10. REGRESSION ANALYSIS [IT2110]

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Numerical Variables??

- *Weight*
- *Height*
- *Temperature etc.*

Paired Numerical Variables??

Paired Variables

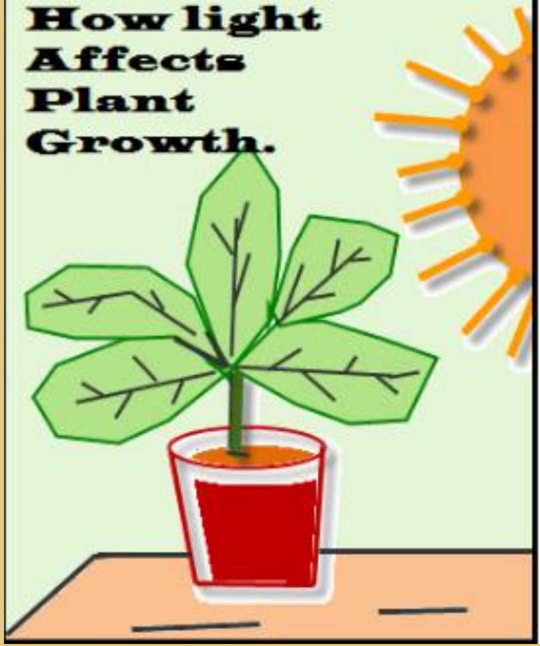
ID_No (Females)	Age	Systolic BP
001	45	151
002	25	138
003	48	143
004	37	140
005	24	136

Unpaired Variables

Age of Females	Systolic BP of Males
45	149
25	150
48	138
37	142
24	139

Dependent Variable??

The variable we wish to explain



Independent Variable??

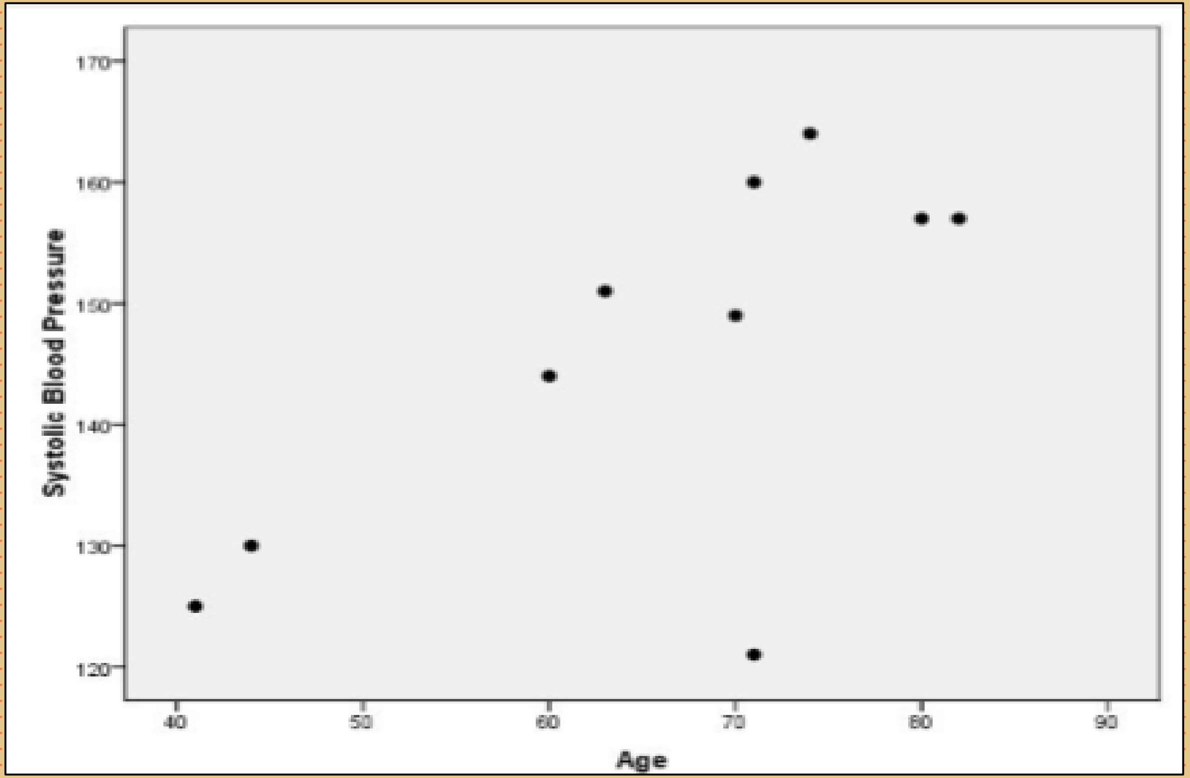
The variable we use to explain the dependent variable

