

Introduction to Logic

Assignment 5 (Part A)

King Mongkut's Institute of Technology Ladkrabang

September 11, 2021

Problem 1

Suppose ϕ is the formula $p \leftrightarrow (q \wedge \neg r)$.

$$(\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r)$$

(a) Find a formula in disjunctive normal form which is logically equivalent to ϕ .

(b) Find a formula in conjunctive normal form which is logically equivalent to ϕ .

$$\neg(\neg p \wedge q \wedge r) \wedge \neg(p \wedge \neg q \wedge r) \wedge \neg(p \wedge q \wedge \neg r) \wedge \neg(p \wedge \neg q \wedge \neg r)$$

$$V(p \wedge q \wedge r)$$

Problem 2

$$\exists (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$$

Determine whether each of the following formula is valid or not. If it is not valid, describe a truth assignment which makes the formula false.

(a) $(r \vee \neg r \vee q) \wedge (s \vee q \vee \neg s) \wedge (p \vee r \vee \neg q) \wedge (s \vee p \vee \neg p)$

$$p = F$$

$$R = F$$

(b) $(\underline{p} \vee q \vee \neg p) \wedge (\underline{s} \vee \neg s \vee q) \wedge (\underline{r} \vee p \vee s \vee \neg r)$

$$Q = T$$

Valid

Problem 3

(a) Suppose ϕ is the conjunction of the following clauses:

unsatisfiable

$$\begin{aligned} & r \vee \neg s \vee t \\ & p \vee \neg r \vee s \\ & \neg t \\ & \neg p \vee \neg q \vee t \\ & s \vee t \\ & \neg p \vee q \end{aligned}$$

$\rightarrow r \vee \neg s \vee \neg q \vee t$
 $\rightarrow \neg r \vee s \vee \neg q \vee t$
 $\rightarrow \neg s \vee t \rightarrow \neg t \vee t \equiv \bot$
 $\rightarrow \neg q \vee t \rightarrow \neg t \vee t \equiv \bot$

Demonstrate the application of the Davis-Putnam algorithm to check whether ϕ is satisfiable or not. If ϕ is satisfiable, describe a truth assignment which makes the formula true.

(b) Suppose ϕ is the conjunction of the following clauses:

$$\text{Satisfiable} \rightarrow \{p, q, r, s\} \models \neg p \vee t, p \vee s \vee r, \neg s \vee t, p \vee \neg q, q \vee \neg r$$
$$\neg p \vee t, p \vee s \vee r, \neg s \vee t \rightarrow \neg p \vee s \vee r$$
$$q \vee \neg r, \neg p \vee s \vee r \rightarrow q \vee \neg r$$
$$p \vee \neg q, q \vee \neg r \rightarrow p \vee \neg q$$
$$q \vee \neg r, p \vee \neg q \rightarrow q \vee \neg r$$
$$\neg s \vee t, p \vee \neg q \rightarrow \neg s \vee t$$
$$q \vee \neg r, \neg s \vee t \rightarrow q \vee \neg r$$

Demonstrate the application of the Davis-Putnam algorithm to check whether ϕ is satisfiable or not. If ϕ is satisfiable, describe a truth assignment which makes the formula true.