Problem 1)

Find $f(12)$

Find the least positive integer n such that $f(n) \geq 1000$

$f(0) = 0$

$f(1) = 1$

$f(n) = 2 \cdot f(n-1) - 1$ for each even integer $n \geq 2$

$f(n) = f(n-1) + f(n-2)$ for each even integer $n \geq 3$

$f(0)$

$f(1) = 1$

$$\begin{aligned} f(2) &= 2 \cdot f(2-1) - 1 \\ &= 2 \cdot f(1) - 1 \\ &= 2(1) - 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} f(3) &= f(3-1) + f(3-2) \\ &= f(2) + f(1) \\ &= 1+1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} f(4) &= 2 \cdot f(4-1) - 1 \\ &= 2 \cdot f(3) - 1 \\ &= 2 \cdot 2 - 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} f(5) &= f(5-1) + f(5-2) \\ &= f(4) + f(3) \\ &= 3+2 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(6) &= 2 \cdot f(6-1) - 1 \\ &= 2 \cdot f(5) - 1 \\ &= 2 \cdot 5 - 1 \\ &= 9 \end{aligned}$$

$$\begin{aligned} f(7) &= f(7-1) + f(7-2) \\ &= f(6) + f(5) \\ &= 9+5 \\ &= 14 \end{aligned}$$

$$\begin{aligned} f(8) &= 2 \cdot f(7) - 1 \\ &= 2 \cdot 14 - 1 \\ &= 28 - 1 \\ &= 27 \end{aligned}$$

$$\begin{aligned} f(9) &= f(8) + f(7) \\ &= 27 + 14 \\ &= 41 \end{aligned}$$

$$\begin{aligned} f(10) &= 2 \cdot f(10-1) - 1 \\ &= 2 \cdot f(9) - 1 \\ &= 2 \cdot 41 - 1 \\ &= 81 \end{aligned}$$

$$\begin{aligned} f(11) &= f(11-1) + f(11-2) \\ &= f(10) + f(9) \\ &= 81 + 41 \\ &= 122 \end{aligned}$$

$$\begin{aligned} f(12) &= 2 \cdot f(12-1) - 1 \\ &= 2 \cdot f(11) - 1 \\ &= 2 \cdot 122 - 1 \\ &= 243 \end{aligned}$$

1.1)

1.2) $$\begin{aligned} f(13) &= f(13-1) + f(13-2) \\ &= f(12) + f(11) \\ &= 243 + 122 \\ &= 365 \end{aligned}$$

$$\begin{aligned} f(14) &= 2 \cdot f(14-1) - 1 \\ &= 2 f(13) - 1 \\ &= 2 \cdot 365 - 1 \\ &= 729 \end{aligned}$$

$$\begin{aligned} f(15) &= f(15-1) + f(15-2) \\ &= f(14) + f(13) \\ &= 729 + 365 \\ &= 1094 \end{aligned}$$

2. assuming that a-b is odd and b-c is even then a-c is odd

$a-b = 2x+1$

$a = 2x+1+b$

$a = 2n+1-b$

$-c = 2n-b$

$c = -2n+b$

$a-c = 2n+1$

$(2n+1-b)-(-2n+b) = 2n+1$

$2n+1-b+2n-b = 2n+1$

$4n-2b+1 = 2n+1$

$n = 2n-b \rightarrow 2(2n-b)+1 = 2n+1$

$2n+1 = 2n+1$

$a-c = 2n+1$

$a-c =$ odd #

2.2 assuming that n^2 is odd

$n^2 = 2a+1$ for some $a \in Z$

$n = \sqrt{2a+1}$

$n = 2a+1$ $\quad n^2 = n$

$2a+1^2 = \sqrt{2a+1}$

$2a+1 = 2a+1$

$n = n$

Then we have $n = 2c+1$ for some $c \in Z$

then n is odd because def 2

If n^2 is odd the n is also odd too #

2.3 For every integer n, n is either odd or even

for $n = 2m$ (even)