How to identify Relationships??

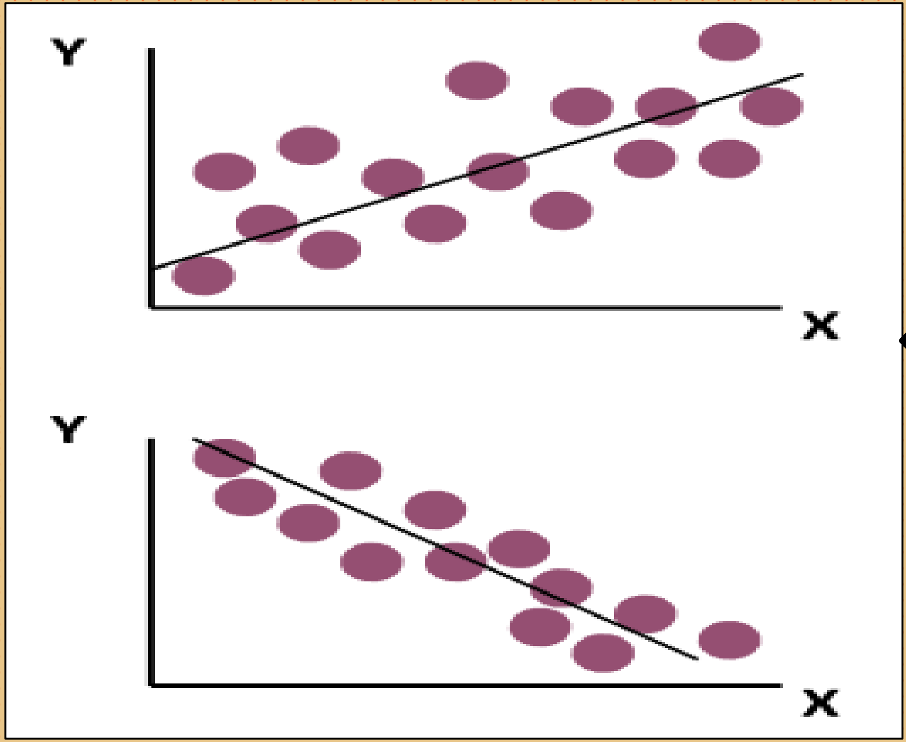
- Basically, we will learn three main methods. They are,
  - *Scatter plot (**Graphical Method)***
  - *Correlation*
  - *Regression Analysis*

