

# **6. CONTINUOUS PROBABILITY DISTRIBUTIONS [IT2110]**

*By Department of Mathematics and Statistics  
Faculty of Humanities and Sciences, SLIIT*

# Random Variables



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graph TD; A[Random Variables] --> B[Discrete Random Variables]; A --> C[Continuous Random Variables];
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Discrete  
Random  
Variables

Continuous  
Random  
Variables

# Continuous Random Variables

- A random variable is said to be continuous, if it can take any value within a range.
- Continuous data are frequently measured in some way rather than counted.
- If  $X$  is a continuous random variable,  $Pr(X=a) = 0$  for any value of  $a$ .

# Examples

- Temperature
- Heart beat of a patient
- Rainfall
- Waiting time for a bus

# PROBABILITY DISTRIBUTIONS

- For continuous random variables, the probability distribution cannot be presented in a tabular form.
- Probability distribution function of a continuous random variable is known as probability density function (*pdf*).
- The area under the p.d.f. gives probability values.

# PDF - DEFINITION

- The function  $f_X(x)$  is a probability density function for the continuous random variable  $X$ , defined over the set of real numbers ( $\mathbb{R}$ ), if
  - $f_X(x) \geq 0$ , for all  $x \in \mathbb{R}$
  - $\int_{-\infty}^{\infty} f_X(x) dx = 1$
  - $Pr(a < X < b) = \int_a^b f_X(x) dx$

# Properties

- Let  $X$  be a continuous random variable with a p.d.f. ( $f_X(x)$ ), defined over the set of real numbers ( $\mathbb{R}$ ).
  - The c.d.f.  $F_X(x) = \Pr(X \leq x) = \int_{-\infty}^x f_X(x) dx$
  - $E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$
  - $V[g(x)] = E[g(x)^2] - \{E[g(x)]\}^2$



