SLIIT

Discover Your Future

IT4130 – Image Understanding and Processing

Lecture 07 – Morphological Image Processing

Sessional Outcomes

- Erosion
- Dilation
- Closing
  - Applications
  - Advantages
- Opening
  - Applications
  - Advantages
- Hit or Miss transform

- Applications of Morphological operations
  1. Boundary extraction
  2. Region Filling
  3. Connected components Extraction

Introduction

- The word *morphology* commonly denotes a branch of biology that deals with the form and structure of animals and plants.
- Therefore, morphological operations are intended to affect the shape of the object

mathematical morphology

We use mathematical morphology as a tool for **extracting image components** that are useful in the representation and description of region shape, such as

- boundaries extraction
- skeletons
- convex hull
- morphological filtering
- thinning
- pruning

Sets

Sets in mathematic morphology represent objects in an image:

- binary image (0 = black, 1 = white) :

the element of the set is the coordinates (x,y) of pixel belong to the object Z^2

- gray-scaled image :

the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels Z^3

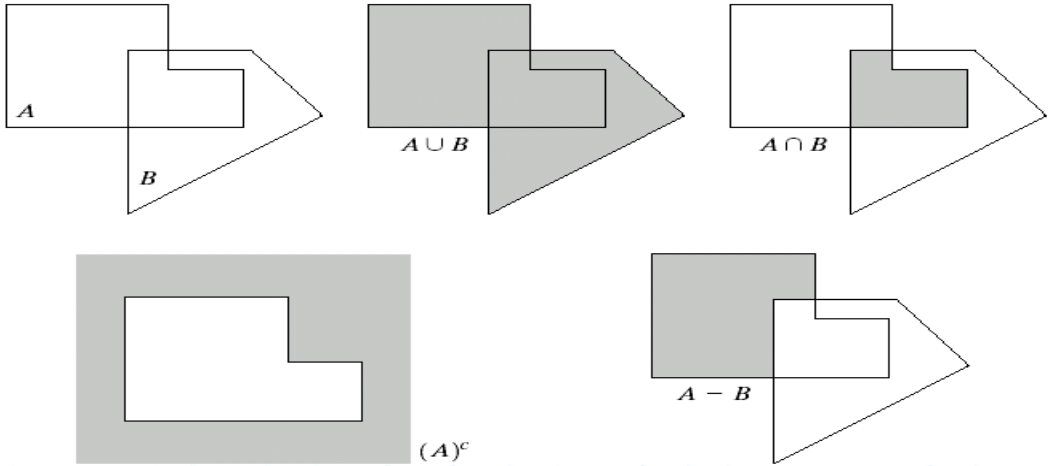
Notations

- A is a set in Z^2 ,
- $a=(a_1,a_2)$ an element of A, $a \in A$ If not, then $a \notin A$
- $\varnothing$: null (empty) set
- A subset of B: $A \subseteq B$
- Union of A and B: $C=A \cup B$
- Intersection of A and B: $D=A \cap B$
- Disjoint sets: $A \cap B= \varnothing$
- Complement of A: $A^c = \{x \mid x \notin A\}$
- Difference of A and B:

$$A-B = \{x \mid x \in A, x \notin B\} = A \cap B^c$$

Operators

Operators by examples:



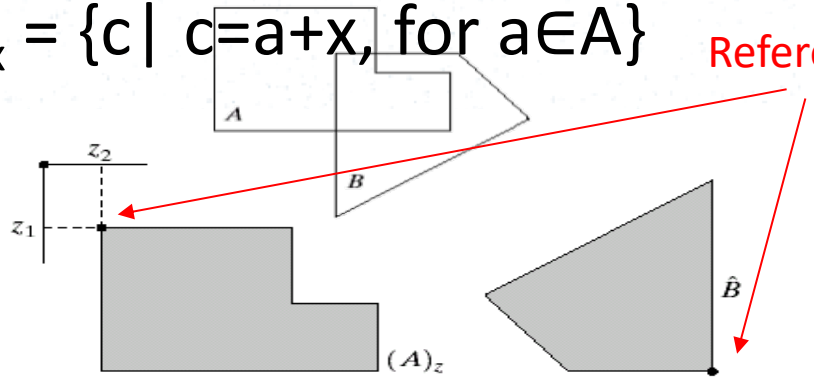
a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflection and Translation

Reflection and Translation by examples:

- Need for a reference point.
- Reflection of B - $\hat{B}= \{x | x=-b, \text{ for } b \in B\}$
- Translation of A by $x=(x_1,x_2)$, denoted by $(A)_x$ is defined as: $(A)_x = \{c | \ c=a+x, \text{ for } a \in A\}$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Logic Operations

Basic Logic Operations:

