

Algebraic MACs and Keyed-Verification Anonymous Credentials

Melissa Chase
Microsoft Research
melissac@microsoft.com

Sarah Meiklejohn
University College London
smeiklej@cs.ucsd.edu

Gregory M. Zaverucha
Microsoft Research
gregz@microsoft.com

ABSTRACT

We consider the problem of constructing anonymous credentials for use in a setting where the issuer of credentials is also the verifier, or more generally where the issuer and verifier have a shared key. In this setting we can use message authentication codes (MACs) instead of public key signatures as the basis for the credential system.

To this end, we construct two algebraic MACs in prime-order groups, along with efficient protocols for issuing credentials, asserting possession of a credential, and proving statements about hidden attributes (e.g., the age of the credential owner). We prove the security of the first scheme in the generic group model, and prove the security of the second scheme—using a dual-system-based approach—under decisional Diffie-Hellman (DDH). Our MACs are of independent interest, as they are the only *uf-cmva-secure* MACs with efficient proofs of knowledge.

Finally, we compare the efficiency of our new systems to two existing constructions of anonymous credentials: U-Prove and Idemix. We show that the performance of the new schemes is competitive with U-Prove (which does not have multi-show unlinkability), and many times faster than Idemix.

1. INTRODUCTION

An anonymous credentials system [14, 9, 11] allows for a landscape in which users can be known in different context by different pseudonyms. For example, a user Alice might be known to Bob under one pseudonym *nym*, and to Carol under a different pseudonym *nym'*. Her behavior under these two pseudonyms should be *unlinkable*, meaning no one can discern that the two pseudonyms belong to the same user, yet she should be able to prove possession of a credential issued to one given pseudonym to any other user, without revealing the pseudonym (e.g., if Carol issued a credential to *nym'*, Alice should nevertheless be able to prove to Bob—using *nym*—that she is in possession of the credential). Beyond proving basic possession of a credential,

Alice may want to furthermore prove that it certifies certain *attributes* about her (e.g., her age).

Many of the potential applications of anonymous credentials involve authentication. For example, a transit authority might issue monthly passes and then check for possession of such a pass when the user boards a bus; similarly, a university might issue keycards to access certain buildings, and then require a user to swipe his card upon entering one of these buildings. For each of these situations, the authority needs to know only that an authorized user is gaining entry, yet current implementations of such access systems allow the authority to learn the patterns of each individual participant (e.g., who is entering which building and when). Anonymous credentials present a solution that simultaneously preserves the integrity of the system through an *unforgeability* guarantee that a user cannot prove possession of credentials he wasn't issued (e.g., cannot gain access to a building without having been granted access) but also preserves the *anonymity* of the individual participants. If the anonymity is preserved throughout multiple presentations of the credential (e.g., each time a user boards the bus), it is said to satisfy *multi-show unlinkability*.

Traditionally, the models for anonymous credentials have assumed that a user must be prepared to prove possession of his credentials to many other participants in the system; as such, all existing constructions are built on top of public-key primitives such as digital signatures. In both of the above examples, however, the issuer and verifier are in fact the same entity; e.g., the transit authority both sells (i.e., issues) monthly passes and verifies them when users enter the system. In these scenarios, and more generally in any setting in which the party controlling access to a resource also manages the accounts of authorized parties, constructions can take advantage of this symmetry to use symmetric-key primitives—which are typically significantly more efficient than their public-key counterparts—to construct a *keyed-verification* credential.

If we define keyed-verification credentials as allowing the issuer and verifier to share access to some secret key, then symmetric-key primitives can be further adopted beyond the setting in which the issuer and verifier are the exact same entity. As long as a user does not need to authenticate himself to *any* other user in the system, the issuer can share a separate secret key with each verifier, and then issue credentials specific to each of these verifiers. In a purely non-interactive setting this might require the issuer to provide credentials for each verifier all at once (which might become

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than the author(s) must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

CCS'14, November 3–7, 2014, Scottsdale, Arizona, USA.

Copyright is held by the owner/author(s). Publication rights licensed to ACM.

ACM 978-1-4503-2957-6/14/11...\$15.00.

<http://dx.doi.org/10.1145/2660267.2660328>

fairly unattractive beyond a small number of verifiers), but in a more online setting the user might request credentials from the issuer as needed. (Interestingly, the blind issuance allows the user to do this in an anonymous fashion, as they can request a credential on the same attributes without the issuer's knowledge.)

More generally, it is also possible to translate a publicly verifiable credential into a more efficient keyed-verification credential with the same attributes and functionality. Another use case is thus as follows: when a user wishes to interact with a new verifier, he first enrolls with the verifier by presenting a traditional credential; the verifier checks this credential, then issues a new credential that only he can verify. Importantly, this protocol does not require that the user reveal the attributes in the traditional credential, or allow the issuer and verifier together to link the credential used during enrollment with a run of the issuance protocol. When the user returns, the efficient keyed-verification credential is used, and it is impossible to link this use with previous presentations of the credential or with the user's initial enrollment. Translating credentials in this manner provides the appealing trade-off that public verifiability is still possible when necessary, but credential use becomes more efficient with repeat verifiers.

Our contributions. In this paper, we introduce keyed verification credentials, which formalize the intuition outlined above. By using message authentication codes (MACs) in place of more traditional public-key signatures, we show that we can achieve performance improvements over existing constructions of anonymous credentials.

In order to integrate nicely with primitives such as zero-knowledge proofs (which are typically needed to construct anonymous credentials), we require an *algebraic MAC*, meaning a MAC constructed using group operations rather than block ciphers or hash functions. In Section 3, we present two such MACs, both constructed in prime-order groups. The first, MAC_{GGM} , is a generalization of a MAC presented by Dodis et al. [18]. While Dodis et al. show that this MAC provides only a very weak notion of security under the DDH assumption, we generalize the scheme to allow for blocks of messages, and then prove it satisfies the standard notion (uf-cmva security) of MAC unforgeability, albeit in the generic group model. The second, MAC_{DDH} , is a new construction which we propose, and which we prove to be uf-cmva secure under the DDH assumption.

Both MACs are of potential independent interest, as they avoid techniques such as collision-resistant hash functions and bit-wise decompositions, which have often been relied upon to construct efficient MACs. MAC_{GGM} is quite efficient; MAC_{DDH} only slightly less efficient and is based on a mild assumption. Interestingly, the proof of security for MAC_{DDH} follows the dual-system approach introduced by Waters [33], which makes it (to the best of our knowledge) the first application of this technique outside of the pairing-based setting.

Next, in Section 4, we present keyed-verification credentials. We first present a formal security model, and then present two constructions, one based on each of our MACs. For each construction, we describe how to efficiently issue and

prove possession of credentials. Our constructions consider credentials certifying many attributes at once to reflect situations with a complex access policy (e.g., in the transit setting, a monthly pass could encode an expiration date, the area of usage, etc.); allow for *blind* issuance of credentials, in which one or more of the attributes may remain hidden from the issuer (e.g., a senior citizen might need to provide their date of birth or other sensitive information in order to receive a discount transit pass, or one might want to transfer attributes from a publicly verifiable credential as discussed above); and allow for presentation of credentials with attributes satisfying a given statement. We then prove the security of our MAC_{DDH} -based construction under DDH. (The second scheme is secure under DDH and the assumption that MAC_{GGM} is secure.)

Finally, in Section 5, we provide a detailed efficiency comparison of our new keyed-verification credentials to U-Prove [8, 26] and Idemix [10, 23], the two most efficient anonymous credential schemes to date. Our comparison indicates that, depending on the parameters of the presentation, our new constructions both have the same or slightly higher cost when compared to U-Prove, and are always many times faster than Idemix (by our estimates, anywhere between 4 and 16 times faster).

Related work. The state of the art in MACs based on number-theoretic assumptions are the schemes by Dodis et al. [18]. (Their paper also contains a survey of prior work.) Of the nine MACs presented in [18], all either (1) satisfy a weaker security notion than uf-cmva, or (2) use hash functions or bitwise decomposition of the message, thus preventing an efficient proof of knowledge. Since our keyed-verification credential constructions require both of these properties, we cannot use these existing MACs directly. Section 3 describes the differences in more detail.

Anonymous credentials were introduced by Chaum [14] as a way to provide individuals more control over the disclosure of personal data. U-Prove [26] is a credential system constructed from a blind version of Schnorr signatures [8]. It is defined in a prime-order group, and is thus very computationally efficient. A U-Prove credential is constructed as a number of tokens, where each token may be used once unlinkably, so the size of U-Prove credentials is linear in the number of unlinkable uses. A recent paper of Baldimtsi and Lysyanskaya [3] presents a construction similar to U-Prove (without multi-show unlinkability), but with a security proof assuming the DDH assumption in the random oracle model (U-Prove does not have a formal proof of security). As the construction of [3] is strictly less efficient than U-Prove (see [3], Table 1), we chose not to include it in our efficiency comparison.

Idemix [23] is based on the Camenisch-Lysyanskaya [11] signature scheme (CL signatures). In terms of performance, Idemix and U-Prove credentials have an opposite trade-off: Idemix credentials have constant size, but are considerably more expensive to present. The computational cost is increased because the underlying signature scheme is constructed in a group where the strong RSA problem (SRSA) is hard. While there are no guidelines for choosing parameters for the strong RSA problem, they must be at least as

large as RSA parameters, e.g., 3072 bits for 128-bit security.¹ With multiple attributes, and advanced presentation proof predicates, this cost quickly becomes too high for lightweight provers such as smartcards [6].

There are also versions of the CL signature scheme defined in bilinear groups [1, 11], and Belenkiy et al. [4] construct anonymous credentials that support delegation. However, the algorithms in this setting are considerably more expensive, and the computational costs of creating a presentation proof and verifying it are still significantly greater than in U-Prove. We will estimate some of the costs of the CL scheme [11] in Section 5 to support this claim. The standardization of cryptographic schemes based on SRSA and bilinear groups also lags further behind prime-order groups, presenting another hurdle to deployment.

Given the trade-offs of each system, our design goal is a credential system with the strengths of U-Prove (efficient presentation and standard parameters), and those of Idemix (constant credential size and multi-show unlinkability).

2. PRELIMINARIES

Notation. We use the notation $x \in_R X$ or $x \xleftarrow{\$} X$ to mean x is chosen uniformly at random from the set X . The notation $\{x_i\}_1^n$, $\sum_{i=1}^n x_i$, and $\prod_{i=1}^n x_i$ are shorthand for $\{x_i\}_{i=1}^n$, $\sum_{i=1}^n x_i$, and $\prod_{i=1}^n x_i$ respectively. This shorthand is used only when the set, sum or product has a single index. The notation \vec{x} is used to denote the vector (x_1, \dots, x_n) , where n will be clear from the context.

We use games in the definition of MAC security and in proofs. A game G has a **MAIN** procedure whose output is the output of the game. $\text{Pr}[G]$ denotes the probability that this output is 1.

