Tute 08

(01) H_0: Hypertension and smoking habit are independant.

H_1: Hypertension and smoking habit are not independant.

	N.Smokers	Moderate Smoker	H. Smakers	Total
Hypertension	21 $E_{11} = \frac{87 \times 69}{180}$ (33.35)	36 $E_{12} = \frac{62 \times 87}{180}$ (29.97)	30 $E_{13} = \frac{49 \times 87}{180}$ (23.68)	87
No hypertension	48 $E_{21} = \frac{93 \times 69}{180}$ (35.65)	26 $E_{22} = \frac{62 \times 93}{180}$ (32.03)	19 $E_{23} = \frac{93 \times 49}{180}$ (25.31)	93
Total	69	62	49	

Under H_0, $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{df}$.

here $df = (3-1) \times (2-1)$
$= \underline{\underline{2}}$.

$$\chi^2 = \frac{(21-33.35)^2}{33.35} + \frac{(36-29.97)^2}{29.97} + \dots\dots + \frac{(19-25.31)^2}{25.31}$$

$= 4.573 + 1.213 + 1.68 + 0.847 + 1.135 + 1.573$

$= \underline{\underline{11.021}}$.

Decision Rule: Reject H_0 if $\chi^2_{cal} > \chi^2_{2, 0.05}$

$\chi^2_{2,0.05} = 5.991$

Decision: Since χ^2_{cal} $(11.021) > \chi^2_{2,0.05}$ we have enough evidance to reject H_0.
at 5% level of Significance.

Conclusion: Hypertension and smoking habits are not indépendant.

02

Education	NO of Children			
	0-1	2-3	over 3	Total
Elementary	14 $E_{11} = 18.68$	37 $E_{12} = 39.84$	32 $E_{13} = 24.49$	83
Secondary	19 $E_{21} = 17.55$	42 $E_{22} = 37.44$	17 $E_{23} = 23.01$	78
College	12 $E_{31} = 8.78$	17 $E_{32} = 18.72$	10 $E_{33} = 11.505$	39
Total	45	96	59	

H_0: family size is independant of the level of education attain by the father.

H_1: family size is not independant of the level of education.

under H_0, $$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{df}$$

here, $df = (3-1) \times (3-1)$
$= 2 \times 2 = \underline{\underline{4}}$

$$\chi^2_{cal} = \frac{(14-18.68)^2}{18.68} + \frac{(37-39.84)^2}{39.84} + \cdots + \frac{(10-11.505)^2}{11.505}$$

$$= 1.173 + 0.202 + 2.301 + 0.120 + 0.555 + 1.570 + 1.181 + 0.158 + 0.197$$

$$= \underline{\underline{7.457}}$$

Decision Rule: Reject H_0 if $\chi^2_{cal} > \chi^2_{4,0.05}$.

$$\chi^2_{4,0.05} = \underline{\underline{9.488}}$$

Decision: Since $\chi^2_{cal} < \chi^2_{4,0.05}$, we don't have enough evidance to reject H_0 at 5% level of Significance.

Conclusion: family size is independant of the level of education attain by the father.

03 H_0: Gender and Satisfaction are independant

H_1: Gender and Satisfaction are not independat.

	Highly Satis	satisfied	No idea	Dissatisfi	H.dis	Total
Male	2 $E_{11}=3.47$	2 $E_{12}=3.96$	15 $E_{13}=14.85$	25 $E_{14}=27.72$	55 $E_{15}=49.01$	99
Female	5 $E_{21}=3.54$	6 $E_{22}=4.04$	15 $E_{23}=15.15$	31 $E_{24}=28.28$	44 $E_{25}=50.00$	101
Total	7	8	30	56	99	

under H_0, $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{df}$.

here $df = (5-1)\times(2-1) = \underline{\underline{4}}$

(Both rules are violating)

Since some cells have Expected values less than 5, we have to join cells.

