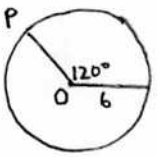
Chiles Mini Mu 12/08/2007 Geometry

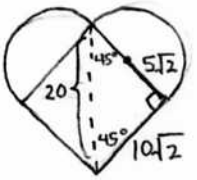
Geometry Superheroes - Circles and Polygons SOLUTIONS

1.  $\underline{2}$ **B**

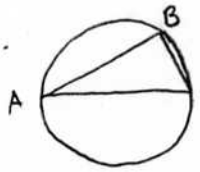
2. The sum of the exterior angles is always 360°. If n is the number of sides of the regular polygon, $\frac{360^\circ}{n} = 1^\circ \Rightarrow n = 360$. **C**

3. 

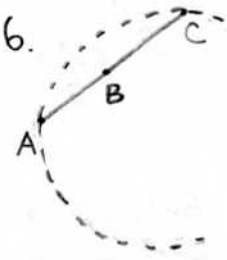
Arc $\overset{\frown}{PQ}$ is $\frac{120^\circ}{360^\circ} = \frac{1}{3}$ of the entire circumference. The circumference is $2(6)\pi = 12\pi$, so $\overset{\frown}{PQ} = \frac{1}{3}(12\pi) = 4\pi$. **B**

4. 

Using 45°-45°-90° relationships, the side of the square has length $\frac{20}{\sqrt{2}} = 10\sqrt{2}$, so the square has area $(10\sqrt{2})^2 = 200$. The radius of the semicircles is $\frac{1}{2}(10\sqrt{2}) = 5\sqrt{2}$. Together, the two semicircles make a circle of area $\pi(5\sqrt{2})^2 = 50\pi$. So the total area is $200 + 50\pi$. **C**

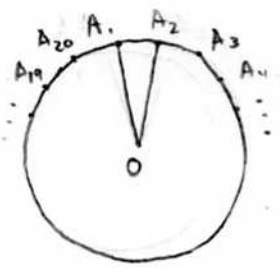
5. 

$\overset{\frown}{AC}$ is a semicircle, so $\overset{\frown}{AC} = 180^\circ$. $\angle ABC$ is an inscribed angle, so $\angle ABC = \frac{1}{2}\overset{\frown}{AC} = 90^\circ$. **C**

6. 

No. If points A, B, and C are collinear, then no circle passes through all three, because a line intersects a circle at at most 2 points. **B**

Note: Love bugs migrated to Florida from the west. The story that they are a failed UF genetics experiment is a popular urban legend.

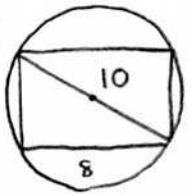
7. 

The vertices of the 20-gon are evenly spaced about the circle, so $\overset{\frown}{A_1A_2}$ represents $\frac{1}{20}$ of the entire circle. So $\angle A_1OA_2 = \overset{\frown}{A_1A_2} = \frac{1}{20}(360^\circ) = 18^\circ$.

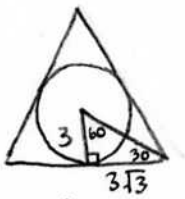
E

Chiles Mini Mu 12/08/2007 Geometry

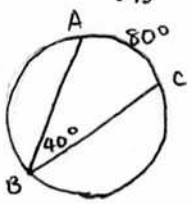
Geometry Superheroes - Circles and Polygons SOLUTIONS

8. 

By the Pythagorean Theorem, the diagonal of the rectangle is $\sqrt{6^2+8^2} = \sqrt{100} = 10$. The diagonal is a diameter, so the radius is 5, and the area is $\pi(5)^2 = 25\pi$. **B**

9. 

Using $30°$-$60°$-$90°$ relationships, we find that half the side length is $3\sqrt{3}$, so the side length is $6\sqrt{3}$. **D**

10. 

