

## Problem 1

This can be solved using contradiction to check if there are bijections or none. If there is then it is countable, if not then it is not countable

a)

give  $n$  to be number of natural numbers then greater than zero and then use  $p$  as power set of  $N$

$$T \in p(2^N) \mid T > 0$$

So a) countable

b)

Assuming  $s$  is subset of the power of natural number and  $S$  is the finite set  
 $p(N) = 0, 1, 4, 9, \dots$

$$P = \{S \in p(N) \mid S \text{ is a finite set}\}$$

So b) countable

## Problem 2

Any real number  $a \leq b$

$$\text{set}[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

$$[-1, 2] = \{-1, 0, 1, 2\} \quad |[-1, 2]| = 4$$

$$[0, 1] = \{0, \frac{1}{3}, \frac{2}{3}, 1\} \quad |[0, 1]| = 4$$

## Problem 3

$$n \in \mathbb{N}$$

Substitute 1 in the number in set when it is even or 0 if the number is odd

#### Problem 4

- a)  $\text{higher}(\text{Google}, \text{Yahoo})$
- b)  $\text{higher}(\text{Google}, \text{Yahoo}) \wedge \text{higher}(\text{Google}, \text{Facebook})$
- c)  $\exists x (\neg \text{link}(x, x))$
- d)  $\forall x (\text{link}(x, \text{Google}) \vee (\text{link}(x, \text{Yahoo}) \rightarrow \text{link}(x, \text{Facebook})))$
- e)  $\forall x (\text{link}(x, \text{Google}))$

#### Problem 5

- a)
  - 1)  $\text{parent}(\text{John}, \text{Mary})$  pre
  - 2)  $\text{parent}(\text{Mary}, \text{Tom})$  pre
  - 3)  $\forall x. \forall y. (\text{parent}(x, y) \rightarrow \text{child}(y, x))$  pre
  - Assuming x as Mary
  - Assuming y as Tom
  - 4)  $\text{parent}(\text{Mary}, \text{Tom}) \rightarrow \text{child}(\text{Tom}, \text{Mary})$   $\forall E, 3$
  - 5)  $\text{child}(\text{Tom}, \text{Mary})$   $\rightarrow E, 2, 4$
  - 6)  $\exists x. \text{child}(x, \text{Mary})$   $\exists I, 4$
- b)
  - 1)  $\text{on}(a, b)$  pre
  - 2)  $\text{on}(b, c)$  pre
  - 3)  $\text{red}(a)$  pre
  - 4)  $\text{blue}(c)$  pre
  - 5)  $\forall x. \neg(\text{red}(x) \wedge \text{blue}(x))$  pre
  - Assuming x as a
  - 6)  $\neg(\text{red}(a) \wedge \text{blue}(a))$   $\forall E, 5$
  - 7)  $\text{red}(a) \wedge \text{blue}(a)$   $\neg E, 6$
  - 8)  $\text{red}(a) \wedge \neg \text{red}(a)$  7
  - 9)  $\exists x. \exists y. (\text{on}(x, y) \wedge \text{red}(x) \wedge \neg \text{red}(y))$   $\exists I, 1, 8$