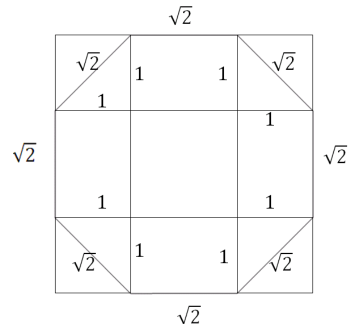
**Geometry – Africa Solutions**

1. Doubling the side length of the square base will quadruple the area of the base. Thus the volume is quadrupled because the height is the same. $\boxed{\text{B) } 4(86{,}682{,}960)}$

2. All of the listed figures have two pairs of congruent sides. $\boxed{\text{E) NOTA}}$

3. The contrapositive of a conditional logically equivalent.
$\boxed{\text{C) “If I do not save my Snapple for later, it rainstoday.”}}$




4. The transformation is called a $\boxed{\text{A) Dilation}}$.

5. The two are $\boxed{\text{B) Similar}}$ .

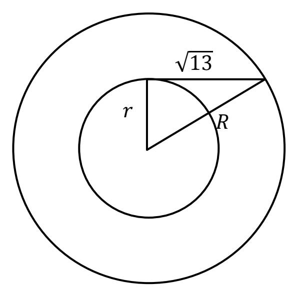
6. We split into 9 sections and add the areas. $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+2+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=4\sqrt{2}+4\left(\frac{1}{2}\right)+2=4+4\sqrt{2}$. Alternatively, we see the large square with side length $2+\sqrt{2}$ and subtract.
$(2+\sqrt{2})^2-4\left(\frac{1}{2}\right)=6+4\sqrt{2}-2$, so the answer is $\boxed{\text{D) } 4+4\sqrt{2}}$.

7. Since $x, y \neq 0$, we multiply the equation by $xy$ on both sides to get $4x+5y=3$. So the slope of the line is $\boxed{\text{B) } -\frac{4}{5}}$ .

8. If you don’t know what an obelisk is or don’t know what the Washington Monument looks like, you can use Euler’s formula for polyhedrons: $vertices + faces = edges + 2$
Using the given information, $9+faces=16+2$. So the number of faces is $\boxed{\text{D) } 9}$

9. It was Euclid that wrote the *Elements*. Euclid is considered the “Father of Geometry”.
$\boxed{\text{C) Euclid}}$

10. I) Counterexample: 2 skew lines.
II) Counterexamples: 2 coincident lines or 2 skew lines
III) Always true. There is no way for 3 point to not be coplanar.
$\boxed{\text{A) III only}}$




11. The longest line segment that can fit in an annulus is a chord of the outer circle that is tangent to the inner circle. Since $\sqrt{52}=2\sqrt{13}$, we can form a right triangle with the radii, legs $\sqrt{13}$ and $r$, and hypotenuse $R$.
So $13+r^2=R^2$ or $R^2-r^2=13$.
The area of the annulus is $\pi R^2-\pi r^2 = \pi(R^2-r^2)=13\pi$ $\boxed{\text{E) NOTA}}$

12. Folding in leg to leg will give the new 45-45-90 triangle the half the area of the original mast. The area of the original mast is $\frac{(100)^2}{2}=5000$. Thus the folded mast has area $\frac{5000}{2}=2500$. Alternatively you can notice the folded mast has legs of $50\sqrt{2}$, so the area would be $\frac{(50\sqrt{2})^2}{2}$ or $\boxed{\text{B) } 2{,}500}$.

13. $19 \times 900 =$ $\boxed{\text{C) } 17{,}100}$

14. Using the given information, we see that all six smaller triangles are equal in area. Thus there area of $\Delta COD =$ area of $\Delta DOB = k$. Alternatively, we can see that all the cevians are medians, so $CD = DB$, and since $\Delta COD$ and $\Delta DOB$ have the same altitude, area of $\Delta COD =$ area of $\Delta DOB =$ $\boxed{\text{A) } k}$.

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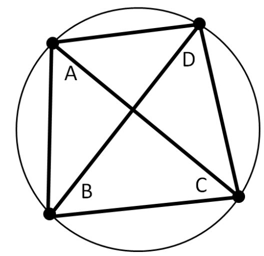