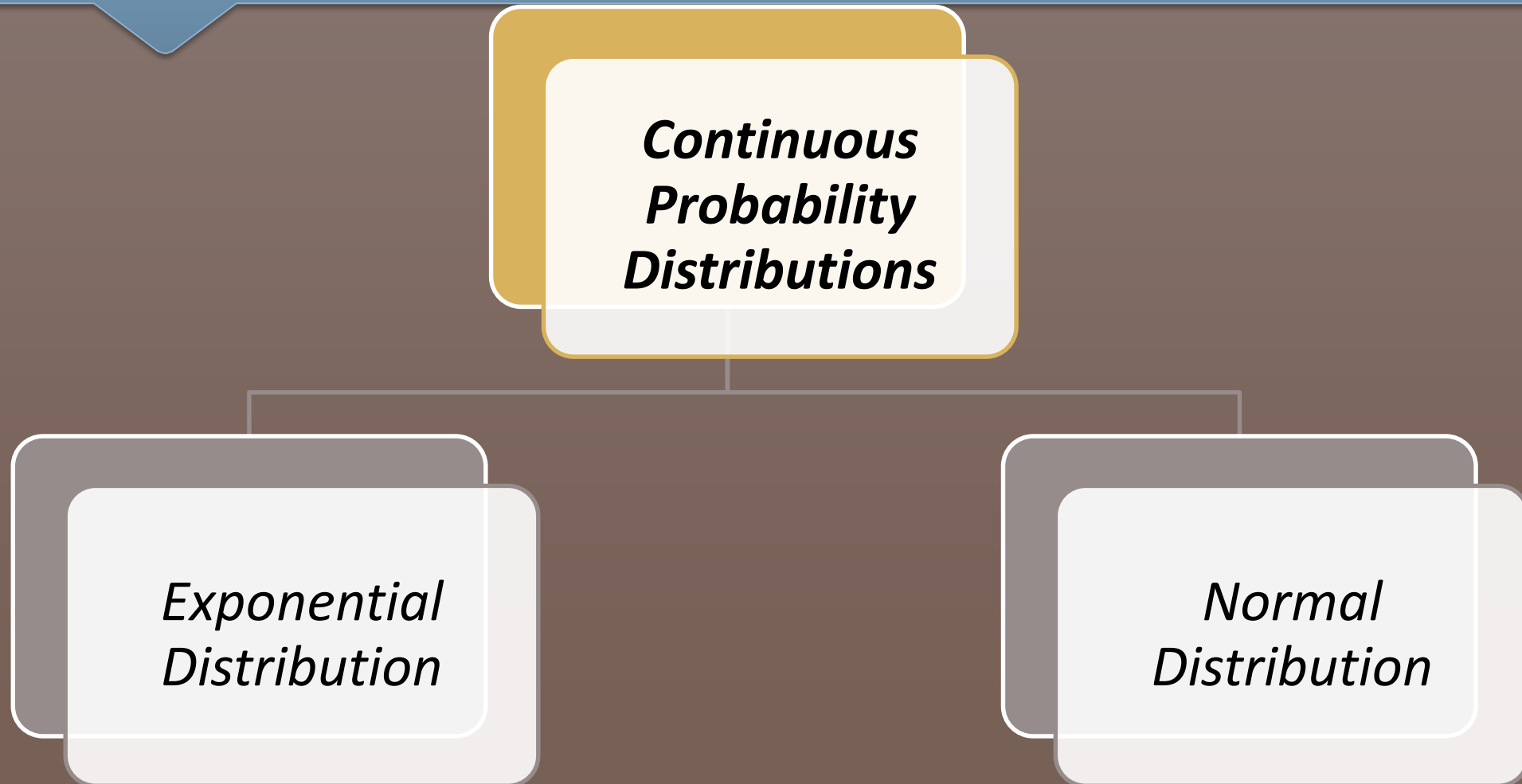
# PDF - Example

Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable  $X$  having the probability density function,

$$f_X(x) = \begin{cases} cx^2 & -1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

- 1) Find the value of  $c$
- 2) Find  $Pr(0 < X \leq 1)$ .
- 3) Find the expected value and the variance.
- 4) Find the c.d.f.

# Continuous Probability Distributions

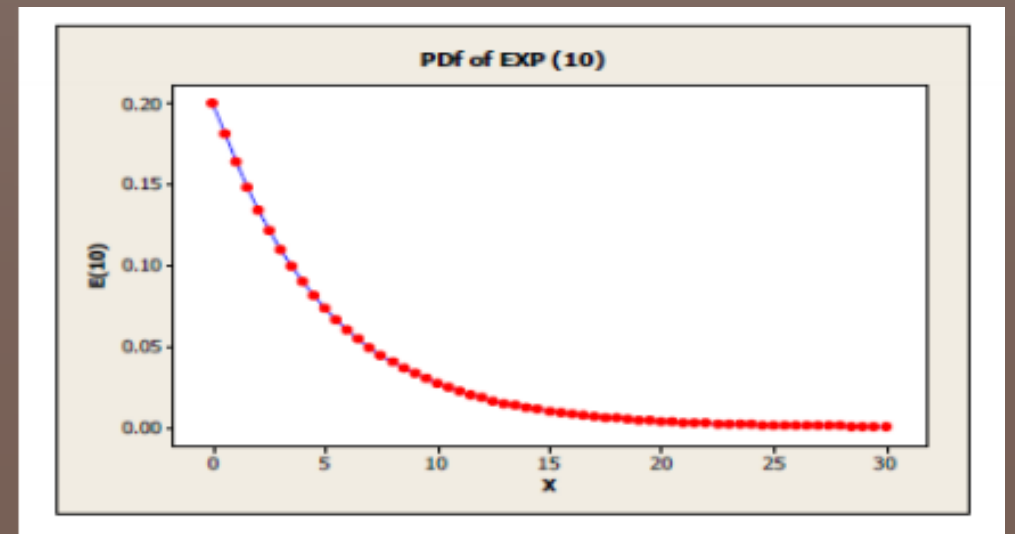


# Continuous Distributions

Exponential Distribution	Normal Distribution / Gaussian Distribution
The distribution is usually used to <b>model life times</b> . (There is a link to the Poisson distribution)	This is most commonly used distribution. This is bell shaped distribution and perfectly symmetric around $\mu$ .
$X \sim \text{Exp}(\lambda)$	$X \sim N(\mu, \sigma^2)$
$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$	$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], -\infty < x < \infty$
$E(X) = 1/\lambda$	$E(X) = \mu$
$V(X) = 1/\lambda^2$	$V(X) = \sigma^2$

# Exponential Distribution

- Widely used in waiting line (or queuing) theory to model the length of time between arrivals in process.
- **Examples:** duration between two customers at Bank ATMs, To model patients entering to an accident ward.



