Introduction to Logic
Assignment 4

King Mongkut's Institute of Technology Ladkrabang

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Problem 1

Suppose $\Gamma = \{p \to (s \lor t), \quad (p \land s) \to q, \quad (t \land \neg q) \to \bot\}$

1.1 Show by means of a truth table that $\Gamma \models p \to q$.

1.2 Show that $\Gamma \vdash p \to q$.

Problem 2

Each passage below contains an argument. For each passage, please do the following:

(a) Write the underlined statements in the passage in propositional logic using the given propositional letters and its specified meaning.
(b) From the formulas you obtained in (a), determine which formulas are the premises and which formula is the conclusion of the argument in the passage.
(c) Based on what you identified as the premises and the conclusion in (b), determine whether the argument is valid or not. If so, provide a derivation of the conclusion from the premises using natural deduction rules. If not, give a truth assignment which makes all the premises true but the conclusion false.

Example. [1]John must not be at home at the moment. [2]If he were at home, his car must be in the garage. But from what I can see, [3]his car is currently not in the garage.

$h =$ John is at home at the moment.
$g =$ John's car is currently in the garage.

Ans.

(a) Statement 1 $= \neg h$
Statement 2 $= h \to g$
Statement 3 $= \neg g$

(b) Premises: $h \to g$, $\neg g$

Suppose $\Gamma = \{p \to (s \lor t),\quad (p \land s) \to q,\quad (t \land \neg q) \to \bot\}$

1.1 Show by means of a truth table that $\Gamma \models p \to q$.

p	q	s	t	$(s \lor t)$	ϕ: $p \to (s \lor t)$	$(p \land s)$	ψ: $(p \land s) \to q$	$\neg q$	$t \land \neg q$	χ: $(t \land \neg q) \to \bot$	$\phi \land \psi \land \chi$	$p \to q$	$(\phi \land \psi \land \chi) \to (p \to q)$
T	T	T	T	T	T	T	T	F	F	T	T	T	T
T	T	T	F	T	T	T	T	F	F	T	T	T	T
T	T	F	T	T	T	F	T	F	F	T	T	T	T
T	T	F	F	F	F	F	T	F	F	T	F	T	T
T	F	T	T	T	T	T	F	T	T	F	F	F	T
T	F	T	F	T	T	T	F	T	F	T	F	F	T
T	F	F	T	T	T	F	T	T	T	F	F	F	T
T	F	F	F	F	F	F	T	T	F	T	F	F	T
F	T	T	T	T	T	F	T	F	F	T	T	T	T
F	T	T	F	T	T	F	T	F	F	T	T	T	T
F	T	F	T	T	T	F	T	F	F	T	T	T	T
F	T	F	F	F	T	F	T	F	F	T	F	T	T
F	F	T	T	T	T	F	T	T	T	F	F	T	T
F	F	T	F	T	T	F	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T	F	F	T	T
F	F	F	F	F	T	F	T	T	F	T	F	T	T

Γ valid.

Suppose $\Gamma = \{p \to (s \vee t), \quad (p \wedge s) \to q, \quad (t \wedge \neg q) \to \bot\}$

1.2 Show that $\Gamma \vdash p \to q$.

1.	$p \to (s \vee t)$	premises
2.	$(p \wedge s) \to q$	
3.	$(t \wedge \neg q) \to \bot$	
4.	p	assumption.
5.	$s \vee t$	MP 1,4
6.	s	assumption.
7.	$p \wedge s$	$\wedge$i 4,6
8.	q	MP 2,7
9.	t	assumption.
10.	$\neg q$	assumption
11.	$t \wedge \neg q$	$\wedge$i 9,10
12.	$\bot$	MP 3,10
13.	q	PBC 10-12.
14.	q	$\vee$e. 5, 6-8, 9-13
15.	$p \to q$	$\to$i 4-14

$\{p \to (s \vee t), (p \wedge s) \to q, (t \wedge \neg q) \to \bot\} \vdash p \to q.$