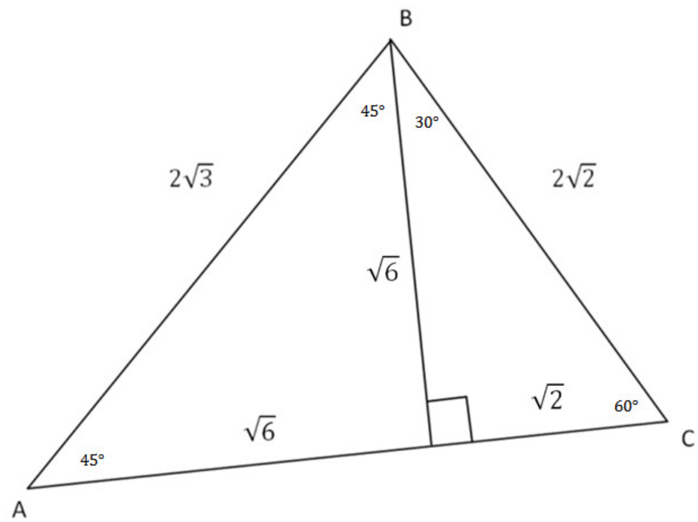

15. Shoe-lacing, we get 19 for the area of the triangle. $\boxed{\text{A) 19}}$
16. Notice that the quadrilateral $ABCD$ is symmetric about $\overleftrightarrow{BD}$. Therefore, $\overline{BD}$ is a diameter of the circle. $BD = 400$.
$ABD$ is then a right triangle,
$AB^2 + AD^2 = BD^2$
$BD = 100\sqrt{13}$ which contradicts itself. Therefore, the figure is impossible $\boxed{\text{E}}$
17. By Triangle Inequality, third side must be between $n$ and $3n$ exclusive. The integral sides can be $n+1, n+2, n+3, \ldots 3n-2, 3n-1$, so there are $2n-1$ possible values. $\boxed{\text{E) NOTA}}$
18. The minute hand is $90°$ from the 6 o'clock position. The hour hand is $1\frac{45}{60} \times 30°$ or $52.5°$ from the 6 o'clock position. Subtracting, we get $37.5°$ as the angle between the hands. The supplement of the complement of $37.5°$ is $180° - (90° - 37.5°)$ or $\boxed{\text{D) } 127.5°}$
19. Since there is a circle inscribed in the 19-gon, the polygon has a center (in this case it happens to be regular). Thus, $Area = apothem \times semiperimeter$ (apothem=inradius). You can think of it as splitting the polygon into 19 triangles and using the $\frac{1}{2}bh$ formula 19 times. Area is $\frac{1}{2} \times 2 \times 12.68$ or just 12.68. Remember, we have to subtract the area of the circle. So the answer is $\boxed{\text{D) } 12.68 - 4\pi}$.
20. We know that geometric mean of two numbers is always less than or equal to the arithmetic mean, with equality when both numbers are same. In other words, in a right triangle, the length altitude (geometric mean) the hypotenuse is always less than or equal to the length of the median (arithmetic) to the hypotenuse. Writing an inequality, $\sqrt{25000x} \leq \frac{25000+x}{2}$. Rearranging, we see that $2\sqrt{25000x} - x \leq 25000$. Notice the left side of the inequality is just the numerical value of the profit, so the maximum profit you can make from this deal is \$25,000. Since we know equality is only reached if $x = 25000$, you can achieve the maximum profit if you first pay the Nigerian prince $\boxed{\text{A) \$25,000}}$. *Warning: Do not actually do this in real life. It's likely a scam.*
21. Euclid used induction to prove that there was an infinite number of prime in his 13-volume *Elements*. $\boxed{\text{B) Induction}}$
22. Direct application of angle bisector theorem. $\frac{25}{x-16} = \frac{20}{16}$, $x - 16 = 20$ $\boxed{\text{D) 36}}$
23. Using Power of a Point Theorem, $4 \times 3 = \frac{12}{5}x$. $x =$ $\boxed{\text{C) 5}}$
24. These are the 3 classical constructions that have puzzled ancient geometers for centuries. They were proven to be impossible in the 1800s. $\boxed{\text{D) I, II, and III}}$
25. The answer is just the sum of the legs of the smallest Pythagorean triple. $3 + 4 =$ $\boxed{\text{A) 7}}$
26. The area is just $\frac{1}{2}AP - \frac{1}{2}ap = k$. $A, a$ are the apothem lengths and $P, p$ the perimeters. Since the outer pentagon is a 2x scale of the smaller pentagon, $\frac{1}{2}A = a$ and $\frac{1}{2}P = p$. Substituting, $\frac{1}{2}AP - \frac{1}{8}AP = k$, $AP = \frac{8}{3}k$. It is give $A = 20$, so $P = \frac{8}{60}k =$ $\boxed{\text{A) } \frac{2}{15}k}$

27. I) Counterexample: a kite; False II) True III) Counterexample: an isosceles trapezoid; $\boxed{\text{C) II only}}$

28. Solving for the three points, we get (2,2), (3,4), and (6,1). The coordinates of the centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, So the centroid is $\left(\frac{11}{3}, \frac{7}{3}\right)$.

$x + y = \frac{18}{3} = \boxed{6 \text{ E) NOTA}}$

29. Notice that $\overline{AC} = \sqrt{6} + \sqrt{2}$ can be split into 2 segments of $\sqrt{6}$ and $\sqrt{2}$. We see that the triangle is actually a 30-60-90 triangle combined with a 45-45-90. So $m\angle B = \boxed{\text{B) } 75°}$




30. The operation shown is a logical conjunction, therefore the symbol should be $\boxed{\text{A) } \land}$