8. STATISTICAL INFERENCE (Part 2) [IT2110]

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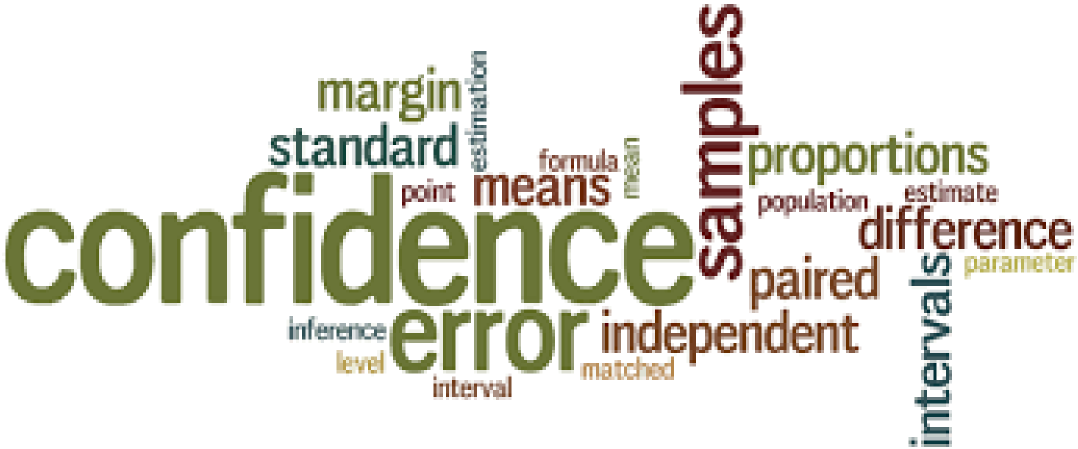
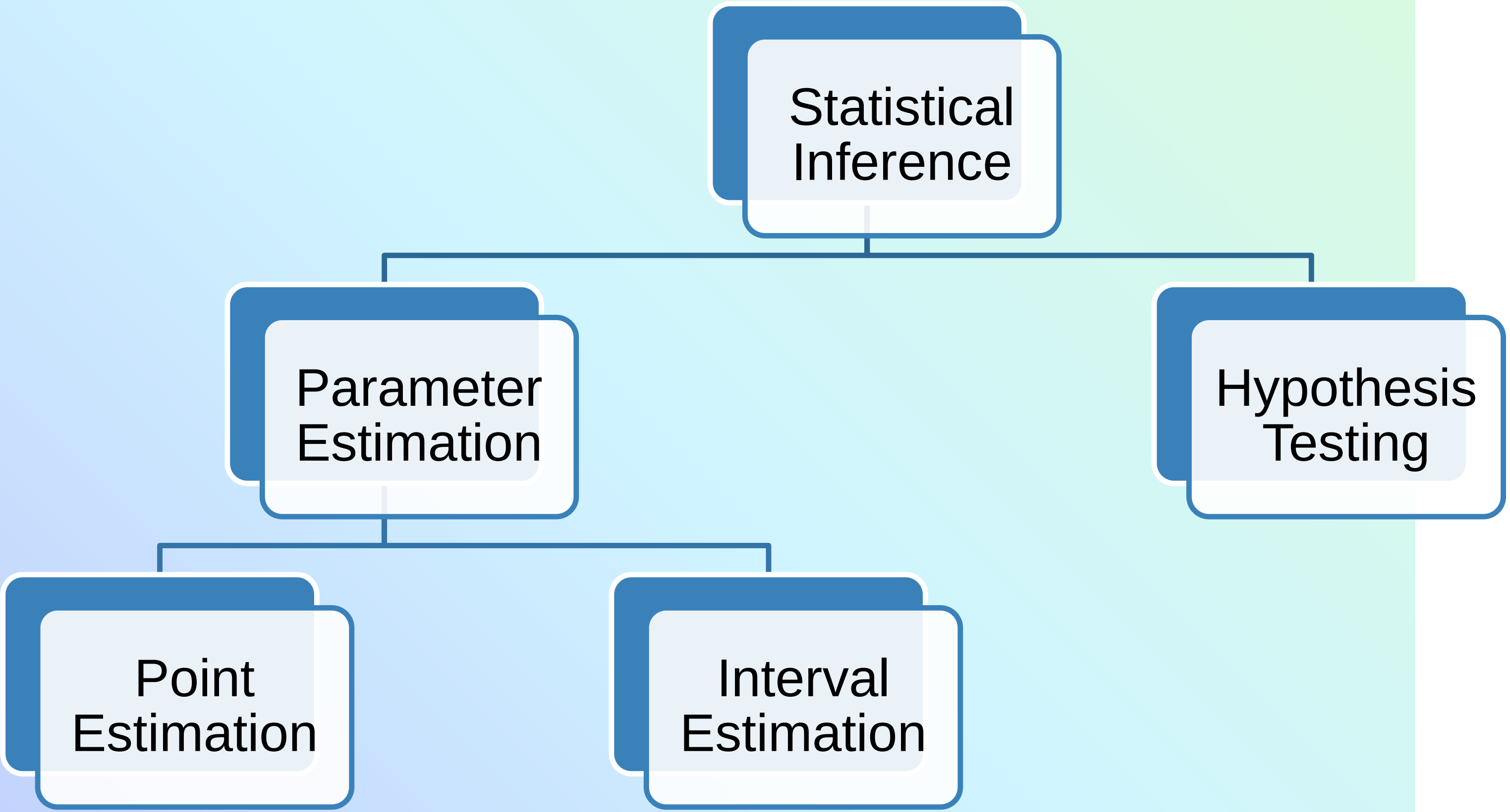
8.2
CONFIDENCE INTERVALS