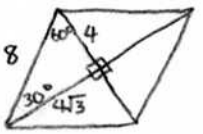
$\overset{\frown}{AC} = 2\angle ABC = 80°$, so $\overset{\frown}{ABC} = 360° - 80° = 280°$. **C**

11. 

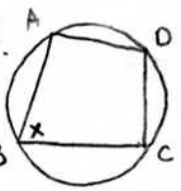
The areas of the large, middle, and small circles are 9π, 4π, and π, respectively. The area inside the middle circle but outside the small circle is $4\pi - \pi = 3\pi$. So the probability is $\frac{\text{Area of desired region}}{\text{Area of entire target}} = \frac{3\pi}{9\pi} = \frac{1}{3}$. **B**

12. 

The radius of the smaller circle is $\frac{1}{2}$ the radius of the larger circle, so the area of the smaller circle is $(\frac{1}{2})^2 = \frac{1}{4}$ the area of the larger circle, or $\frac{1}{4}A$. The desired area is $A - \frac{1}{4}A = \frac{3}{4}A$. **D**

13. 

Drawing the diagonals of the rhombus, we form four congruent $30°$-$60°$-$90°$ triangles. The legs have lengths 4 and $4\sqrt{3}$, so the sum of the diagonal lengths is $2(4) + 2(4\sqrt{3}) = 8 + 8\sqrt{3}$. **D**

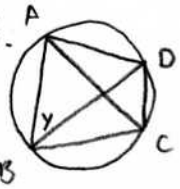
14. 

$\overset{\frown}{ADC} = 2\angle ABC = 2x$

$\overset{\frown}{ABC} = 360° - \overset{\frown}{ADC} = 360° - 2x$

$\angle CDA = \frac{1}{2}\overset{\frown}{ABC} = 180° - x$ **D**

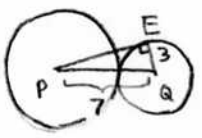
Geometry Superheroes - Circles and Polygons SOLUTIONS

15. 

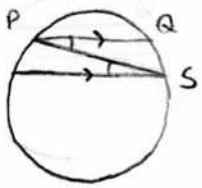
$\angle ABD$ and $\angle ACD$ are both inscribed angles that subtend the same arc, so they have equal measures. $\angle ACD = y$.

B

16. Each of the pieces has side length 1. The area of an equilateral triangle is $\frac{(1)^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$. The area of a square is $(1)^2 = 1$. The regular hexagon can be split into 6 equilateral triangles  , so its area is $6\left(\frac{\sqrt{3}}{4}\right) = \frac{3\sqrt{3}}{2}$. The total area is $6\left(\frac{\sqrt{3}}{4}\right) + 6(1) + \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} + 6 + \frac{3\sqrt{3}}{2} = 6 + 3\sqrt{3}$. **B**

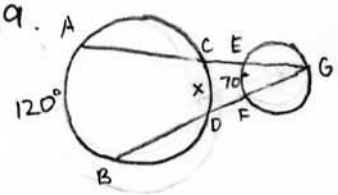
17. 

The radius QE is perpendicular to the tangent at E. $PQ = 4 + 3 = 7$, so by the Pythagorean Theorem, $PE = \sqrt{7^2 - 3^2} = \sqrt{40} = 2\sqrt{10}$. **B**

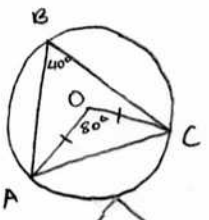
18 

If we draw transversal PS, we see that $\angle QPS = \angle PSR$ (alternating interior angles). Therefore, $\overset{\frown}{PR} = \overset{\frown}{QS}$, because their inscribed angles are equal, so $\overset{\frown}{QS} = 10°$.

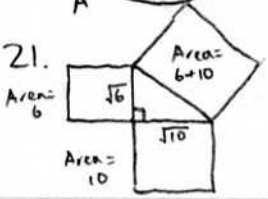
E

19. 