$n(n^2-1)(n+2) = 2m(4m^2-1)2m+2) = 2m(4m^2-1)$

$2(m+1) = 4m$

for $n = 2m+1$ (odd)

$n(n^2-1)(n+2) = (2m+1)((2m+1)^2-1)(2m+1+2)$

$= (2m+1)((4m^2+4m+1)-1)(2m+3) = 4m$

Problem 3)

Prove $1^3+2^3+\dots+n^3=\left[\frac{n(n+1)}{2}\right]^2$

$$f(n)=1^3+2^3+\dots+n^3$$

LHS	RHS
$f(1)=1^3$	$\left[\frac{1(1+1)}{2}\right]^2$
$=1$	$=\left[\frac{2}{2}\right]^2$
	$=1^2$
	$=1$

Basis case

$1=1$ ✓

$$f(n)=\left[\frac{n(n+1)}{2}\right]^2$$

Suppose ≥ 1, assuming $f(n)=\left[\frac{n(n+1)}{2}\right]^2$ is true
show that $f(n+1)=\left[\frac{n+1(n+1+1)}{2}\right]^2$ is also true

$$f(n+1)=1^3+2^3+\dots+n^2+(n+1)^3$$

$f(n)$

$$=f(n)+(n+1)$$

$$=\left[\frac{n(n+1)}{2}\right]^2+(n+1)^3$$

$$=\left[\frac{n^4+6n^3+13n^2}{4}\right]+3n+1$$

$$=\left[\frac{(n+1)(n+2)}{2}\right]^2$$

$$=\left[\frac{(n+1)[(n+1)+1]}{2}\right]^2$$

$$=\left[\frac{n(n+1)}{2}\right]^2$$ It is the same as $f(n)$

Problem 4)

1) $n = 0$; $h(0) = 0 = 2^0 - 0 - 1$

$= 1 - 0 - 1$

$= 0$

2) Assuming that $h(n) = 2^n - n - 1$ for some $n \geq 1$

$h(n+1) = 2^{n+1} - (n+1) - 1$

$= 2 \cdot 2^n - n - 2$

$h(n+1) = 2 \cdot h(n+1-1) + n + 1 - 1$

$= 2\, h(n) + n$

$= 2 \cdot (2^n - n - 1) + n$

$= 2^{n+1} - 2n - 2 + n$

$= 2^{n+1} - n - 2$ #

Problem 5)

5.1) likes (¬(John, Mary), Tom)

5.2) ∀y likes (athlete (x), Mary)

5.3) ∃x∀y likes(smart(x), athelete (y))

5.4) ∀x likes(Mary, ¬ (likes(athelete (x), athelete (x)))

5.5) likes (Mary, athlete (John))

Problem 7)

7.1) $(\forall x.\ smart(x)) \wedge (\forall y.\ athlete(y))$ and $\forall x.\ (smart(x) \wedge athlete(x))$

Domain = { Jared, Tom, Jake }

smart = { Jared, Jake }

athlete = { Jared, Tom }

Formula $\forall x.\ (smart(x) \wedge athlete(x))$ is true but both $\forall x.\ smart(x))$ and $\forall y.\ athlete(y)$ are false

7.2) $(\exists x.\ smart(x)) \wedge (\exists y.\ athlete(y))$ and $\exists x.\ (smart(x) \wedge athlete(x))$

Show $(\exists x.\ smart(x)) \wedge (\exists y.\ athlete(y))$ implies $\exists x.\ (smart(x) \wedge athlete(x))$

1.	$\exists x [(smart(x)] \wedge \exists y [athlete(y)]$	assume
2.	$\exists x [smart(x)]$	$\wedge E$, 1
3.	$\exists y [athlete(y)]$	$\wedge E$, 1
	$y := x$	
4.	$athlete(x)$	$\exists E$, 3
5.	$smart(x)$	$\exists E$, 2
6.	$smart(x) \wedge athlete(x)$	$\wedge I$ 2,5
7.	$\exists x.\ (smart(x) \wedge athlete(x))$	$\exists I$, 6