

BSc (Hons) in Information Technology Year 2

Data Structures and Algorithms – IT2070

Tutorial 9 – Heaps

Semester 2, 2023

Question 1

- a) What is a binary tree?
- b) Show that the relationship between height (h) of a Full Binary Tree and the number of nodes (n) is given by $n = 2^{h+1} - 1$.

Question 2

- a) The following are the algorithms for Heap sort, Max Build Heap and Max_Heapify.

HEAPSORT(A)

1. BUILD_HEAP[A]
 2. for $i = A.length$ down to 2
 3. Exchange $A[1]$ with $A[i]$
 4. $A.heap_size = A.heap_size - 1$;
 5. MAX_HEAPIFY(A, 1)
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MAX_BUILD_HEAP (A)

1. $A.heap_size = A.length$
 2. for $i = \lfloor A.length/2 \rfloor$ downto 1
 3. MAX_HEAPIFY(A, i)
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MAX_HEAPIFY (A, i)

1. $l = LEFT_CHILD(i)$;
 2. $r = RIGHT_CHILD(i)$;
 3. if $l \leq A.heap_size$ and $A[l] > A[i]$
 4. then $largest = l$;
 5. else $largest = i$;
 6. if $r \leq A.heap_size$ and $A[r] > A[largest]$
 7. then $largest = r$;
 8. if $largest \neq i$
 9. then exchange $A[i]$ with $A[largest]$
 10. MAX_HEAPIFY (A, $largest$)
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Illustrate the operations of the Heap sort for the array **A** of elements given below. (For the purpose of illustration, assign the values only once to the given algorithm and use diagrammatic way to reach the answer.)

1	2	3	4	5	6	7	8
4	20	30	1	50	60	0	80

b) We can compute the upper bound on the running time of BUILD-HEAP as follows.

$$T(n) = \sum_{h=0}^{\lceil \lg n \rceil} \left\lceil \frac{n}{2^{h+1}} \right\rceil O(h) \quad \text{Briefly explain two components of the above}$$

equation.