Mystery Test

Introduction:

This test centers around a young mathematician named Timothy B. Wolfe. Timothy was known to his friends as Tim, and that is what we will call him as well. Tim was a student at Chiles High School, not unlike some of your test proctors and other helpers at Chiles Mini Mu.

The following set of problems will retrace one of Tim's greatest adventures. I will describe what Tim saw and did, and you, the test taker, will attempt to solve the problems he encountered. Let's see if you can match wits with Tim!

N.B.:

The abbreviation "N.B." stands for *Nota Bene*, which is Latin for "note well" and is usually used to indicate something important that should be noted. For this test, make sure you read everything! There is some great information on the last page of this test. You should also read all directions, including the following:

General Directions and Rules

1. You will have 45 minutes to answer all 30 questions on the following pages. Be sure to check out all the pages on this test!

2. Scoring will be 5 points for a correct answer, 1 point for an answer left blank, and 0 points for an incorrect answer.

3. No calculators or other aids are allowed on this test. "Other aids" includes computers, smart watches, cell phones, other humans, pet tigers, teddy bears, and other related appliances.

4. For most questions, answer choice E will be "NOTA", meaning "None Of These Answers". Only select E if you believe none of the other answers are correct, or if you believe the question is flawed in some way.

5. Every question should be answered in the corresponding space to which the question is labeled. As an example, if a question reads "23. Evaluate 1+2+3+…" then the correct answer should be bubbled in spot number 23 on your scantron, regardless of where it appears on the test.

X. If a question does not exist, and is labeled as such, please bubble D.

7. Good luck and have fun!

Portions of this test were inspired by previous Mystery Tests given at Mu Alpha Theta national conventions.

Our story begins on April 1st, 2014 (a Tuesday). Tim was working on a particularly nasty Algebra II homework assignment assigned by Mr. Friedlander. One of the questions read as follows:

24. What is the best description of the conic section defined by the equation $3x^2 + 4y^2 = 7$?

A. Line B. Circle C. Ellipse D. Hyperbola E. NOTA

Tim, being a studious student, knew that he had taken great notes in class. He rummaged through his backpack but unfortunately couldn't seem to find them. Not having his notes, and being a procrastinator (not unlike most other high school students), he decided to go out for a bike ride.

25. Tim's bike routes usually ended up at his best friend Mitt's house. Suppose Tim's bike has a top speed of 5 meters per second. If the distance between Mitt's house and Tim's house is 2 kilometers, how many seconds would it take for Tim to travel to Mitt's house given that Tim takes a direct path and travels at his top speed for the entire trip?

A. 200 B. 400 C. 600 D. 750 E. NOTA

Unfortunately, Tim never made it to Mitt's house. He wasn't paying enough attention to the road while biking; while going down a large hill he didn't brake in time and ended up crashing into a barrier at the bottom of the slope. (Good thing he was wearing a helmet.) After the dust cleared and he picked himself up, Tim noticed an open manhole cover. But this was no ordinary manhole cover; it was shaped like a trapezoid with bases of 2 ft and 3 ft and a height of 4 feet.

26. Given the dimensions above, what is the area of this manhole cover, in square feet?

A. 10 B. 12 C. 20 D. 24 E. NOTA

Tim tried to look into the hole when a very strong gust of wind blew him into the hole.

27. Suppose that the hole was 100 feet deep. Tim's depth after falling can be modeled by the equation $d = 16t^2$ where d represents Tim's depth below the surface (in feet) and t represents the time after Tim started falling (in seconds). How long did it take Tim to reach the bottom of the hole?

A. 1 second B. 1.5 seconds C. 2 seconds D. 2.5 seconds E. NOTA

After falling through the hole, Tim picked himself up (again). That manhole cover didn't look very normal and it definitely didn't lead anywhere normal, so far as Tim could tell. However, it was quite dark underground and Tim couldn't see much but Tim's vision slowly adjusted to the dark.

28. In the first minute, Tim could see only 1.5625% of what was around him but each minute, as his eyes adjusted, the amount he could see doubled. (For example, in the second minute, Tim could see 3.125% of what was around him.) During what minute could Tim see everything (100%) around him?

A. 5 B. 6 C. 7 D. 8 E. NOTA

When Tim's eyes finally adjusted to the darkness, he noticed markings on the wall next to him. Upon closer examination, the writing on the wall read:

Greetings adventurous one,

We welcome you to our secret training grounds! Within these rooms you will find many questions to test your mathematical prowess. We encourage you to read and work quickly and carefully. After completing our training, we hope you emerge as a better, smarter mathematician ready to tackle any other problems that you may happen to cross paths with.