Define point G as shown, and let $\overset{\frown}{CD} = x$. $\angle EGF = \frac{1}{2}\overset{\frown}{EF} = 35°$. It also equals $\frac{\overset{\frown}{AB} - \overset{\frown}{CD}}{2}$, so $\frac{120° - x}{2} = 35°$. Solving, we get $x = 50°$. **B**

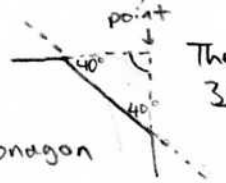
20. 

$\overset{\frown}{AC} = 2\angle ABC = 80°$, so $\angle AOC = 80°$. The radii of a circle are equal, so triangle OAC is isosceles. Therefore, $\angle OAC = \frac{1}{2}(180° - \angle AOC) = 50°$. **C**

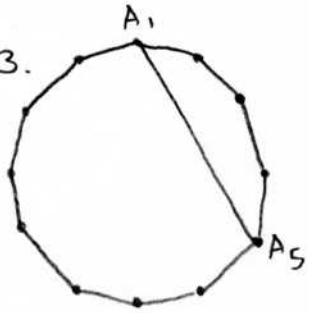
21. 

The square of one side of the triangle is equal to the sum of the squares of the other two, so the triangle is right. The legs are the side lengths of the original squares, $\sqrt{6}$ and $\sqrt{10}$, so the area is $\frac{1}{2}(\sqrt{6})(\sqrt{10}) = \sqrt{15}$. **A**

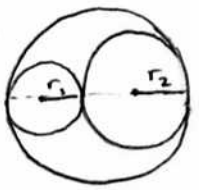
Geometry Superheroes - Circles and Polygons SOLUTIONS

22. 

The exterior angles of a regular nonagon measure $\frac{360°}{9} = 40°$. Using the triangle shown to the left, the desired angle is $180° - 2(40°) = 100°$. **A**

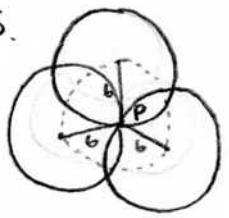
23. 

Each diagonal that crosses A_1A_5 has one endpoint on the left side of A_1A_5 and the other on the right side. There are 7 vertices on the left and 3 on the right, so there are $(7)(3) = 21$ total combinations. **A**

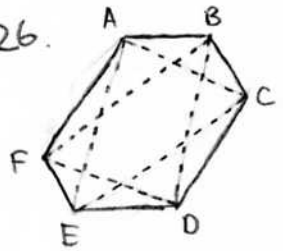
24. 

Let r_1 and r_2 be the radii of the two smaller circles. The diameter of the largest circle is $2r_1 + 2r_2$, so its radius is $\frac{1}{2}(2r_1 + 2r_2) = r_1 + r_2$. Hence, the area inside the largest circle but outside the smaller circles is

$\pi(r_1+r_2)^2 - \pi r_1^2 - \pi r_2^2 = \pi r_1^2 + 2\pi r_1 r_2 + \pi r_2^2 - \pi r_1^2 - \pi r_2^2 = 2\pi r_1 r_2$. This is equal to 64π, so $2\pi r_1 r_2 = 64\pi \Rightarrow r_1 r_2 = 32$. **C**

25. 

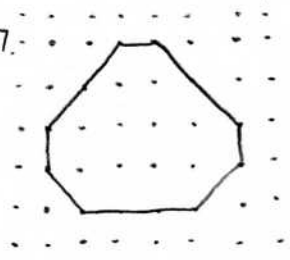
Notice that the centers of all three circles are each 6 units away from P, so they all lie on the circle centered at P with radius 6. Only 1 circle can pass through three given points, so this is the desired circle. It has radius 6. **B**

26. 

