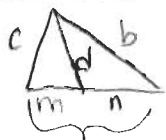# Solutions - Mini Mu 2007: Triangles
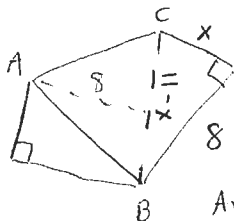
(1) Stewart's Theorem

$cnc + bmb =$
$dad + man$

$10 \cdot 2 \cdot 10 + 8 \cdot 4 \cdot 8$
$d^2 \cdot 6 + 2 \cdot 4 \cdot 6$
$2\sqrt{17}$

(2)

$64 + x^2 = (11-x)^2$
$x = \frac{57}{22}$

$\text{Area} = (8)(11 - \frac{57}{22}) \cdot \frac{1}{2}$
$= \frac{370}{11}$

(3) $\triangle AEB \sim \triangle DEC$

$AB = \frac{15}{2}$

$\triangle FDB \sim \triangle FAC$

$\frac{DF}{AF} = \frac{5}{7} \qquad AF = \frac{7}{5} DF$

$CF = 15 + DF = \frac{7}{5}(\frac{7}{5}DF + \frac{15}{2})$

$DF = \frac{75}{16}$

(4) $BF = \frac{15}{2} + AF = \frac{15}{2} + \frac{7}{5} \cdot \frac{75}{16}$

$= (\frac{15}{2} + \frac{105}{16}) \cdot 16 = 225$

(5) $\overset{\frown}{EA} = x$

$\frac{360 - 2x}{2} = 50 \qquad x = 130$

$\triangle EOD \cong \triangle DOF \cong \triangle FOB \cong \triangle BOA$

$\frac{130}{2} = 65$
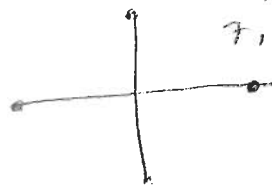
(6) $\triangle DBC \sim \triangle EAC$

$\frac{4}{8} = \frac{6}{8+x} \qquad x = 4$

$V = \frac{1}{3}(9)\pi(1^2) = 36\pi$

(7)

$15, 20, 25 \Rightarrow (15,20); (20,15); (-15,20)$
$7, 24, 25 \qquad (-20,15), (-15,-20);$
$\qquad\qquad (-20,-15); (15,-20);$
$\qquad\qquad (20,-15) \Rightarrow 8$

$(25,0); (0,25); (-25,0); (0,-25) = 4$

$8 + 4 + 8 = 20$ possible locations

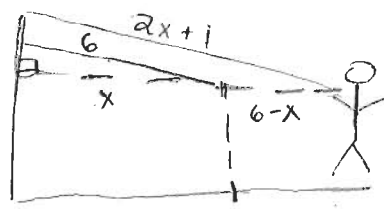**(8)** $20^2 + 21^2 = 29^2$

Circumference $= 10\pi$

revolutions $= \frac{29}{10\pi}$

(9)

$\frac{6}{x} = \frac{2x+1}{6}$

$2y^2 + x = 36$

$x = 4$

(10) centroid

$(\frac{3+8+13}{3}, \frac{6+1+8}{3}) \cdot (8, 5)$

(11)

3	6
8	1
13	8
3	6

$3 - 48 = -45$
$64 - 13 = 51$
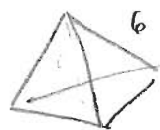$78 - 24 = +54$
$\overline{\qquad\qquad 60}/2 = 30$

(12) Angle of reflection = angle of incidence

$126°$

(13)
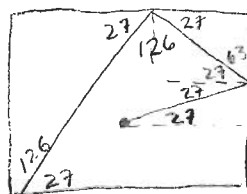
Volume of tetrahedron:
$\frac{s^3\sqrt{2}}{12} = \frac{6^3\sqrt{2}}{12} = 18\sqrt{2}$

Test and Solutions by: Clara Vu

(14)

55 | 11/2 | X | 80−X

$$\frac{\frac{11}{2}}{80-x} = \frac{55}{80}$$ X = 72

(15)

9 | 25 | X−9 | y | 13
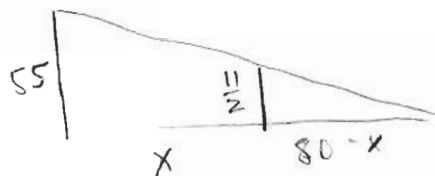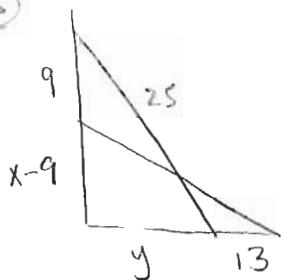
$$x^2 + y^2 = (x-9)^2 + (y+13)^2$$
$$18x - 26y = 250$$
$$3x + 26y = 254$$
$$y = 7$$