Zero-Knowledge Proofs. The protocols that comprise our credential system make use of zero-knowledge (ZK) proofs to prove knowledge of, and relations between, discrete logarithms. We abstract these protocols with a notation introduced by Camenisch and Stadler [12]. Proofs are expressed with the notation

$$\text{PK}\{(x, y, \dots) : \text{statements about } x, y, \dots\}$$

where (x, y, \dots) are secrets (discrete logarithms) which satisfy *statements*. The prover asserts knowledge of (x, y, \dots) , and all other values in the protocol are public.

There are many choices to implement these protocols, especially since the types of statements required by our protocols are relatively simple (knowledge of a representation and proof of logarithm equality). In particular, all the statements we prove can be captured by efficient sigma protocols.

For our application, we need a proof system that is zero knowledge and satisfies a weak form of online extraction [20]. We propose two approaches to instantiate the proof system. The first is to use the Damgård protocol [17], which converts any sigma protocol into a three-round interactive

¹Note the optimizations that apply to the RSA signing operation are available only to the issuer in Idemix, not the user or verifiers, as in that case the group order is unknown and exponents must be large to satisfy privacy requirements.

zero-knowledge proof of knowledge secure under concurrent composition. This protocol requires trusted parameters, but this restriction can be omitted in the random oracle model. The second option is to make the assumption that Fiat-Shamir based proofs [19] in the random oracle model satisfy the required extraction property. For more discussion and a more precise description of the necessary extraction property, see the full version[2].

Parameter Generation. Some of the parameters of our constructions include a group element h , chosen such that $\log_g h$ is unknown, where g is a generator of the group. In practice, this can be done by deterministically deriving h from arbitrary public information using a cryptographic hash function. All protocol participants may then verify that h was derived correctly by repeating the derivation process. One such derivation procedure is specified in [26]. Formally, we model this as a trusted setup algorithm which generates g, h where $\log_g h$ is unknown to all parties.

Cryptographic Assumptions. The *decisional Diffie-Hellman problem* (DDH) is the following: Let G be a cyclic group of prime order p with generator g and let $a, b, c \in_R \mathbb{F}_p$; given $(A = g^a, B = g^b, C) \in G^3$, determine whether $C = g^{ab}$ or $C = g^c$. The DDH assumption is that this problem is intractable for all polynomial time adversaries when p is exponential.

For some of our constructions we will also give security results in the *generic group model* (GGM). Intractability results in this model essentially mean that problems are intractable provided the adversary only performs a series of group operations. The GGM was first used by Shoup to prove lower bounds on DDH and related problems [7, 29].

Concrete examples of groups that are thought to satisfy these assumptions are certain elliptic curve groups over \mathbb{F}_p , such as those standardized by NIST in [25].

3. MACS IN PRIME-ORDER GROUPS

In this section we present two MACs constructed using a cyclic group of prime order. Both schemes use the same system parameters, created with the following algorithm.

Setup(1^k). Choose a group G with order p , where p is a k -bit prime. Let g and h be generators of G such that $\log_g h$ is unknown. The system parameters are $\text{params} := (G, p, g, h)$.

In addition to the **Setup** algorithm, MACs have a key generation function **KeyGen**, a MAC function **MAC** that produces an authentication tag on a message, and a verify function **Verify** that verifies a tag is valid with respect to a key and message. While we do not include it as an explicit parameter, the **MAC** and **Verify** functions are assumed to have params . This could easily be captured by including it in the secret key; we omit it to simplify the descriptions. The message space of both schemes is \mathbb{F}_p^n , where $n > 0$ is a parameter.

We say that $(\text{Setup}, \text{KeyGen}, \text{MAC}, \text{Verify})$ is a secure MAC if it is existentially unforgeable under chosen message attack, given a verification oracle (defined as uf-cmva in [18]). We

augment the definition slightly to guarantee security even when the signer publishes some parameters $iparams$ associated with his secret key. In our application to anonymous credentials, $iparams$ are the issuer parameters and we use them to implement an efficient presentation protocol.

DEFINITION 1 (UF-CMVA SECURITY). *For a MAC (Setup, KeyGen, MAC, Verify), define $\text{Adv}_{mac,A}^{uf-cmva}(k) = \Pr[\mathcal{G}_{uf-cmva}^A(k)]$, where $\mathcal{G}_{uf-cmva}^A(k)$ is defined as follows:*

MAIN $\mathcal{G}_{uf-cmva}^A(k)$
 $Q \leftarrow \emptyset$; $params \xleftarrow{\$} \text{Setup}(1^k)$;
 $(iparams, sk) \xleftarrow{\$} \text{KeyGen}(params)$
 $(m, \sigma) \xleftarrow{\$} A^{\text{MAC}, \text{VERIFY}}(params, iparams)$
 return $(m \notin Q) \wedge (\text{Verify}(sk, m, \sigma) = 1)$

Procedure $\text{MAC}_{sk}(m)$
 $Q \leftarrow Q \cup \{m\}$
 return $\text{MAC}(sk, m)$

Procedure $\text{VERIFY}_{sk}(m, \sigma)$
 return $\text{Verify}(sk, m)$

Then the MAC is uf-cmva secure if $\text{Adv}_{mac,A}^{uf-cmva}(k) < \nu(k)$ for all PPT adversaries A , where ν is a negligible function.

A stronger security notion for MACs is sometimes used, where A may win by outputting (m, σ) , even if $m \in Q$, provided σ was not output by the MAC oracle for m . The schemes we present were expressly designed *not* to provide this type of security, to allow tags to be *re-randomized* (or blinded) and thus allow for more efficient zero-knowledge proofs of possession of a MAC.

3.1 The MAC_{GGM} Construction

Our first MAC is a generalization of a scheme due to Dodis et al. [18]. The original MAC works in a cyclic group G of prime order p , and the secret key is a pair $(x_0, x_1) \in \mathbb{F}_p^2$. To compute the MAC of a message $m \in \mathbb{F}_p$, choose $u \in_R G$, and compute $(u, u^{mx_1+x_0})$ as the tag. To verify a tag (u, u') for a message m , check whether $u^{mx_1+x_0} = u'$.

We extend the scheme to support n attributes, where the secret key becomes (x_0, x_1, \dots, x_n) and tags are computed as $(u, u^{x_1 m_1 + \dots + x_n m_n + x_0})$. Note that m_1, \dots, m_n are n messages, each in \mathbb{F}_p , rather than the binary decomposition of a single message m . We refer to this scheme as MAC_{GGM} (the single message and binary message schemes were respectively called $\text{MAC}_{\text{hwhPRF}}$ and $\text{MAC}_{\text{WhwPRF}}$ in [18]). **KeyGen** has an optional step that is required only when MAC_{GGM} is used for keyed-verification credentials.

In what follows (including for MAC_{DDH}), we use $\vec{m} = (m_1, \dots, m_n)$ to mean a list of n messages in \mathbb{F}_p , and use $H_x(\vec{m}) := x_0 + \sum_i x_i m_i$.

KeyGen($params$): Choose a secret key $sk := \vec{x} \in_R \mathbb{F}_p^{n+1}$. Optionally, choose $\tilde{x}_0 \in_R \mathbb{F}_p$ and compute $C_{x_0} := g^{x_0} h^{\tilde{x}_0}$ and $(X_1 := h^{x_1}, \dots, X_n := h^{x_n})$, and publish the *issuer parameters*, denoted $iparams := (C_{x_0}, X_1, \dots, X_n)$.

MAC(sk, \vec{m}): Choose $u \in_R G \setminus \{1\}$ and compute the tag $\sigma = (u, u')$, where $u' := u^{H_x(\vec{m})}$.

Verify(sk, \vec{m}, σ): Parse $\sigma = (u, u') \in G^2$. Accept if $u \neq 1$ and $u' = u^{H_x(\vec{m})}$.

Dodis et al. [18] prove that under the DDH assumption, MAC_{GGM} is suf-cma secure when $n = 1$. In this definition, security is called *selective unforgeability*, because the attacker must select the message he will use in a forgery *before* seeing any tags, and is not allowed verification queries. For our credential system, however, we require uf-cmva security. (Selective unforgeability gives only limited protection against misbehaving adversaries, and verification queries are inherent in anonymous credentials as the adversary is always able to present credentials and observe the verifier's reaction.)

We stress that Dodis et al. give no evidence that MAC_{GGM} is not in fact uf-cmva secure. Rather, it appears that their proof technique does not extend to also prove security under the stronger definition. A simple (but inefficient) reduction exists between uf-cma and suf-cma. A uf-cma adversary is transformed into a suf-cma adversary by an algorithm which guesses the message to be forged by the uf-cma adversary. The success probability of the new adversary is $\epsilon/|M|$ where M is the message space of the scheme, and ϵ is the success probability of the uf-cma adversary. If the size of M is constrained, the loss in security may be acceptable (i.e., it may be acceptable to use an suf-cma-secure scheme). This may be of use in our application, in the very limited setting where credentials contain a small number of attributes from a small set, known to the issuer, and where during presentation the user is required to prove that all the attributes in his credential are within this set.

To ensure security in the more realistic case of unconstrained messages (attributes), and when verification queries are allowed (as in a credential system), we prove that MAC_{GGM} is uf-cmva secure in the generic group model. Additionally, we include $iparams$ in our analysis. A proof of the following theorem is given in the full version of this paper [2].

THEOREM 2. *In the generic group model, a uf-cmva adversary attacking the MAC_{GGM} scheme, succeeding with non-negligible probability, performs $\Omega(\sqrt{p})$ group operations.*

3.2 The MAC_{DDH} Construction

In this section, we describe another MAC construction, called MAC_{DDH} . Recall that $params$ are created by $\text{Setup}(1^k)$ and are assumed to be available to all algorithms, that $\vec{m} = (m_1, \dots, m_n)$ is a list of n messages in \mathbb{F}_p , and that the optional step in **KeyGen** is required only when MAC_{DDH} is used for keyed-verification credentials.

KeyGen($params$): Pick $z, x_0, y_0, \dots, x_n, y_n \xleftarrow{\$} \mathbb{F}_p$. Output $sk := (\vec{x}, \vec{y}, z)$. Optionally, choose $\tilde{x}_0, \tilde{y}_0, \tilde{z} \in_R \mathbb{F}_p$ and compute $C_{x_0} := g^{x_0} h^{\tilde{x}_0}$, $C_{y_0} := g^{y_0} h^{\tilde{y}_0}$, $C_z := g^z h^{\tilde{z}}$, compute $X_i := h^{x_i}$ and $Y_i := h^{y_i}$ for each $i \in \{1, \dots, n\}$, and publish $iparams := (C_{x_0}, C_{y_0}, C_z, \vec{X}, \vec{Y})$.²

²Since C_{x_0}, C_{y_0}, C_z are statistically independent of sk and the rest of the parameters, we can ignore them for the unforgeability proof.

