim \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow$

MP

$$\begin{array}{l} p \rightarrow Q \\ p \\ \hline Q \end{array} \qquad \begin{array}{l} \sim p \rightarrow \sim Q \\ \sim P \\ \hline \sim Q \end{array}$$

MT

$$\begin{array}{l} p \rightarrow Q \\ \sim Q \\ \hline \sim p \end{array}$$

ID

$$\begin{array}{l} p \\ \sim p \\ \hline \perp \end{array}$$

DD

$\vee \quad \wedge \quad \leftrightarrow \quad \emptyset r \quad \sim$

Disjunct Syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline q \end{array}$$

MTP 1

$$\begin{array}{l} A \vee B \\ \sim A \\ \hline B \end{array}$$

MTP 2

$$\begin{array}{l} A \vee B \\ \sim B \\ \hline A \end{array}$$

Sets

Set theory

$\mathbb{N}$ - natural numbers

$\mathbb{Z}$ - integers

$\mathbb{Q}$ - rational numbers

V - vowels

R - real numbers

elements and cardinality

$\in$ - is an element of

$\notin$ - is not an element of

$| \ |$ - cardinality (size) ~ $A = \{1, 2, 3\} \Rightarrow |A| = 3$

cartesian product (ordered pair)

$x = \{0, 1, 2\}$ $\quad y = \{0, 1\}$

$x \times y = \{(0,0), (0,1), (1,0), (1,1), (2,0), (2,1)\}$

$|A \times B|$ = size of A × size of B

$\emptyset \times A = \emptyset \Rightarrow |\emptyset \times A| = |\emptyset| \cdot |A| = 0 \cdot$ size of A $= 0$

Subset and Power set

$\subseteq$ - subset

$\subset$ - proper subset

$A \subseteq B \Rightarrow$ A is a subset of B

$A \subset B \Rightarrow$ A must be strictly smaller than B

$P(A)$ - power set of A (set containing all possible subset of A)

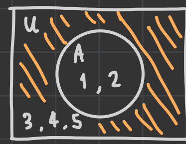
$A = \{a, b\}$ $\quad P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$|A| = n$ then $|P(A)| = 2^{|A|} = 2^n \leftarrow$ size of $P(A)$

$p(\emptyset) = \{\emptyset\} \quad \therefore |p(\emptyset)| = 1$

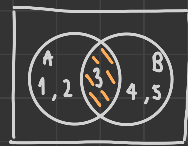
$p(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\} \quad \therefore |p(\{\emptyset\})| = 2$

Set operation



Compliment => $\bar{A} = \{a \in U \mid a \notin A\}$

$\bar{A} = \{3, 4, 5\}$



Intersect => Occur in both A and B

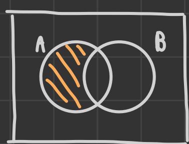
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A \cap B = \{3\}$

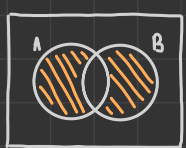
Union => in A or B

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Difference => $A - B = \{x \mid x \in A \text{ and } x \notin B\}$



Symmetric difference => $x \in A$ xor $x \in B$



$\{2, 4, 6\} \oplus \{2, 3, 4\} = \{6, 3\}$

$A \oplus B \equiv (A - B) \cup (B - A)$

Well - Ordering Principle

- Any non empty subset of $\mathbb{N}$ has a least element

$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$

$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$

Laws

$A \cup \emptyset = A \quad A \cap \emptyset = \emptyset$

Indempotent Law $\quad A \cup A = A \quad A \cap A = A$

Communicative Law $\quad A \cup B = B \cup A \quad A \cap B = B \cap A$

Associative Law $\quad (A \cup B) \cup C = A \cup (B \cup C) \quad (A \cap B) \cap C = A \cap (B \cap C)$

Distributive Law $\quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Absorption Law $\quad A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

U = universal set

$A \cup U = U \quad A \cap U = A$

$\bar{\bar{A}} = A$

De Morgan's Law $\quad \overline{A \cup B} = \bar{A} \cap \bar{B} \quad \overline{A \cap B} = \bar{A} \cup \bar{B}$

Complement Law $\quad A \cup \bar{A} = U \quad A \cap \bar{A} = \emptyset$

Relation

=> relation is a subset of cartesian product

n-ary relation $\quad R \subseteq A_1 \times \ldots \times A_n = R(a_1, \ldots, a_n) = (a_1, \ldots, a_n) \in R$

2-ary = binary relation $\quad R \subseteq A \times B = R(a,b)$ infix notation

$aRb = (a,b) \in R$

1-ary = unary relation (subset of some set)

