Introduction to Logic
Assignment 8

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Definition 1 (Even integers) *An integer n is* even *if and only if $n = 2a$ for some integer $a \in \mathbb{Z}$.*

Definition 2 (Odd integers) *An integer n is* odd *if and only if $n = 2a + 1$ for some integer $a \in \mathbb{Z}$.*

Definition 3 *Two integers are said to have the* same parity *if they are both even or they are both odd; otherwise they are said to have* opposite parity.

Definition 4 (Division) *Suppose a and b are integers. We say that a* divides b, *written $a|b$, if $b = ca$ for some integer c. In this case we also say that a is a* divisor *of b, and that b is a* multiple *of a.*

Definition 5 (Quotients and Remainders) *For any integer $a > 0$ and any integer b, if $b = ca + r$ for some integers c and r such that $0 \le r < a$, then c is called the* quotient *of the division of b by a and r is called the* remainder *of the division of b by a.*

Definition 6 (Rational numbers) *A real number x is said to be* rational *if and only if $x = \frac{a}{b}$ for some integers a, b where $b \neq 0$. A real number that is* not *rational is called an* irrational number.

Fact 1 *An integer n is odd if and only if it is* not *even.*

Problem 1

Prove or disprove each of the following statements.

(a) For any integers a and b, if a and b are odd, then ab is odd.

(b) For any integers a, b, and c, if $a|b$ and $a|c$, then $a|(b+c)$.

(c) If n is an integer, then $n^2 - 3n + 9$ is odd. (Try proof by case)

(d) For any real numbers x and y, if $x + y$ is irrational then either x or y must be irrational.