MAC(sk, \vec{m}): Pick $r \xleftarrow{\$} \mathbb{F}_p$ and set $\sigma_w := g^r$, $\sigma_x := g^{rH_x(\vec{m})}$, $\sigma_y := g^{rH_y(\vec{m})}$, and $\sigma_z := g^{rz}$. Output $(\sigma_w, \sigma_x, \sigma_y, \sigma_z)$.

Verify(sk, \vec{m}, σ): Parse $\sigma = (\sigma_w, \sigma_x, \sigma_y, \sigma_z) \in G^4$. Check that $\sigma_w \neq 1$, $\sigma_x = \sigma_w^{H_x(\vec{m})}$, $\sigma_y = \sigma_w^{H_y(\vec{m})}$, and $\sigma_z = \sigma_w^z$. Accept if these checks pass and reject otherwise.

THEOREM 3. *If the DDH assumption holds in G , then MAC_{DDH} is uf-cmva secure.*

Our proof takes inspiration both from the dual system technique introduced by Waters [33], and from the twin Diffie-Hellman techniques of Cash, Kiltz, and Shoup[13]. Roughly, we use the twin DH approach to argue that we can switch to a verification procedure which does not require the full discrete logarithms \vec{x}, \vec{y}, z . As in [13] this allows us to reduce to DDH even in the presence of this verification oracle. Then we use the dual system approach to gradually add noise into the MACs, while still ensuring that our reduction can verify that the adversary's forgery is correctly formed. Finally we arrive at a game where part of the secret key is statistically hidden, so we can guarantee that the adversary can produce a correctly formed forgery only with negligible probability.

PROOF. Let A be a PT adversary playing game $\mathbf{G}_{\text{uf-cmva}}^A(k)$ that makes q_m MAC queries and q_v VERIFY queries, where $q_m = q_m(k)$ and $q_v = q_v(k)$ for polynomials $q_m(\cdot)$ and $q_v(\cdot)$. Define two games, $\mathbf{G}_0^A(k)$ and $\mathbf{G}_1^A(k)$ as follows:

- $\mathbf{G}_0^A(k)$: Pick additional values $s, t, v_0, \dots, v_n \xleftarrow{\$} \mathbb{F}_p$ and include these in the secret key sk . If $iparams$ are to be generated, compute $X_i = g^{x_i} h^{v_i}$ and $Y_i = g^{y_i} X_i^{-s}$ for $i \in \{1, \dots, n\}$. In the $\text{MAC}_{sk}(\vec{m})$ procedure, continue to pick a random value $r \xleftarrow{\$} \mathbb{F}_p$, but compute instead $\sigma_w \leftarrow h^r$, $\sigma_x \leftarrow (g^{H_x(\vec{m})} h^{H_v(\vec{m})})^r$, $\sigma_y \leftarrow (g^{H_y(\vec{m}) - sH_x(\vec{m})} h^{-sH_v(\vec{m})})^r$, and $\sigma_z \leftarrow (g^z h^{-t})^r$. In the $\text{VERIFY}_{sk}(\vec{m}, \sigma)$ procedure, check instead that $((\sigma_y \sigma_x^s)^z = (\sigma_z \sigma_w^t)^{H_y(\vec{m})}) \wedge (\sigma_w \neq 1)$, and at the end of the game check that $((\sigma_y \sigma_x^s)^z = (\sigma_z \sigma_w^t)^{H_y(\vec{m}^*)}) \wedge ((\sigma_w^{-H_v(\vec{m}^*)} \sigma_x)^{H_y(\vec{m}^*)} = (\sigma_x^s \sigma_y)^{H_x(\vec{m}^*)}) \wedge (\sigma_w \neq 1)$.
- $\mathbf{G}_1^A(k)$: Continue to run the setup and the verification procedure (both in the $\text{VERIFY}_{sk}(\vec{m}, \sigma)$ algorithm and at the end of the game) as in $\mathbf{G}_0^A(k)$. In $\text{MAC}_{sk}(\vec{m})$, pick $r, \omega, \chi \xleftarrow{\$} \mathbb{F}_p$ and compute $\sigma_w \leftarrow h^\omega$, $\sigma_x \leftarrow h^\chi$, $\sigma_y \leftarrow g^{rH_y(\vec{m})} h^{-\chi s}$, and $\sigma_z \leftarrow g^{zr} h^{-\omega t}$.

Now, we provide a PT adversary B and negligible functions $\nu_0(\cdot)$ and $\nu(\cdot)$ such that

$$\mathbf{Adv}_{\text{mac}, A}^{\text{uf-cmva}}(k) \leq q_v \nu_0(k) + q_m \mathbf{Adv}_B^{\text{ddh}}(k) + \nu(k)$$

for all $k \in \mathbb{N}$, from which the theorem follows. To do this, we build B , $\nu_0(\cdot)$, and $\nu(\cdot)$ such that for all $k \in \mathbb{N}$ we have

$$\Pr[\mathbf{G}_{\text{uf-cmva}}^A(k)] - \Pr[\mathbf{G}_0^A(k)] \leq q_v \nu_0(k) \quad (1)$$

$$\Pr[\mathbf{G}_0^A(k)] - \Pr[\mathbf{G}_1^A(k)] \leq q_m \mathbf{Adv}_B^{\text{ddh}}(k) \quad (2)$$

$$\Pr[\mathbf{G}_1^A(k)] \leq \nu(k). \quad (3)$$

We then have

$$\begin{aligned} \mathbf{Adv}_{\text{mac}, A}^{\text{uf-cmva}}(k) &= \Pr[\mathbf{G}_{\text{uf-cmva}}^A(k)] \\ &= (\Pr[\mathbf{G}_{\text{uf-cmva}}^A(k)] - \Pr[\mathbf{G}_0^A(k)]) \\ &\quad + (\Pr[\mathbf{G}_0^A(k)] - \Pr[\mathbf{G}_1^A(k)]) + \Pr[\mathbf{G}_1^A(k)] \\ &\leq q_v \nu_0(k) + q_m \mathbf{Adv}_B^{\text{ddh}}(k) + \nu(k). \end{aligned}$$

To first prove Equation 1, we consider a modified version of $\mathbf{G}_{\text{uf-cmva}}^A(k)$ as an intermediate game: rather than pick \vec{x} and \vec{y} randomly, pick $x'_i, y'_i, v_i \xleftarrow{\$} \mathbb{F}_p$, let $\beta = \log_g(h)$, and set $x_i := x'_i/\beta + v_i$ and $y_i := y'_i/\beta - s x_i$ for all i , $0 \leq i \leq n$; furthermore, rather than pick $z \xleftarrow{\$} \mathbb{F}_p$, pick $z' \xleftarrow{\$} \mathbb{F}_p$ and set $z := z'/\beta - t$. In the MAC oracle, use $r := r'\beta$ for $r' \xleftarrow{\$} \mathbb{F}_p$ rather than $r \xleftarrow{\$} \mathbb{F}_p$. These values are distributed identically to the values used in $\mathbf{G}_{\text{uf-cmva}}^A(k)$, so the distribution in this modified game is identical. Furthermore, we have

$$\begin{aligned} \sigma'_w &= g^r = g^{r'\beta} = h^{r'} \\ \sigma'_x &= g^{rH_x(\vec{m})} = g^{r'\beta(H_{x'}(\vec{m})/\beta + H_v(\vec{m}))} = (g^{H_{x'}(\vec{m})} h^{H_v(\vec{m})})^{r'} \\ \sigma'_y &= g^{rH_y(\vec{m})} = g^{r'\beta((H_{y'}(\vec{m}) - sH_{x'}(\vec{m}))/\beta - sH_v(\vec{m}))} \\ &= (g^{H_{y'}(\vec{m}) - sH_{x'}(\vec{m})} h^{-sH_v(\vec{m})})^{r'} \\ \sigma'_z &= g^{rz} = g^{r'\beta(z'/\beta - t)} = (g^{z'} h^{-t})^{r'}, \end{aligned}$$

so the MAC responses in the modified game are identical to those in $\mathbf{G}_0^A(k)$.

To address the changes in verification, we proceed through a series of hybrids: define $\mathbf{H}_i^A(k)$ to be a game in which the first i VERIFY queries are answered using Verify, the last $q_v - i$ are answered using the verification procedure defined in $\mathbf{G}_0^A(k)$ (referred to in the sequel as **SimVerify**), and the verification at the end is considered the $q_v + 1$ st query; then $\mathbf{H}_{q_v+1}^A(k)$ is the intermediate game and $\mathbf{H}_0^A(k)$ is $\mathbf{G}_0^A(k)$. To transition from $\mathbf{H}_i^A(k)$ to $\mathbf{H}_{i-1}^A(k)$ for $i < q_v + 1$, we therefore need only consider the i -th query (\vec{m}, σ) (as the two games are identical both before and after this query), and the probability that $\text{Verify}(sk, \vec{m}, \sigma) \neq \text{SimVerify}(sk, \vec{m}, \sigma)$; i.e., that $\mathbf{H}_{i-1}^A(k)$ and $\mathbf{H}_i^A(k)$ produce different responses on the i -th query. We refer to this event as E_i , and show that $\Pr[E_i] \leq \nu_0(k)$ for a negligible function $\nu_0(k)$.

If $\text{Verify}(sk, \vec{m}, \sigma) = \text{accept}$ then

$$\begin{aligned} (\sigma_y \sigma_x^s)^{z'} &= (\sigma_w^{H_y(\vec{m})} \sigma_w^{sH_x(\vec{m})})^{z'} \\ &= (\sigma_w^{(H_{y'}(\vec{m}) - sH_{x'}(\vec{m}))/\beta - sH_v(\vec{m})} \sigma_w^{s(H_{x'}(\vec{m})/\beta + H_v(\vec{m}))})^{z'} \\ &= (\sigma_w^{H_{y'}(\vec{m})/\beta})^{\beta(z+t)} \\ &= (\sigma_w^z \sigma_w^t)^{H_{y'}(\vec{m})} \\ &= (\sigma_z \sigma_w^t)^{H_{y'}(\vec{m})}, \end{aligned}$$

so $\text{SimVerify}(sk, \vec{m}, \sigma) = \text{accept}$ and event E_i happens only if $\text{SimVerify}(sk, \vec{m}, \sigma) = \text{accept}$ but $\text{Verify}(sk, \vec{m}, \sigma) = \text{reject}$. Consider R_x, R_y , and R_z such that $\sigma_x = R_x \sigma_w^{H_x(\vec{m})}$, $\sigma_y = R_y \sigma_w^{H_y(\vec{m})}$, and $\sigma_z = R_z \sigma_w^z$; then Verify accepts only if $R_x = R_y = R_z = 1$. If $\text{SimVerify}(sk, \vec{m}, \sigma) = \text{accept}$ then, since

$H_y(\vec{m}) + sH_x(\vec{m}) = H_{y'}(\vec{m})/\beta$ and $z' = \beta(z + t)$, we have

$$\begin{aligned} (\sigma_y \sigma_x^s)^{z'} &= (\sigma_z \sigma_w^t)^{H_{y'}(\vec{m})} \\ (R_y \sigma_w^{H_y(\vec{m})} R_x^s \sigma_w^{sH_x(\vec{m})})^{z'} &= (R_z \sigma_w^z \sigma_w^t)^{H_{y'}(\vec{m})} \\ (R_x^s R_y)^{\beta(z+t)} \sigma_w^{(z+t)H_{y'}(\vec{m})} &= R_z^{H_{y'}(\vec{m})} \sigma_w^{(z+t)H_{y'}(\vec{m})} \\ (R_x^s R_y)^{z'} &= R_z^{H_{y'}(\vec{m})}. \end{aligned}$$

As up until the i -th query A , the values of z' and y' are information theoretically hidden, it has at most a negligible probability (in fact, probability $1/2^k$) of coming up with R_x , R_y , and R_z that satisfy this equation but such that it is not the case that $R_x^s R_y = R_z = 1$. Similarly, we can argue that the value of s is information theoretically hidden from A 's view so far, so the probability that $R_x^s R_y = 1$ but it is not the case that $R_x = R_y = 1$ is also negligible. We therefore have that $\Pr[E_i] \leq \nu_0(k)$ for a negligible function $\nu_0(\cdot)$, for all queries to the VERIFY oracle.