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

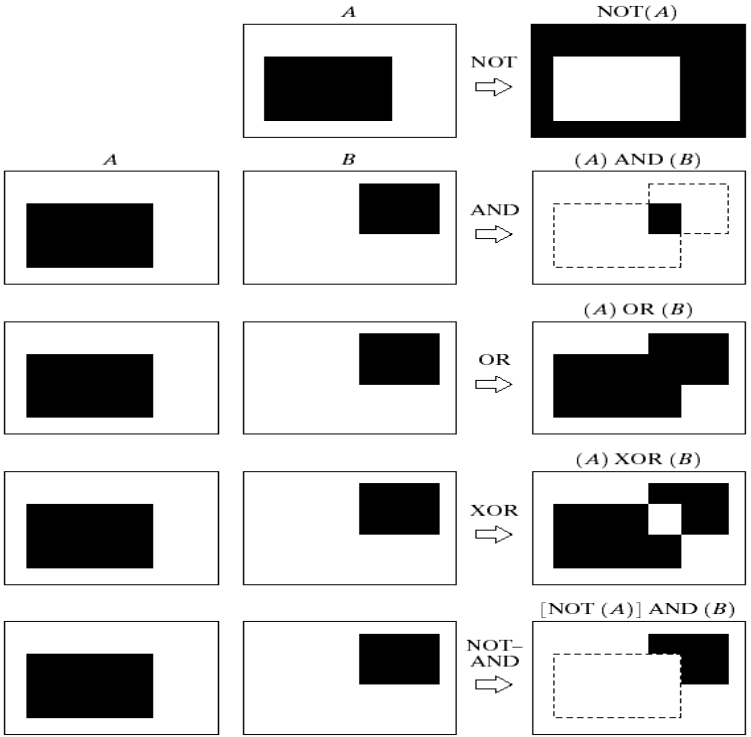


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring element

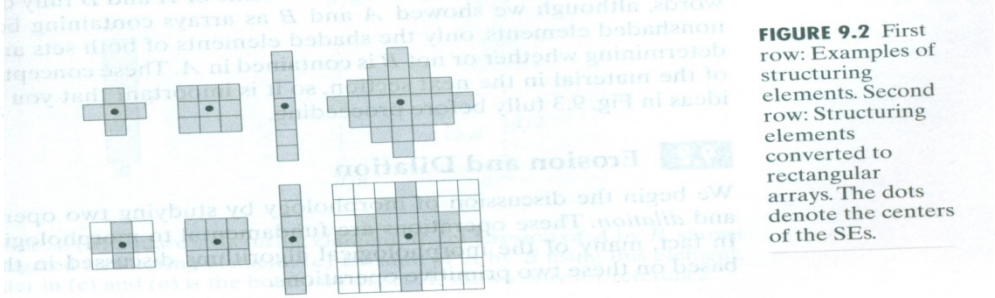
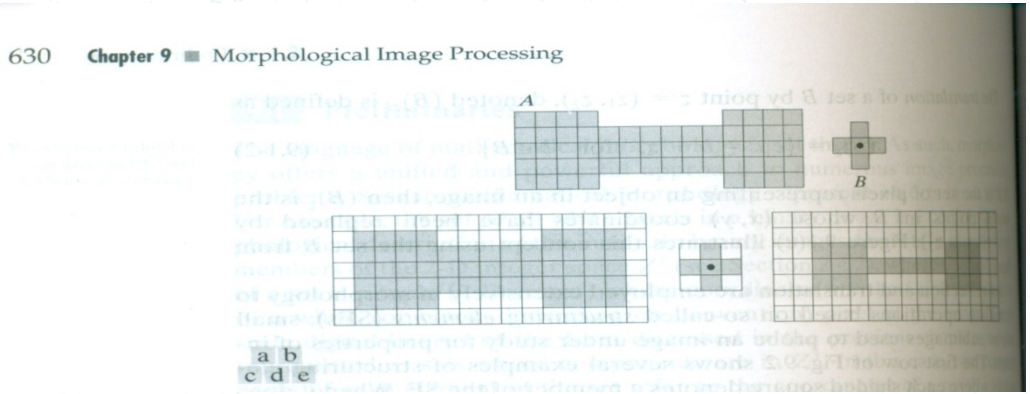


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

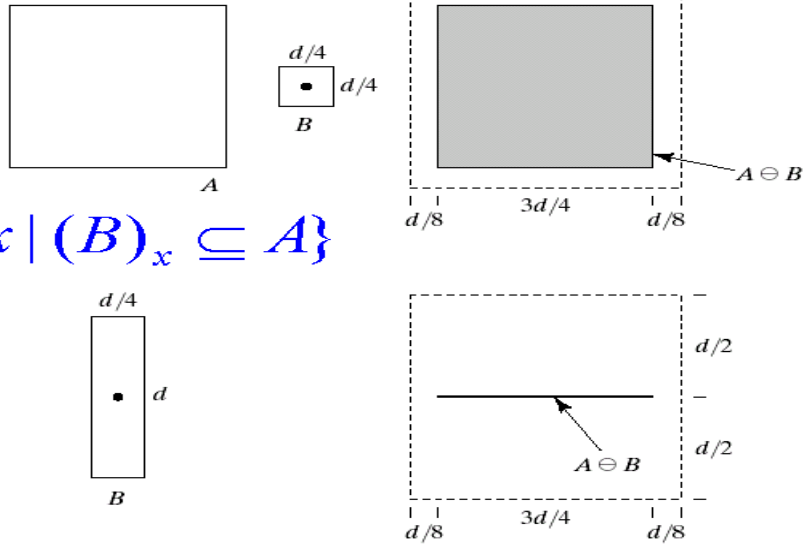


Morphological Operators I

- Erosion:

$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

  i.e. the erosion of A by B is the set of all points x such that B, translated by x, is contained in A.
  - It is typically applied to binary image, but there are versions that work on gray scale image.
  - The basic effect of the operator on a binary image is to erode away the boundaries of regions of foreground pixels (*i.e.* white pixels, typically).
  - Thus areas of foreground pixels shrink in size, and holes within those areas become larger.
- General effect
  - If all the elements of the structuring element are positive, the output image tends to be darker than the input
  - The effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself.



$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Erosion

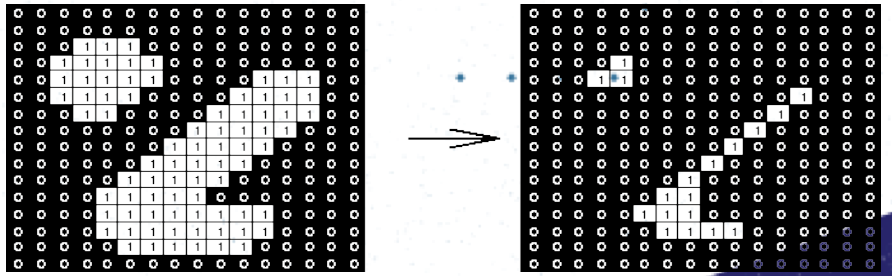
Suppose that the structuring element is a 3×3 square .

Note that in subsequent diagrams, foreground pixels are represented by 1's and background pixels by 0's.

The structuring element is now superimposed over each foreground pixel (input pixel) in the image. If all the pixels below the structuring element are foreground pixels then the input pixel retains it’s value. But if any of the pixels is a background pixel then the input pixel gets the background pixel value.

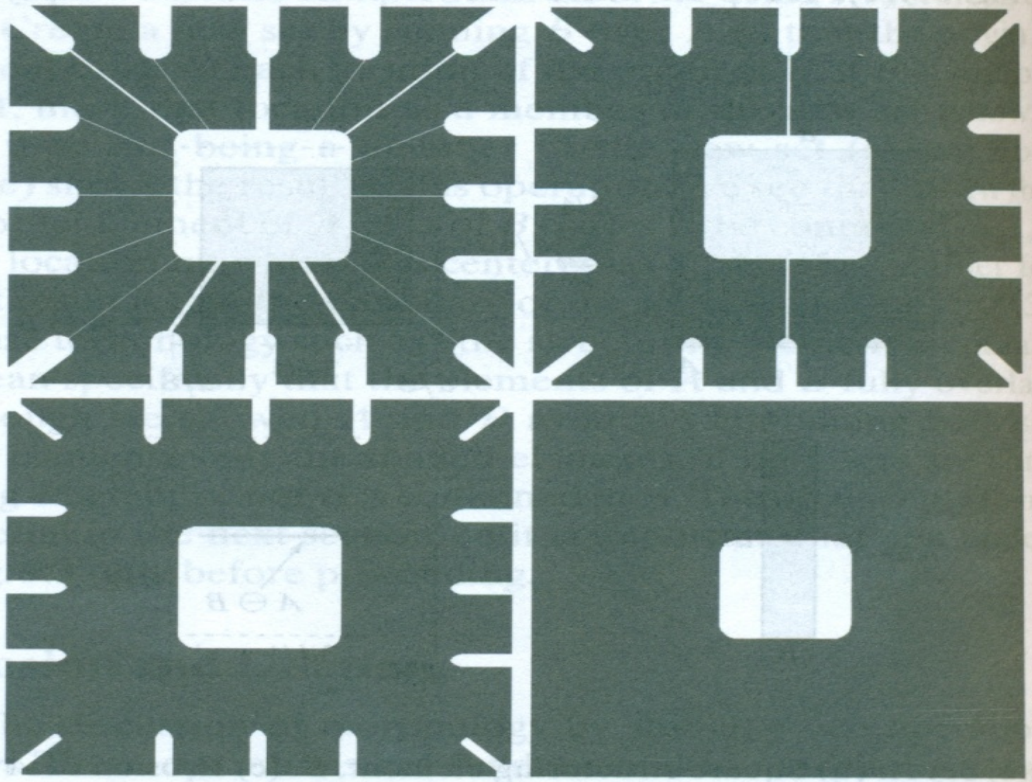
1	1	1
1	1	1
1	1	1

Structuring element



a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.



Morphological Operators II

Dilation:

1. Obtaining the reflection of B about its origin and then shifting this reflection by z
2. The dilation of A b $A \oplus B = \left\{ z \middle| \left(\hat{B} \right)_z \cap A \neq \varnothing \right\} = \left\{ z \middle| \left(\hat{B} \right)_z \cap A \subseteq A \right\}$ displacements such that B and A overlap by at least one nonzero element…

(B is the structuring element in dilation)

Relation to Convolution mask:

- Flipping
- Overlapping
- nonlinear

- Dilation is typically applied to binary image, but there are versions that work on gray scale image.