	Table 1: Some Logical Equivalences	
	Equivalences	Name
E1	$\phi \land \top \equiv \phi$	Identity Laws
E2	$\phi \lor \bot \equiv \phi$	
E3	$\phi \land \bot \equiv \bot$	Domination Laws
E4	$\phi \lor \top \equiv \top$	
E5	$\phi \land \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \lor \neg\phi \equiv \top$	
E7	$\phi \land \phi \equiv \phi$	Idempotent Laws
E8	$\phi \lor \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \land \psi \equiv \psi \land \phi$	Commutative Laws
E11	$\phi \lor \psi \equiv \psi \lor \phi$	
E12	$\phi \land (\psi \land \chi) \equiv (\phi \land \psi) \land \chi$	Associative Laws
E13	$\phi \lor (\psi \lor \chi) \equiv (\phi \lor \psi) \lor \chi$	
E14	$\phi \land (\psi \lor \chi) \equiv (\phi \land \psi) \lor (\phi \land \chi)$	Distributive Laws
E15	$\phi \lor (\psi \land \chi) \equiv (\phi \lor \psi) \land (\phi \lor \chi)$	
E16	$\neg(\phi \land \psi) \equiv \neg\phi \lor \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \lor \psi) \equiv \neg\phi \land \neg\psi$	
E18	$\phi \land (\phi \lor \psi) \equiv \phi$	Absorption Laws
E19	$\phi \lor (\phi \land \psi) \equiv \phi$	
E20	$\phi \to \psi \equiv \neg\phi \lor \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \to \psi) \land (\psi \to \phi)$	

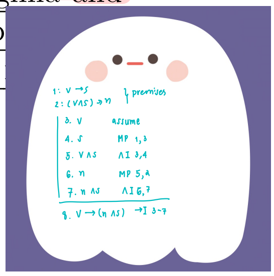
Conclusion: $\neg h$

(c) The argument is valid.

$1 : h \to g$	premise
$2 : \neg g$	premise
$3 : \neg h$	$MT, 1, 2$

$v \to s$ $\quad$ $v \land s \to n$

2.1 [1]If Virginia supports independence, then so do the southern colonies. [2]If Virginia and the southern colonies support independence, then the northern colonies will also supp pendence. Therefore, [3]Virginia's supporting of independence is sufficient for both the and the southern colonies to do the same.



$v =$ Virginia supports independence.
$s =$ The southern colonies support independence.
$n =$ The northern colonies support independence.

1 : $v \to s$
2 : $(v \land s) \to n$ } premises
3 : $v \to (s \land n)$ $\to$ conclusion.

2.2 [1]The deaths were caused either by overdoses of heroin or by bad quality heroin. [2]If the former, the victims would have shown the usual overdose symptoms. However, [3]they did not exhibit these symptoms. Therefore, we may conclude that [4]the deaths were caused by bad quality heroin.

$o =$ The deaths were caused by overdoses of heroin.
$b =$ The deaths were caused by bad quality heroin.
$s =$ The victims showed the usual overdose symptoms.

$o \lor b$
$o \to s$ } pre
$\neg s$
con: b

1: $o \lor b$
2: $o \to s$
3: $\neg s$
4. $\neg o$ MT 2,3
5: b $\lor$e 1,4

2.3 [1]If I eat the cake, the cake will make me larger or smaller. [2]If it makes me larger, I can reach the key. [3]If it makes me smaller, I can creep under the door. [4]I can get into the garden if I can reach the key or creep under the door. So [5]if I eat the cake, I can get into the garden.

$e =$ I eat the cake.
$l =$ The cake makes me larger.
$s =$ The cake makes me smaller.
$k =$ I can reach the key.
$d =$ I can creep under the door.
$g =$ I can get into the garden.