To finally address the case of $i = q_v + 1$, in which we additionally check that $(\sigma_w^{-H_v(\vec{m}^*)} \sigma_x)^{H_{y'}(\vec{m}^*)} = (\sigma_x^s \sigma_y)^{H_x(\vec{m}^*)}$, we observe that if $\text{Verify}(sk, \vec{m}^*, \sigma^*) = \text{accept}$ then

$$\begin{aligned} (\sigma_w^{-H_v(\vec{m}^*)} \sigma_x)^{H_{y'}(\vec{m}^*)} &= (\sigma_w^{-H_v(\vec{m}^*)} \sigma_w^{H_x(\vec{m}^*)})^{H_{y'}(\vec{m}^*)} \\ &= (\sigma_w^{-H_v(\vec{m}^*)} \sigma_w^{H_{x'}(\vec{m}^*)/\beta + H_v(\vec{m}^*)})^{H_{y'}(\vec{m}^*)} \\ &= (\sigma_w^{H_{y'}(\vec{m}^*)/\beta})^{H_{x'}(\vec{m}^*)} \\ &= (\sigma_w^{sH_x(\vec{m})} \sigma_w^{H_{y'}(\vec{m}^*)/\beta - sH_x(\vec{m})})^{H_{x'}(\vec{m}^*)} \\ &= (\sigma_x^s \sigma_w^{H_{y'}(\vec{m}^*)})^{H_{x'}(\vec{m}^*)} \\ &= (\sigma_x^s \sigma_y)^{H_{x'}(\vec{m}^*)}. \end{aligned}$$

This means that if Verify accepts \vec{m}^*, σ^* , then the final verification query in $G_0^A(k)$ will also accept. Finally, since the final verification check in $G_0^A(k)$ includes verifying that $(\sigma_y \sigma_x^s)^{z'} = (\sigma_z \sigma_w^t)^{H_{y'}(\vec{m})}$, the argument above implies that if this check succeeds then Verify would also accept. This proves Equation 1.

We now prove Equation 2. To do this, we consider a series of hybrids: in each hybrid $H_i^A(k)$, the first i queries use the MAC values from $G_1^A(k)$, and the last $q_m - i$ use the values from $G_0^A(k)$; $H_0^A(k)$ is then equivalent to $G_0^A(k)$, and $H_{q_m}^A(k)$ is equivalent to $G_1^A(k)$. To argue that $H_i^A(k)$ is indistinguishable from $H_{i-1}^A(k)$, we cannot proceed in a single step. Instead, we gradually change the value of the i -th query across two additional games $H_{i,1}^A(k)$ and $H_{i,2}^A(k)$ that proceed as follows:

- $H_{i,1}^A(k)$: In the $\text{MAC}_{sk}(\vec{m})$ procedure, if $j < i$ then compute σ as specified in $G_1^A(k)$, and if $j > i$ compute σ as specified in $G_0^A(k)$. If $j = i$, pick $r, \omega \xleftarrow{\$} \mathbb{F}_p$ and compute $\sigma_w \leftarrow h^\omega$, $\sigma_x \leftarrow g^{rH_x(\vec{m})} h^{\omega H_v(\vec{m})}$, $\sigma_y \leftarrow g^{r(H_y(\vec{m}) - sH_x(\vec{m}))} h^{-s\omega H_v(\vec{m})}$, and $\sigma_z \leftarrow g^{zr} h^{-\omega t}$.
- $H_{i,2}^A(k)$: In the $\text{MAC}_{sk}(\vec{m})$ procedure, continue to behave as in $H_{i,1}^A(k)$ in the cases that $j < i$ or $j > i$. If $j = i$, then pick $r, \omega, \chi \xleftarrow{\$} \mathbb{F}_p$, and continue to compute σ_w and σ_z as in $H_{i,1}^A(k)$, but compute $\sigma_x \leftarrow g^{rH_x(\vec{m})} h^{rH_v(\vec{m}) + \chi}$ and $\sigma_y \leftarrow g^{rH_y(\vec{m})} \sigma_x^{-s}$.

We then show that

$$\Pr[H_{i,1}^A(k)] - \Pr[H_{i-1}^A(k)] \leq \mathbf{Adv}_B^{\text{ddh}}(k) \quad (2.1)$$

$$\Pr[H_{i,2}^A(k)] - \Pr[H_{i,1}^A(k)] = 0 \quad (2.2)$$

$$\Pr[H_i^A(k)] - \Pr[H_{i,2}^A(k)] = 0 \quad (2.3)$$

for all $k \in \mathbb{N}$, from which Equation 2 follows.

To prove Equation 2.1, the construction of B is as follows:

$B(p, G, g, A, B, C)$
 $Q \leftarrow \emptyset; h \leftarrow A;$
 $x_0, \dots, x_n, v_0, \dots, v_n, y_0, \dots, y_n, s, z, t \leftarrow \mathbb{F}_p$
 $X_i \leftarrow g^{x_i} h^{v_i}; Y_i \leftarrow g^{y_i} X_i^{-s} \forall i \in \{1, \dots, n\};$
 $iparams \leftarrow (\vec{X}, \vec{Y})$
 $(\vec{m}^*, \sigma^*) \xleftarrow{\$} A^{\text{SIMMAC, VERIFY}}((p, G, g, h), iparams)$
 $b \leftarrow ((\sigma_y \sigma_x^s)^z = (\sigma_z \sigma_w^t)^{H_{y'}(\vec{m}^*)})$
 $\wedge ((\sigma_w^{-H_v(\vec{m}^*)} \sigma_x)^{H_{y'}(\vec{m}^*)} = (\sigma_x^s \sigma_y)^{H_x(\vec{m}^*)}) \wedge (\sigma_w \neq 1)$
 return $b' = ((\vec{m}^* \notin Q) \wedge b)$

Procedure $\text{SIMMAC}_{sk}(\vec{m})$

On queries $j < i$ or $j > i$: use $\vec{x}, \vec{v}, \vec{y}, s, z, t, h$ to compute σ as in $G_1^A(k)$ or $G_0^A(k)$ respectively.
 On query i : compute $\sigma_w \leftarrow C$; $\sigma_x \leftarrow B^{H_x(\vec{m})} C^{H_v(\vec{m})}$;
 $\sigma_y \leftarrow B^{H_y(\vec{m}) - sH_x(\vec{m})} C^{-sH_v(\vec{m})}$; $\sigma_z \leftarrow B^z C^{-t}$
 and return $(\sigma_w, \sigma_x, \sigma_y, \sigma_z)$

Procedure $\text{VERIFY}_{sk}(\vec{m}, \sigma)$

return $((\sigma_y \sigma_x^s)^z = (\sigma_x \sigma_w^t)^{H_{y'}(\vec{m})}) \wedge (\sigma_w \neq 1)$

To see that B successfully simulates the MAC oracle, observe that if we implicitly use $r = b$ then, if $C = g^{ab}$, on the i th query we have

$$\begin{aligned} \sigma_w &= g^{ab} = h^b = h^r \\ \sigma_x &= g^{bH_x(\vec{m})} g^{abH_v(\vec{m})} = (g^{H_x(\vec{m})} h^{H_v(\vec{m})})^r \\ \sigma_y &= g^{b(H_y(\vec{m}) - sH_x(\vec{m}))} g^{-absH_v(\vec{m})} \\ &= (g^{H_y(\vec{m}) - sH_x(\vec{m})} h^{-sH_v(\vec{m})})^r \\ \sigma_z &= g^{bz} g^{-abt} = (g^z h^{-t})^r, \end{aligned}$$

which are distributed identically to the values in $H_{i-1}^A(k)$. If instead C is random, then in particular we can write it as $C = h^\omega$ for some $\omega \xleftarrow{\$} \mathbb{F}_p$. In this case

$$\begin{aligned} \sigma_w &= h^\omega \\ \sigma_x &= g^{rH_x(\vec{m})} h^{\omega H_v(\vec{m})} \\ \sigma_y &= g^{r(H_y(\vec{m}) - sH_x(\vec{m}))} h^{-s\omega H_v(\vec{m})} \\ \sigma_z &= g^{zr} h^{-\omega t}, \end{aligned}$$

which are distributed identically to the values in $H_{i,1}^A(k)$.

To prove Equation 2.2, we remind ourselves of the transition: in both games, the first $i - 1$ queries are answered using h^{χ_j} for $\chi_j \xleftarrow{\$} \mathbb{F}_p$ and the last $q_m - i$ queries are answered using $(g^{H_x(\vec{m}_j)} h^{H_v(\vec{m}_j)})^{r_j}$ for $r_j \xleftarrow{\$} \mathbb{F}_p$. The i -th query then uses either $g^{r_i H_x(\vec{m}_i)} h^{\chi_i}$ (in $H_{i,2}^A(k)$) or $g^{r_i H_x(\vec{m}_i)} h^{\omega H_v(\vec{m}_i)}$ (in $H_{i,1}^A(k)$); if we can argue that the value of $H_v(\vec{m}_i)$ is

independent of any other values in the game, then in particular $H_v(\vec{m}_i)$ could take on any value and the distribution over these two values is identical. To do this, we first observe that **SimVerify**, and thus **VERIFY**, is independent of \vec{v} . The first $i - 1$ MAC responses are also independent of \vec{v} , so we must prove only two properties: (1) the value of $(g^{H_x(\vec{m}_j)} h^{H_v(\vec{m}_j)})^{r_j}$ in the last $q_m - i$ MAC responses is independent of \vec{v} , and (2) the value of $H_v(\vec{m}_i)$ is independent of the value of $H_v(\vec{m}^*)$, and thus changing the i -th query does not affect the distribution at the end of the game.

