June 2022 PS

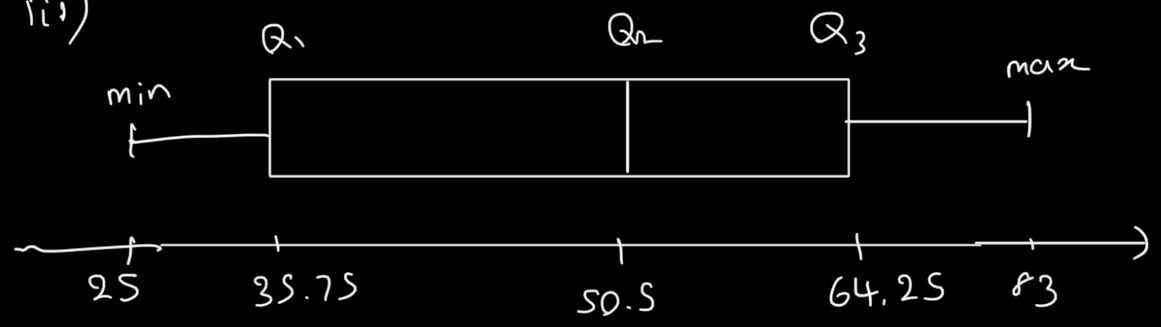
Q1) a) i) Q_1 – $31 \times 0.25 = 7.75^{th}$ place $\rightarrow$ 35.75
Q_2 – $31 \times 0.5 = 15.5^{th}$ place $\rightarrow$ 50.5
Q_3 – $31 \times 0.75 = 23.25^{th}$ place $\rightarrow$ 64.25
min – 25
max – 83

ii) IQR = 28.5
lower bound = $35.75 - \underbrace{1.5 \times IQR}_{42.75} = -7$

upper bound = $64.25 + 42.75 = 107$

there are no outliers in the data set.

iii)



iv) There are no outliers and the data is somewhat normally distributed with a slight negative skewness.

b) i) tossing a coin will create a mutually exclusive event.

ii) The event of getting any 1 outcome from the faces 1, 2, 3, 4, 5 and 6 is collectively exhaustive

Q2) a) n = 12 X - words typed per minute

$\alpha = 0.025$ $\quad \bar{x} = 68.4$ $\quad s = 7.3$

$$\bar{x} - t_{(n-1),\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \; , \; \bar{x} + t_{(n-1),\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$68.4 - 2.201 \times \frac{7.3}{\sqrt{12}} \; , \; 68.4 + 2.201 \times \frac{7.3}{\sqrt{12}}$$

$$63.761 \; , \; 73.038$$

$$(64 \; , \; 73)$$

we can conclude that the true average is between 64 and 73 at 95% level of confidence.

b) x - content of containers of lubricant

$\bar{x} = 10.56$ $\quad s = 1.246$

$n = 35$

$\mu = 10$ $\quad x \sim N(\mu, \sigma^2)$

$\alpha = 0.5$

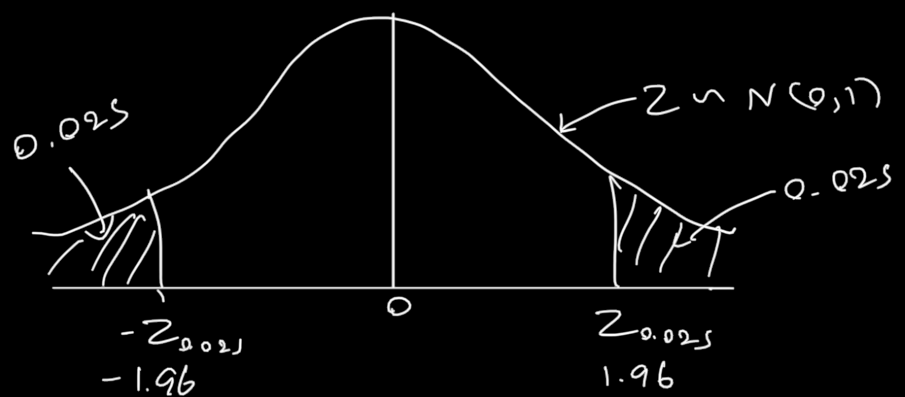
$H_0: \mu = 10$ $\quad H_1: \mu \neq 10$

this is a two tailed test

under H_0,

test statistic $= \dfrac{\bar{x} - \mu}{s/\sqrt{n}} \sim N(0,1)$

$$T = 2.65891 = 2.66$$



decision rule

reject H_0 if $T > z_{0.025}$ or $T < -z_{0.025}$

since $T(2.85491) > z_{0.025}(1.96)$

H_0 is reject under 5% level of significance.

∴ we can conclude that there is not enough evidence to prove the claim.