Scatter Plot

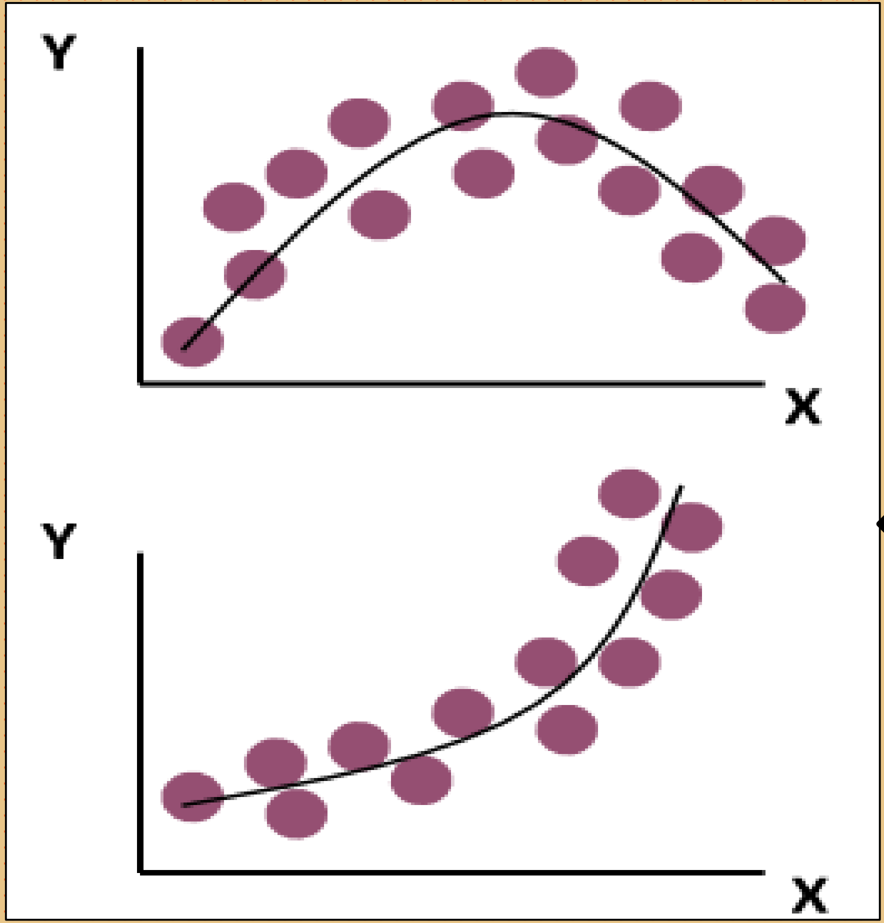
Age	*Systolic BP*
63	151
70	149
74	164
82	157
60	144
44	130
80	157
71	160
71	121
41	125