To first prove this latter property, we observe that $H_v(\cdot)$ is a pairwise independent function. As the winning conditions of the game require that $m^* \notin Q$ and thus $\vec{m}_i \neq \vec{m}^*$, this means that for any $\alpha_1, \alpha_2 \in \mathbb{F}_p$, $\Pr[H_v(\vec{m}_i) = \alpha_1 \wedge H_v(\vec{m}^*) = \alpha_2] = 1/p^2$, which in turn implies that the values of $H_v(\vec{m}_i)$ and $H_v(\vec{m}^*)$ are independent as desired.

To prove the former property, we perform a similar argument to that in the proof of Equation 1: consider a modified game in which $x_i = x'_i - \beta v_i$; then for the last $q_m - i$ queries,

$$(g^{H_x(\vec{m}_j)} h^{H_v(\vec{m}_j)})^{r_j} = g^{r_j(H_x(\vec{m}_j) + \beta H_v(\vec{m}_j))} = g^{r_j(H_{x'}(\vec{m}_j) - \beta H_v(\vec{m}_j) + \beta H_v(\vec{m}_j))} = g^{r_j H_{x'}(\vec{m}_j)},$$

so these values information-theoretically hide \vec{v} . For the i -th query, however, if we use $g^{r_i H_x(\vec{m}_i)} h^{\omega_i H_v(\vec{m}_i)}$ then for σ_x we have

$$g^{r_i(H_{x'}(\vec{m}_i) - \beta H_v(\vec{m}_i))} g^{\beta \omega_i H_v(\vec{m}_i)} = g^{r_i H_{x'}(\vec{m}_i)} h^{(\omega_i - r_i) H_v(\vec{m}_i)},$$

which, given that as we have argued $H_v(\vec{m}_i)$ is independent of all other values, will be distributed identically to $g^{r_i H_{x'}(\vec{m}_i)} h^\chi = g^{r_i H_x(\vec{m}_i)} h^{r_i H_v(\vec{m}_i) + \chi}$ for $\chi \leftarrow \mathbb{F}_p$. Thus, this is identical to the value in $H_{i,2}^A(k)$, and we can argue about σ_y similarly.

To prove Equation 2.3, we consider a modified version of $H_i^A(k)$ in which, rather than pick $\chi \leftarrow \mathbb{F}_p$, pick $\chi' \leftarrow \mathbb{F}_p$ and set $\chi := r H_x(\vec{m})/\beta + r H_v(\vec{m}) + \chi'$. Then the distribution over χ is still uniformly random and thus identical to the distribution in $H_i^A(k)$, and

$$\begin{aligned} h^\chi &= h^{r H_x(\vec{m})/\beta + r H_v(\vec{m}) + \chi'} \\ &= g^{\beta(r H_x(\vec{m})/\beta + r H_v(\vec{m}) + \chi')} \\ &= g^{r H_x(\vec{m})} h^{r H_v(\vec{m}) + \chi'}, \end{aligned}$$

so the distribution over σ_x in the modified game is identical to that in $H_{i,2}^A(k)$ (and, consequently, so is the distribution over σ_y).

Finally, we prove Equation 3. If the forgery (\vec{m}^*, σ^*) output by A passes verification, then $(\sigma_w^{-H_v(\vec{m}^*)} \sigma_x)^{H_y(\vec{m}^*)} = (\sigma_x^s \sigma_y)^{H_x(\vec{m}^*)}$ by definition. Since x_0 and v_0 are not used in any values given to A and thus $H_x(\vec{m}^*)$ and $H_v(\vec{m}^*)$ are information-theoretically hidden, however, A has a negligible probability of producing (\vec{m}^*, σ^*) such that this equality holds and $\sigma_w \neq 1$, meaning the probability that it passes final verification is bounded by $\nu(k)$ for a negligible function $\nu(\cdot)$. \square

4. KEYED-VERIFICATION CREDENTIALS

In this section we first describe the set of algorithms that form a keyed-verification credential scheme. We then informally describe the desired security and privacy properties (formal definitions are in the full version [2]), present constructions of keyed-verification credentials based on MAC_{GGM} and MAC_{DDH} , and prove our MAC_{DDH} -based construction secure assuming DDH. The proof of security for our MAC_{GGM} -based construction is a trivial simplification of the MAC_{DDH} -based proof, so we omit it.

A keyed-verification credential system consists of the following algorithms:

Setup(1^k) defines the system parameters $params$. We will assume that $params$ is available to all algorithms, and that all parties have assurance it was created correctly.

CredKeygen($params$) is run by the issuer on input $params$ to generate a secret key sk and (public) issuer parameters $iparams$.

BlindIssue(sk, S) \leftrightarrow **BlindObtain**($iparams, (m_1, \dots, m_n)$) is a potentially interactive protocol where a user can obtain a credential on attributes (m_1, \dots, m_n) from an issuer who is only given some subset S of those attributes.

Show($iparams, cred, (m_1, \dots, m_n), \phi$) \leftrightarrow **ShowVerify**(sk, ϕ) is an interactive protocol between a user and a verifier. **Show** is run by a user to generate a proof of possession π of a credential $cred$ certifying some set of attributes (m_1, \dots, m_n) satisfying a set of statements ϕ under the key corresponding to $iparams$, and **ShowVerify** is run by the verifier in possession of sk to verify proof π claiming knowledge of a credential satisfying the statements ϕ .

While we defined our presentation protocol in terms of a single credential, we could generalize our definitions and constructions to allow the user to prove relationships between attributes across multiple credentials that he owns. We chose the above variant because it allows for fairly simple definitions, yet still allows us to consider properties of a credential scheme as it would be used.

Note that the standard approach of requiring that the **Show** protocol be a proof of knowledge of a credential cannot be directly applied here because the verifier must know the issuer secret key in order to verify the credential. This is somewhat similar to a designated verifier proof [24], but it has the additional complication that the statement (validity of the credential) depends on the verifier's secret key.

4.1 Security properties

A keyed-verification credential system should have the following security properties (defined formally in [2]). Informally, *correctness* requires that every credential generated by **Issue** for attribute set $\{m_1, \dots, m_n\}$ can be used to generate a proof for any statement satisfied by that attribute set. *Unforgeability* requires that an adversary cannot produce an accepting proof for a statement ϕ that is not satisfied by any of the attribute sets for which it has received credentials. *Anonymity* requires that the proofs produced by **Show** reveal nothing more than the statement being proved. *Blind*

issuance requires that **BlindIssue**, **BlindObtain** define a secure two-party protocol for generating credentials on the user's attributes. Finally, *key-parameter consistency* requires that the probability that an adversary can find two secret keys that correspond to the same set of issuer parameters is negligible; this guarantees that the issuer cannot use different secret keys with different users and thus compromise their anonymity.

4.2 Keyed-verification credentials from DDH

We now give a construction of a keyed-verification credential system from an algebraic MAC, as presented in Section 3. We describe each protocol first in the abstract, and then using the specific algorithms for $\text{MAC}_{\text{DDH}} = (\text{Setup}_{\text{DDH}}, \text{KeyGen}_{\text{DDH}}, \text{MAC}_{\text{DDH}}, \text{Verify}_{\text{DDH}})$. (The MAC_{GGM} algorithms are a straightforward simplification of the ones here, and we present them separately in the full version [2].)

We first define the following setup algorithms for the credential system. Briefly, **Setup** runs the MAC setup to generate system parameters for the MAC scheme, and **CredKeygen** runs the MAC key generation to generate a MAC key which will be the issuer secret key and additional values *iparams* which will be used as the issuer's public key.

Setup(1^k): Output $(G, p, g, h) \xleftarrow{\$} \text{Setup}_{\text{DDH}}(1^k)$.

CredKeygen(*params*): Compute first the MAC keys as $((\vec{X}, \vec{Y}), (\vec{x}, \vec{y}, z)) \xleftarrow{\$} \text{KeyGen}_{\text{DDH}}(\text{params})$. Now commit to the secret values by picking $\tilde{x}, \tilde{y}, \tilde{z} \in_R \mathbb{F}_p$ and forming commitments $C_{x_0} := g^{x_0} h^{\tilde{x}}$, $C_{y_0} := g^{y_0} h^{\tilde{y}}$, and $C_z := g^z h^{\tilde{z}}$. Output *iparams* = $(\vec{X}, \vec{Y}, C_{x_0}, C_{y_0}, C_z)$ and $sk = (\vec{x}, \vec{y}, z, \tilde{x}, \tilde{y}, \tilde{z})$.

Issuance. Briefly, to issue a credential on a set of attributes, when all the attributes are known to the issuer, the issuer will generate a MAC on those attributes, and prove that it has done so correctly. More specifically, to issue a credential with the attributes $\vec{m} \in \mathbb{F}_q^n$, the issuer computes $\sigma = (\sigma_w, \sigma_x, \sigma_y, \sigma_z) \xleftarrow{\$} \text{MAC}_{\text{DDH}}(sk, \vec{m})$, and returns σ to the user with a proof π that proves knowledge of the secret key needed to compute σ . For MAC_{DDH} ,

$$\begin{aligned} \pi &:= \text{PK}\{(\vec{x}, \vec{y}, z, \tilde{x}, \tilde{y}, \tilde{z}) : \text{Verify}_{\text{DDH}}(sk, \vec{m}, \sigma) \\ &\wedge C_{x_0} = g^{x_0} h^{\tilde{x}} \wedge C_{y_0} = g^{y_0} h^{\tilde{y}} \wedge C_z = g^z h^{\tilde{z}} \\ &\wedge X_i = h^{x_i} \wedge Y_i = h^{y_i} \forall i \in \{1, \dots, n\}\}. \end{aligned}$$

The proof π proves that the credential is well-formed with respect to the system and issuer parameters. If this proof verifies, the user outputs σ ; otherwise it outputs \perp .

If some subset $\mathcal{H} \subseteq \{1, \dots, n\}$ of the attributes must be hidden, we can proceed as follows: the user can generate encryption keys and then encrypt each of the hidden attributes. Now, rather than sending the attributes to the issuer, the user can send these ciphertexts along with a proof of knowledge of the plaintext. If the encryption scheme has the appropriate homomorphic properties, the issuer can apply them to form an encryption of the MAC, along with a proof that this encryption has been generated correctly. Upon receiving this proof and ciphertext, the user rejects if the proof does not verify, and otherwise decrypts the ciphertext to obtain the credential.