Binary Relation

reflexive for all $a \in A$ aRa

symmetric for all $a,b \in A$ aRb implies bRa

antisymmetric for all $a,b \in A$ aRb and bRa implies $a=b$

transitive for all $a,b,c \in A$ aRb and bRc implies aRc

Identity Relation

$$Id_A = \{(a,a) \mid a \in A\}$$

Inverse Relation

$$R^{-1} = \{(y,x) \in B \times A \mid (x,y) \in R\}$$

Composition

$$Q \circ R = \{(a,c) \in A \times C \mid \text{for some } b \in B,\ aRb \text{ and } bQc\}$$

Q is Relation from B to C R is Relation from A to B

$R \subseteq A \times C = R \circ R$

$A = \{1,2,3\}$

Associativity of composition

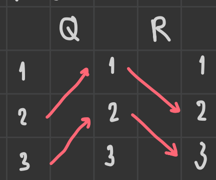
$R = \{(1,2),(2,3)\}$

Any relation R, Q, P

$Q = \{(2,1),(3,2)\}$

$(R \circ Q) \circ P = R \circ (Q \circ P)$

$R \circ Q$



$R \circ Q =$

$\{(2,2),(3,3)\}$

Closure

- There exists a unique smallest reflexive relation on A containing R, called the reflexive closure of R.
- There exists a unique smallest symmetric relation on A containing R, called the symmetric closure of R.
- There exists a unique smallest transitive relation on A containing R, called the transitive closure of R.

$R \cup Id_A$ is a reflexive closure of R

$R \cup R^{-1}$ is a symmetric closure of R

Function

no two distinct pair have the same first component

Domain & Range

$Dom(f)$

$Ran(f)$

$B = \{(1,2), (2,3)\}$

$Dom(B) = \{1,2\}$ $Ran(B) = \{2,3\}$

Onto mapping

$f: A \to B$ if and only if $f \subseteq A \times B$ and $Dom(f) = A$

$A = \{1,2,3\}$ $B \in \mathbb{N}$ $f = \{(1,2),(2,3),(3,4)\}$

↳ $Dom(f) = A$ & $f \subseteq A \times B$

Composite function

$f: x \to y$ $g: y \to z$ $h: x \to z$

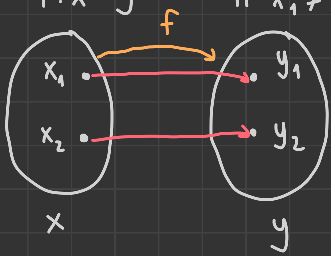
$h = g \circ f = \underbrace{g(\underbrace{f(x)}_{x \to y})}_{y \to z} \to x \to z$

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[rrrr, bend right, "k"'] & y \arrow[r, "g"] & z \arrow[r, "h"] & w \arrow[r, "j"] & v \end{tikzcd}$$

$k = j \circ (h \circ (g \circ f))$
$= j(h(g(f(x))))$
$= j(h(g(y)))$
$= j(h(z))$
$= j(w)$
$= v$

Injective (One to One)

$f: x \to y$ if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$



Show $f(x) = 3x - 2$ is injective

$x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2)$

$f(x_1) = f(x_2) \rightarrow x_1 = x_2$ Contra positive

$f(x_1) = f(x_2)$ Ass.

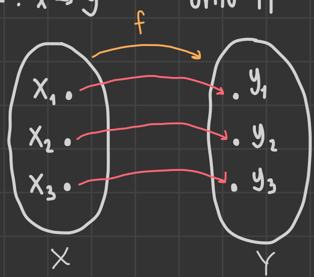
$3x_1 - 2 = 3x_2 - 2$

$3x_1 = 3x_2$

$x_1 = x_2$ $\therefore$ injective

Surjective (onto)

$f: x \to y$ onto if $\forall y \in Y$, $\forall x \in X$ such that $f(x) = y$



Show $f(x) = 5x + 2$ is surjective for $\forall x \in \mathbb{R}$, $\forall x \in \mathbb{Z}$

$y = f(x)$

$y = 5x + 2$

$y - 2 = 5x$

$\frac{y-2}{5} = x$

$y = 0 \quad x = -\frac{2}{5}$

$y = 1 \quad x = -\frac{1}{5}$

if $f: \mathbb{R} \to \mathbb{R}$
f is surjective

$f: \mathbb{Z} \to \mathbb{Z}$

$y = 7 \rightarrow x = 1$

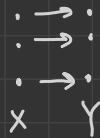
$y = 5 \rightarrow x = \frac{3}{5} \} \notin \mathbb{Z}$

not surjective

Bijective (Both injective and surjective)

if $|X| \neq |Y|$ not bijective

$f: X \to Y$ Each $x \in X$ maps to exactly one unique $y \in Y$



Inverse

$f: X \to Y \qquad f^{-1}: Y \to X$

$f(x) = y \qquad f^{-1}(y) = x$

$$\begin{tikzcd} x_1 \arrow[r, bend left, "f"] & y_1 \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