1. $e \to (l \lor s)$
2. $l \to k$
3. $s \to d$
4. $(k \lor d) \to g$ } p
5. $e \to g$

1. $e \to (l \lor s)$
2. $l \to k$
3. $s \to d$
4. $(k \lor d) \to g$ } p

5. e assumption
6. $l \lor s$ MP 1,5
7. l assume
8. k MP 2,7
9. $k \lor d$ $\lor$i$_1$ 8
10 s assume
11. d MP 3,7'
12. $k \lor d$ $\lor$i$_2$ 11
13. $k \lor d$ $\lor$e 6, 7-9, 10-12
14. g MP 4,13
15. $e \to g$ $\to$i 5-14

2.4 [1]Either Alex or David (or both) is a thief. [2]If Alex is a thief, then Bob is also a thief. And [3]if Bob is a thief, so is Calvin. [4]If Alex and David are both thieves, then Calvin is also a thief. Therefore, [5]if David is a thief, so is Calvin.

$a =$ Alex is a thief.
$b =$ Bob is a thief.
$c =$ Calvin is a thief.
$d =$ David is a thief.

1. $a \lor d$ (F, T)
2. $a \to b$ (F, F)
3. $b \to c$ (F, F)
4. $(a \land d) \to c$ } P
5. $d \to c$ (T, F)

If a, b, c is false and d is true, make conclusion false and all premises true.

a	b	c	d	$a \lor d$	$a \to b$	$b \to c$	$a \land d$	$(a \land d) \to c$	$d \to c$
F	F	F	T	T	T	T	F	T	F

premises- $\quad$ conclusion.

2.5 [1]If Cain married his sister, his marriage was incestuous. [2]If he did not marry his sister, then Adam and Eve were not the progenitors of the entire human race. It follows that [3]if Adam and Eve were the progenitors of the whole human race, then Cain's marriage was incestuous.

$s =$ Cain married his sister.
$i =$ Cain's marriage was incestuous.
$p =$ Adam and Eve were the progenitors of the entire human race.

2.6 [1]If Japan is to reduce its huge trade surplus, then it must either convince its citizens to spend more or it must move its manufacturing facilities to other countries. [2]It is not the case that Japan will either increase its imports or convince its citizens to spend more. Furthermore, [3]it is not the case that Japan will either allow foreign companies to compete fairly or move its manufacturing facilities to other countries. Therefore, [4]Japan will not reduce its huge trade surplus.

$s =$ Japan will reduce its huge trade surplus.
$i =$ Japan will increase its imports.
$c =$ Japan will convince its citizens to spend more.
$f =$ Japan will allow foreign companies to compete fairly.
$m =$ Japan will move its manufacturing facilities to other countries.

2.7 [1]Watson has reddish dirt on his boots. [2]He wouldn't have that if he had not been to the Post Office this morning. [3]If he had been to the Post Office but did not mail a letter this morning, then either he bought some stamps or sent a telegram. [4]If he mailed a letter this morning, then he would have written a letter this morning. [5]He wouldn't buy some stamps unless he ran out of stamps. Therefore, [6]if Watson didn't write a letter this morning and didn't run out of stamps, then he must have sent a telegram this morning.

$d =$ Watson has reddish dirt on his boots.
$p =$ Watson went to the Post Office this morning.
$m =$ Watson mailed a letter this morning.
$s =$ Watson bought some stamps this morning.
$t =$ Watson sent a telegram this morning.
$w =$ Watson wrote a letter this morning.
$o =$ Watson ran out of stamps.

2.8 [1]If there is evil and God does not know it, then God is not omniscient. [2]If there is evil and God knows it but he is unable to prevent it, then God is not omnipotent. [3]If there is evil and God knows it and is able to prevent it but is unwilling to do so, then God is not supremely good. [4]If God exists and there is evil, then either God does not know it or he is unable or unwilling to prevent it. [4]If God exists then he is omnipotent, omniscient, and supremely good. It follows that [5]either there is no evil or there is no God (or both). (Yu Kam Por)

$e =$ There is evil.
$g =$ There is God.
$k =$ God knows that there is evil.
$a =$ God is able to prevent evil.
$w =$ God is willing to prevent evil.
$c =$ God is omniscient.
$p =$ God is omnipotent.
$s =$ God is supremely good.