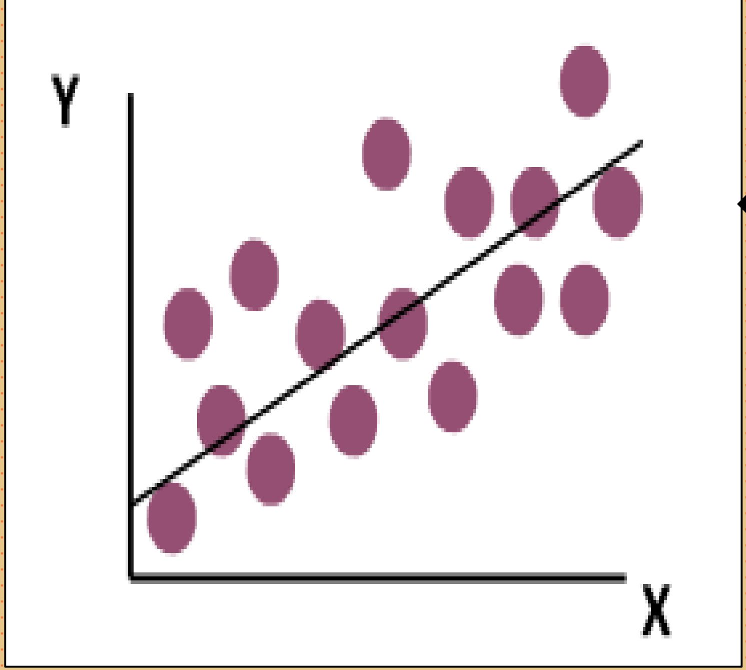
Types of Relationships



Linear Relationships

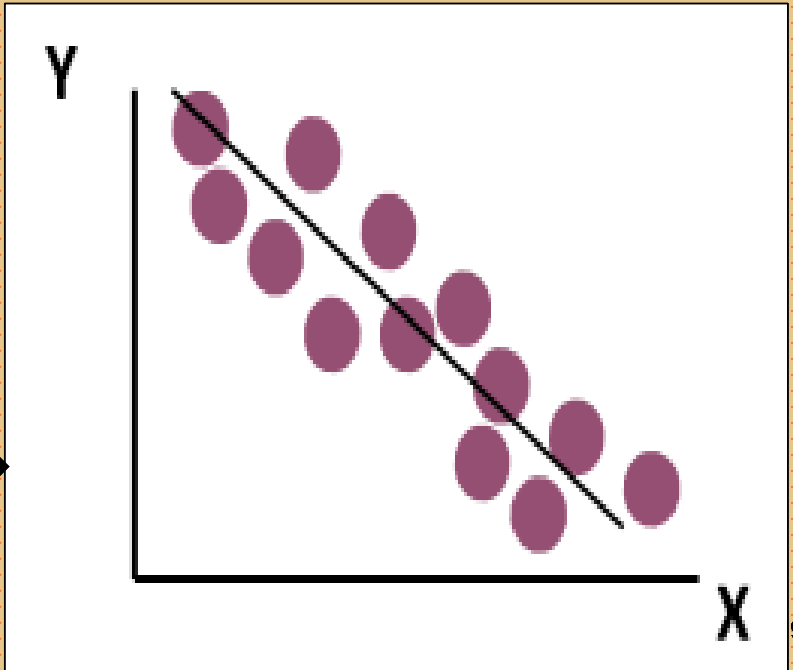


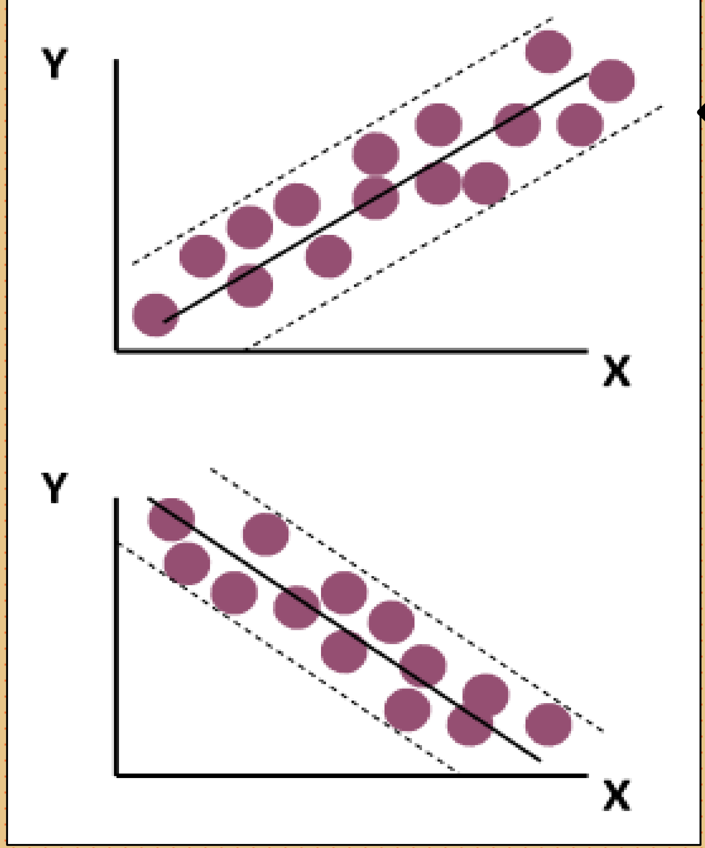
Non-Linear Relationships