Concretely, the user generates an ElGamal keypair $(d, \gamma = g^d)$ and encrypts each hidden attribute m_i as $E_i = (g^{r_i}, g^{m_i \gamma^{r_i}})$ for $r_i \in_R \mathbb{F}_p$. Upon receiving these ciphertexts (along with the proof of knowledge of $\{r_i, m_i\}_{i \in \mathcal{H}}$), the issuer chooses $b \xleftarrow{\$} \mathbb{F}_p$, computes $\sigma_w = g^b$, $\sigma_z = \sigma_w^z$, and uses the homomorphic properties of ElGamal to generate an encryption E_x of $\sigma_x = g^{b x_0} \prod_1^n (g^{m_i})^{b x_i}$ and an encryption E_y of $\sigma_y = g^{b y_0} \prod_1^n (g^{m_i})^{b y_i}$. It then randomizes these to obtain E'_x (by picking $r_x \xleftarrow{\$} \mathbb{F}_p$ and multiplying in an encryption $(g^{r_x}, g^{0 \gamma^{r_x}})$ of 0) and E'_y . It sends $(\sigma_w, \sigma_z, E'_x, E'_y)$ to the user and gives a proof that these values have been generated correctly with respect to $(\vec{X}, \vec{Y}, C_{x_0}, C_{y_0}, C_z)$ (i.e., a proof of knowledge of the appropriate $(\tilde{x}, \tilde{y}, z, \tilde{x}, \tilde{y}, \tilde{z}, b, r_x, r_y)$). If the proof does not verify, the user outputs \perp . Otherwise, the user decrypts E'_x and E'_y to get σ_x and σ_y respectively, and outputs $\sigma = (\sigma_w, \sigma_x, \sigma_y, \sigma_z)$.

Credential translation. In addition to proving that the ciphertexts E_i are well formed, the user can include proofs about the attributes the ciphertexts encrypt. For example, the user may prove that some of the attributes m_i are the same as in another credential, such as one that is more expensive to use (e.g., an Idemix credential), or one that cannot be presented multiple times unlinkably (e.g., a U-Prove credential).

Credential presentation. Here we present a construction for **Show** and **ShowVerify**. We focus on how to show that the credential certifies attributes matching those in a given set of commitments. Once commitments have been established, the user can prove a large variety of statements about the committed values, using e.g. the techniques in [8].

Roughly, the protocol proceeds as follows: The prover forms Pedersen commitments to the MAC, and the messages, and proves that these were computed correctly. Using the homomorphic property of the commitments, the verifier is able to recompute the MAC and compare it to the committed value. However, because of the randomness in the commitments, there will be some additional terms in the MAC the verifier computes; the user will use \vec{X}, \vec{Y} from *iparams* to compute exactly those values necessary to cancel these terms, and again prove that these values are correctly computed.

Show(*params*, *iparams*, ϕ , *cred*, $\{m_i\}_i^n$): The prover chooses

$r, r_x, r_y, z_1, \dots, z_n \xleftarrow{\$} \mathbb{F}_p$ and parses *cred* = $(\sigma_w, \sigma_x, \sigma_y, \sigma_z)$. It first randomizes the credential by computing $\sigma_w = \sigma_w^r$, $\sigma_x = \sigma_x^r$, $\sigma_y = \sigma_y^r$, and $\sigma_z = \sigma_z^r$, and then computes $\{C_{m_i} := \sigma_w^{m_i} h^{z_i}\}_{i=1}^n$, $C_{\sigma_x} := \sigma_x g^{r_x}$, $C_{\sigma_y} := \sigma_y g^{r_y}$,

$$V_x := g^{-r_x} \prod_{i=1}^n X_i^{z_i}, \text{ and } V_y := g^{-r_y} \prod_{i=1}^n Y_i^{z_i}.$$

It then sends $\sigma = (\sigma_w, \sigma_z, C_{\sigma_x}, C_{\sigma_y}, V_x, V_y, \{C_{m_i}\}_i^n)$ along with a proof of knowledge π , which it computes as

$$\begin{aligned} \pi &= \text{PK}\{\vec{m}, \vec{z}, -r_x, -r_y : \phi(m_1, \dots, m_n) = 1 \\ &\wedge C_{m_i} = \sigma_w^{m_i} h^{z_i} \forall i \in \{1, \dots, n\} \\ &\wedge V_x = g^{-r_x} \prod_{i=1}^n X_i^{z_i} \wedge V_y = g^{-r_y} \prod_{i=1}^n Y_i^{z_i}\}. \end{aligned}$$

ShowVerify(*params*, *iparams*, ϕ , $\{x_i, y_i\}_i^n$, z , σ , π): The verifier parses $\sigma = (\sigma_w, \sigma_y, \sigma_z, V_x, V_y, \{C_{m_i}\}_i^n, C_{\sigma_x}, C_{\sigma_y})$ and verifies that

$$V_x = \frac{\sigma_w^{x_0} \prod_{i=1}^n C_{m_i}^{x_i}}{C_{\sigma_x}} \quad \text{and} \quad V_y = \frac{\sigma_w^{y_0} \prod_{i=1}^n C_{m_i}^{y_i}}{C_{\sigma_y}}.$$

It then verifies the proof π . If the proof is valid and if $\sigma_z = \sigma_w^z$ it accepts and outputs $(C_{m_1}, \dots, C_{m_n})$, and otherwise it rejects and outputs \perp .

Security. A proof of the following theorem can be found in Appendix A.

THEOREM 4. *If DDH holds and the proof system is a ZK proof of knowledge with the strengthened extraction property³, the algorithms (CredKeygen, Issue, CredVerify, ShowVerify, BlindIssue, BlindObtain) defined above make up a secure keyed-verification credential system.*

Intuitively, credential unforgeability follows from the unforgeability of the MAC (which is based on DDH); credential anonymity follows from the zero knowledge property of the proofs; blind issuance follows from the extractability of the proofs and the IND-CPA security of the encryption scheme (which, for ElGamal, follows from DDH); and key-parameter consistency follows from the binding property of the commitment scheme (which, for Pedersen commitments, follows from the discrete log assumption, which is implied by DDH).

5. EFFICIENCY

In this section we compare the efficiency of our new schemes to U-Prove and Idemix. We focus on the computational cost of creating a presentation proof, as this operation typically must be done by the largest range of devices. We consider the MAC_{GGM} - and MAC_{DDH} -based schemes, where the proof system is implemented with Fiat-Shamir (full details for MAC_{GGM} are given in [2], and MAC_{DDH} is very similar). Using the interactive proof system from [17] will have essentially the same computational cost. Complete descriptions of Idemix and U-Prove are available in [23] and [26] respectively. As the recent scheme of Baldimtsi and Lysyanskaya [3], is strictly less efficient than U-Prove, we omit it from our comparison. For the bilinear CL signature schemes [1, 11], detailed specifications and parameter choices are not available; we made a rough estimate assuming the scheme was implemented with a 256-bit elliptic curve group, and that a pairing requires 1ms on our benchmark hardware. We ignored the cost of arithmetic in the target group (i.e., G_T the range of the pairing), thus underestimating the cost.

Credential Size. Table 1 shows the size of a credential in all four schemes, both asymptotically, and for a concrete choice of parameters. The parameter s is the number of times the credential may be shown unlinkably (which is relevant for U-Prove). The size only counts the cryptographic components of the credential, the metadata and attribute values are assumed to be the same for all systems. The overhead of MAC_{GGM} is the lowest, followed by MAC_{DDH} , which is the size of a single U-Prove token. The size of SRSA group elements makes Idemix credentials larger than MAC_{GGM} and

MAC_{DDH} , and Idemix credentials are smaller than U-Prove credentials once $s > 5$.

	Credential size for s shows, n attributes	
	Asymptotic	Concrete (in bits)
U-Prove	$O(s)$	1024s
Idemix	$O(1)$	5369
Bilinear CL	$O(n)$	$512n + 768$
MAC_{GGM}	$O(1)$	512
MAC_{DDH}	$O(1)$	1024

Table 1: Comparison of credential sizes of U-Prove, Idemix, MAC_{GGM} and MAC_{DDH} . The number of times the credential may be shown is denoted s . U-Prove, bilinear CL, MAC_{GGM} and MAC_{DDH} use a 256-bit elliptic curve group. Idemix uses a 2048-bit modulus.

Computation Cost for Presentation. We estimate the cost of creating a presentation proof and compare the four schemes. Our estimate is formed by counting the number of multi-exponentiations required to create a presentation proof. We use the notation $\ell\text{-exp}$ to denote computing the product of ℓ powers. To realistically estimate the performance of Idemix, the bitlengths of the exponents must also be considered, so we use the notation $\ell\text{-exp}(b_1, \dots, b_\ell)$ to denote the product of ℓ powers when the bitlengths of the exponents are b_1, \dots, b_ℓ . These bitlengths are calculated from the Idemix specification [23]. For U-Prove, MAC_{GGM} and MAC_{DDH} the bitlength of the exponent is always the length of the group order (256-bits in our comparison).

Table 2 gives the number of multi-exponentiations in terms of three parameters: n is the number of attributes in a credential, r is the number of *revealed* attributes in a presentation proof, and c is the number of *committed* attributes. (As discussed in section 4, we focus on the part of the protocol which proves that certified attributes match those in a set of commitments; the rest of the presentation is essentially the same in all of the schemes we consider. We differentiate between revealed and committed attributes because one can simplify the protocols in the case where attributes will be directly revealed. For details see the full version [2].) For each committed attribute m , a separate Pedersen commitment is output. As a further comparison, Table 2 includes the time required to compute these multi-exponentiations for a given choice of parameters (n, c, r) . Our multi-exponentiation implementation in G uses the NIST 256-bit elliptic curve, and for Idemix uses the parameters in [23]. The benchmarks were computed on an Intel Xeon CPU (E31230, quad core, 3.2 GHz) on an HP Z210 workstation running Windows 7 (64-bit). The times are in milliseconds, and are the average of 100 runs.

The times given in Table 2 show that the new schemes are competitive with U-Prove, especially when most of the attributes are committed, and that they are much faster than Idemix. In particular, in the first benchmark (when $(n, c, r) = (10, 2, 2)$), MAC_{GGM} is 6.28 times faster than Idemix, and MAC_{DDH} is 4.7 times faster than Idemix. Compared to U-Prove, MAC_{GGM} and MAC_{DDH} are 3.4 and 4.5 times slower, much less than the 21.2 times slowdown for Idemix.

³See discussion in 2 and the full version[2].

	Number of exponentiations	Time (in ms) when $(n, c, r) =$ $(10, 2, 2)$ $(10, 10, 0)$	
U-Prove	1 $(n - r + 1)$ -exp, 2c 2-exp	3.38	12.43
MAC _{GGM}	3 1-exp, 1 $(n - r + 1)$ -exp 2 $(n - r)$ 2-exp	11.42	13.93
MAC _{DDH}	6 1-exp, 2 $(n - r + 2)$ -exp 2 $(n - r + 1)$ 2-exp	15.31	18.10
Idemix	1 1-exp(2048) c 2-exp(256, 2046) c 2-exp(592, 2385) 1 $(n - r + 2)$ -exp(456, 3060, 592, ..., 592)	71.72	226.79
Bilinear CL	3 + n pairings (3 + n) 1-exp, 2c 2-exp	20.98	28.32

Table 2: Comparison of estimated presentation proof generation cost. U-Prove, bilinear CL, MAC_{GGM} and MAC_{DDH} use 256-bit elliptic curve parameters, and Idemix uses a 2048-bit modulus.