(16) 30-60-90

80 | 40 | 40√3 | 30 | √20√3 | 60

(17)

N, 90°, 50°, 40°, 20°, 60, X, E
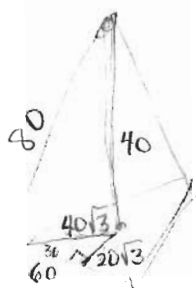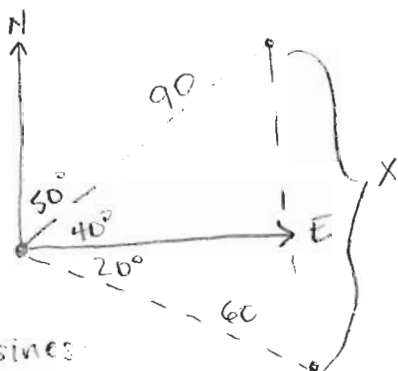
Law of cosines

$$x^2 = 90^2 + 60^2 - 2(90)(60)\cos 60°$$
$$= 8100 + 3600 - 10800 \cdot \frac{1}{2}$$
$$= 6300$$
$$x = 30\sqrt{7}$$

(18) To be retraceable, network must have ≤ 2 odd points.

(19)

S_2 30 S_1
60 30
20 13
5
B_1 30 60
10 B_2
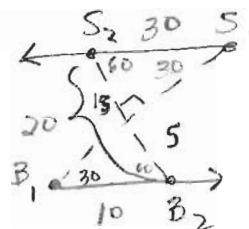
$$\frac{10 \text{ mi}}{0.5 \text{ ho}} = 20 \text{ mph}$$

(20)

$$AD^2 = 100^2 + 50^2 \Rightarrow AD = 50\sqrt{5}$$
$$BC = AC - AB$$

$$\frac{AC}{50\sqrt{5}} = \frac{50}{100} \qquad AC = 25\sqrt{5}$$

$$\frac{AB}{50\sqrt{5}} = \frac{20}{100} \qquad AB = 10\sqrt{5}$$

$$BC = 25\sqrt{5} - 10\sqrt{5} = 15\sqrt{5}$$

(21)

$$CF = \sqrt{AC^2 - AF^2} = 25$$
$$Area = \left(\frac{25+50}{2}\right)(50) = 1875$$

(22) 45-45-90

X√2, x

smallest hypotenuse = 8

(23)

$$4\sqrt{2} + 8 + 8\sqrt{2} + 16 + 16\sqrt{2} + 32 + 32\sqrt{2} + 64 +$$
$$64\sqrt{2} - 4\sqrt{2} = 120 + 120\sqrt{2}$$

(24) $r = 4\sqrt{3}$

$$SA = (8\sqrt{3}\,\pi)(10) + 2(4\sqrt{3})^2 \pi - 2\left(\frac{12^2\sqrt{3}}{4}\right)$$
$$+ 3(12)(10) = 80\pi\sqrt{3} + 96\pi - 72\sqrt{3}$$
$$+ 360$$

$$8\sqrt{3}(10\pi - 9) + 24(4\pi + 15)$$
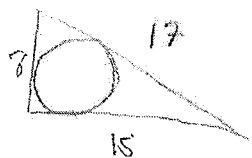$$a = 10 \quad b = -9 \quad c = 4 \quad d = 15$$
$$10 + 9 + 4 + 15 = 8$$

(25) First one is true

B — 7-24-25 is example of a triple of Pythagoras

C   8-15-17 is example of a triple of Plato

I   hypotenuse always odd

(26)

17, 8, 15 triangle

$A = \frac{8 \cdot 15}{2} = 60$

inradius $= \frac{8 + 15 \cdot 17}{2} = 3$

$\frac{9\pi}{60} = \frac{3\pi}{20}$
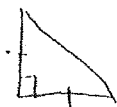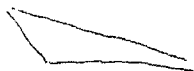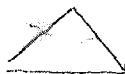
(27) circumcenter

(28) 3

isosceles right   scalene obtuse   equilateral acute

(29) $R = \frac{a \cdot b \cdot c}{Area \cdot 4} = \frac{28 \cdot 45 \cdot 53}{(\frac{28 \cdot 45}{2})(4)}$

$= \frac{53}{2}$ $2r - 3 = 47$

(30) Central angle of $1° \Rightarrow$

360 slices

Minimum angle creates

Maximum number of slices