Positive Linear Relationship

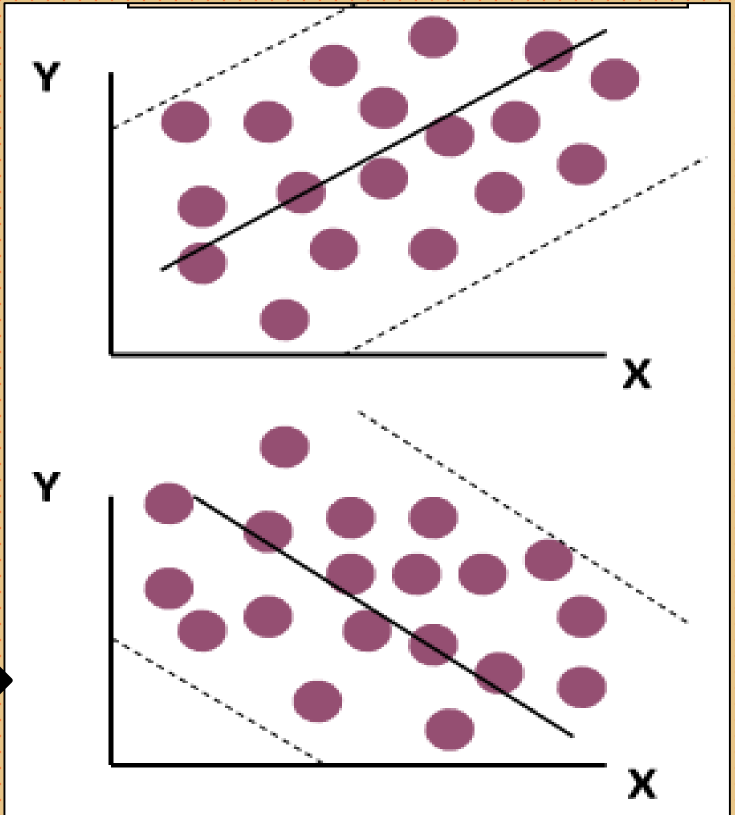
Negative Linear Relationship

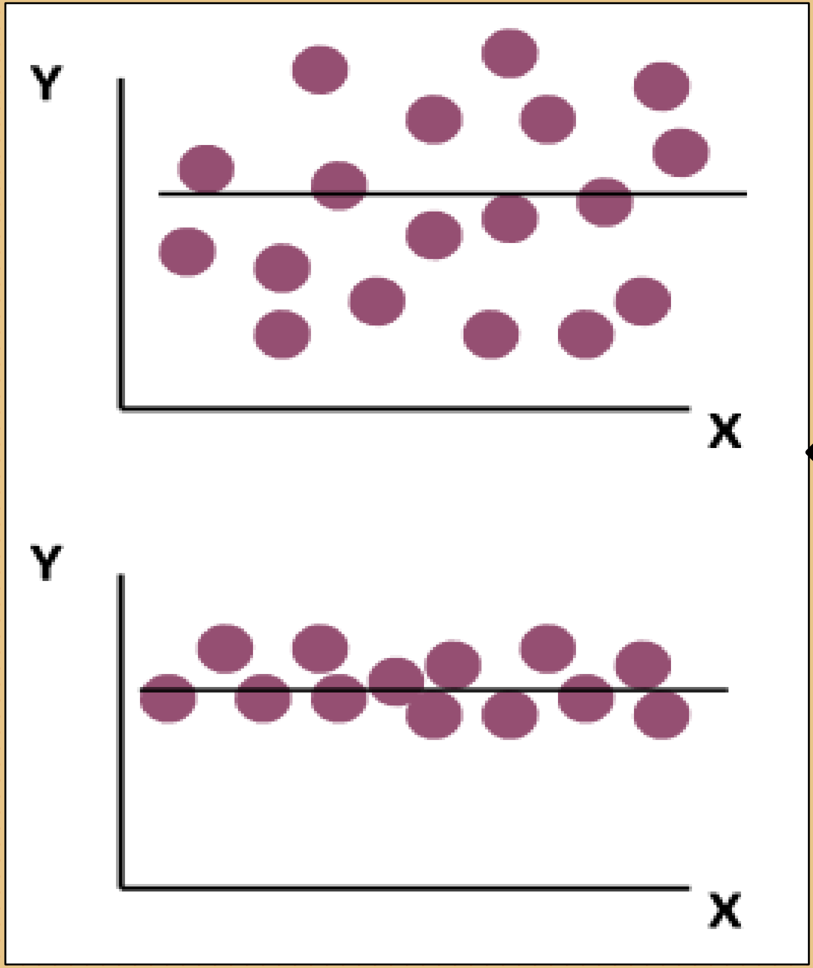




Strong Relationships

Weak Relationships





No Relationships

Correlation??

