Introduction to Logic
Midterm Examination, Semester 1/2021

2 Oct 2021, 13.30-15.00

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Problem 1 (5 pts)

The passage below contains an argument.

> [1]The government must save Thai airways from bankruptcy. [2]It is the pride of our nation. More importantly, [3]its failing would make our tourism industry collapse.

Identify the premises, the conclusion, and the hidden premise(s) (if any).

Example. [1]Boxing causes injury, so [2]it is not a sport we should encourage.
Statement 2 is the conclusion. Statement 1 is a premise. The hidden premise is

> We should not encourage a sport that causes injury.

Problem 2 (10 pts)

Each passage below contains an argument. Draw a diagram showing the inferential relationship among the statements in the passage. If a statement is redundant or plays no role in the argument, do not include them in the diagram.

2.1 [1]Proteins are discovered not invented. [2]Inventions are patentable but discoveries are not. Hence, [3]the patenting of proteins is simply flawed.

2.2 [1]The Big Bang theory is being regarded as wrong. [2]According to this theory, the universe began with the Big Bang, a huge explosion occurring 20 billion years ago. The problem is [3]astronomers have found a huge cluster of galaxies that is too big to have been formed in 20 billion years. Based on recent data, it is now known that [4]galaxies form vast ribbons stretching billions of light years and [5]are seperated by empty spaces spanning hundreds of millions of light years. Because [6]galaxies travel much slower than the speed of light, these facts imply that [7]such a large cluster of galaxies must have taken at least 100 billion years to form, five times as long as the time since the Big Bang presumably occurred.

Problem 3 (9 pts)

Each passage below contains a compound statement. Write each statement below as a formula in propositional logic using the given propositional letters and their specified meaning.

Example. "If you have not paid your tuition fee, you will not be allowed to graduate."

$p =$ You have paid your tuition fee.
$g =$ You are allowed to graduate.

Ans. $\neg p \to \neg g$

3.1 "Our constitution neither acknowledges nor tolerates racisms."

$a =$ Our constitution acknowledges racisms.
$t =$ Our constitution tolerates racisms.

3.2 "The defendant will receive probation provided that he/she cooperates with the attorney."

$p =$ The defendant will receive probation.
$c =$ The defendant cooperates with the attorney.

3.3 "All of these are equivalent: (a) S is the empty set; (b) $\overline{S}$ is the universal set; and (c) S is a subset of every set."

$a = S$ is the empty set.
$b = \overline{S}$ is the universal set.
$c = S$ is a subset of every set.

Problem 4 (5 pts)

Rewrite the following code fragment into an equivalent one without the `else` statement.

```
if(x > 1) {
    if(y > 1)
        printf("a");
    else
        printf("b");
} else {
    printf("c");
    if(y > 1)
        printf("d");
}
```

Problem 5 (10 pts)

For each formula below, check whether it is satisfiable or not. If the formula is satisfiable, give a truth assignment which makes the formula true. If not, show that it is unsatisfiable.

5.1 $(p \wedge q \wedge \neg p \wedge r) \vee (\neg p \wedge s \wedge \neg q \wedge \neg s) \vee (r \wedge \neg p \wedge \neg q \wedge p) \vee \neg q$

5.2 $(p \vee \neg q \vee r) \wedge (p \vee q) \wedge (r \vee \neg q \vee \neg s) \wedge (\neg p \vee s) \wedge (\neg r \vee \neg q) \wedge (\neg s \vee q)$

Problem 6 (10 pts)

For each pair of formulas below, either show that the two formulas are logically equivalent or describe a truth assignment which makes one formula true and the other formula false.

6.1 $p \leftrightarrow (q \leftrightarrow r)$ and $(p \leftrightarrow q) \leftrightarrow r$

6.2 $\neg p \vee (q \vee (\neg r \vee s))$ and $(p \wedge r) \rightarrow (q \vee s)$

Problem 7 (10 pts)

Draw a reduced OBDD for the formula $(p \rightarrow q) \rightarrow (p \rightarrow r)$.

Problem 8 (20 pts)

Each passage below contains an argument. For each passage, please do the following:

(a) Write the underlined statements in the passage in propositional logic using the given propositional letters and its specified meaning.
(b) From the formulas you obtained in (a), determine which formulas are the premises and which formula is the conclusion of the argument in the passage.
(c) Based on what you identified as the premises and the conclusion in (b), determine whether the argument is valid or not. If so, provide a derivation of the conclusion from the premises using natural deduction rules. If <u>not</u>, give a truth assignment which makes all the premises true but the conclusion false.

Example. "[1]<u>John must not be at home at the moment</u>. [2]<u>If he were at home, his car must be in the garage</u>. But from what I can see, [3]<u>his car is currently not in the garage</u>.

$h =$ John is at home at the moment.
$g =$ John's car is currently in the garage.

Ans.

