8. STATISTICAL INFERENCE [IT2110]

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- In most researches, we collect data through a sample survey over a census.

- Statistical inference is used when sample survey is conducted over a census.

- *Inference:* A conclusion reached on the basis of evidence and reasoning.

  - Oxford University Press -

- *Statistical Inference:* Drawing conclusions about population parameters by using sample statistics.

PARAMETER ESTIMATION

Parameter Estimation

- In distribution theory we assumed that distribution parameters are known.

- But practically they should be found or estimated.

- If estimated parameters are wrong, all calculated probabilities will be inaccurate.

- Estimation can be done in two methods.

  ➢ *Point estimation*

  ➢ *Interval estimation*

Parameter Estimation

- Point estimation gives a single estimated value for the parameter.
- Interval estimation gives a range of values (interval) as the estimate.
- There are many point and interval estimation methods with their own criteria for use.
- Some interval estimates will be discussed later in this chapter.

HYPOTHESIS TESTING
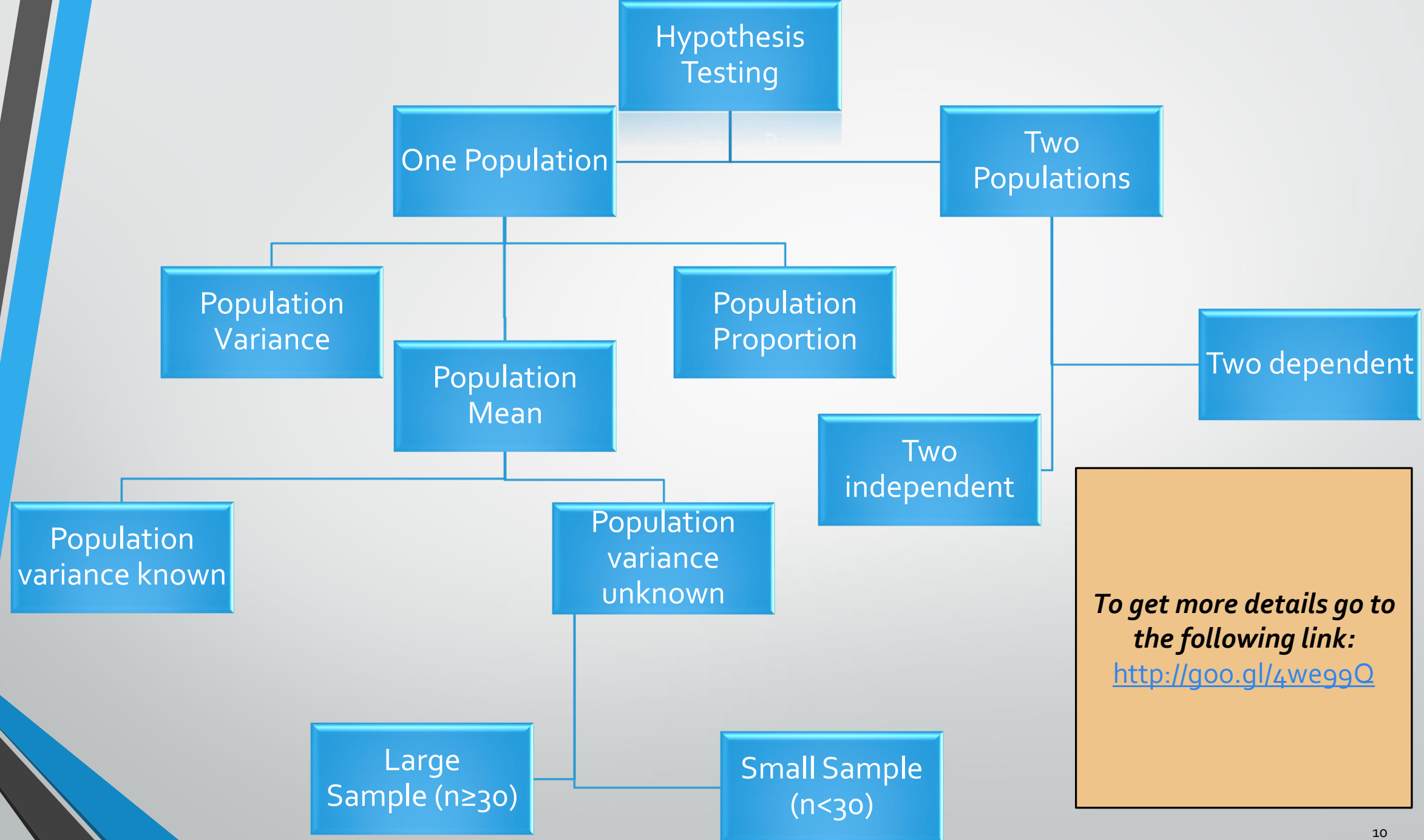
Hypothesis Testing

- ***Hypothesis:*** A supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation.

  -Oxford University Press-

- Hypothesis testing is all about checking whether assumptions (research hypothesis) are correct.

- These assumption should be regarding population parameters.

Major Steps under Hypothesis Testing

1. Define the hypothesis (H_0 & H_1)
2. Test statistic and its distribution
3. Define the significance level (α)
4. Define the rejection region.
5. Conduct the test (Calculate test statistic value)
6. Conclusion

- There are various cases under hypothesis testing. The test statistic that you should use depends on the case.
- In this session, we will discuss the hypothesis testing for one population mean.
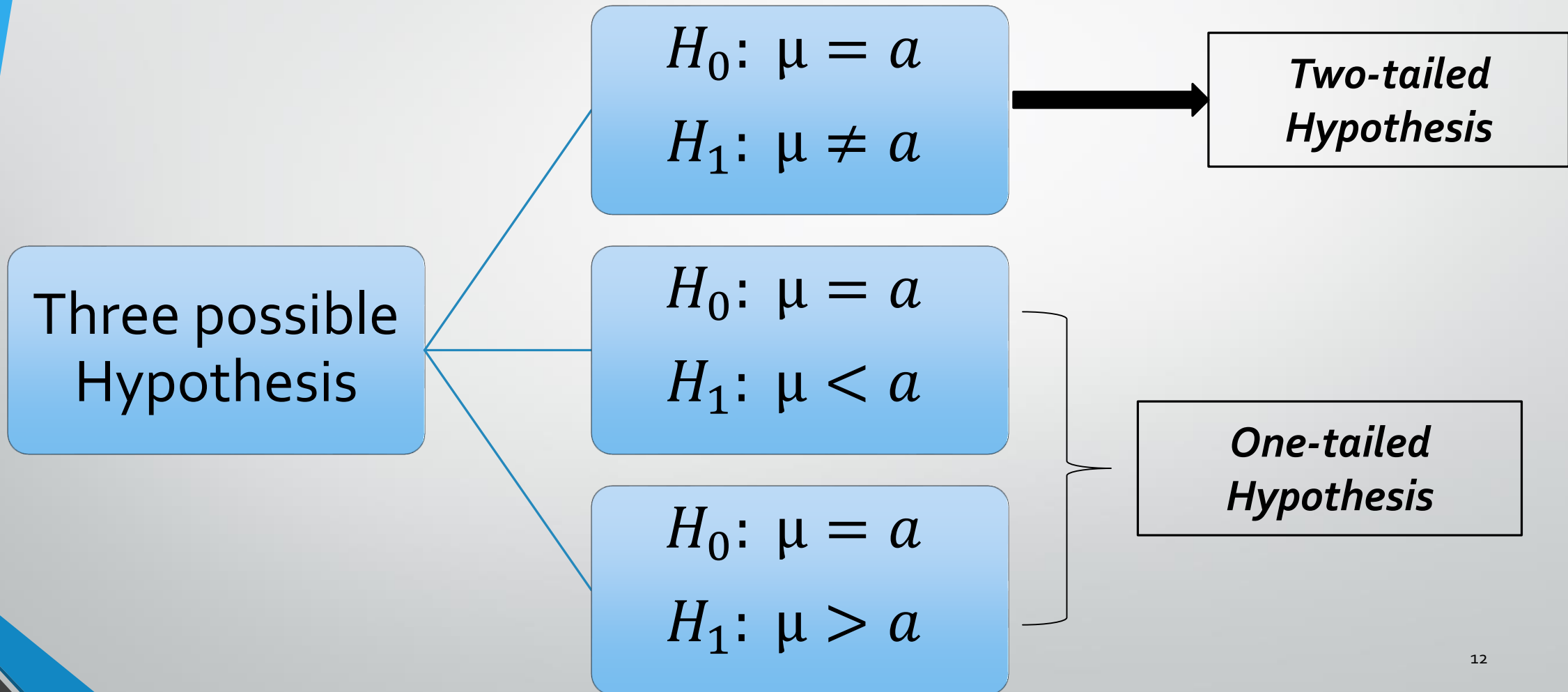
Defining Hypothesis

