7/10

$\forall x \Rightarrow$ for every x

$\exists x \Rightarrow$ for some x

1) Everyone like him/her self

$\Rightarrow \forall x \; likes(x, x)$

2) Everyone like someone

$\Rightarrow \forall x \exists y \; likes(x, y)$

3) Someone like everyone

$\Rightarrow \exists x \forall y \; likes(x, y)$

4) There is a cat that Mary likes

$\Rightarrow \exists x \left(cat(x) \land likes(Mary, x) \right)$

5) Mary likes herself

$\Rightarrow likes(Mary, Mary)$

6) x dad is a cat

$\Rightarrow cat(dad(x))$

7) Cats dad are also cat

$\Rightarrow \forall x \left(cat(x) \rightarrow cat(dad(x)) \right)$

Formular → can be T or F

↓ reffered

statement

predicate

term

cat(felix) ⇒ felix is a cat

likes(Mary, x) ⇒ Mary likes x

cat(dad(felix)) ⇒ Felix's dad is a cat

for all x cat(x) ⇒ Every x is a cat

Felix = dad(Tom) ⇒ Felix is Tom's dad

Term → cannot be T or F

↓ reffered

object

felix → name

dad(felix) → Felix's dad

constant

function

— x —

Order

$$\boxed{\neg, \; \exists x, \; \forall x, \; \land, \lor, \rightarrow, \leftrightarrow}$$

$\Rightarrow \exists x \; p(x) \land q(x)$

means $(\exists x \; p(x)) \land q(x)$

$\Rightarrow \phi_1 \rightarrow \phi_2 \rightarrow \phi_3$

means $(\phi_1 \rightarrow (\phi_2 \rightarrow \phi_3))$

$\Rightarrow \phi_1 \land \phi_2 \land \phi_3$

means $((\phi_1 \land \phi_2) \land \phi_3)$

$p(x) \rightarrow q(f(x), y) \lor r(John)$:

sub formular ⇒ $p(x)$, $q(f(x), y)$, $r(John)$, $q(f(x), y) \lor r(John)$,

$p(x) \rightarrow q(f(x), y) \lor r(John)$

atomic formular ⇒ don't have any subformular other than itself

⇓

$$\boxed{\top, \bot, t_1 = t_2, P(t_1, t_2, \ldots t_n)}$$

$t_1 \neq t_2$ ✗ $\Rightarrow \neg(t_1 = t_2)$

$\forall x \ likes(x, y)$

bounded by

free occurence

$\forall x \ (likes(x, y) \rightarrow \exists y \ cat(y))$

bounded

free

bounded

Prefix notation (in front)

$+(x, y)$

$square(x, y)$

$power(x, y)$

Infix notation (middle)

$(x + y)$

(x / y)

—x—

6 Rules of First order Logic

Rule 1: $\forall E$ (Universal Elimination)

$$\frac{\forall x \ p(x)}{p(t)}$$

→ True

→ any term

Ex:- $\{\forall x \ (cat(x) \rightarrow animal(x)), \ cat(Felix)\}$

1)	$\forall x \ (cat(x) \rightarrow animal(x))$	Premise	(everthing is a cat)
2)	$cat(Felix)$	Premise	(Felix is a cat)
3)	$cat(Felix) \rightarrow animal(Felix)$	$\forall E$ 1	(x = Felix)
4)	$animal(Felix)$	$\rightarrow E$ 2,3	(Felix is animal)

Ex:- $animal(felix)$

1)	$\forall x \ cat(x)$	Premises
2)	$\forall x \ (cat(x) \rightarrow animal(x))$	Premises
3)	$cat(Felix)$	$\forall E$, 1
4)	$cat(Felix) \rightarrow animal(Felix)$	$\forall E$, 2
5)	$animal(Felix)$	MV, 3,4

Rule 2: $\forall I$ (Universal Introduction) (Generalization rule)

Ex:- $\forall x \ animal(x)$ (Proof every x is cat then every x is animal)

1)	$\forall x \ cat(x)$	Premises
2)	$\forall x \ (cat(x) \rightarrow animal(x))$	Premises
	Suppose x_0 is any object	
3)	$Cat(x_0)$	$\forall E$, 1
4)	$Cat(x_0) \rightarrow animal(x_0)$	$\forall E$, 2
5)	$animal(x_0)$	MP, 3,4
	$\forall x \ animal(x)$	$\forall I$, 3-5

Ex:- Everything is a cat and is white, everthing is white

1)	$\forall x (cat(x) \wedge white(x))$	Premises
	Suppose x_1 is any object	
2)	$Cat(x_1) \wedge white(x_1)$	$\forall E$, 1
3)	$white(x_1)$	$\wedge E$, 2
4)	$\forall x \ white(x)$	$\forall I$, 2-3

Rule 3: $\exists I$

$$\frac{cat(\_\_)}{\exists z . cat(z)}$$

Ex:-

1) $\forall x \ likes(x, Mary)$ Premises
2) $\exists y \forall x \ likes(x, y)$ $\exists I, 1$

Not same

Ex:-

1) $\forall x \ like(x, Mary)$ Premises
2) $\underbrace{\exists x \forall x \ likes(x, x)}_{\text{Every one likes themselves}}$ $\exists I, 1$

Rule 4: $\exists E$

Ex: $\forall y \exists x \ likes(y, x)$ (Every one likes someone)

1) $\exists x \forall y \ likes(y, x)$ Premises (There is someone that everyone likes)