*Do not be afraid of what lies ahead! Be **bold**!*

"What a strange message!" Tim thought. He shrugged and continued down a corridor. There, problems began to appear on the wall:

It is always necessary to know the basics, for without a solid foundation, higher math is impossible.

1. Which of these numbers is composite?

A. 1 B. 2 C. 3 D. 4 E. NOTA

2. Simplify $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$.

A. $1\frac{1}{2}$ B. 2 C. $2\frac{1}{12}$ D. $4\frac{1}{6}$ E. NOTA

3. What is the smallest positive number that is both a perfect square and a perfect cube?

A. 8 B. 16 C. 64 D. 81 E. NOTA

4. What is the name for a number that cannot be written as a fraction with integer numerator and denominator?

A. Complex B. Irrational C. Rational D. Unnatural E. NOTA

***What** we know today is influenced by the past, and many of our problems can be solved using techniques others before us found.*

5. Using Euler's formula, how many vertices are on a solid having 15 edges and 7 faces? (Euler's formula says that for any 3D solid, the number of vertices plus the number of faces gives the number of edges plus two.)

A. 22 B. 20 C. 10 D. 8 E. NOTA

6. The great mathematician Gauss found that the formula for the sum of the first n positive integers was $\frac{n(n+1)}{2}$. Using the work of Gauss, what is the sum of the first 999 positive integers?

A. 498500 B. 499000 C. 499500 D. 500500 E. NOTA

*Mathematics **is** everywhere, if you know where to look!*

7. Our food can be quite geometric! Suppose a donut can be unwrapped into a cylinder with a radius of 1 cm and a height of 5 cm. What is the volume of this unwrapped donut?

A. 5 cm^3 B. 5π cm^3 C. 10π cm^3 D. 25π cm^3 E. NOTA

8. One day, you take a trip to a friend's house, 24 miles away. You start off biking at 12 mph, but halfway there, your bike breaks and you are forced to walk the rest of the way at 4 mph. What was your average speed for this trip?

A. 6 B. 8 C. 9 D. 10 E. NOTA

9. You must be able to recognize magic in the mathematics around you. Which of the following is a valid "magic square"?

A.

1	8	5
7	4	2
6	3	9

B.

1	2	3
4	5	6
7	8	9

C.

1	3	5
7	9	8
6	4	2

D.

8	1	6
3	5	7
4	9	2

E. NOTA

10. Logic is a type of math! Suppose that if a person doesn't wear polo shirts, then that person isn't cool. Furthermore, if a person does wear polo shirts, then that person is cool. If Tim is cool, can we say for sure that Tim wears polo shirts? (A person either wears polo shirts or doesn't wear polo shirts; there aren't any other options.)

A. Yes, we know for sure that Tim wears polo shirts.
B. No, we can't know for sure that Tim wears polo shirts.
C. Read answer D but don't bubble C or D. Don't dispute because of these answers either!
D. Hexagonal numbers are those numbers of the form $n(2n-1)$.
E. NOTA

11. While one must know his or her math, it is also important to know what is not mathematical. Which of the following is not a mathematical term?

A. Complex Number B. Hashtag C. Hyperbola
D. Row-Echelon Form E. NOTA

Great mathematicians do not fear large numbers!

12. If $G = 2^4$ and $M = 5^4$ then how many digits does the number $G \cdot M$ have?

A. 3 B. 4 C. 5 D. 8 E. NOTA

13. Suppose $A = \frac{2^7 - 2^6}{2}$ and $B = 32$. What is the relationship between A and B?

A. $A < B$ B. $A = B$ C. $A > B$
D. Cannot be determined E. NOTA

Your travels may bring you to places far and wide.

14. You must follow the rules, wherever you are. Though the order of operations is standard, you never know if they may change in exotic places. Evaluate $1 \times 3 + 5 \div 2 + 6 - 4$ using reverse order of operations (addition and subtraction first, followed by multiplication and division).

A. -3 B. 2 C. 6 D. 7.5 E. NOTA

15. Strange lands may have strange operations. Mathematicians in Oddlandia have created the operation ■ such that $x \blacksquare y = x - y + x \cdot y - 2$. What is $1 \blacksquare 10$? (Use normal order of operations now.)