2.5 [1]If Cain married his sister, his marriage was incestuous. [2]If he did not marry his sister, then Adam and Eve were not the progenitors of the entire human race. It follows that [3]if Adam and Eve were the progenitors of the whole human race, then Cain's marriage was incestuous.

$s =$ Cain married his sister.
$i =$ Cain's marriage was incestuous.
$p =$ Adam and Eve were the progenitors of the entire human race.

1. $s \to i$
2. $\neg s \to \neg p$

} Premises

3. $p \to i$ → Conclusion.

1.	$s \to i$	Premises
2.	$\neg s \to \neg p$	
3.	p	assumption.
4.	$\neg\neg p$	$\neg\neg$I 3
5.	$\neg\neg s$	MT 2,4
6.	s	$\neg\neg$E 5
7	i	MP 1,6
8	$p \to i$	$\to$i 3-7

2.6 [1]If Japan is to reduce its huge trade surplus, then it must either convince its citizens to spend more or it must move its manufacturing facilities to other countries. [2]It is not the case that Japan will either increase its imports or convince its citizens to spend more. Furthermore, [3]it is not the case that Japan will either allow foreign companies to compete fairly or move its manufacturing facilities to other countries. Therefore, [4]Japan will not reduce its huge trade surplus.

$s =$ Japan will reduce its huge trade surplus.
$i =$ Japan will increase its imports.
$c =$ Japan will convince its citizens to spend more.
$f =$ Japan will allow foreign companies to compete fairly.
$m =$ Japan will move its manufacturing facilities to other countries.

1.	$s \to (c \vee m)$	Premises
2	$\neg(i \vee c)$	
3.	$\neg(f \vee m)$	
4.	s	assumption
5.	$c \vee m$	MP 1,4
6.	c	Ass
7.	$i \vee c$	$\vee$i 6.
8.	$\bot$	$\neg$E 2,7
	m	Ass
	$f \vee m$	$\vee$i 6
	$\bot$	$\neg$E 2,7
9.	$\bot$	$\vee$E 5, 6-8
10.	$\neg s$	$\neg$I 4-9

1. $s \to (c \vee m)$
2. $\neg(i \vee c)$
3. $\neg(f \vee m)$

} Premises

4. $\neg s$ → conclusion.

2.7 [1]Watson has reddish dirt on his boots. [2]He wouldn't have that if he had not been to the Post Office this morning. [3]If he had been to the Post Office but did not mail a letter this morning, then either he bought some stamps or sent a telegram. [4]If he mailed a letter this morning, then he would have written a letter this morning. [5]He wouldn't buy some stamps unless he ran out of stamps. Therefore, [6]if Watson didn't write a letter this morning and didn't run out of stamps, then he must have sent a telegram this morning.

$d =$ Watson has reddish dirt on his boots.
$p =$ Watson went to the Post Office this morning.
$m =$ Watson mailed a letter this morning.
$s =$ Watson bought some stamps this morning.
$t =$ Watson sent a telegram this morning.
$w =$ Watson wrote a letter this morning.
$o =$ Watson ran out of stamps.

not A unless B

If not B then A

1. d
2. $\neg p \to \neg d$
3. $(p \land \neg m) \to (s \lor t)$
4. $m \to w$
5. $\neg o \to \neg s$

Premises

6. $(\neg w \land \neg o) \to t$

Conclusion

1. d
2. $\neg p \to \neg d$
3. $(p \land \neg m) \to (s \lor t)$
4. $m \to w$
5. $\neg o \to \neg s$