- The assumption should be clearly stated in order to test.

- Two statements, null hypothesis (H_0) and an alternative hypothesis (H_1 or H_a) are used for that.

- H_0 and H_1 can be considered as opposites of each other.

- The statement with the equal (=) should always come to H_0. Usually if a claim is made, it is selected for H_1.

Defining Hypothesis

Three possible Hypothesis

$$H_0: \mu = a$$
$$H_1: \mu \neq a$$

Two-tailed Hypothesis

$$H_0: \mu = a$$
$$H_1: \mu < a$$

$$H_0: \mu = a$$
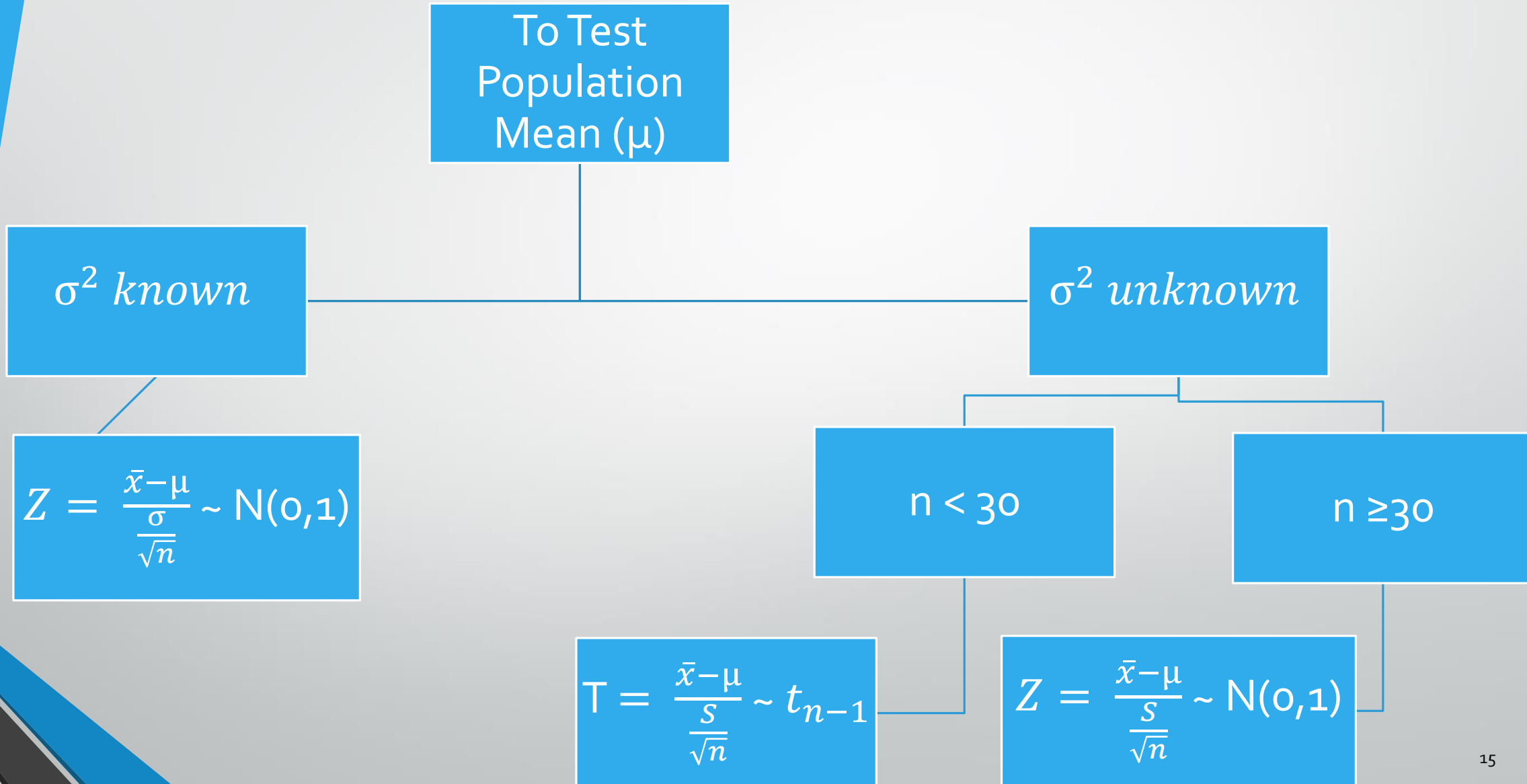$$H_1: \mu > a$$

One-tailed Hypothesis

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Examples

1) In a coin tossing experiment, it should be found whether

   a) it's fair coin or not.

   b) it's biased in favor of heads.

   c) it's biased in favor of tails.

2) A company that manufactures cars claims that the gas mileage for its new line of hybrid cars, on the average, is 60 miles per gallon (mpg) with a standard deviation of 4 mpg. It was also found out that the mpg was normally distributed. A random sample of 16 cars yielded a mean of 57 miles per gallon. Is the company's claim about the mean gas mileage per gallon of its cars, correct?

Test Statistic

- ***Recap:*** A function of observable r.v.s that does not depend on any unknown parameters is called a statistic.

- A test statistic is a quantity associated with the sample.

- The test statistic will depend on the ***parameter of interest*** as well as the ***characteristics of the population***.

- We assume that the assumption (H_0) is correct and find a sampling distribution for the test statistic.

Test Statistic & Distribution

To Test Population Mean (μ)

$\sigma^2\ known$

$\sigma^2\ unknown$

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$$

n < 30

n ≥30

$$T = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

$$Z = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} \sim N(0,1)$$

Test Statistic [For μ - When σ^2 known]

- **Recap:** Let $X_1, \ldots, X_n$ be a random sample of size n from a Normal population with mean μ and variance σ^2. Then,