A. 17 B. 1 C. 0 D. -1 E. NOTA

The *world around us is filled with patterns, and many are mathematical.*

16. What is the name of the famous sequence $1, 1, 2, 3, 5, 8, 13, 21$ …?

A. Catalan Numbers B. Eisenstein Sequence C. Euler Numbers
D. Pascal's Sequence E. NOTA

17. A geometric sequence is one where each term is multiplied by the same number to get the next term. For example, the sequence $3, 6, 12, 24, 48$ … is a geometric sequence because each term is multiplied by 2 to get the next term in the sequence. Is the sequence $1, -2, 4, -8, 16$ … a geometric sequence?

A. Yes B. No C. Cannot be determined
D. Only on Saturdays E. NOTA

18. Hexagonal numbers are defined using a particular formula. Which of these is not a hexagonal number?

A. 1 B. 6 C. 15 D. 28 E. NOTA

~~19. What comes next in the following sequence: 2, 3, 5, 9, 17, ….~~ Question 19 does not exist.

Mathematics is an underlying ***theme*** *in many* ***of*** *our games.*

20. If you pick two different cards from a standard deck of 52 cards (no jokers) without replacement, which of the following is most likely? (For this question: Jacks, Queens, and Kings are face cards while 2 through 10 are number cards. Ace is neither.)

A. Picking a heart and then the Ace of spades
B. Picking two clubs
C. Picking a face card diamond and then a number card heart
D. Picking a spade and then a diamond
E. NOTA

Many math problems are simpler than they seem, if you know the shortcut!

21. What is $97 \cdot 103$?

A. 9981 B. 9991 C. 9997 D. 10009 E. NOTA

22. Let $G =$ the GCF of 24 and 32, and let $L =$ the LCM of 24 and 32. What is $G \cdot L$?

A. 96 B. 192 C. 384 D. 768 E. NOTA

23. Suppose there is a painting company, with many painters that work at the same rate. If 3 painters can paint 3 houses in 3 days, how many days will it take for 6 painters to paint 6 houses?

A. 6 B. 12 C. 24 D. 36 E. NOTA

After finishing question 23, a door suddenly appeared in the wall next to Tim. He walked through, and discovered a ladder leading right back up to the hole he fell through. He walked his bike home and decided to take a nice nap before tackling his homework again.

The writer of this test would like to congratulate you on your successful completion of the 2014 ***Mini Mu*** *Mystery Test! A lot of effort was put into this test to make it interesting, fun, and challenging for the different levels of students taking the test.*

The writer of ***this*** *test would also like to remind all students that mathematics is a lifelong enjoyment that doesn't simply end at the conclusion of math class or a math competition. You can begin your mathematical journey by participating in Mini Mu each* ***year****; once you've graduated from this competition, it would be awesome if you helped out at Chiles Mini Mu or your own math competitions hosted by your school.*

By the way, row-echelon form is a form that a matrix can take, and it's a valid mathematical term. Trust me. (I thought I'd share.)

Before you go, I, the test writer, have some final questions for you:

29. Look carefully! The variables X, Y, and Z appear on this test somewhere. What is X + Y + Z?

A. 12 B. 19 C. 24 D. 42 E. NOTA

30. Were you **bold** enough? Answer the hidden question on this test.

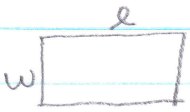
A. Mini Mu B. Games C. Math D. Awesomeness E. NOTA

Tim Wolfe
3/27/14

NOTE: The test writer would like to provide some of Tim's notes to help the test taker. The information below may or may not help with the taking of the Mystery Test.

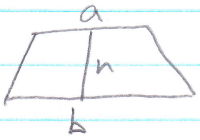
Algebra 2 Notes

Geometry review



$A = \ell w$

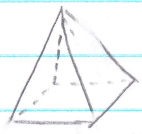
$A = \pi r^2$
$C = 2\pi r$



$A = \left(\frac{a+b}{2}\right) h$

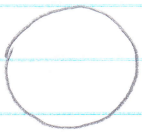
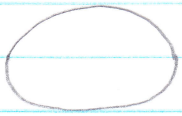
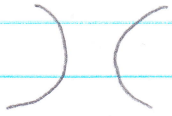
$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$



$V = \frac{1}{3}$ base $\times$ height

Conic Sections

name	shape	equation	
circle		$x^2 + y^2 =$ something	
ellipse		$ax^2 + by^2 =$ something	$a>0, b>0, a \neq b$
parabola		$y = ax^2 +$ something $x = ay^2 +$ something	opens up/down opens left/right
hyperbola		$ax^2 - by^2 =$ something or $ay^2 - bx^2 =$ something	$a>0, b>0$ $a>0, b>0$