Introduction to Logic
Assignment 3

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Problem 1

Determine whether each sentence below can be translated into a formula in Propositional Logic using the given propositional letters and their specified meanings. If so, provide a formula that has the closest meaning to the sentence; otherwise, state that there is no translation.

Example. "If you have not paid your tuition fee, you will not be allowed to graduate."

$p =$ You have paid your tuition fee.
$g =$ You are allowed to graduate.

Ans. Yes. $\neg p \to \neg g$

(a) "Sweden and Norway will both not adopt the Euro."

$s =$ Sweden will adopt the Euro.
$n =$ Norway will adopt the Euro. $\neg s \land \neg n$

(b) "Sweden and Norway will not both adopt the Euro."

$s =$ Sweden will adopt the Euro.
$n =$ Norway will adopt the Euro. $\neg(s \land n)$

(c) "Our leader doesn't dye his hair, use makeup, or wear a wig."

$h =$ Our leader dyes his hairs.
$m =$ Our leader uses makeup. $\neg h \land \neg m \land \neg w$
$w =$ Our leader wears a wig.

(d) "By signing this document, you agree to the terms and conditions of this software."

$s =$ You sign this document.
$a =$ You agree to the terms and conditions of this software.

$s \to a$

(e) "Unless I see it with my own eyes, and hear it with my own ears, I never will believe it." *(Charles Dicken)*

$s =$ I see it with my own eyes.
$h =$ I hear it with my own ears.
$b =$ I believe it.

$\neg b \to (s \land h)$

(f) "The message was sent from an unknown system but it was not scanned for viruses."

$u =$ The message was sent from an unknown system.
$s =$ The message was scanned for virus.

$u \to \neg s$

(g) "Access is granted whenever the user has paid the subscription fee and enters a valid password."

$a =$ Access is granted.
$f =$ The user has paid the subscription fee.
$p =$ The user enters a valid password.

$f \land p \to a$

(h) "John has a belief that both Mary and Tom lied."

$j =$ John has a belief.
$m =$ Mary lied.
$t =$ Tom lied.

$j \to (m \land t)$

(i) "Being affiliated with a major political party is not sufficient for you to become the President of the United States."

$a =$ You are affiliated with a major political party.
$p =$ You are becoming the President of the United States.

$\neg (a \to p)$

(j) "High public debt and a sharp rise in consumer prices are necessary and sufficient conditions for an economic crises to happen in the country."

$d =$ The country has high public debt.
$p =$ There is a sharp rise in consumer prices in the country.
$c =$ There is going to be an economic crisis in the country.

$(p \land c) \to d$

(k) John and Mary are friends.

$j =$ John is a friend.
$m =$ Mary is a friend.

No

Problem 2

Suppose SE-Rocks is a popular rock band at KMITL, whose members are the following students in the Software Engineering program: Alex, Beth, and Carl. Let p_1, p_2, p_3, q_1, q_2 and q_3 be the following propositions:

p_1 : Alex is a lead singer.

p_2 : Beth is a lead singer.

p_3 : Carl is a lead singer.

q_1 : Alex plays guitar.

q_2 : Beth plays guitar.

q_3 : Carl plays guitar.

Write the following propositions about the band using p_1, p_2, p_3, q_1, q_2, q_3 and logical connectives.

(a) Beth does not play guitar and Carl is not a lead singer. $\neg q2 \land \neg p3$

(b) Neither Beth nor Carl is a lead singer. $\neg(p2 \lor q3)$

(c) The band's lead singers also play guitar. $(p1 \land q1) \land (p2 \land q2) \land (p3 \land q3)$

(d) There is one lead singer in the band. $p1 \land \neg p2 \land \neg p3$

(e) At least two members of the band play guitar. $q1 \land q2$

Problem 3

Rewrite the following formulas by inserting all the omitted parentheses.

(a) $p \lor \neg q \lor r \land p \lor q \land \neg r$ $(p \lor \neg q) \lor (r \land p) \lor (q \land \neg r)$

(b) $p \land \neg q \to p \lor q$ $(p \lor \neg q) \to (p \lor q)$

Problem 4

Suppose $\phi = ((p \land q) \to r) \leftrightarrow ((p \to r) \land (q \to r))$

(a) Describe a truth assignment which makes ϕ **true**.

(b) Describe a truth assignment which makes ϕ **false**.

Problem 5

Show by means of a truth table that the formulas $p \leftrightarrow q$ and $(p \lor \neg q) \land (\neg p \lor q)$ are logically equivalent.

p	q	$\neg p$	$\neg q$	$\neg p \lor q$	$p \lor \neg q$	$(p \lor \neg q) \land (\neg p \lor q)$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	F	F	F
F	F	T	T	T	T	T	T

Problem 6

Table 1 lists some well-known logical equivalences in propositional logic.

Theorem 1 (Replacement Theorem) *Suppose ϕ is a formula and ψ is a subformula of ϕ. And suppose ψ' is a formula such that $\psi \equiv \psi'$. If ϕ' denotes the formula resulted from replacing an occurrence of ψ in ϕ by ψ', then $\phi \equiv \phi'$.*

The Replacement Theorem allows us to convert a formula into an equivalent one by replacing some subformula ψ in the original formula by any formula equivalent to ψ. The following example shows that the formulas $\neg(\neg p \wedge \neg q)$ and $p \vee q$ are logically equivalent by using repeated applications of the Replacement Theorem and the logical equivalences in Table 1.

Example 1

$$\begin{aligned} \neg(\neg p \wedge \neg q) &\equiv \neg(\neg p) \vee \neg(\neg q) && \textit{by E16} \\ &\equiv p \vee \neg(\neg q) && \textit{by E9} \\ &\equiv p \vee q && \textit{by E9} \end{aligned}$$

By applying the Replacement Theorem and the logical equivalences listed in Table 1, show (as in the previous example) that each pair of formulas below are logically equivalent.

(a) $\neg(p \to q)$ and $p \wedge \neg q$

(b) $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $(\neg p \vee q) \wedge (p \vee \neg q)$

(c) $p \to (q \to r)$ and $q \to (p \to r)$

(d) $(q \vee \neg p) \to (q \wedge p)$ and $p \wedge (r \to p)$

a) $\neg(p \to q) = \neg(\neg p \vee q)$ by E20
$= p \vee \neg q$ by E17

c) $p \to (q \to r) = p \to (\neg q \vee r)$ by E20
$= \neg p \vee \neg q \vee r$ by E20
$= \neg q \vee \neg q \vee r$ by E7
$= q \to (p \to r)$ by E20

b)

d)