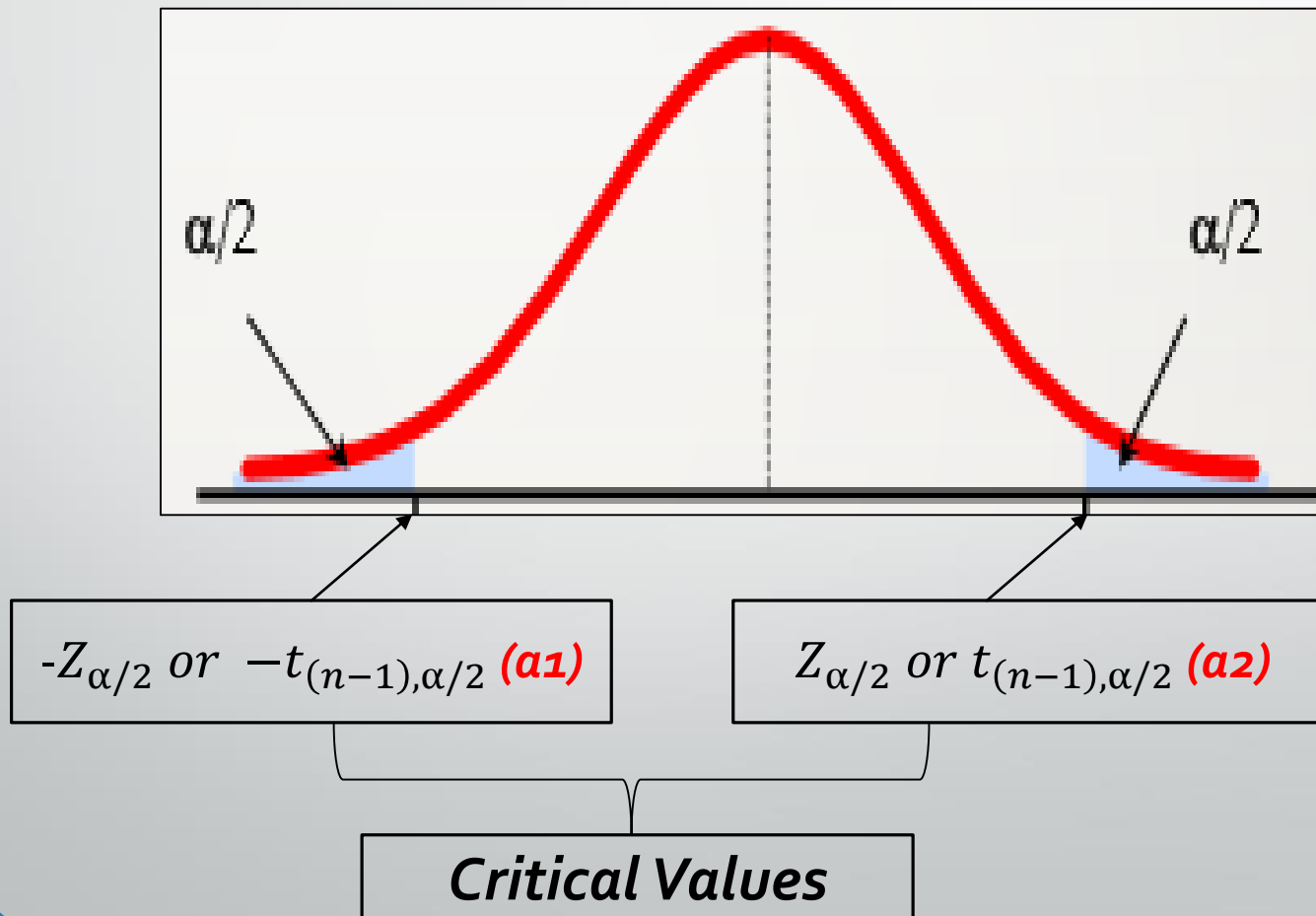$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

- Then,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

- If the hypothesis