Statistical Inference
Parameter Estimation
Hypothesis Testing
Point Estimation
Interval Estimation

Introduction

- Estimates will differ from the true parameter values by varying amounts depending on the samples obtained.
- Point estimates do not convey any measure of reliability.

Interval Estimation

- Interval estimation states that a ***population parameter*** is ***within two values*** (an interval) with a ***certain probability (Confidence Level)***.

- Interval Estimation is also known as ***Confidence Interval***.

- For a good interval estimate,
  - *The probability that the parameter is within the interval should be high.*
  - *The length of the interval should be small.*

- A confidence level for the interval should be defined first.

$$\boldsymbol{Confidence\ Level = 1 - \alpha}$$

where $\boldsymbol{\alpha}$ is the significance level discussed in hypothesis testing.

- Let L and U be the lower and upper confidence limits for a parameter θ based on a random sample $X_1,\ldots,X_n$. Both L and U are functions of the sample. We can write the interval estimate of θ as,

$$Pr\ (L \leq \theta \leq U) = 1\text{-}\alpha$$

- ***Interpretation:***

  We are $(1-\alpha)$ % confident that the true parameter θ is located in the interval (L, U).

- In this session, we will discuss confidence intervals for population mean (μ) only.

Confidence Intervals (CI)

- CI for μ
  - σ^2 *known*
    - ($\bar{x}$ - $Z_{\alpha\%/2}(\sigma/\sqrt{n})$, $\bar{x}$ + $Z_{\alpha\%/2}(\sigma/\sqrt{n})$)
  - σ^2 *unknown*
    - n ≥30
      - ($\bar{x}$ - $Z_{\alpha\%/2}(S/\sqrt{n})$, $\bar{x}$ + $Z_{\alpha\%/2}(S/\sqrt{n})$)
    - n < 30
      - ($\bar{x}$ - $t_{(n-1),\alpha\%/2}(S/\sqrt{n})$, $\bar{x}$ + $t_{(n-1),\alpha\%/2}(S/\sqrt{n})$)

Example:

A company that manufactures cars claims that the gas mileage for its new line of hybrid cars, has a standard deviation of 4 mpg. It was also found out that the mpg was normally distributed. A random sample of 16 cars yielded a mean of 57 miles per gallon. What is the interval estimation for the population mean at a 95% confidence level?

Thanks!

Any questions?