- *This measures* ***strength*** *and the* ***direction*** *of the* ***linear relationship*** *between two numerical variables.*

- *Correlation is a* ***value*** *in between* ***-1 & +1****.*

Population → ρ

Sample → r

- *This is also known as* ***Pearson product-moment correlation coefficient****.*

Sample correlation coefficient (r),

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2]\,[n(\sum y^2) - (\sum y)^2]}}$$

$r = -1$ → ***Perfect Negative Linear Relationship***

$r = 0$ → ***No Linear Relationship***

$r = +1$ → ***Perfect Positive Linear Relationship***

Exercise:

In the pursuit of finding whether the age affects the systolic blood pressure of females, the following data were observed from 10 randomly selected females between ages 40 and 82.

Age	Systolic BP
63	151
70	149
74	164
82	157
60	144
44	130
80	157
71	160
71	121
41	125

Correlation – Hypothesis Testing

- A hypothesis test can be carried out to find whether the population correlation is zero.

- $H_0: \rho = 0$ *Vs.* $H_1: \rho \neq 0$

- Under H_0,

$$T = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$$

Regression??

- The process of ***finding a mathematical equation*** that ***best fits the noisy data*** is known as ***regression analysis***.

- In this session, only ***Simple Linear Regression models*** are discussed.

- The ***primary usage*** of a regression model is ***prediction***.

Simple Linear Regression Model

$$\boxed{Y = \alpha + \beta X + \varepsilon}$$

- α - y Intercept
- β - Regression Coefficient (Slope)
- ε - Random Error

- This model is defined for population data.
- Should be careful when making predictions outside the observed range.

- α and β in the regression model are population characteristics which cannot be measured straightaway.

- Therefore, they should be estimated by using sample data.

- Estimated regression model would be as follows.

$$\hat{y} = \hat{\alpha} + \hat{\beta}X$$

$$\hat{\beta} = b = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$$

$$\hat{\alpha} = a = \bar{y} - b\bar{x}$$

Significance of Regression Coefficient

- A hypothesis test can be carried out to find whether the true slope (β) is actually zero (This is same as testing whether the regression
- model is significant).

- An ***ANOVA*** table is used to evaluate the ***test statistic*** for this test.

ANOVA Table

Model	Sum of Squares (SS)	Df (Degrees of Freedom)	Mean Sum of Square (MSS)	F Statistic	P Value
Regression	SSR	1	MSSR	F Statistic	
Error / Residual	SSE	n-2	MSSE		
Total	SST	n-1			

- $SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$
- $SSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
- $SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$
- $SST = SSR + SSE$
- $MSSR = SSR /1$
- $MSSE = SSE/(n-2)$
- $F\ Statistic = MSSR/MSSE$
- $P\ value = \Pr(F > F_{Cal})$

Coefficient of Determination (R^2)

- One way to measure the strength of the relationship between the response variable (*y*) and the predictor variable (*x*) is to calculate coefficient of determination.
- This refers to the proportion of the total variation that is explained by the linear regression of *y* on *x*. In other words, R^2 is percentage of variation of *Y* explained by the *X* variable in the fitted model.

$$\boxed{R^2 = \frac{SSR * 100}{SST}}$$

Regression Assumptions

- The model is linear in parameters
- $E(\varepsilon_i) = 0$ (Mean of residuals is zero)
- $V(\varepsilon_i) = \sigma^2$ (Variance of residuals are constant)
- The residuals (ε_i) are normally distributed.
- The residuals (ε_i) are independent.

Remember that, ***neither correlation nor regression imply any causation*** between variables.

Thanks!

Any questions?