

Nattheetarn P.

64011474 SE

Introduction to Logic

Assignment 5 (Part A)

King Mongkut's Institute of Technology Ladkrabang

September 11, 2021

p	q	r	$\neg r$	$q \wedge \neg r$	$p \leftrightarrow (q \wedge \neg r)$
0	0	0	1	0	1
0	0	1	0	0	1
0	1	0	1	1	0
1	0	0	1	0	0
0	1	1	0	0	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	0	0	0

Problem 1

Suppose ϕ is the formula $p \leftrightarrow (q \wedge \neg r)$.

- (a) Find a formula in disjunctive normal form which is logically equivalent to ϕ . ↗
- (b) Find a formula in conjunctive normal form which is logically equivalent to ϕ .

$$(p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$

$$\neg(p \wedge q \wedge \neg r) \wedge \neg(\neg p \wedge \neg q \wedge \neg r) \wedge \neg(p \wedge q \wedge r) \wedge \neg(\neg p \wedge q \wedge r)$$

Problem 2

$$\equiv (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$$

Determine whether each of the following formula is **valid** or not. If it is not valid, describe a truth assignment which makes the formula false.

- (a) $(r \vee \neg r \vee q) \wedge (s \vee \neg s) \wedge (p \vee r \vee \neg q) \wedge (s \vee \neg s \vee \neg p) \rightarrow p \equiv F$
- (b) $(p \vee q \vee \neg p) \wedge (s \vee \neg s \vee q) \wedge (r \vee p \vee s \vee \neg r) \rightarrow p \equiv F$
 $q \equiv T$

↘ valid

Problem 3

- (a) Suppose ϕ is the conjunction of the following clauses:

$$\begin{aligned} & r \vee \neg s \vee t \\ & p \vee \neg r \vee \neg s \\ & \neg t \\ & \neg p \vee \neg q \vee t \\ & s \vee t \\ & \neg p \vee q \end{aligned}$$

Demonstrate the application of the Davis-Putnam algorithm to check whether ϕ is satisfiable or not. If ϕ is satisfiable, describe a truth assignment which makes the formula true.

(b) Suppose ϕ is the conjunction of the following clauses:

$$\begin{aligned}\neg p \vee t \\ p \vee s \vee r \\ \neg s \vee t \\ p \vee \neg q \\ q \vee \neg r\end{aligned}$$

Demonstrate the application of the Davis-Putnam algorithm to check whether ϕ is satisfiable or not. If ϕ is satisfiable, describe a truth assignment which makes the formula true.

$$\begin{aligned}
 &r \vee \neg s \vee t \\
 &p \vee \neg r \vee \neg s \\
 &\neg t \\
 &\neg p \vee \neg q \vee t \\
 &s \vee t \\
 &\neg p \vee q
 \end{aligned}$$

$$\begin{aligned}
 \Pi &= p, q, r, s, t \\
 B_p & \text{ ① } pqr, r, pqr, pqr \rightarrow \text{② } pqr, pqr \\
 B_t & \text{ ③ } pqr, r, pqr \rightarrow pqr, pqr \\
 B_r & \text{ ④ } pqr, r, pqr \rightarrow pqr, pqr \\
 B_s & \text{ ⑤ } pqr, r, pqr \rightarrow pqr, pqr \\
 B_t & \text{ ⑥ } pqr, r, pqr \rightarrow pqr, pqr \\
 &\hookrightarrow pqr, pqr \equiv \text{unsatisfiable}
 \end{aligned}$$

$$\begin{aligned}
 &\neg p \vee t \quad / \\
 &p \vee s \vee r \quad / \\
 &\neg s \vee t \\
 &p \vee \neg q \quad / \\
 &q \vee \neg r
 \end{aligned}$$

$$\begin{aligned}
 \Pi &= p, q, r, s, t \\
 B_p & \text{ ① } pqr, pqr, pqr \rightarrow pqr, pqr \\
 B_q & \text{ ② } pqr, pqr, pqr \rightarrow pqr, pqr \\
 B_r & \text{ ③ } pqr, pqr, pqr \rightarrow pqr, pqr \\
 B_s & \text{ ④ } pqr, pqr, pqr \rightarrow pqr, pqr \\
 B_t & \text{ ⑤ } pqr, pqr, pqr \rightarrow pqr, pqr
 \end{aligned}$$

Satisfiable $\rightarrow p, q, r, s, t \equiv T$