Dilation

Suppose that X is the set of coordinates corresponding to the input binary image, and that K is the set of coordinates for the structuring element.

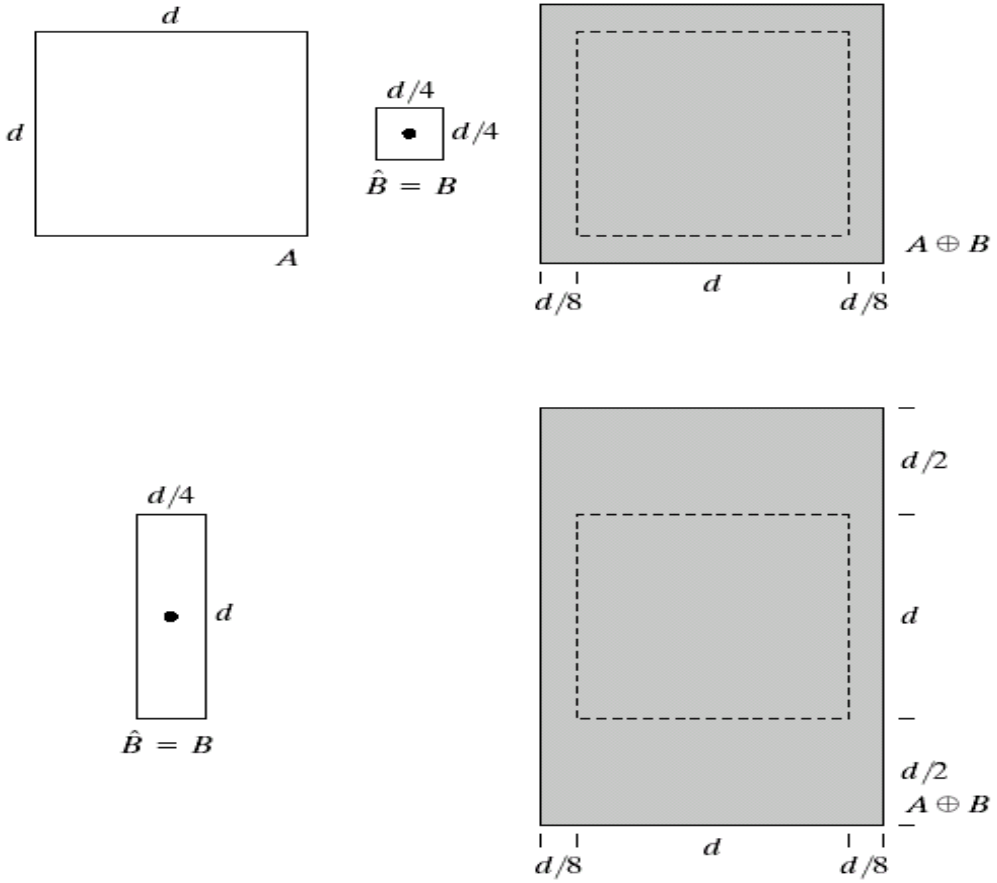
Let K_x denote the translation of K so that its origin is at x.

Then the dilation of X by K is simply the set of all points x such that the intersection of K_x with X is non-empty.

Dilation by example:

a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Dilation

Suppose that the structuring element is a 3×3 square .

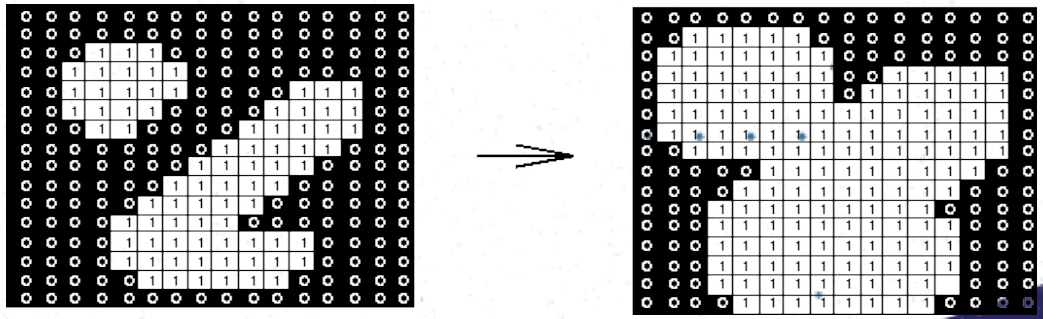
Note that in subsequent diagrams, foreground pixels are represented by 1's and background pixels by 0's.

To compute the dilation of a binary input image by this structuring element, we superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.

If the center pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value.

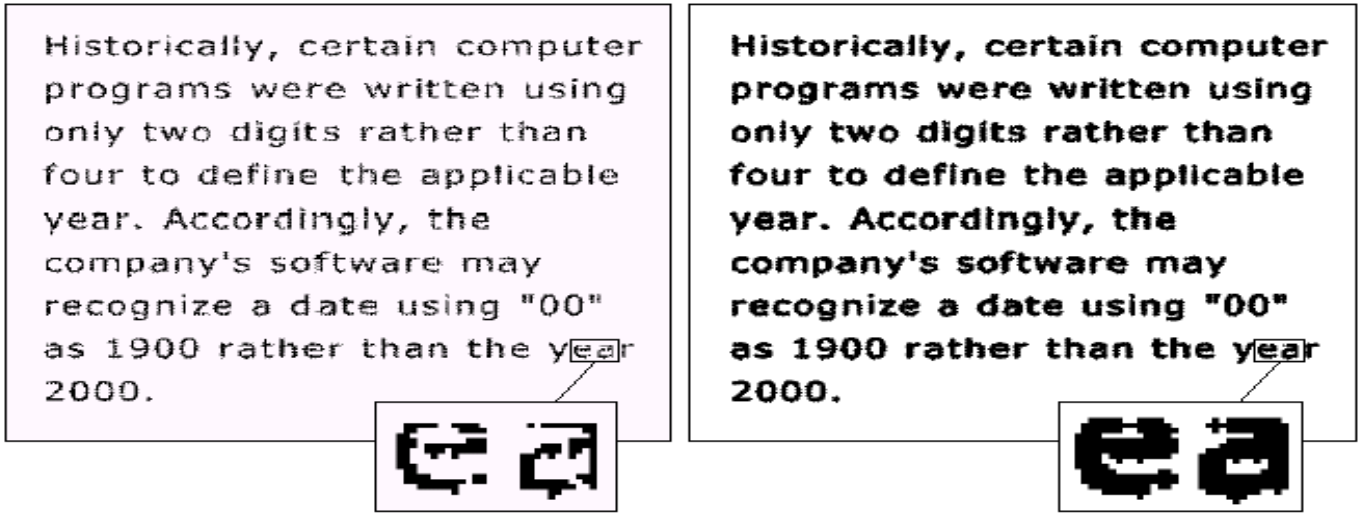
1	1	1
1	1	1
1	1	1

Structuring element



Dilation

- Dilation is typically applied to binary image, but there are versions that work on gray scale image.
- The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (*i.e.* white pixels, typically).
- Thus areas of foreground pixels grow in size while holes within those regions become smaller.
- General effect
  - If all the values of the structuring element are positive, the output image tends to be brighter than the input
  - Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation.