is, $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$, then under H_0,

$$\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

Rejection Region [**For μ**]
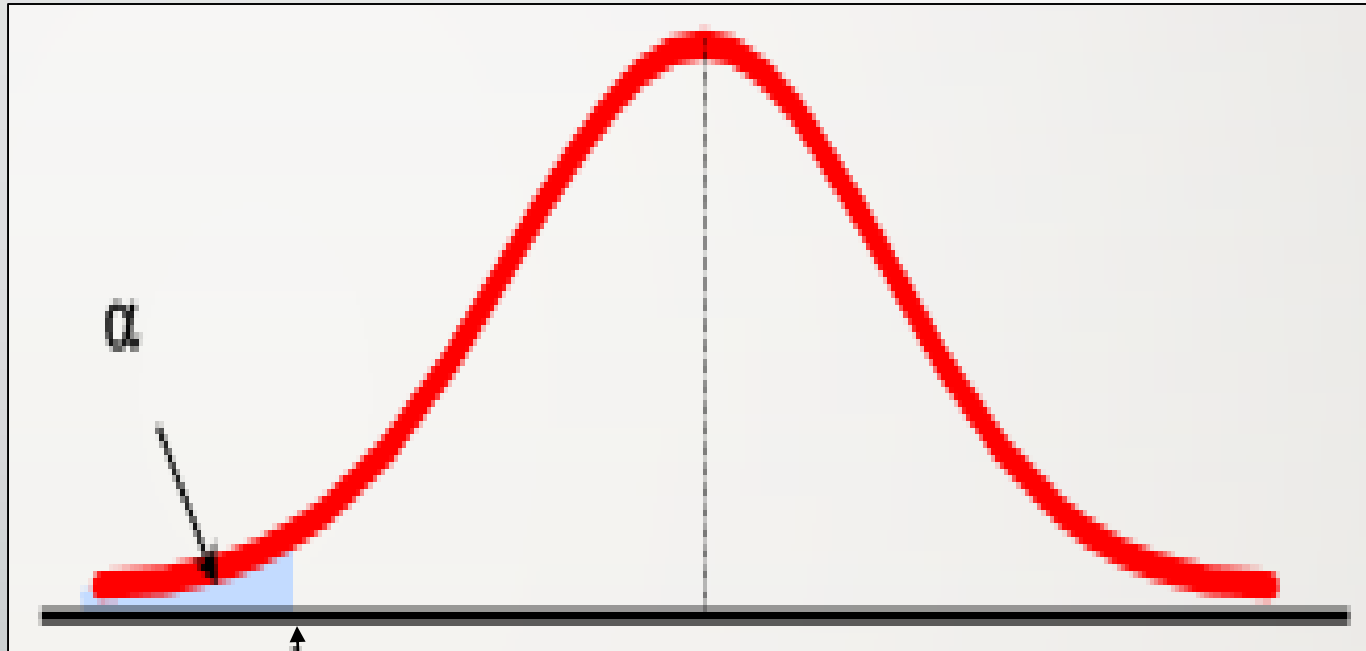
For a two-tailed hypothesis