In the second benchmark, when $(n, c, r) = (10, 10, 0)$, the performance of U-Prove, MAC_{GGM} and MAC_{DDH} are very similar. MAC_{GGM} and MAC_{DDH} are only 1.04 and 1.5 times slower than U-Prove. Idemix is 18.2, 16.3 and 12.5 times slower than U-Prove, MAC_{GGM} and MAC_{DDH}, respectively.

Discussion. These performance estimates show that the new schemes do provide a considerable performance advantage when compared to Idemix, and a small decrease compared to U-Prove. The other protocols, namely issuance and verification, have similar relative performance (for the user and issuer). In the case of issuance, our new schemes are expected to have slightly higher computational cost than issuing a single U-Prove token, but with one less round of interaction (when implemented with Fiat-Shamir proofs). When issuing multiple tokens, MAC_{GGM} and MAC_{DDH} have the best performance. In all protocols, the cost of verification is within a small factor of the cost of proof generation.

We note some limitations of our comparison. First, the comparison is limited to applications where the issuer and verifier share a key; otherwise MAC_{GGM} and MAC_{DDH} are not applicable. The parameter set used for Idemix is not believed to provide 128-bit security, so this favors Idemix in the comparison. For RSA, a 3072-bit modulus is required for 128-bit security, and for strong RSA we are unaware of any published guidance on choosing the modulus size. (Idemix would need *at least* a 3072-bit modulus for 128-bit security.) Another limitation is our choices of (n, c, r) , which will be different across applications. Once an application is fixed, optimizations may be possible, such as creating a single commitment to multiple attributes, or re-using the same commitment in multiple presentations (e.g., when the commitment is used as a pseudonym).

Acknowledgments

This work was done in part while Sarah Meiklejohn was an intern at MSR Redmond and in part while she was a graduate student at UC San Diego, funded by CNS-1237264 and support from the UCSD Center for Networked Systems (CNS).

6. REFERENCES

- [1] M. Au, W. Susilo, and Y. Mu. Constant-size dynamic k-TAA. *Proceedings of SCN 2006, LNCS 4116* (2006), 111-125.
- [2] M. Chase, S. Meiklejohn, and G. Zaverucha. Full version of this paper. IACR Cryptology ePrint Archive, Report 2013/516.
- [3] F. Baldimtsi and A. Lysyanskaya. Anonymous Credentials Light. *Proceedings of ACM CCS 2013*, ACM Press, (2013). To appear.
- [4] M. Belenkiy, J. Camenisch, M. Chase, M. Kohlweiss, A. Lysyanskaya and H. Shacham. Randomizable proofs and delegatable anonymous credentials. *Proceedings of CRYPTO 2009, LNCS 5677*, (2009), 108-125.
- [5] M. Bellare and O. Goldreich. On Defining Proofs of Knowledge. *Proceedings of CRYPTO 1992, LNCS 740* (1993), 390-420
- [6] P. Bichsel, J. Camenisch, T. Groß, and V. Shoup. Anonymous Credentials on a Standard Java Card. *Proceedings ACM CCS 2009*, ACM Press, (2009) 600-610.
- [7] D. Boneh. The Decision Diffie-Hellman Problem. *Proceedings of ANTS-III, LNCS 1423* (1998), 48-63.
- [8] S. Brands. *Rethinking Public Key Infrastructures and Digital Certificates*. The MIT Press, August 2000.
- [9] J. Camenisch and A. Lysyanskaya. An Efficient System for Non-Transferable Anonymous Credentials with Optional Anonymity Revocation. *Proceedings of EUROCRYPT 2001, LNCS 2045* (2001), 93-118.
- [10] J. Camenisch and A. Lysyanskaya. A Signature Scheme with Efficient Protocols. *Proceedings of SCN 2003, LNCS 2576* (2003), 268-289.
- [11] J. Camenisch and A. Lysyanskaya. Signature Schemes and Anonymous Credentials from Bilinear Maps. *Proceedings of CRYPTO 2004, LNCS 3152* (2004), 56-72.
- [12] J. Camenisch and M. Stadler. Proof Systems for General Statements About Discrete Logarithms. Technical Report TR 260 (1997), Institute for Theoretical Computer Science, ETH Zurich.
- [13] D. Cash, E. Kiltz, and V. Shoup. The Twin Diffie-Hellman Problem and Applications *Proceedings of Eurocrypt 2008, LNCS 4965* (2008), 127-145.
- [14] D. Chaum. Security without Identification: Transaction Systems to Make Big Brother Obsolete. *Communications of the ACM 28(10)* (1985), 1030-1044.
- [15] J. Chen, H. Lim, S. Ling, H. Wang, and H. Wee. Shorter IBE and Signatures via Asymmetric Pairings. *Proceedings of Pairing 2012, LNCS 7708*, (2012), 122-140.
- [16] R. Cramer, R. Gennaro, and B. Schoenmakers. A Secure and Optimally Efficient Multi-Authority Election Scheme. *Proceedings of EUROCRYPT'97, LNCS 1233*, (1997), 103-118.
- [17] I. Damgård. Efficient Concurrent Zero-Knowledge in the Auxiliary String Model. *Proceedings of EUROCRYPT 2000, LNCS 1807* (2000), 418-430.
- [18] Y. Dodis, E. Kiltz, K. Pietrzak, D. Wichs. Message Authentication, Revisited. *Proceedings of EUROCRYPT'12, LNCS 7237* (2012), 355-374.
- [19] A. Fiat and A. Shamir. How to Prove Yourself: Practical Solutions to Identification and Signature Problems. *Proceedings of CRYPTO 1986, LNCS 263* (1987), 186-194.

- [20] M. Fischlin. Communication-Efficient Non-Interactive Proofs of Knowledge with Online Extractors. *Proceedings of CRYPTO 2005, LNCS 3621* (2005), 152–168.
- [21] M. Gurbush, A. Lewko, A. O’Neill, and B. Waters Dual Form Signatures: An Approach for Proving Security from Static Assumptions. *Proceedings of ASIACRYPT 2012, LNCS 7658* (2012), 25–42.
- [22] O. Goldreich. *The Foundations of Cryptography - Volume 2 Basic Applications*. Cambridge University Press, New York, 2004.
- [23] IBM. Specification of the Identity Mixer Cryptographic Library (Revised version 2.3.0). IBM Research Report RZ 3730, April 2010.
- [24] M. Jakobsson, K. Sako, and R. Impagliazzo. Designated Verifier Proofs and Their Applications. *Proceedings of EUROCRYPT 1996, LNCS 1070* (1996) 143–154.
- [25] NIST. *FIPS 186-3: Digital Signature Standard (DSS)*, Federal Information Processing Standards Publication (2009).
- [26] C. Paquin and G. Zaverucha. U-Prove Cryptographic Specification V1.1 (Revision 2). April 2013. Available online: www.microsoft.com/uprove.
- [27] C. Schnorr. Efficient Signature Generation by Smart Cards. *Journal of Cryptology* 4 (1991), 161–174.
- [28] C. Schnorr. Security of Blind Discrete Log Signatures Against Interactive Attacks. *ICICS 2001, LNCS 2229* (2001), 1–12.
- [29] V. Shoup. Lower Bounds for Discrete Logarithms and Related Problems. *Proceedings of EUROCRYPT’97, LNCS 1233* (1997), 256–266.
- [30] Y. Tsounis and M. Yung On the security of ElGamal based encryption. *Proceedings of PKC 1998, LNCS 1431* (1998), 117–134.
- [31] N. Smart. The Exact Security of ECIES in the Generic Group Model. *Proceedings of Cryptography and Coding, IMA Int. Conf., LNCS 2260* (2001), 73–84.
- [32] R. Steinfeld, J. Pieprzyk and H. Wang. How to strengthen any weakly unforgeable signature into a strongly unforgeable signature. *Proceedings of CT-RSA 2007, LNCS 4377* (2007), 357–371.
- [33] B. Waters. Dual System Encryption: Realizing Fully Secure IBE and HIBE under Simple Assumptions. *Proceedings of CRYPTO 2009, LNCS 5677* (2009), 619–636.

APPENDIX

A. A PROOF OF THEOREM 4

We present the following algorithms, which we use when we prove security of the scheme. **Issue** defines the distribution of honestly generated credentials, and **CredVerify** defines whether or not a given credential is valid.

Issue($sk, (m_1, \dots, m_n)$): Output $cred \xleftarrow{\$} \text{MAC}_{\text{DDH}}(sk, (m_1, \dots, m_n))$.

CredVerify($sk, (m_1, \dots, m_n), cred$): Output the result of $\text{Verify}_{\text{DDH}}(sk, (m_1, \dots, m_n), cred)$.

PROOF. We show that these algorithms satisfy correctness, unforgeability, anonymity, and blind issuance.

Correctness. For correctness we need to show two properties. First, that any credential generated as defined in **Issue** is accepted by **CredVerify**, which follows directly from correctness of the MAC. The second is that if a credential is generated as defined by **Issue** and then used in **Show**, then it is accepted. To see this, consider the following:

Issue($sk, (m_1, \dots, m_n)$) generates credentials of the form $(u, u^{x_0 + \sum_1^n x_i m_i}, u^{y_0 + \sum_1^n y_i m_i}, u^z)$. If both **Show** and **ShowVerify** are executed honestly, then the proof π is accepted by completeness of the proof system. Also, the honest **Show** com-

putes

$$\begin{aligned} V_x &= \frac{\sigma_w^{x_0} \prod_1^n C_{m_i}^{x_i}}{C_{\sigma_x}} = \frac{\sigma_w^{x_0} \prod_1^n \sigma_w^{m_i x_i} h^{x_i w_i}}{\sigma_x g^{r_x}} \\ &= \frac{\sigma_w^{x_0} \prod_1^n \sigma_w^{m_i x_i}}{\sigma_x g^{r_x}} \prod_{i=1}^n X_i^{w_i} \\ &= \frac{u^{x_0} \prod_1^n u^{m_i x_i}}{g^{r_x} u^{x_0 + \sum_1^n x_i m_i}} \prod_{i=1}^n X_i^{w_i} = g^{-r_x} \prod_{i=1}^n X_i^{w_i} \end{aligned}$$

so the verifier’s check on V_x succeeds. A similar equality holds for V_y . Finally, since **Issue** produces $\sigma_z = u^z$, the verifier’s final check succeeds and the verifier will accept.