a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

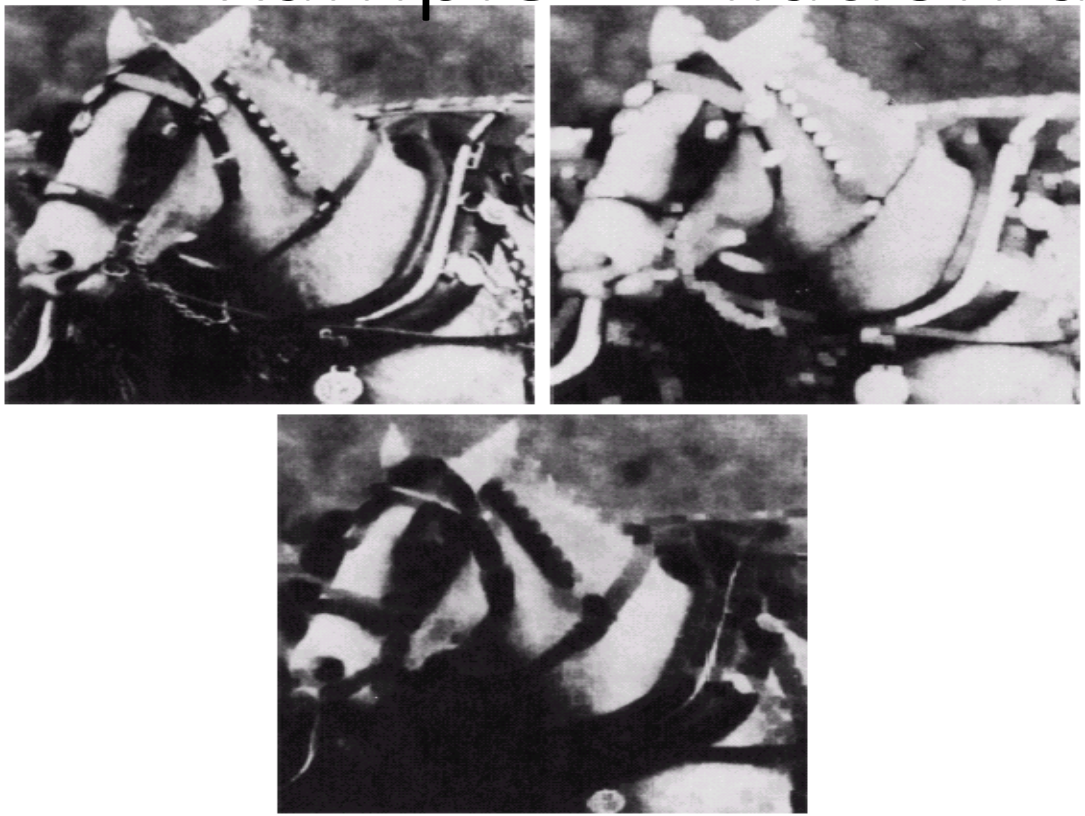
0	1	0
1	1	1
0	1	0

- The basic effect of the operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (*i.e.* white pixels, typically).

Dilation

- Summary effects of dilation:
  - Expand/enlarge objects in the image
  - Fill gaps or bays of insufficient width
  - Fill small holes of sufficiently small size
  - Connects objects separated by a distance less than the size of the window

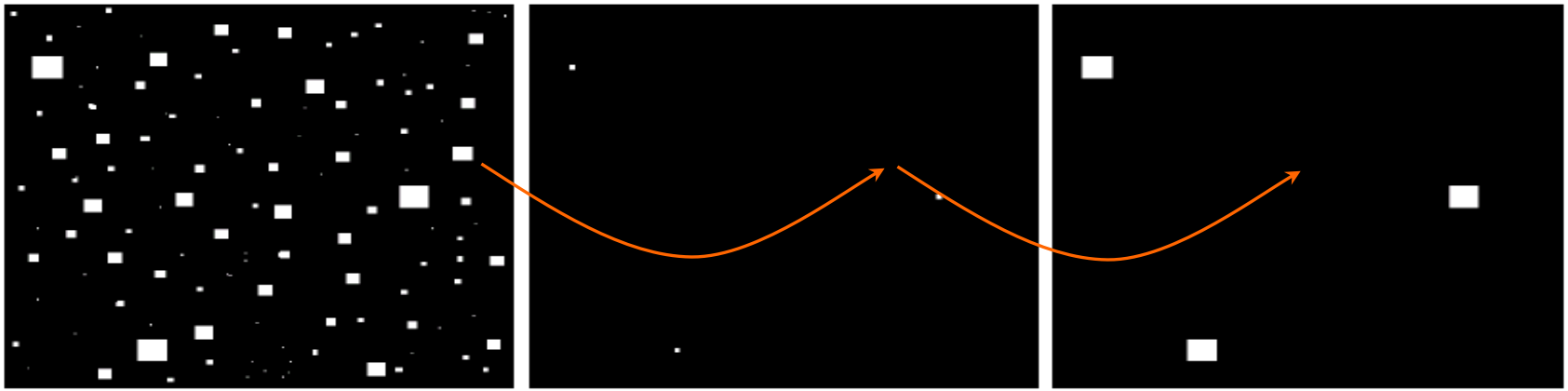
Example - Dilation and Erosion



a b
c

FIGURE 9.29
(a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Example – Erosion followed by Dilation



a b c

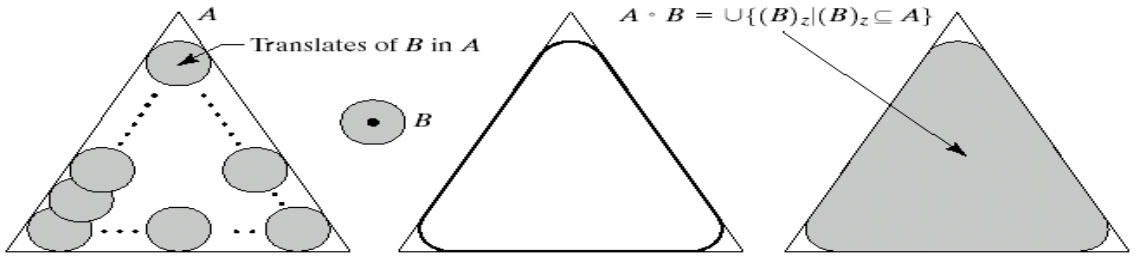
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening

- Opening
  - An erosion followed by a dilation using the ***same structuring element*** for both operations.
  - Dilation expands and Erosion shrinks.

$$A \circ B = (A \ominus B) \oplus B = \bigcup \{(B_z) | (B_z) \subseteq A\}$$

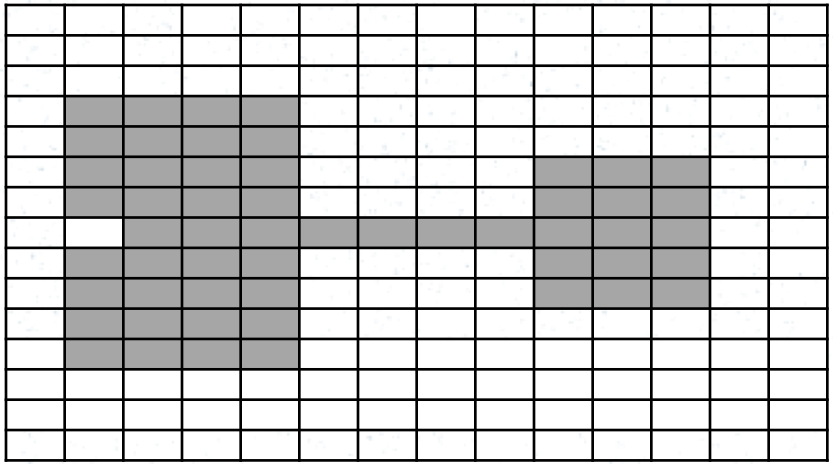
- Smoothens contours
- Breaks narrow isthmuses
- Remove