$\alpha/2$

$\alpha/2$

$-Z_{\alpha/2}$ or $-t_{(n-1),\alpha/2}$ **(a1)**

$Z_{\alpha/2}$ or $t_{(n-1),\alpha/2}$ **(a2)**

Critical Values

Reject H_0 if $Z_{cal} \geq$ **a2**
OR if $Z_{cal} \leq$ **a1**

__For a one-tailed hypothesis__

$$H_0 : \mu = a$$
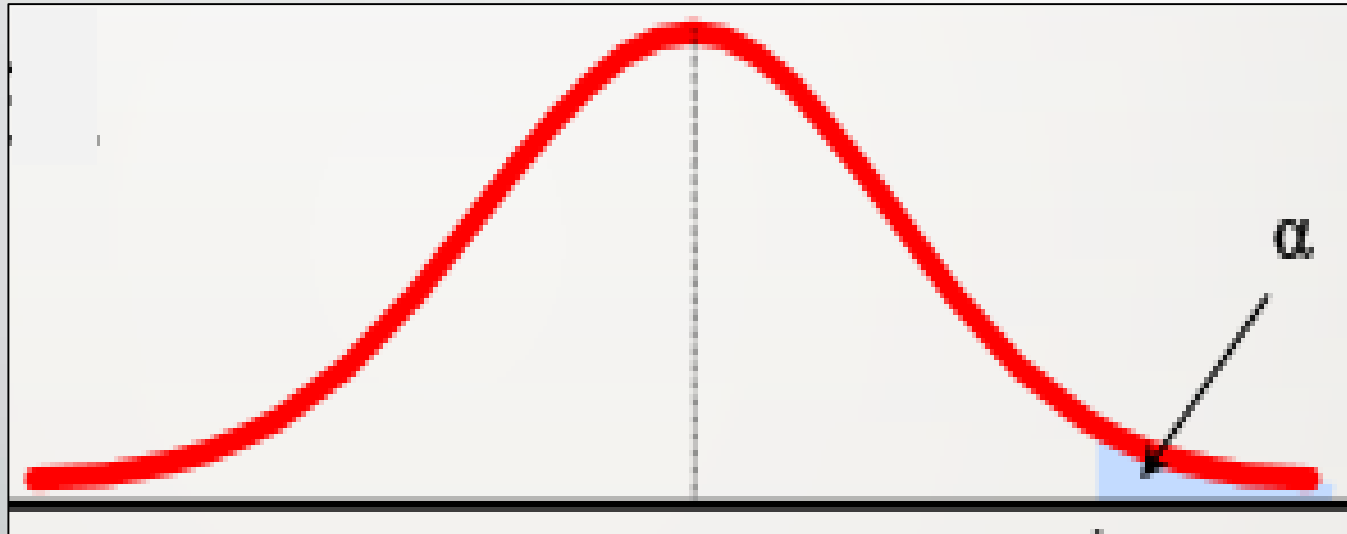$$H_1 : \mu < a$$

α

$-Z_\alpha \ or \ -t_{(n-1),\alpha}$ *(a1)* ← **Critical Value**

Reject H_0 if $Z_{cal} <$ **a1**

For a one-tailed hypothesis

$$H_0 : \mu = a$$
$$H_1 : \mu > a$$

α

Reject H_0 if $Z_{cal} >$ ***a1***

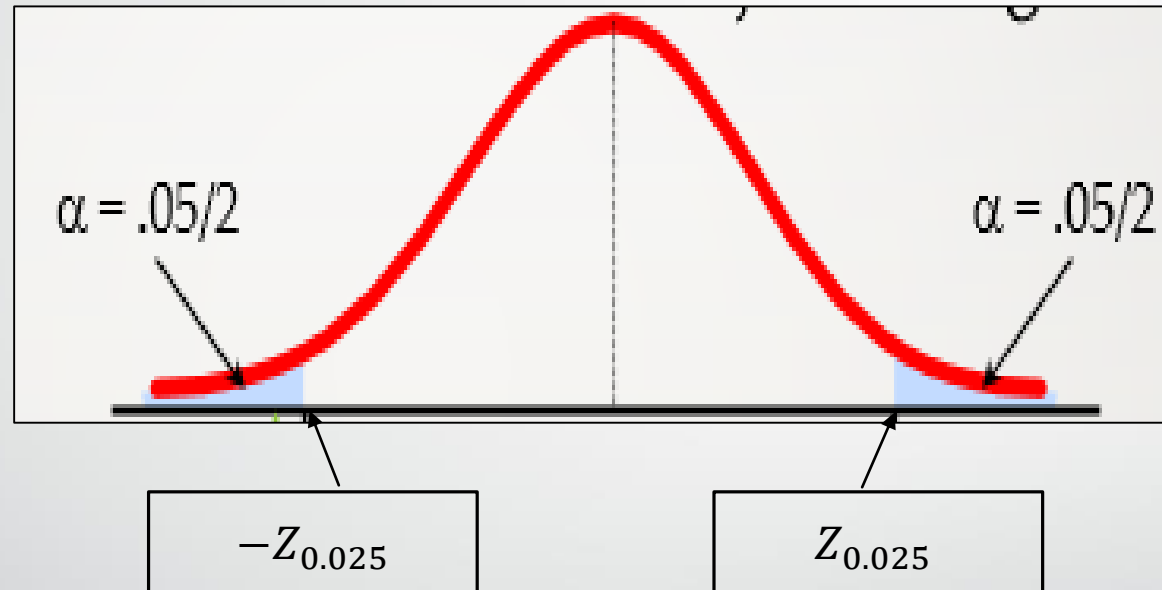***Critical Value*** → $Z_\alpha \ or \ t_{(n-1),\alpha}$ ***(a1)***

Example 02:

- $H_0: \mu = 60$
  $H_1: \mu \neq 60$

  Two-tailed hypothesis

- **Test Statistic**: Under H_0,

$$Z = \frac{\bar{x} - 60}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

- *Consider 5% level of significance.*

- *Rejection Region:*

$\alpha = .05/2$ $\alpha = .05/2$

$-Z_{0.025}$ $Z_{0.025}$

Reject H_0 if $Z_{cal} > Z_{0.025}$ **OR** if $Z_{cal} < -Z_{0.025}$

$Z_{0.025} = 1.96$

- **Test:**

$$\bar{x} = 57 \quad , \sigma = 4 \ \& \ n = 16$$

Then,
$$Z_{Cal} = \frac{\bar{x} - 60}{\frac{\sigma}{\sqrt{n}}}$$

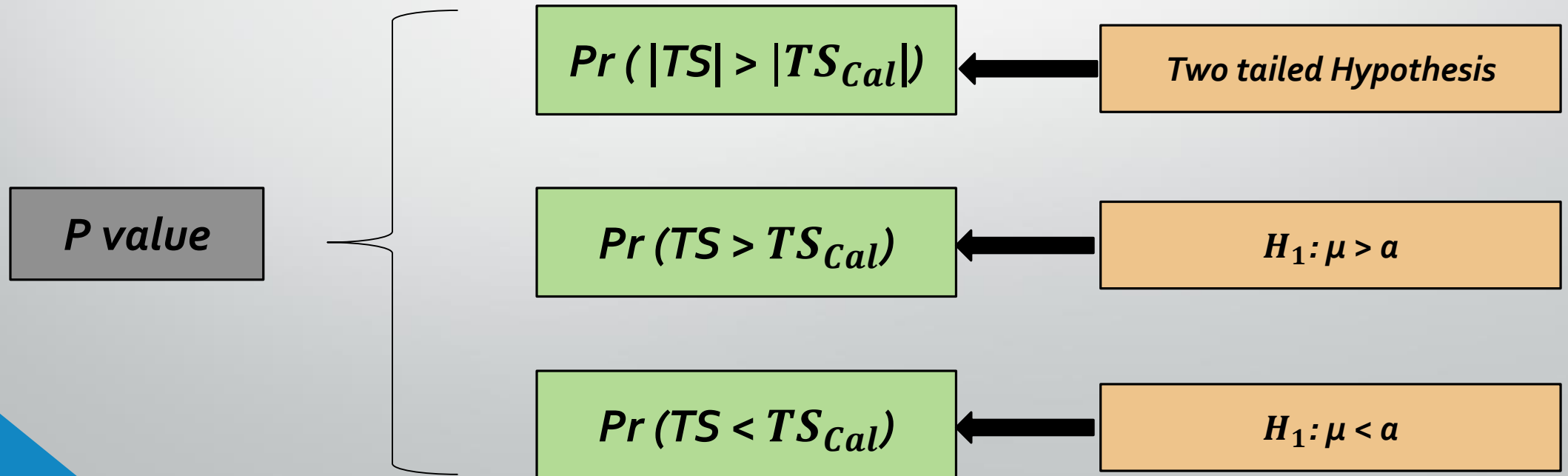$$Z_{Cal} = \frac{57 - 60}{\frac{4}{\sqrt{16}}}$$

$$Z_{Cal} = -3$$

- **Conclusion:**

Since Z_{Cal} = -3 < -1.96 = $Z_{0.025}$, we reject H_0 at 5% level of significance. Therefore, we can conclude that company's claim about the mean gas mileage per gallon of its cars is incorrect.

P value Approach

- This is an alternative way of get the decision in hypothesis testing.
- ***P value:*** *The probability of obtaining a test statistic which is more extreme than observed test statistic value given when H_0 is true.*

	$Pr\ (\ \lvert TS \rvert > \lvert TS_{Cal} \rvert)$	**Two tailed Hypothesis**
P value	$Pr\ (TS > TS_{Cal})$	$H_1 : \mu > a$
	$Pr\ (TS < TS_{Cal})$	$H_1 : \mu < a$

- *For any test,*

$$\text{If p value} \leq \text{significance level } (\alpha) \longrightarrow \text{Reject } H_0$$

$$\text{If p value} > \text{significance level } (\alpha) \longrightarrow \text{Do not Reject } H_0$$

- P value is a measure of the strength of evidence in the data against $H0$

- This is the smallest value of α for which $H0$ can be rejected and *actual risk of committing type I error*.

- P value also know as *observed significance level*.

Errors in Hypothesis Testing

Statistical Decision	True State of the Null Hypothesis	
	H_0 is True	H_0 is False
Reject H_0	Type I Error	Correct
Do not Reject H_0	Correct	Type II Error

Pr (Type I Error) = Pr (Reject H_0|H_0 true) = α
Pr (Type II Error) = Pr (Do not Reject H_0|H_0 false) = β

THANKS!

Any questions?