	highly satisfied + Satisfied	No idea	Dissatisfied	H. dissatisfied	
Male	4 $E_{11}=7.425$	15 $E_{12}=14.85$	25 $E_{13}=27.72$	55 $E_{14}=49.01$	99
Female	11 $E_{21}=7.58$	15 $E_{22}=15.15$	31 $E_{23}=28.28$	44 $E_{24}=50.00$	101
	15	30	56	99	200

under H_0, $X = \sum_{i=1}^{n} \frac{(O-E_i)^2}{E_i} \sim \chi^2_{df}$

here $df = (4-1) \times (2-1)$

$= 3 \times 1$

$= \underline{\underline{3}}$.

$$\chi^2_{cal} = \sum_{i=1}^{8} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(4-7.425)^2}{7.425} + \frac{(15-14.85)^2}{14.85} + \dots + \frac{(44-50)^2}{50}$$

$$= 1.580 + 0.002 + 0.267 + 0.732 + 1.063 + 0.001 + 0.262 + 0.72$$

$$= \underline{\underline{4.627}}$$

Decision Rule: Reject H_0 if $\chi_{cal} > \chi_{3,0.05}$.

$\chi_{3,0.05} = 7.815$

Decision: Since $\chi_{cal} < \chi_{3,0.05}$, we don't have enough evidance to reject H_0.

Conclusion: Gender and satisfaction are independant.

H_0: No. of germination follows a binomial distribution.

H_1: No. of germination doesn't follow a binomial distribution.

No. of Seeds germinating	No. of pots	$P(X=x_i)$.
0	12	$12/100 = 0.12$
1	24	$24/100 = 0.24$
2	39	$39/100 = 0.39$
3	22	$22/100 = 0.22$
4	3	$3/100 = 0.03$
	100	

* First of all, we have to estimate a value of Success probability(

Expected value $= \sum P \times x_i$

$$= 0 \times 0.12 + 1 \times 0.24 + 2 \times 0.39 + 3 \times 0.22 + 4 \times 0.03$$

$$= \underline{\underline{1.8}}.$$

we know, in a binomial dis^n,

Expected value $= n \times \hat{\theta}$

$$1.8 = 4 \times \hat{\theta}.$$

$$\underline{\underline{0.45 = \hat{\theta}}}$$

Therefore, $X \sim \text{bin}(4, 0.45)$

Now, we have to calculate the expected frequencies.

No. of Seeds germinating	$P(X=x_i)$	$E_i = P(X=x_i)\times 100$
0	$4C0 \times 0.45^0 \times 0.55^4 = 0.092$	9.2
1	$4C1 \times 0.45^1 \times 0.55^3 = 0.299$	29.9
2	$4C2 \times 0.45^2 \times 0.55^2 = 0.368$	36.8
3	$4C3 \times 0.45^3 \times 0.55^1 = 0.200$	20.0
4	$4C4 \times 0.45^4 \times 0.55^0 = 0.041$	4.1

unde H_0,

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_{(\text{no of classes} - \text{no. of Parameter} - 1)}$$

here, df = no of classe − no of parameters − 1

$= 5-1-1$

$= \underline{\underline{3}}$.

$$\chi^2_{cal} = \frac{(12-9.2)^2}{9.2} + \frac{(24-29.9)^2}{29.9} + \cdots + \frac{(3-4.1)^2}{4.1}$$

$= 0.852 + 1.164 + 0.132 + 0.2 + 0.295$

$= \underline{\underline{2.643}}$

Decision Rule: Reject H_0 if $\chi^2_{cal} > \chi^2_{3,0.05}$.

here $\chi^2_{3,0.05} = 7.815$

Decision: Since $\chi^2_{cal} < \chi^2_{3,0.05}$, we dont have enough evidence to reject H_0 at 5% level of significance.

Conclusion: No. of germination follows a binomial distribution with with Success Proba. of 0.45.