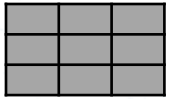


a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

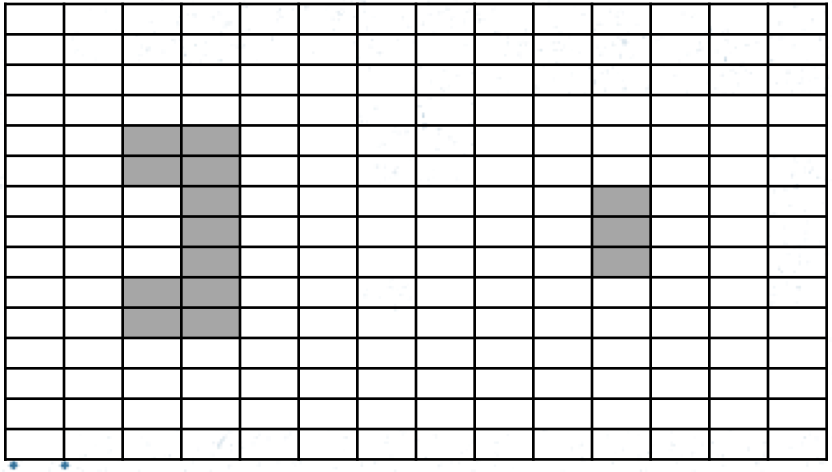
Example I



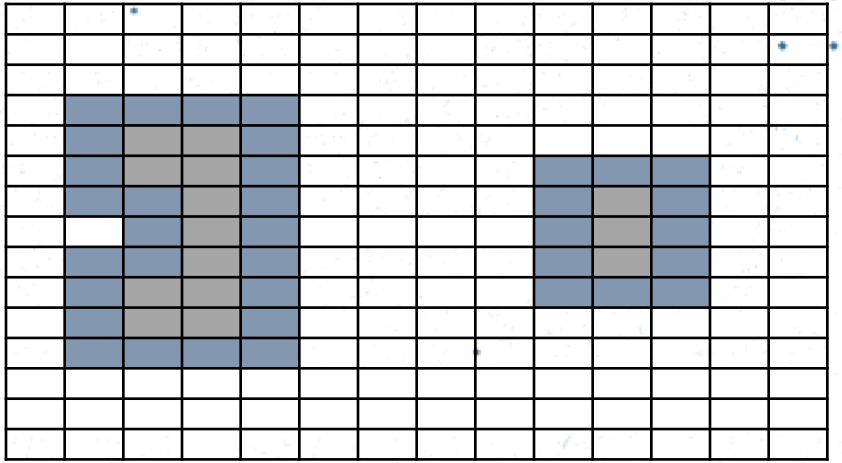


Opening

- Erosion



- Dilation



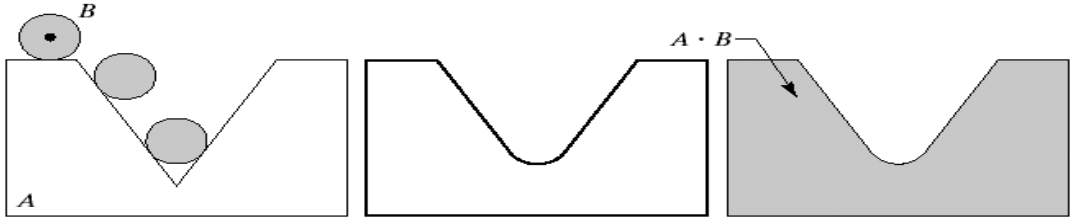
Closing

- **Closing:**

  A Dilation followed by an erosion using the ***same structuring element*** for both operations.

$$A \circ B = (A \oplus B) \ominus B$$

- Smooth contour
- Fuse narrow breaks, and long thin gulfs.
- Remove small holes, and fill gaps.

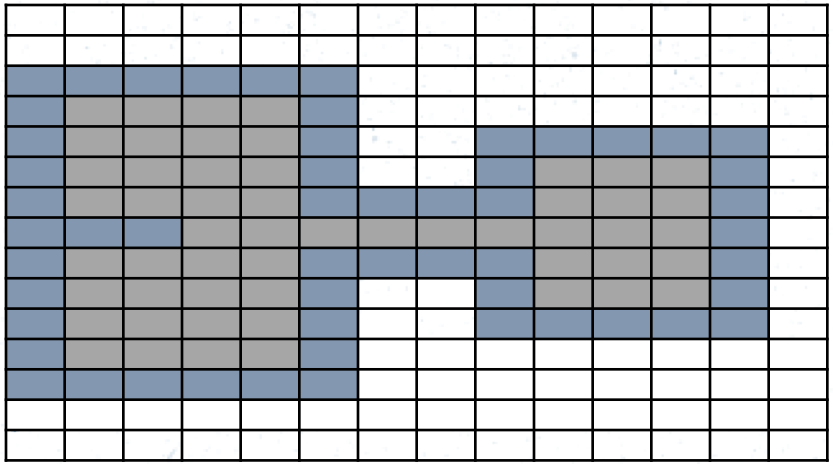


a b c

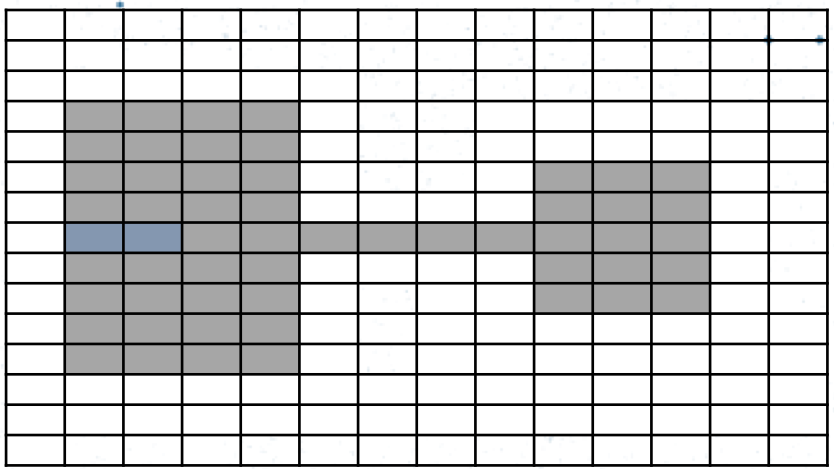
FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Closing