# Exponential Distribution - Example

- 1) The time, in hours, during which an electrical generator is operational is a random variable that follows an exponential distribution with a mean of 160. What is the probability that a generator of this type will be operational for,
- a) Less than 40 hours?
  - b) Between 60 and 160 hours?
  - c) More than 200 hours?

# Standard Normal Distribution

- Normal distribution with  $\mu=0$  and  $\sigma^2=1$  is known as the Standard Normal Distribution.
- Evaluating probabilities with Normal requires complex integration.
- To simplify the procedure, statistical tables are defined.
- But, tables for each combination of  $\mu$  and  $\sigma^2$  cannot be created.
- So, tables are only for the standard normal distribution.

# Normal $\longrightarrow$ Standard Normal

If  $X \sim N(\mu, \sigma^2)$ , Then

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

# Normal Distribution - Examples

- 1) For  $Z \sim N(0, 1)$ , calculate  $Pr(Z \geq 1.13)$ .
- 2) For  $X \sim N(5, 4)$ , calculate  $Pr(-2.5 < X < 1.13)$ .
- 3) The actual marks for FCS of Metro students revealed that they were normally distributed with a mean mark of 45 and a standard deviation of 22. What is the probability that a randomly chosen student will pass? (Assume that pass mark is 45)



# Approximating Binomial Probabilities

## Normal Distribution

- For  $X \sim \text{Bin}(n, p)$  this approximation can be used if  $n$  is large and  $p$  is moderate.
- A general rule can be defined as,  $np$  and  $n(1 - p)$  is greater than 5.
- Can be approximated with a r.v. with a distribution  $N(np, np(1 - p))$ .
- A continuity correction is needed because a discrete distribution is approximated with a continuous distribution.

# Continuity Correction

- If  $X \sim \text{Bin}(n, p)$  is approximated with a r.v.  $Y \sim N(np, np(1 - p))$ ,
  - $\Pr(X \leq a) = P(Y < a+0.5)$
  - $\Pr(X \geq a) = P(Y > a-0.5)$
  - $\Pr(X < a) = P(Y < a-0.5)$
  - $\Pr(X > a) = P(Y > a+0.5)$
  - $\Pr(X = a) = P(a-0.5 < Y < a+0.5)$

# Example

Suppose that a sample of  $n = 1,600$  tires of the same type are obtained at random from an ongoing production process in which 8% of all such tires produced are defective. What is the probability that in such a sample 150 or fewer tires will be defective?

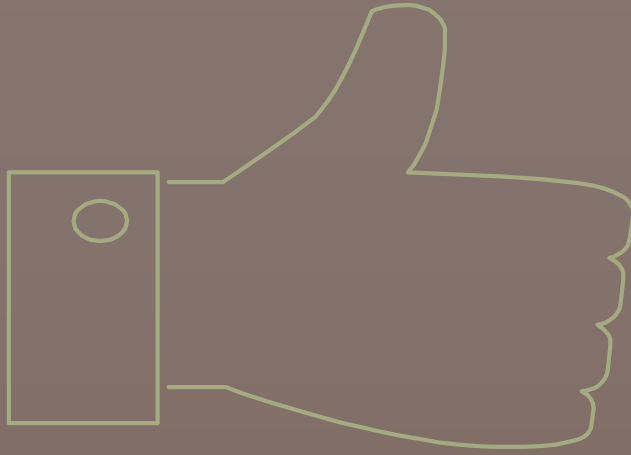
# Approximating Poisson Probabilities

## Normal Distribution

- If  $X \sim \text{Poisson}(\lambda)$  then if  $\lambda$  is greater than 20, the approximation can be used.
- Can be approximated with a r.v. with a distribution  $N(\lambda, \lambda)$ .
- A continuity correction is needed because a discrete distribution is approximated with a continuous distribution (just as in the case of the Binomial to Normal approximation).

# Example

The annual number of earthquakes registering at least 2.5 on the Richter Scale and having an epicenter within 40 miles of down town Memphis follows a Poisson distribution with mean 22.5. What is the probability that at least 25 such earthquakes will strike next year?



# THANKS!

**Any questions?**