Inductive definition

- Systematic way to define infinite set by describing how set can be construct from base element then apply rule to generate more element

Rule

Suppose Prop is set of proposition letter

Form contain all finite sequence that can be construct by applying rule

1. Prop $\subseteq$ Form
2. $\top \in$ Form and $\bot \in$ Form
3. If $\phi \in$ Form, then $(\neg\phi) \in$ Form.
4. If $\phi \in$ Form and $\psi \in$ Form, then $(\phi \land \psi) \in$ Form.
5. If $\phi \in$ Form and $\psi \in$ Form, then $(\phi \lor \psi) \in$ Form.
6. If $\phi \in$ Form and $\psi \in$ Form, then $(\phi \to \psi) \in$ Form.
7. If $\phi \in$ Form and $\psi \in$ Form, then $(\phi \leftrightarrow \psi) \in$ Form.

Recursive Definition

A^* denote ϵ = empty string

LEN
1. $\text{len}(\epsilon) = 0$
2. $\text{len}(w \cdot x) = \text{len}(w) + 1$, for any $w \in A^*$ and $x \in A$

DBL
1. $\text{double}(\epsilon) = \epsilon$
2. $\text{double}(w \cdot x) = \text{double}(w) \cdot xx$, for any $w \in A^*$ and $x \in A$

FAC
1. $\text{fac}(0) = 1$
2. $\text{fac}(n+1) = (n+1) * \text{fac}(n)$, for any $n \in \mathbb{N}$

SUP

1. $super(0,n) = 1$, for any $n \in \mathbb{N}$
2. $super(m,0) = 1$, for any $m \in \mathbb{N}$ where $m > 0$
3. $super(m+1, n+1) = (m+1) * (n+1) * super(m,n)$ for any $m,n \in \mathbb{N}$

Well define

Circular def CIR

1. $f(0,0) = 1$
2. $f(m+1, n+1) = 2 * f(n+1, m+1)$ for any $m,n \in \mathbb{N}$

Ambiguous def AM

1. $g(0) = 1$
2. $g(n+1) = 10 * g(n)$, for any $n \in \mathbb{N}$
3. $g(n+2) = 20 * g(n)$, for any $n \in \mathbb{N}$

Incomplete def IN

* can't find when m is odd

1. $h(0) = 1$
2. $h(n+2) = 2 * h(n)$, for any $n \in \mathbb{N}$

Diverging def DI

* make problem larger

1. $p(0) = 0$
2. $p(n+1) = (n+1) * p(n+2)$, for any $n \in \mathbb{N}$

First Order Logic

Propositional logic - simple logic for reasoning about proposition

Predicate logic

Predicate - property of object or relations e.g. likes(x)

Constant - symbolic representation of specific object e.g. Tom

Variable - symbol of unspecified object eg. x, y

First order logic

Term - finite string

Construt rule :
- Each variable is a term
- Each constant symbol is a term
- if f is a function of arity k and $(t_1, \dots, t_k)$ are terms, then $f(t_1, \dots, t_k)$ is a term

Formula - finite string

Construct rule :
- T and $\perp$ are formula
- if t_1 and t_2 are term then $t_1 = t_2$ are formula
- if p is a predicate symbol of arity k and $t_1, \dots, t_k$ are terms then $p(t_1, \dots, t_2)$ is formula
- if $\emptyset$ is a formula then so is $(\neg \emptyset)$
- if $\emptyset$ and ψ is a formulas

Symbol

equality : =

L. connectives : $\top \perp \wedge \vee \rightarrow \leftarrow \sim$

quantifiers : $\forall x$, $\exists x$ for each x

parenthesis and comma : ( ) ,

L. connectives precedence

$\neg \; \exists x \; \forall x \; \wedge \; \vee \; \rightarrow \; \leftrightarrow$

high low

then so is $(\phi \land \psi)$, $(\phi \lor \psi)$, $(\phi \to \psi)$
$(\phi \leftrightarrow \psi)$

- if x is a variable and ϕ is a formula then $(\forall x.\, \phi)$, $(\exists x.\, \phi)$ is a formula