Suppose y_0 is any object

Let x_0 be object such that $\forall y \ likes(y, x_0)$

2) $\forall y \ likes(y, x_0)$ Ass
3) $likes(y_0, x_0)$ $\forall E, 2$
4) $\exists x \ likes(y_0, x)$ $\exists I, 3$

5) $\exists x \ likes(y_0, x)$ $\exists E, 1, 2\text{-}4$

6) $\forall y \exists x \ likes(y, x)$ $\forall I, 2\text{-}5$

Rule 5: Substitution rule

Ex:

$$\frac{\begin{array}{c} cat(felix) \\ felix = dad(Tom) \end{array}}{cat(dad(Tom))}$$

Felix is a cat
Felix is Tom's dad
Tom's dad is a cat

Rule 6: =I (Identity rule)

Ex: $t = t$

$$\frac{t = John}{John = John}$$

Ex: likes (dad(Tom), Felix)

1) $\forall x$ likes (x, x) — Premises
2) felix = dad Tom — Premises
3) likes (dad(Tom), dad(Tom)) — $\forall E$, 1
4) likes (dad(Tom), felix) — Subst, 2, 3

Ex:-

Given:- $\forall x\,[cat(x)]$
$\forall x\,[cat(x) \rightarrow animal(x)]$

Goal:- $\forall x\,[animal(x)]$

1) $\forall x\,[cat(x)]$
2) $\forall x\,[cat(x) \rightarrow animal(x)]$

3)	sk1	ass
4)	cat(sk1)	$\forall E$ 1
5)	cat(sk1) → animal(sk1)	$\forall E$ 2
6)	animal(sk1)	$\rightarrow E(4,5)$

7) $\forall x\,[animal(x)]$ — $\forall I(3,6)$

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not in first order logic

Q) There is exactly one cat (There exists a unique cat) $(\exists ! x \ cat(x))$

unique

$\Rightarrow \underbrace{\exists x \ cat(x)} \wedge \underbrace{\forall x \forall y \, [cat(x) \wedge cat(y) \rightarrow x = y]}$

There is at least one cat

For every x & y, if x is cat & y is cat x should be same object as y

Mostly if all x $(\forall x)$ use $\rightarrow$ (implies) if some x $(\exists x)$ use $\wedge$ (and)

$\Rightarrow \exists x \, [cat(x) \wedge \forall y \, (cat(y) \rightarrow y = x)]$

For some of x, if x is a cat and for all of y if y is a cat, then y should be same as x

Q) Mary likes cats

$\forall x \, [cat(x) \rightarrow likes(Mary, x)]$ $\Rightarrow$ For all x if x is a cat then mary likes it

$\forall x \ likes(Mary, cat(x)) \rightarrow$ Error: means Mary likes that x is a cat

$likes(Mary, \underbrace{\forall x \ cat(x)}_{\text{everything is a cat}}) \rightarrow$ Error

Q) Mary likes some cat

$\exists x \, [cat(x) \wedge likes(Mary, x)]$

$\exists x \, [cat(x) \rightarrow likes(Mary, x)] \Rightarrow$ X

10/18

If x is odd, then x^2 is odd

Ans) Ass x is odd

By def. $\rightarrow x = 2a+1$; some a is interger

$x^2 = (2a+1)^2$

$x^2 = 4a^2+4a+1$

$= 2(2a^2+2a)+1$

$x^2 = 2b+1$; some b is interger

$\therefore x^2 =$ odd

If x is odd then $x+1$ is even

Ans)

Ass x is odd

by def. $x = 2n+1$; some n

$x+1 = 2n+1+1$

$= 2n+2$

$= 2(n+1)$

$x+1 = 2b$; some b

by def of even interger $x+1$ is even

$\therefore$ if $x =$ odd then $x+1 =$ even #

-x-

Q If $7x+9$ is even then x is odd

by direct proof

Ass $7x+9$ is even

By def. of even interger $\Rightarrow 7x+9 = 2n$; for some n

$6x+x+9 = 2n$

$x = 2n-6x-9$

$x = 2n-6x-10+1$

$= 2(n-3x-5)+1$

$x = 2b+1$; for some b

by def. of odd interger $x =$ odd

$\therefore x$ is odd

$\therefore$ If $7x+9$ is even, x is odd #

by contraposition proof

Ass x is not odd

From fact 9 $\rightarrow$ x is even

From def. of even $\Rightarrow$ $x = 2n$; for some n

$7x = 14n$

$7x+9 = 14n+9$

$= 14n+8+1$

$= 2(7n+4)+1$

$= 2b+1$; some b

$\therefore 7x+9 = 2b+1$

Hence $7x+9$ is odd

by fact 9; $7x+9$ is not even

Then $7x+9$ is not even

$\therefore$ If $7x+9$ is even, then x is odd #

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Q) $p \to q$, if p then q

p implies q

q if p

Proof Method 1 :- Direct proof

First Assume P

Then Proof q

$\therefore$ If P then q

$$\boxed{\begin{array}{l} \text{Ass} \quad P \\ \quad\quad \vdots \\ \quad\quad q \\ \therefore \text{ If } p \text{ then } q \end{array}}$$

Proof Method 2 :- Contrapositive Proof

First Assume not q ($\neg q$)

Then Proof not p ($\neg p$)

$\therefore$ If p then q

$$\boxed{\begin{array}{l} \text{Ass} \quad \neg q \\ \quad\quad \vdots \\ \quad\quad \neg p \\ \therefore \text{ If } p \text{ then } q \end{array}}$$