Introduction to Logic
Assignment 9

Faculty of Engineering, KMITL

Problem 1

Let f be the function on $\mathbb{N}$ defined recursively as follows:

$$\begin{aligned} &f(0) = 1 \\ &f(1) = 2 \\ &f(2) = 3 \\ &f(n) = (3 \cdot f(n-1) + 2 \cdot f(n-2) + f(n-3)) \bmod 7, \qquad && n \geq 3 \end{aligned}$$

Compute $f(12)$.

Note: In the above equations, $x \bmod y$ is the remainder of the division of x by y.

Problem 2

Prove by mathematical induction that $3|(5^{2n} - 1)$ for every integer $n \geq 0$.

Problem 3

Suppose f is a function on $\mathbb{N}$ defined recursively by

$$\begin{aligned} f(0) &= 1 \\ f(n+1) &= 2f(n) + n - 1, \qquad && n \geq 0 \end{aligned}$$

Prove by mathematical induction that $f(n) = 2^n - n$, for all integers $n \geq 0$.

Problem 4

Prove by mathematical induction that, for all integers $n \geq 1$,

$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1.$$

Note that, in the above equation, $x \cdot x!$ means $x \cdot (x!)$.

Problem 5

Recall the following rules in Calculus:

$$\frac{d}{dx}(x) = 1$$
$$\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u),$$

for any differentiable functions u and v on x.

Using the above rules, prove by mathematical induction that, for any integer $n \geq 1$,

$$\frac{d}{dx}x^n = nx^{n-1}$$

Problem 6

Suppose we have unlimited supplies of 3-baht postal stamps and 5-baht postal stamps. Apply the *well-ordering principle* to prove that any postage cost of 8 Baht or more can be paid using only 3-baht stamps and 5-baht stamps.