(a) Statement 1 $= \neg h$
Statement 2 $= h \rightarrow g$
Statement 3 $= \neg g$

(b) Premises: $h \rightarrow g$, $\neg g$
Conclusion: $\neg h$

(c) The argument is valid.

$1 : h \rightarrow g$	premise
$2 : \neg g$	premise
$3 : \neg h$	$MT, 1, 2$

8.1 "[1]The victim was right-handed. [2]If the victim committed suicide and was right-handed, she would not have wounds on the left of her head. [4]Hence, if there are wounds on the left of the victim's head, she did not commit suicide.

$r =$ The victim was right-handed.
$w =$ There are wounds on the left of the victim's head.
$s =$ The victim committed suicide.

8.2 "[1]You should not stay up all night to study for the exam. [2]If you stay up all night to study for the exam, you will be tired in the morning. And [3]if you are tired in the morning and the exam is difficult, you will not be able to do well on the exam. Obviously, [4]if you stay up all night to study for the exam and still not be able to do well on the exam, then you should not do that.

$s =$ You should stay up all night to study for the exam.
$u =$ You stay up all night to study for the exam.
$t =$ You are tired in the morning.
$d =$ The exam is difficult.
$w =$ You are able to do well on the exam.

Problem 9 (20 pts)

Imagine a fictional island where two types of inhabitants, called the *knights* and the *knaves*, are living. A knight always tells the truth, whereas a knave always tells lies (i.e. the opposite of the truth). Each inhabitant is of one of these two types, but unfortunately it is not clear which type he/she is. When you visited this island, you met 5 inhabitants on the island, namely A, B, C, D, and E. Below is the transcript from your conversation with some of these inhabitants.

- A said "Both C and D are knights."
- B said "If E is a knight, then so is A."
- C said "Either B or E or both are knaves."
- D said "E is a knave if and only if C is."

You are then asked to determine whether each of the 5 inhabitants is a knight or a knave. Luckily, you are in possession of a highly-efficient SAT solver program, which can determine whether a formula in CNF is satisfiable or not. Explain in detail how you can utilize your SAT solver to solve this.

Hint: Introduce the following propositional symbols a, b, c, d, and e which mean that A, B, C, D, and E, respectively, are *knights*.

Problem 10 (10 pts)

Suppose A, B, C, D, and E are the sets given by:

$$\begin{aligned} A &= \{0,1,2\} \\ B &= \{-5,1,3,6,10\} \\ C &= \{x \in \mathbb{Z} \,|\, 0 < x \leq 20 \text{ and } x \text{ is even}\} \\ D &= \{x \in \mathbb{Z} \,|\, x = y - z \text{ for some y and z in A}\} \\ E &= \{2x+1 \in \mathbb{Z} \,|\, x \in A\} \end{aligned}$$

List all the members of each of the following sets.

10.1 $B \cup C$

10.2 $\wp(A)$

10.3 D

10.4 $A \times E$

10.5 $\wp(\wp(A \cap B))$

Problem 11 (10 pts)

Suppose $A = \{x \in \mathbb{Z} \,|\, -25 \leq x \leq 25\}$. Let P be the following binary relation:

$$P = \{(x,y) \in A \times A \,|\, y = x^2\}$$

11.1 List all the members of P.

11.2 List all the members of $P \circ P$.

Problem 12 (10 pts)

A binary relation R on a non-empty set A is said to be transitive if and only if

$$xRy \text{ and } yRz \text{ implies } xRz, \text{ for all } x,y,z \in A$$

The *transitive closure* of a binary relation R on A is the *smallest* transitive relation on A that includes R.

Find the transitive closure of the following relation on $\mathbb{N}$:

$$R = \{(1,3),(2,1),(3,4),(4,2),(4,5)\}.$$

——————— This is the end of the exam paper. ———————

Table 1: Some Logical Equivalences		
	Equivalences	Name
E1	$\phi \wedge \top \equiv \phi$	Identity Laws
E2	$\phi \vee \bot \equiv \phi$	
E3	$\phi \wedge \bot \equiv \bot$	Domination Laws
E4	$\phi \vee \top \equiv \top$	
E5	$\phi \wedge \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \vee \neg\phi \equiv \top$	
E7	$\phi \wedge \phi \equiv \phi$	Idempotent Laws
E8	$\phi \vee \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \wedge \psi \equiv \psi \wedge \phi$	Commutative Laws
E11	$\phi \vee \psi \equiv \psi \vee \phi$	
E12	$\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$	Associative Laws
E13	$\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$	
E14	$\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$	Distributive Laws
E15	$\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$	
E16	$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	
E18	$\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption Laws
E19	$\phi \vee (\phi \wedge \psi) \equiv \phi$	
E20	$\phi \to \psi \equiv \neg\phi \vee \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \to \psi) \wedge (\psi \to \phi)$	

Table 1 lists some well-known logical equivalences in propositional logic.