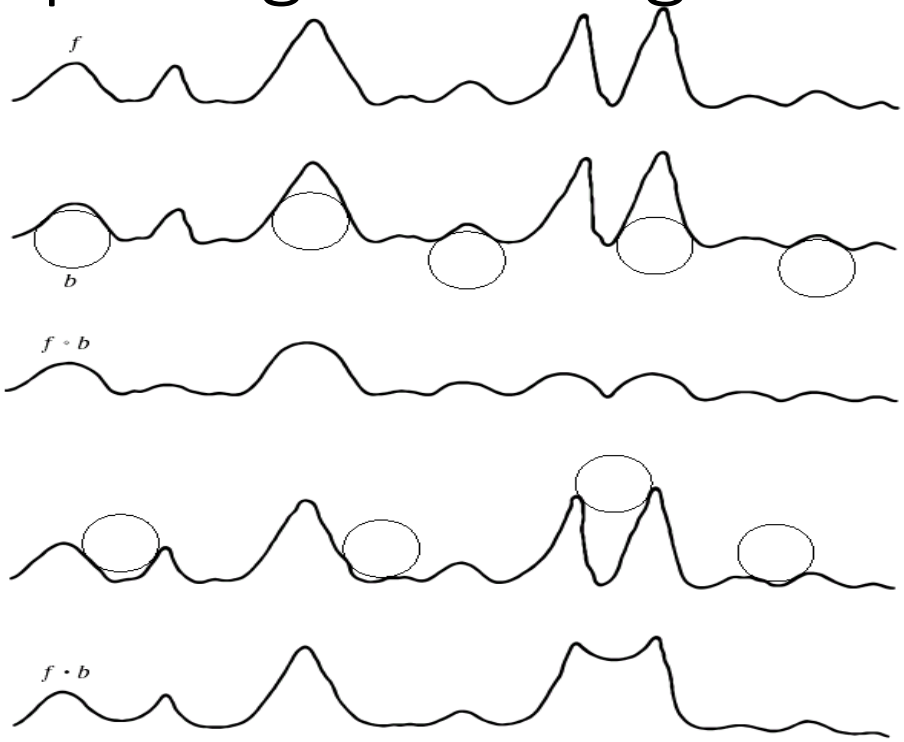
- Dilation



- Erosion



Opening & Closing– effect on peaks



a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Opening

- Opening
  - The structuring element is rolled underside the surface of f
  - All the peaks that are narrow with respect to the diameter of the structuring element will be reduced in amplitude and sharpness
  - So, opening is used to remove small light details, while leaving the overall gray levels and larger bright features relatively undisturbed.
  - The initial erosion removes the details, but it also darkens the image.
  - The subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion.

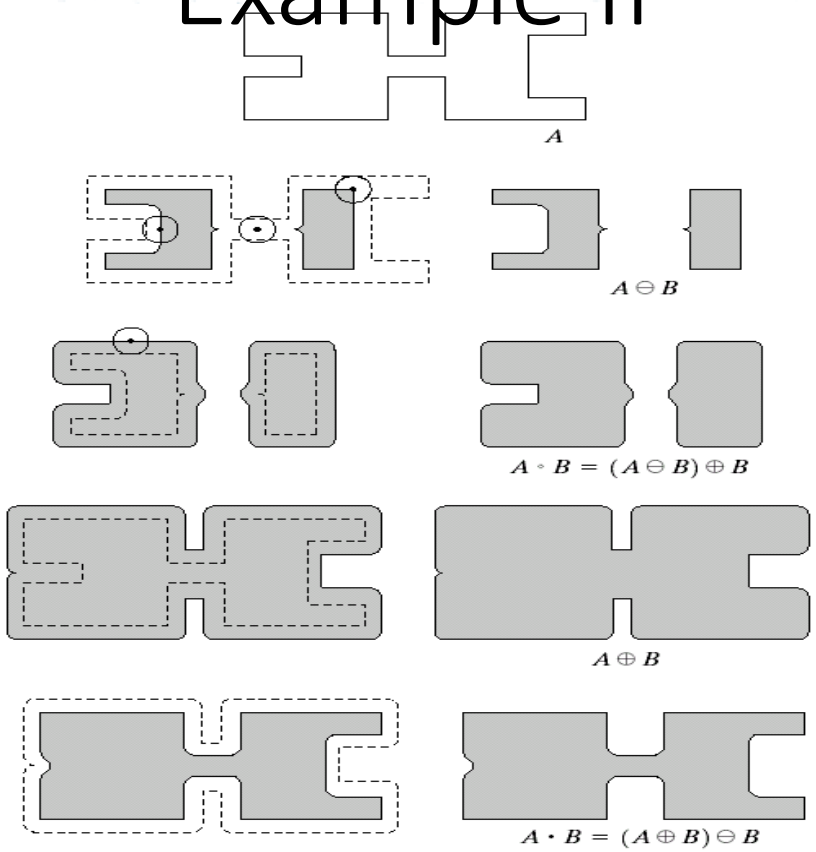
Closing

- Closing
  - The structuring element is rolled on top of the surface of f
  - Peaks essentially are left in their original form (assume that their separation at the narrowest points exceeds the diameter of the structuring element)
  - So, closing is used to remove small dark details, while leaving bright features relatively undisturbed.
  - The initial dilation removes the dark details and brightens the image
  - The subsequent erosion darkens the image without reintroducing the details totally removed by dilation

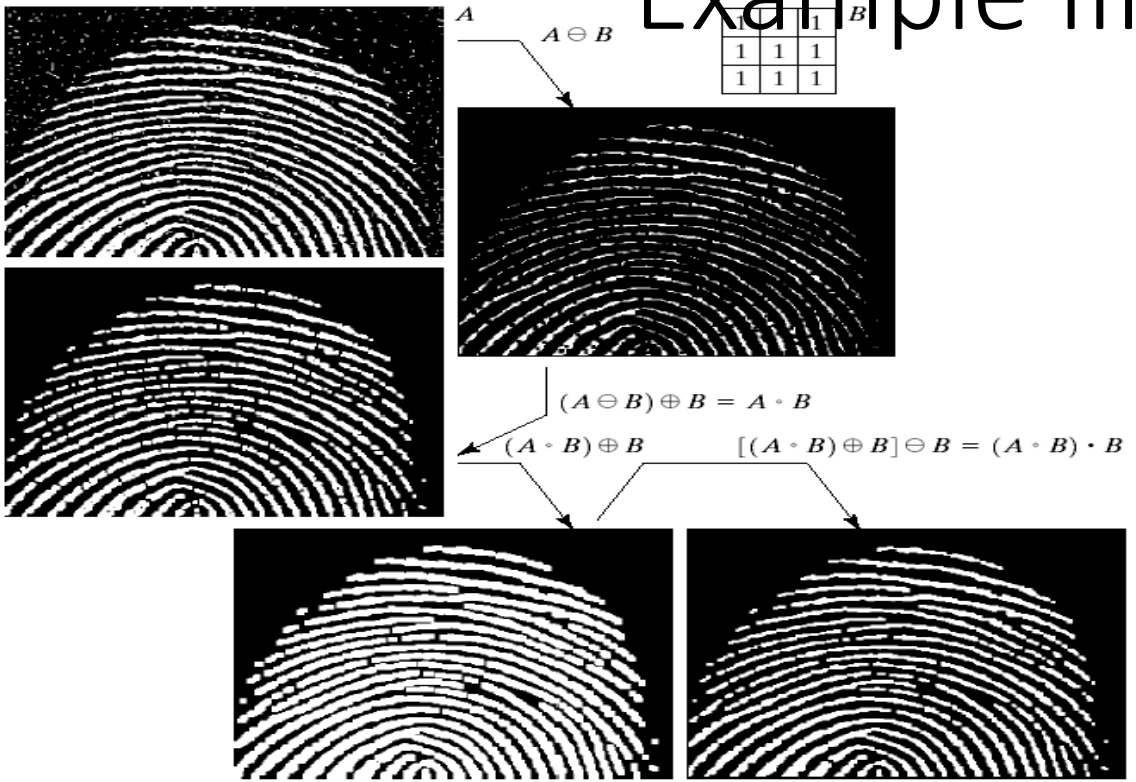
Example II

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Example III



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss

- Hit-or-Miss Transform

The hit-or-miss transform is a general binary morphological operation that can be used to look for particular patterns of foreground and background pixels in an image.

The hit-and-miss transform is a basic tool for shape detection.