Q3) a)

$\alpha = 0.05$

		A	B	C	total
City 1	O	210	221	95	526
	E	221.446	208.822	95.732	
City 2	O	211	176	87	476
	E	200.396	188.972	86.632	
total		421	397	182	1009

$$E_{ij} = \frac{i\text{th row total} \times j\text{th column total}}{\text{grand total}}$$

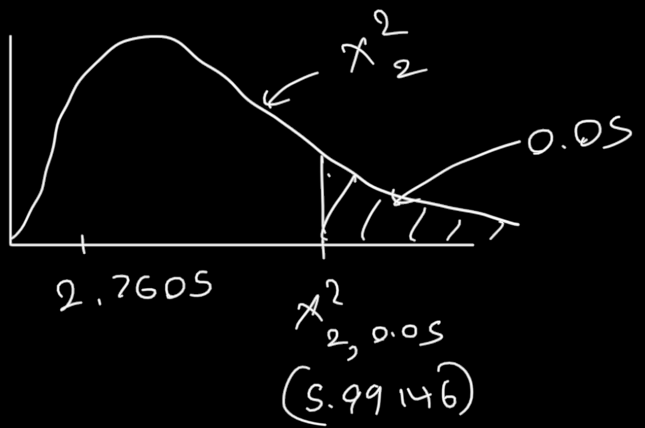
H_0: sentiment is independent from city
H_1: sentiment is not independent from city

under H_0,
Test statistic $\chi^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\frac{[O_{ij} - E_{ij}]^2}{E_{ij}} \right) \sim \chi^2_2$

$T = 0.59161 + 0.21016 + 0.00559$

$+ 0.56111 + 0.89046 + 0.00156$

$$T = 2.76052 = 2.7605$$



decision rule

reject h_0 if Test statistic $> \chi^2_{2,0.05}$

since Test stat (2.7605) $< \chi^2_{2,0.05}$ (5.99146)

H_0 is not rejected under 5% level of significance.

∴ There is enough evidence to show that sentiment and city are independent.

Q4) $n = 25$

a) The data shows a positive linear relation

b) $A = 1$
$B = n-2$
$C = n-1$

$$36.7 = \frac{D}{n-2} \quad \therefore \quad D = 844.1$$

$$E = 2863.20 + D = 3707.3$$

$$F = 2863.20/1 = 2863.20$$

$$G = F/36.7 = 78.0163$$

c) $Y = \hat{\alpha} + \hat{\beta} X$

$\hat{\alpha}$: intercept, $\hat{\beta}$: regression coefficient

$\therefore \quad Y = 45.805 + 1.107^{*} X$

d) $$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{\left(n(\Sigma x^2) - (\Sigma x)^2\right) * \left(n(\Sigma y^2) - (\Sigma y)^2\right)}}$$

$= \ldots$

e) $H_0: \rho = 0 \qquad H_1: \rho \neq 0$

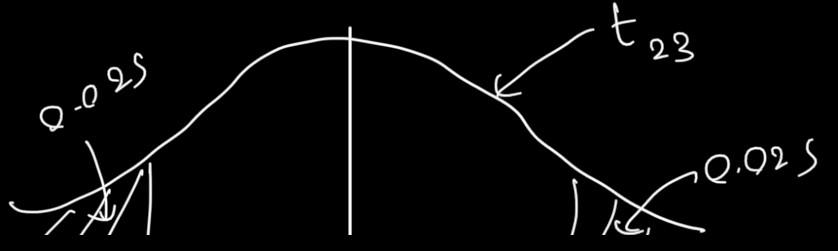
this is a two tailed test

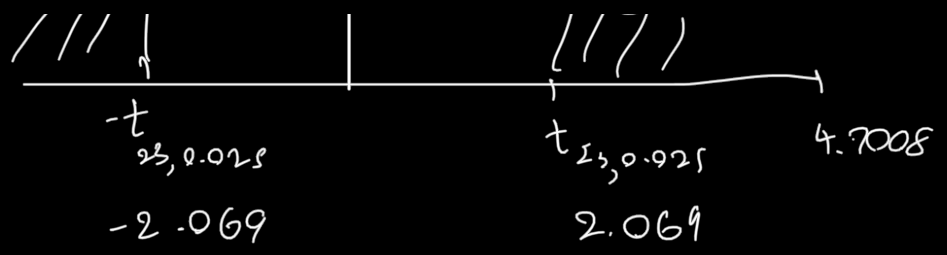
$\alpha = 0.05$

under H_0, Test statistic: $T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$

* assumed $r = 0.7$

$= 4.7008$





decision rule

reject H_0 if $T > t_{23,0.025}$ or

$T < -t_{23,0.025}$

since $T(4.7008) > t_{23,0.025}(2.069)$

h_0 is rejected under 5% level of significance.

∴ there is enough evidence to show that arm strength and dynamic lift are correlated.