Since $\overline{AB}$ and $\overline{DE}$ are equal and parallel, quadrilateral ABDE is a parallelogram, so $AE = BD$. We similarly show that $DF = AC = 5$ and $FB = CE = 8$. Thus, the perimeter of BDF is $BD + DF + FB = 6 + 5 + 8 = 19$.

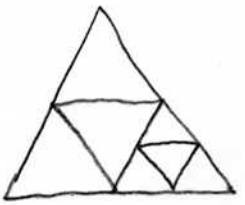
C

Geometry Superheroes - Circles and Polygons SOLUTIONS

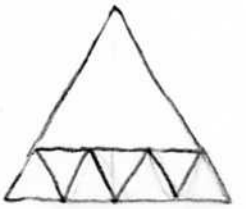
27. 

All the line segments drawn are sides or diagonals of unit squares. Therefore, the only angles we can make are $45°$, $90°$, and $135°$. If our polygon has n sides, then the sum of its angles is at most $135n$. But the sum of the angles is $180(n-2)$, so $180(n-2) \le 135n \Rightarrow 45n \le 360 \Rightarrow n \le 8$. It is indeed possible to make an 8-sided polygon, so 8 is the answer. **C**

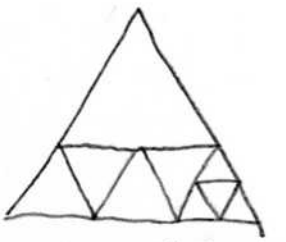
28. $n=7$, 8, and 9 are all possible:



For $n=7$, start with 4 equilateral triangles, then divide one of them into 4.

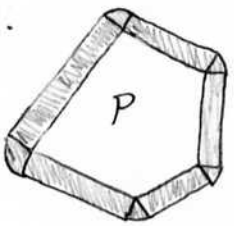


$n=8$: Similar to $n=6$.



$n=9$: Start with 6 triangles, then divide one of them into 4.

D. Note: If we can get k triangles, then we can get $k+3$ triangles by splitting one into 4. Since we can get 6, 7, and 8, we can get anything beyond that.

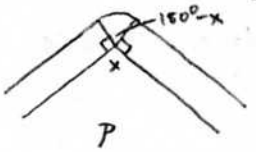
29. 

The region Q consists of

(1) rectangular strips whose lengths are the sides of P, and whose widths are 1.

(2) circular sectors "in between" the rectangular strips; these are centered at the vertices of P and have radius 1.

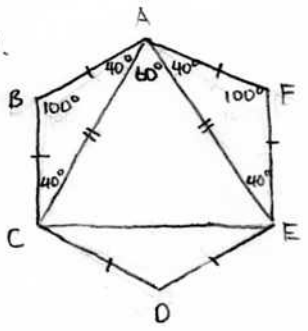
We first find the area of the rectangles. Each rectangle has area equal to the side length it is along. Therefore, the sum of the areas of the rectangles equals the sum of the side lengths of P, which is p.



Now, the sectors. Choose a sector, and let the angle of P at that vertex be x. The angle of the sector is $360°-90°-90°-x = 180°-x$; that is, it is equal to the exterior angle at that point. Since the sum of the exterior angles is $360°$, if we combine all the sectors, we get a full circle of area $\pi(1)^2=\pi$. The total area is $p+\pi$. **A**

Geometry Superheroes - Circles and Polygons SOLUTIONS

30.



By SAS, triangles ABC and AFE are congruent, so $AC = AE$. $\triangle ABC$ is isosceles, so $\angle CAB = \frac{1}{2}(180° - \angle ABC) = 40°$, and similarly, $\angle FAE = 40°$. Thus, $\angle EAC = \angle FAB - \angle FAE - \angle CAB = 140° - 40° - 40° = 60°$. Since $AC = AE$ and $\angle EAC = 60°$, $\triangle ACE$ is equilateral (by SAS, it is congruent to an equilateral triangle). Thus, $CE = AC$, so triangles ABC and CDE are congruent by SSS. Hence, $\angle CDE = \angle ABC = 100°$. [A]