Unforgeability. Here we prove unforgeability when A is given credentials generated by **Issue**. Blind issuance then shows that this also holds with the blind issuance protocol. We have shown (Theorem 3) that MAC_{DDH} is unforgeable under DDH. Suppose there exists an adversary A who can break the unforgeability property of our credential system. Then we can construct an algorithm B that breaks unforgeability of MAC_{DDH} as follows:

B receives $params, iparams_{\text{DDH}}$ and chooses random $C_{x_0}, C_{y_0}, C_z \xleftarrow{\$} \mathbb{F}_p$. It then sends $params, iparams = (iparams_{\text{DDH}}, C_{x_0}, C_{y_0}, C_z)$ to A .

When A queries the **Issue** oracle, B forwards the query to its **MAC** oracle and returns the resulting tag.

When A queries the **ShowVerify** oracle: A sends $\sigma_w, \sigma_z, C_{m_1}, \dots, C_{m_n}, C_{\sigma_x}, C_{\sigma_y}, V_x, V_y$, and gives a proof π . If the proof π is invalid, B returns \perp . Otherwise B runs the proof of knowledge extractor to extract $\{m_i\}_1^n, r_x, r_y$. Then it computes $\sigma_x = C_{\sigma_x} g^{-r_x}$ and $\sigma_y = C_{\sigma_y} g^{-r_y}$. Finally, it queries its **Verify** oracle with $(m_1, \dots, m_n), (\sigma_w, \sigma_x, \sigma_y, \sigma_z)$, and outputs the result.

In the final show protocol, B again extracts $\{m_i\}_1^n, r_x, r_y$, and outputs $(m_1, \dots, m_n), (\sigma_w, C_{\sigma_x} g^{-r_x}, C_{\sigma_y} g^{-r_y}, \sigma_z)$ as its forgery.

First, note that B ’s response to **Issue** queries is identical to the honest **Issue** algorithm. Then, we argue that its response to **ShowVerify** queries is also with overwhelming probability identical to the output of the honest algorithm. To see this, note that the proof of knowledge property guarantees that the extractor succeeds in producing a valid witness with all but negligible probability. Furthermore, if the extractor gives valid $\{m_i\}_1^n, r_x, r_y$ then

$$\begin{aligned} V_x &= \frac{\sigma_w^{x_0} \prod_1^n C_{m_i}^{x_i}}{C_{\sigma_x}} \\ \iff g^{-r_x} \prod_1^n X_i^{w_i} &= \frac{\sigma_w^{x_0} \prod_1^n (\sigma_w^{m_i} h^{w_i})^{x_i}}{C_{\sigma_x}} \\ \iff g^{-r_x} \prod_1^n (h^{x_i})^{w_i} &= \frac{\sigma_w^{x_0 + \sum_1^n m_i x_i} \prod_1^n h^{w_i x_i}}{C_{\sigma_x}} \\ \iff C_{\sigma_x} g^{-r_x} &= \sigma_w^{x_0 + \sum_1^n m_i x_i} \end{aligned}$$

Similarly $V_y = \frac{C_{r_y} \sigma_w^{y_0} \prod_1^n C_{m_i}^{y_i}}{C_{\sigma_y}}$ if and only if $C_{\sigma_y} g^{-r_y} = \sigma_w^{y_0 + \sum_1^n y_i m_i}$. The final check that the honest verifier makes guarantees that $\sigma_z = \sigma_w^z$. Thus, the honest verifier algorithm accepts if and only if $(\sigma_w, C_{\sigma_x} g^{-r_x}, C_{\sigma_y} g^{-r_y}, \sigma_z)$ would be accepted by $\text{Verify}_{\text{DDH}}$ for message (m_1, \dots, m_n) .

Similarly, we can argue that B extracts a valid MAC from the final show protocol whenever ShowVerify would have output 1. Thus, if A can cause ShowVerify to accept for some statement ϕ that is not satisfied by any of the attribute sets queried to Issue , then B extracts a new message (m_1, \dots, m_n) and a valid tag for that message.

Anonymity. Suppose the user is trying to prove that he has a credential for attributes satisfying some statement ϕ . Then we want to show that there exists an algorithm SimShow that for the adversary is indistinguishable from Show , but that only takes as input the statement ϕ and the secret key sk . Let $\phi \in \Phi$ and $(m_1, \dots, m_n) \in \mathcal{U}$ be such that $\phi(m_1, \dots, m_n) = 1$. Let $(iparams, sk)$ be in the range of CredKeygen , and let $cred$ be such that $\text{CredVerify}(sk, cred, (m_1, \dots, m_n)) = 1$.

Then $\text{SimShow}(sk, \phi)$ behaves as follows: It chooses random values $\sigma_w, C_{\sigma_x}, C_{\sigma_y}, C_{m_1}, \dots, C_{m_n} \xleftarrow{\$} G$. It then uses $\{x_i, y_i\}_0^n, z$ from sk to compute $\sigma_z = \sigma_w^z, V_x = \frac{\sigma_w^{x_0} \prod_1^n C_{m_i}^{x_i}}{C_{\sigma_x}}$, and $V_y = \frac{\sigma_w^{y_0} \prod_1^n C_{m_i}^{y_i}}{C_{\sigma_y}}$. It runs A with these values as the first message, and then simulates the proof of knowledge, and outputs whatever A outputs at the end of the proof.

First note $C_{\sigma_x}, C_{\sigma_y}, C_{m_1}, \dots, C_{m_n}$ are distributed identically to those produced by Show . Next, note that for any $cred$ such that $\text{CredVerify}(sk, cred, (m_1, \dots, m_n)) = 1$, randomizing the credential produces the same distribution as choosing random σ_w and computing $\sigma_x = \sigma_w^{x_0 + \sum_1^n x_i m_i}$, $\sigma_y = \sigma_w^{y_0 + \sum_1^n y_i m_i}$, and $\sigma_z = \sigma_w^z$ for the values $z, \{x_i, y_i\}_0^n$ in sk . Thus, σ_w, σ_z is also distributed identically to those produced by Show .

Finally, note that if we define $r_x, r_y, \{w_i\}_i$ to be the values such that $C_{\sigma_x} = \sigma_w^{x_0 + \sum_1^n x_i m_i} g^{r_x}$, $C_{\sigma_y} = \sigma_w^{y_0 + \sum_1^n y_i m_i} g^{r_y}$, and $C_{m_i} = u^{m_i} h^{w_i}$ for the random values $C_{\sigma_x}, C_{\sigma_y}, C_{m_1}, \dots, C_{m_n}$ chosen by SimShow , then the calculation above in the proof of correctness shows that the V_x, V_y that SimShow computes is identical to those that the honest Show would have produced.

By the zero knowledge property of the proof of knowledge, we conclude that the resulting view is indistinguishable from that produced by the adversary interacting with Show .

Blind issuance. First, we consider the setting where all of the attributes are known to the issuer and we use the simpler algorithm. Consider the case where the user is corrupt. Then our 2PC simulator on shared input $(S, iparams)$ receives the user's list of attributes (m_1, \dots, m_n) and forward it to the functionality. The functionality returns "attribute error" if $S \neq (m_1, \dots, m_n)$ and otherwise it returns $cred$. If the error does not occur, the 2PC simulator then sends $cred$ and runs the proof of knowledge ZK simulator to simulate

the proof of correctness for $cred$. By zero knowledge, this is indistinguishable from the real world.

Next, we consider the case where the issuer is corrupt. In this case our 2PC simulator receives $cred = (\sigma_w, \sigma_x, \sigma_y, \sigma_z)$ from the issuer and runs the verifier for the proof system. If the proof accepts, it runs the proof of knowledge extractor to extract $sk = (\{x_i\}_0^n, \{y_i\}_0^n, \tilde{x}, \tilde{y}, \tilde{z})$ and $r = \sigma_w$. It sends (sk, r) to the ideal functionality. By the proof of knowledge property, the credential sent in the real world is $\sigma_w, \sigma_w^{x_0} \prod_1^n (\sigma_w^{m_i})^{x_i}, \sigma_y = \sigma_w^{y_0} \prod_1^n (\sigma_w^{m_i})^{y_i}, \sigma_z = \sigma_w^z$ which is exactly what would be produced by the ideal functionality on input the (sk, r) described above.

Then, we consider the more complex algorithm which allows hidden attributes. Consider the case where the user is corrupt. Then our 2PC simulator on shared input $(S, iparams)$ receives the user's list of ciphertexts (E_1, \dots, E_n) , and run the verification for the proof of knowledge. If the proof accepts, it then uses the proof of knowledge extractor to extract $\{m_i\}_{i \in \mathcal{H}}$ and send it along with the set S to the functionality. The functionality returns $cred = (\sigma_w, \sigma_x, \sigma_y, \sigma_z)$. The 2PC simulator then computes an encryption E'_x of σ_x and an encryption E'_y of σ_y , send $(\sigma_w, \sigma_z, E'_x, E'_y)$ to the user, and uses the ZK simulator to simulate the correctness proof. Note that in the real BlindIssue protocol, if E_1, \dots, E_n are encryptions of g^{m_1}, \dots, g^{m_n} , then the resulting E_x, E_y is distributed identically to a fresh encryption of $\sigma_w^{x_0} \prod_1^n (\sigma_w^{m_i})^{x_i}, \sigma_w^{y_0} \prod_1^n (\sigma_w^{m_i})^{y_i}$. Thus, these are identical to what the simulator produces.

Next, we consider the case where the issuer is corrupt. In this case our 2PC simulator generates encryptions E_i of 1 for all $i \in \mathcal{H}$, sends them to A , and simulates the proof. It then receives $cred = (\sigma_w, \sigma_x, \sigma_y, \sigma_z)$ from A and runs the verification of the proof of knowledge; if the proof is accepting, it runs the proof of knowledge extractor to extract $sk = (\{x_i\}_0^n, \{y_i\}_0^n, \tilde{x}, \tilde{y}, \tilde{z})$ and $r = \sigma_w$. It sends (sk, r) to the ideal functionality. To see that this is indistinguishable from the real game, consider the following series of games. The first game G_1 is identical to the real game, except that instead of computing σ_x, σ_y by decrypting the ciphertexts E_x, E_y , we run the proof of knowledge extractor to extract sk, r and use those to form the credential by running Issue . By the proof of knowledge property and correctness and homomorphic properties of the encryption scheme, the credential sent in the real world is $(\sigma_w, \sigma_w^{x_0} \prod_1^n (\sigma_w^{m_i})^{x_i}, \sigma_w^{y_0} \prod_1^n (\sigma_w^{m_i})^{y_i}, \sigma_w^z, m_n)$ which is exactly what would be produced by the ideal functionality on input the (sk, r) described above. Next, in game G_2 we replace the proof of knowledge of the messages in E_i with a simulated proof - by zero knowledge this is indistinguishable. Finally, we note that the only difference between this game and the simulated game is that E_i is generated as an encryption of g^{m_i} rather than 1; thus the two games are indistinguishable by CPA-security of Elgamal encryption (which follows from DDH [30]).

Key-parameter consistency. This follows under the discrete log assumption from the binding property of the Pedersen commitment scheme. (Note that the discrete log assumption is implied by DDH.) \square