Premises

6.	$\neg w \land \neg o$	assumption.
7.	$\neg w$	$\land e$ 6
8.	$\neg m$	MT 4,7
9.	$\neg\neg d$	$\neg\neg i$ 1
10.	$\neg\neg p$	MT 2,9
11	p	$\neg\neg e$ 10
12.	$p \land \neg m$	$\land i$ 8,11
13.	$s \lor t$	MP 3,12
14.	$\neg o$	$\land e$ 6
15.	$\neg s$	MP 5,14
16	t	$\lor e$ 13,15
17	$(\neg w \land \neg o) \to t$	$\to i$ 6-16

2.8 [1]If there is evil and God does not know it, then God is not omniscient. [2]If there is evil and God knows it but he is unable to prevent it, then God is not omnipotent. [3]If there is evil and God knows it and is able to prevent it but is unwilling to do so, then God is not supremely good. [4]If God exists and there is evil, then either God does not know it or he is unable or unwilling to prevent it. [4]If God exists then he is omnipotent, omniscient, and supremely good. It follows that [5]either there is no evil or there is no God (or both). (Yu Kam Por)

$e =$ There is evil.
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$w =$ God is willing to prevent evil.
$c =$ God is omniscient.
$p =$ God is omnipotent.
$s =$ God is supremely good.

$$\left.\begin{array}{ll} 1. & (e \land \neg k) \to \neg c \\ 2. & (e \land k \land \neg a) \to \neg p \\ 3. & (e \land k \land a \land \neg w) \to \neg s \\ 4. & (g \land e) \to (\neg k \lor \neg w) \\ 5. & g \to (p \land c \land s) \end{array}\right\} \text{premises}$$

6. $\neg e \lor \neg g$

1	$e \wedge \neg k \to \neg c$	*given*				
2	$e \wedge k \wedge \neg a \to \neg p$	*given*				
3	$e \wedge k \wedge a \wedge \neg w \to \neg s$	*given*				
4	$g \wedge e \to \neg k \vee \neg a \vee \neg w$	*given*				
5	$g \to p \wedge c \wedge s$	*given*				
6	$g \vee \neg g$	*LEM*				
7	g	$< ass >$	$\neg g$	$< ass >$		
8	$p \wedge c \wedge s$	$\to E(5,7)$	$\neg e \vee \neg g$	$\vee I(7)$		
9	p	$\wedge E(8)$				
10	c	$\wedge E(8)$				
11	s	$\wedge E(8)$				
12	e	$< ass >$				
13	$\neg k$	$< ass >$				
14	$e \wedge \neg k$	$\wedge I(12,13)$				
15	$\neg c$	$\to E(1,14)$				
16	$\bot$	$\bot I(10,15)$				
17	k	$PBC(13-16)$				
18	$\neg a$	$< ass >$				
19	$e \wedge k$	$\wedge I(12,17)$				
20	$e \wedge k \wedge \neg a$	$\wedge I(18,19)$				
21	$\neg p$	$\to E(2,20)$				
22	$\bot$	$\bot I(9,21)$				
23	a	$PBC(18-22)$				
24	$g \wedge e$	$\wedge I(7,12)$				
25	$\neg k \vee \neg a \vee \neg w$	$\to E(4,24)$				
26	$\neg k$	$< ass >$	$\neg a$	$< ass >$	$\neg w$	$< ass >$
27	$\bot$	$\bot I(17,26)$	$\bot$	$\bot I(23,26)$	$e \wedge k$	$\wedge I(12,17)$
28					$e \wedge k \wedge a$	$\wedge I(23,27)$
29					$e \wedge k \wedge a \wedge \neg w$	$\wedge I(26,28)$
30					$\neg s$	$\to E(3,29)$
31					$\bot$	$\bot I(11,30)$
32	$\bot$	$\vee E(25,26-31)$				
33	$\neg e$	$\neg I(12-32)$				
34	$\neg e \vee \neg g$	$\vee I(33)$				
35	$\neg e \vee \neg g$	$\vee E(6,7-34)$				