Introduction to Logic
Assignment 3

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Problem 1

Determine whether each sentence below can be translated into a formula in Propositional Logic using the given propositional letters and their specified meanings. If so, provide a formula that has the closest meaning to the sentence; otherwise, state that there is no translation.

Example. "If you have not paid your tuition fee, you will not be allowed to graduate."

$p =$ You have paid your tuition fee.
$g =$ You are allowed to graduate.

Ans. Yes. $\neg p \rightarrow \neg g$

(a) "Sweden and Norway will both not adopt the Euro."

$s =$ Sweden will adopt the Euro.
$n =$ Norway will adopt the Euro.

(b) "Sweden and Norway will not both adopt the Euro."

$s =$ Sweden will adopt the Euro.
$n =$ Norway will adopt the Euro.

(c) "Our leader doesn't dye his hair, use makeup, or wear a wig."

$h =$ Our leader dyes his hairs.
$m =$ Our leader uses makeup.
$w =$ Our leader wears a wig.

(d) "By signing this document, you agree to the terms and conditions of this software."

$s =$ You sign this document.
$a =$ You agree to the terms and conditions of this software.

(e) "Unless I see it with my own eyes, and hear it with my own ears, I never will believe it." *(Charles Dicken)*

$s =$ I see it with my own eyes.
$h =$ I hear it with my own ears.
$b =$ I believe it.

(f) "The message was sent from an unknown system but it was not scanned for viruses."

$u =$ The message was sent from an unknown system.
$s =$ The message was scanned for virus.

(g) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

$a =$ Access is granted.
$f =$ The user has paid the subscription fee.
$p =$ The user enters a valid password.

(h) "John has a belief that both Mary and Tom lied."

$j =$ John has a belief.
$m =$ Mary lied.
$t =$ Tom lied.

(i) "Being affiliated with a major political party is not sufficient for you to become the President of the United States."

$a =$ You are affiliated with a major political party.
$p =$ You are becoming the President of the United States.

(j) "High public debt and a sharp rise in consumer prices are necessary and sufficient conditions for an economic crises to happen in the country."

$d =$ The country has high public debt.
$p =$ There is a sharp rise in consumer prices in the country.
$c =$ There is going to be an economic crisis in the country.

(k) John and Mary are friends.

$j =$ John is a friend.
$m =$ Mary is a friend.

Problem 2

Suppose SE-Rocks is a popular rock band at KMITL, whose members are the following students in the Software Engineering program: Alex, Beth, and Carl. Let p_1, p_2, p_3, q_1, q_2 and q_3 be the following propositions:

p_1 : Alex is a lead singer.

p_2 : Beth is a lead singer.

p_3 : Carl is a lead singer.

q_1 : Alex plays guitar.

q_2 : Beth plays guitar.

q_3 : Carl plays guitar.

Write the following propositions about the band using p_1, p_2, p_3, q_1, q_2, q_3 and logical connectives.

(a) Beth does not play guitar and Carl is not a lead singer.

(b) Neither Beth nor Carl is a lead singer.

(c) The band's lead singers also play guitar.

(d) There is one lead singer in the band.

(e) At least two members of the band play guitar.

Problem 3

Rewrite the following formulas by inserting all the omitted parentheses.

(a) $p \vee \neg q \vee r \wedge p \vee q \wedge \neg r$

(b) $p \wedge \neg q \rightarrow p \vee q$

Problem 4

Suppose $\phi = ((p \wedge q) \rightarrow r) \leftrightarrow ((p \rightarrow r) \wedge (q \rightarrow r))$

(a) Describe a truth assignment which makes ϕ **true**.

(b) Describe a truth assignment which makes ϕ **false**.

Problem 5

Show by means of a truth table that the formulas $p \leftrightarrow q$ and $(p \vee \neg q) \wedge (\neg p \vee q)$ are logically equivalent.

Problem 6

Table 1 lists some well-known logical equivalences in propositional logic.

Theorem 1 (Replacement Theorem) *Suppose ϕ is a formula and ψ is a subformula of ϕ. And suppose ψ' is a formula such that $\psi \equiv \psi'$. If ϕ' denotes the formula resulted from replacing an occurrence of ψ in ϕ by ψ', then $\phi \equiv \phi'$.*

The Replacement Theorem allows us to convert a formula into an equivalent one by replacing some subformula ψ in the original formula by any formula equivalent to ψ. The following example shows that the formulas $\neg(\neg p \wedge \neg q)$ and $p \vee q$ are logically equivalent by using repeated applications of the Replacement Theorem and the logical equivalences in Table 1.

Example 1

$$\begin{aligned} \neg(\neg p \wedge \neg q) &\equiv \neg(\neg p) \vee \neg(\neg q) && \textit{by E16} \\ &\equiv p \vee \neg(\neg q) && \textit{by E9} \\ &\equiv p \vee q && \textit{by E9} \end{aligned}$$

By applying the Replacement Theorem and the logical equivalences listed in Table 1, show (as in the previous example) that each pair of formulas below are logically equivalent.

(a) $\neg(p \to q)$ and $p \wedge \neg q$

(b) $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $(\neg p \vee q) \wedge (p \vee \neg q)$

(c) $p \to (q \to r)$ and $q \to (p \to r)$

(d) $(q \vee \neg p) \to (q \wedge p)$ and $p \wedge (r \to p)$

Table 1: Some Logical Equivalences		
	Equivalences	Name
E1	$\phi \wedge \top \equiv \phi$	Identity Laws
E2	$\phi \vee \bot \equiv \phi$	
E3	$\phi \wedge \bot \equiv \bot$	Domination Laws
E4	$\phi \vee \top \equiv \top$	
E5	$\phi \wedge \neg\phi \equiv \bot$	Complement Laws
E6	$\phi \vee \neg\phi \equiv \top$	
E7	$\phi \wedge \phi \equiv \phi$	Idempotent Laws
E8	$\phi \vee \phi \equiv \phi$	
E9	$\neg(\neg\phi) \equiv \phi$	Double Negation Law
E10	$\phi \wedge \psi \equiv \psi \wedge \phi$	Commutative Laws
E11	$\phi \vee \psi \equiv \psi \vee \phi$	
E12	$\phi \wedge (\psi \wedge \chi) \equiv (\phi \wedge \psi) \wedge \chi$	Associative Laws
E13	$\phi \vee (\psi \vee \chi) \equiv (\phi \vee \psi) \vee \chi$	
E14	$\phi \wedge (\psi \vee \chi) \equiv (\phi \wedge \psi) \vee (\phi \wedge \chi)$	Distributive Laws
E15	$\phi \vee (\psi \wedge \chi) \equiv (\phi \vee \psi) \wedge (\phi \vee \chi)$	
E16	$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi$	De Morgan's Laws
E17	$\neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$	
E18	$\phi \wedge (\phi \vee \psi) \equiv \phi$	Absorption Laws
E19	$\phi \vee (\phi \wedge \psi) \equiv \phi$	
E20	$\phi \to \psi \equiv \neg\phi \vee \psi$	
E21	$\phi \leftrightarrow \psi \equiv (\phi \to \psi) \wedge (\psi \to \phi)$	