Concept: To detect a shape:

- **Hit object**
- **Miss background**

Hit-or-Miss

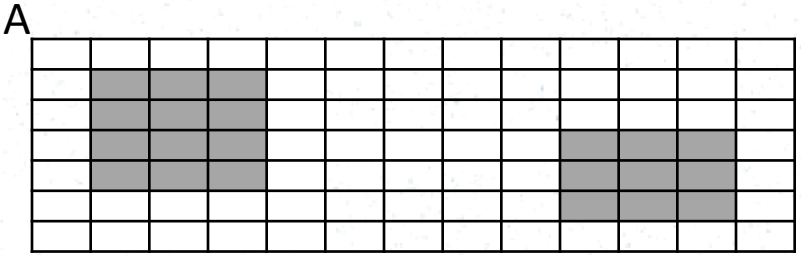
- Hit-or-Miss Transform
  - The structural elements used for Hit-or-miss transforms are an extension to the ones used with dilation, erosion etc.
  - The structural elements can contain both foreground and background pixels, rather than just foreground pixels, i.e. both ones and zeros.
  - The structuring element is superimposed over each pixel in the input image, and if an exact match is found between the foreground and background pixels in the structuring element and the image, the input pixel lying below the origin of the structuring element is set to the foreground pixel value. If it does not match, the input pixel is replaced by the boundary pixel value.

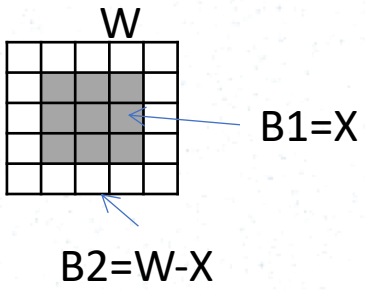
Hit-or-Miss

The hit-or-miss transform is defined as:

Let B ={B1, B2}, where B1 is the set formed from elements of B associated with an object and B2 is the set of elements of B associated with the corresponding background, where B1 and B2 are disjoint.

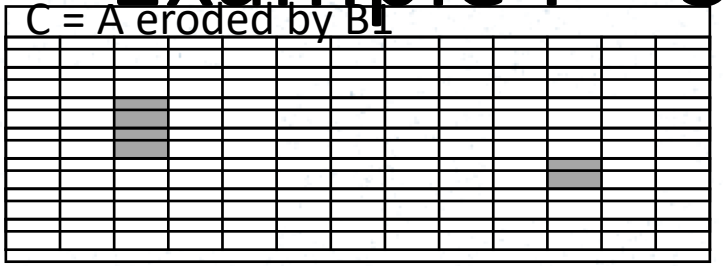
Example I – step 1



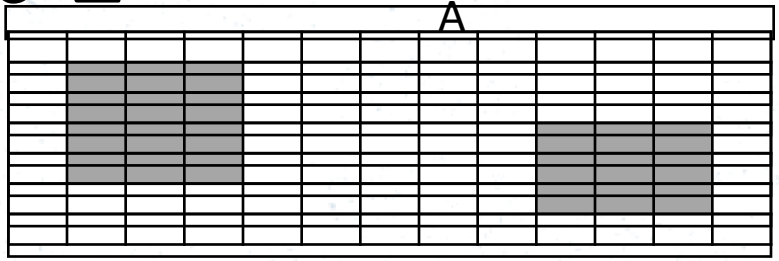


Example I – Step 2

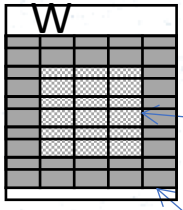
C = A eroded by B1



A



W

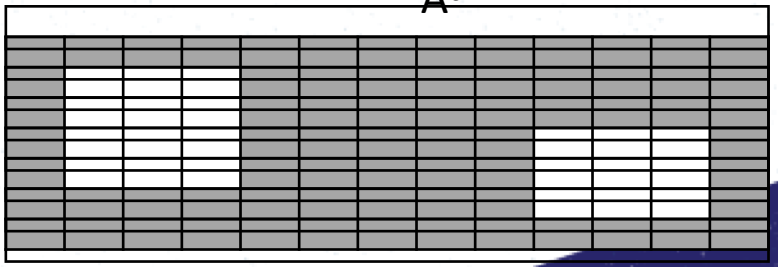


B1=X

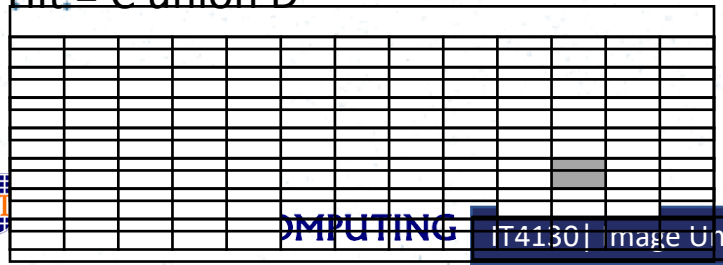
B2=W-X

D = A^c eroded by B2

A^c



Hit = C union D



Example II

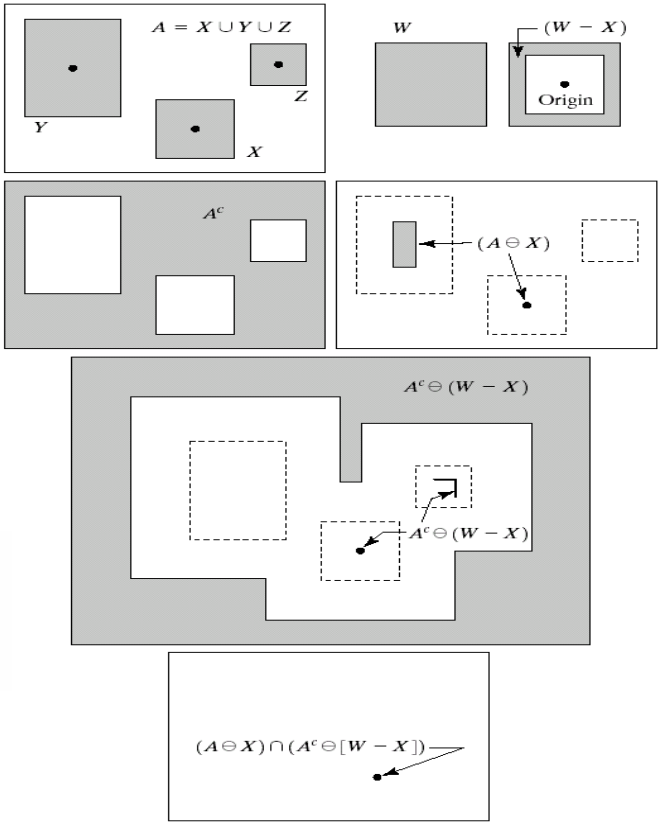
Hit-and-Miss Transform

$$A * B = (A \Theta X) \cap \left[A^c \Theta (W - X) \right]$$

$$A * B = (A \Theta B_1) \cap \left[A^c \Theta B_2 \right]$$

B1: Object related

B2: Background related



a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Applications

1. Boundary Extraction

$$\beta(A) = A - (A \Theta B)$$

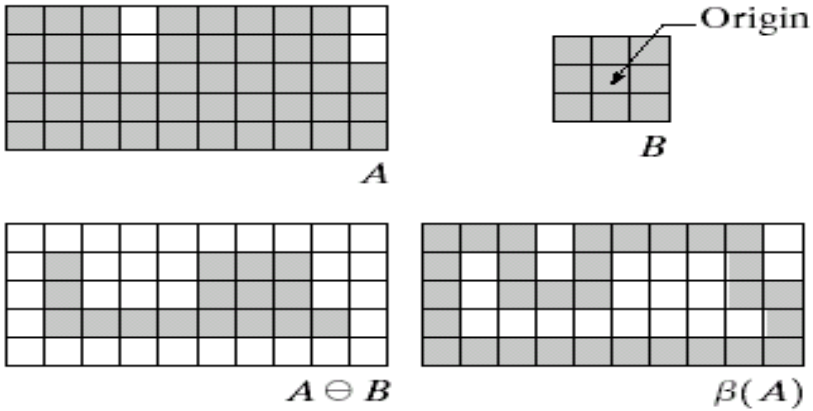
- The boundary of a set X is obtained by first eroding X by structuring element K and then taking the set difference of X and it's erosion.
- The resultant image after subtracting the eroded image from the original image has the boundary of the objects extracted. The thickness of the boundary depends on the size of the structuring element.

Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Applications
2. Region Filling:

- Start from *p* inside boundary.

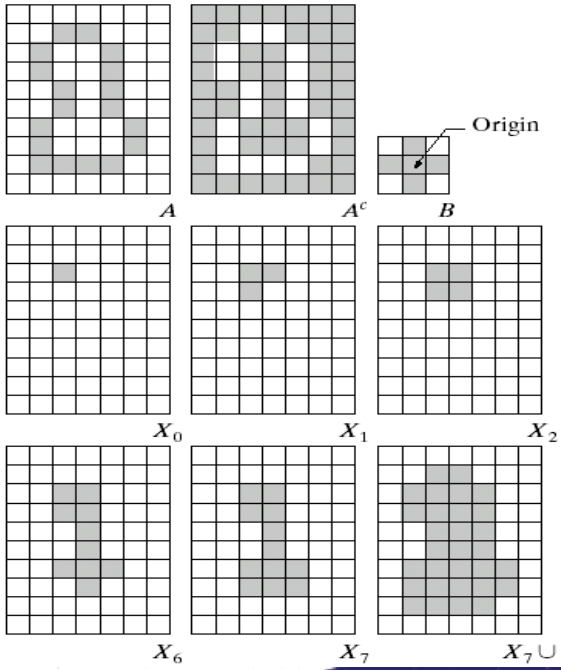
$$X_0 = p$$

$$X_k = (X_k \oplus B) \cap A^c$$

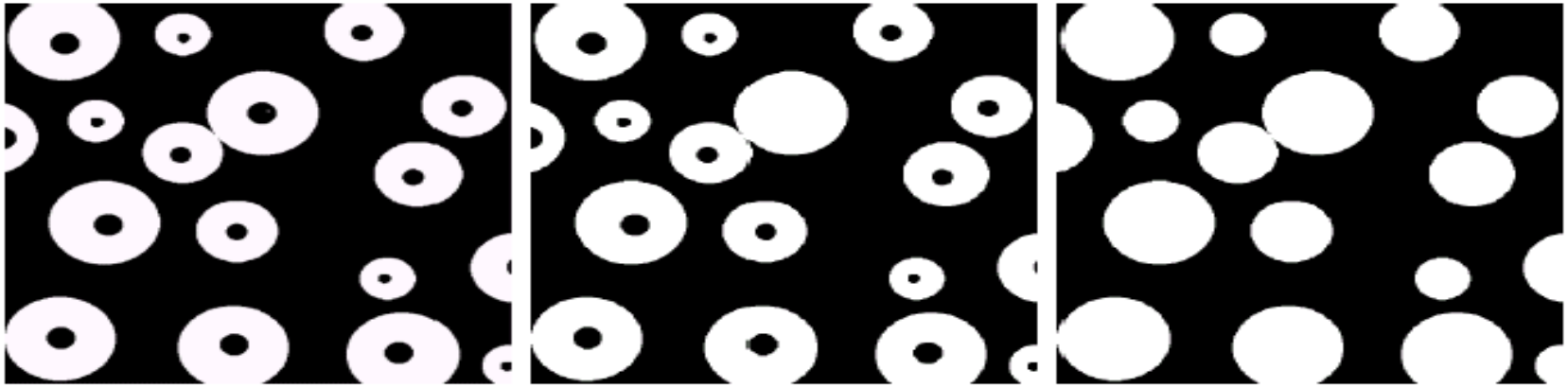
$$\text{Until: } X_{k+1} = X_k$$

a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



Semi-automated to cancel reflection effect



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Applications

3. Connected components Extraction:

Start from *p* belong to desired region.

$$X_0 = p$$

$$X_k = (X_k \oplus B) \cap A$$

Until: $X_{k+1} = X_k$

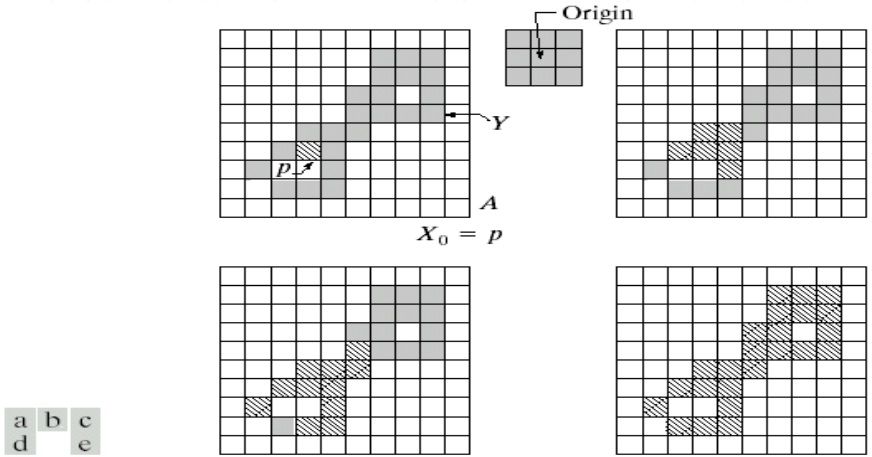
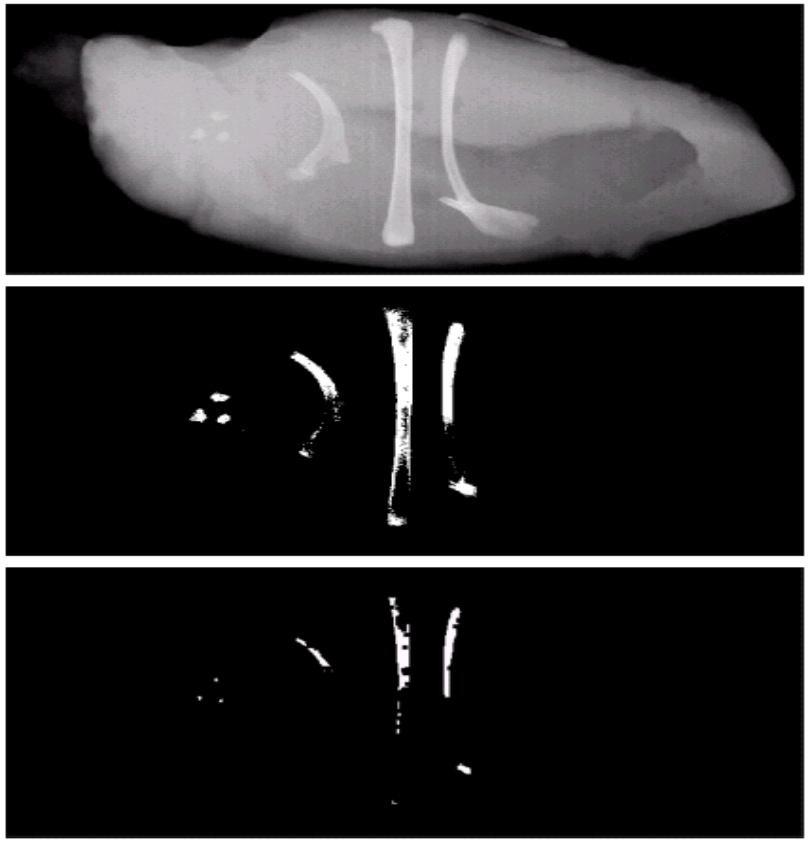


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

References

1. Digital Image Processing by R. C. Gonzales and R. E. Woods, Addison-Wesley Publishing Company, 1992.