1. C The only 2 integer solutions are -1 and 1.
2. B Factoring the left side, we get $(x+13)(x-5) \leq 0$. From this, we get 3 distinct regions, $x < -13, -13 \leq x \leq 5, and\ x > 5$. From these regions, the LHS is only $\leq$ 0 when $-13 \leq x \leq 5$. From this region, the positive integers are 1,2,3,4,5.
3. B Moving everything to one side, we get $x^4 - 2x^3 - 19x^2 + 68x - 60 = 0$. Looking at the equation, we see that $x = 2$ is a root. Using synthetic division, we get $x^3 - 19x + 30$. Again, x=2 is a root, so using synthetic division, we get $x^2 + 2x - 15$. This is $(x-3)(x+5)$. So, our original equation factors to $(x-2)^3(x-3)(x+5)$. So, the sum of the distinct roots is 2+3+-5=0.
4. D By using our double angle equivalence, we get this is $2\sin(\theta)\cos(\theta) = \cos(\theta)$. Moving everything to the LHS, we get $2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$. Factoring, we get $\cos(\theta)\,(2\sin(\theta) - 1) = 0$. For this to equal 0, either $\cos(\theta) = 0$, $2\sin(\theta) - 1 = 0$, or both. $\cos(\theta) = 0$ gives us $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$. $2\sin(\theta) - 1 = 0$ when $\sin(\theta) = \frac{1}{2}$. This gives us $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$. Since there is no overlap, we sum all of the thetas up, giving us 3π
5. B These equations are the same as $3^{2x+y} = 3^6$ and $2^{3x-y} = 2^{16}$. From this, we get $2x + y = 6$ and $3x - y = 14$. Adding both equations, we get $5x = 20$ so $x = 4$. Plugging this back into the first equation, we get $y = 6 - 2x = 6 - 8 = -2$. So, $x + y = 4 - 2 = 2$.
6. D From the question, we know that there is some a, b where $(a + bi)^2 = 40 + 42i$. Expanding the LHS, we get $a^2 + 2abi - b^2 = 40 + 42i$. Matching the real and imaginary components, we get $a^2 - b^2 = 40$ and $2ab = 42$. From the second equation, dividing both sides by $2a$ gives us $b = \frac{21}{a}$. Substituting this back into the first equation gives $a^2 - \frac{441}{a^2} = 40$. Moving everything to the LHS and multiplying by a^2 gives us $a^4 - 40a^2 - 441$. This factors to $(a^2 - 49)(a^2 + 9) = 0$ This is $(a-7)(a+7)(a^2+9) = 0$. Since $a > 0$, the only solution is $a = 7$. To find b, we get $b = \frac{21}{a} = \frac{21}{7} = 3$. So, $a + b = 10$.
7. B To do this, we need to turn everything into the same base, in this case base 2. So, we get $\log_2(x+3) + \frac{\log_2(x^2+8x+16)}{2}$=1. Since $x^2 + 8x + 16 = (x+4)^2$, which means $\frac{\log_2(x^2+8x+16)}{2} = \frac{2\log_2(x+4)}{2} = \log_2(x+4)$. This means, our original equation is $\log_2(x+3) + \log_2(x+4) = 1$ Using the properties of log addition, we get $\log_2(x+3)(x+4) = 1$. This turns into $x^2 + 7x + 12 = 2$ so $x^2 + 7x + 10 = 0$. Factoring, we get $(x+5)(x+2) = 0$ which have solutions $x = -5, -2$. However, in our original equation, we have a $\log_2 x + 3$. When $x = -5, x + 3$ is negative, which is not defined in logs. So, it is extraneous, so the only solution is $x = -2$.
8. C Since 24 is even, there are 2*24 petals. So, there are 48 petals.
9. B Each term of the sum will have the form $(x_n)^2 + 2nx_n + 4n^2$. When summing for all n, the sum of all the $(x_n)^2$ will be 195 and $4n^2$ will be 364 no matter the order of $x_1, x_2, \dots x_6$. So, the only thing we need to consider is the sum of all the $2nx_n$. So, to minimize the sum, we need to minimize $\sum_{n=1}^{6} 4nx_n$. To minimize, nx_n, we can see

this as multiplying elements of two sets, one which contains elements 1,2,4,5,7,10 and one which contains elements 4,8,12,16,20,24. By the rearrangement inequality, we get the minimum sum is $(4 * 10 + 8 * 7 + 12 * 5 + 16 * 4 + 20 * 2 + 24) = 284$. Summing up all the terms, we get $284 + 195 + 364 = 843$.

10. A The polynomial that contains the reciprocal of the roots of this polynomial as roots is a polynomial where all the coefficients are reversed, $x^4 + x^3 - 5x^2 - 7x + 10$. The sum of the roots is -1 so the sum of the reciprocal of the roots is -1.

11. D Say $\lambda = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then, from matrix multiplication, we get

$$a + 3c = 23$$

$$-2a + 5c = 31$$

$$b + 3d = 7$$

$$-2b + 5d = -3$$

Multiplying the first equation by 2 and adding it to the second, you get $11c = 77$ so $c = 7$. Plugging this back into the first equation, you get $a = 2$. Multiplying the third equation by 2 and adding it to the fourth, you get $11d = 11$ so $d = 1$. Plugging this into the third equation, you get $b = 4$. So, the sum of the elements is 14

12. A From the question, we get 3 equations:

$$\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} = 2$$

$$\sqrt[3]{abc} = 6$$

$$\frac{a + b + c}{3} = 10$$

Simplifying the third equation, we get $a + b + c = 30$. Squaring both sides, we get $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = 900$. From this, we get

$$a^2 + b^2 + c^2 = 900 - 2(ab + bc + ac)$$

Simplifying the first equation, we get $\frac{3}{\frac{bc+ac+ab}{abc}} = \frac{3abc}{bc+ac+ab} = 2$. Multiplying both sides by $bc + ac + ab$, we get $3abc = 2(bc + ac + ab)$. From the second equation, by cubing both sides, we get $abc = 216$. So, substituting this into the second equation, we get $3(216) = 648 = 2(ab + ac + ab)$. Substituting this back into our first equation, we get $a^2 + b^2 + c^2 = 900 - 684 = 252$

13. D This equation turns into $-6\sin^2(\theta) = -\sin(\theta) - 4$. So, we get $6\sin^2(\theta) + \sin(\theta) + 4 = 0$. This is not factorable, so by the quadratic formula, we get $\sin(\theta) = \frac{-1\pm\sqrt{97}}{12}$. Since, $-1 < \frac{-1-\sqrt{97}}{12} < \frac{-1+\sqrt{97}}{12} < 1$, both solutions are valid. We can't actually solve for the exact values of θ by hand, but we can find the sum. When $\sin(\theta) = \frac{-1+\sqrt{97}}{12}$, we know the two angles that fall in the first and second

quadrant. The angles are $\frac{\arcsin(-1+\sqrt{97})}{12}$ and $\pi - \frac{\arcsin(-1+\sqrt{97})}{12}$. The sum of these is π. When $\sin(\theta) = \frac{-1-\sqrt{97}}{12}$. We know the angles fall in the third and fourth quadrant. The angles are $2\pi - \arcsin\left(\frac{-1-\sqrt{97}}{12}\right)$ and $\pi - \arcsin\left(\frac{-1-\sqrt{97}}{12}\right)$. These sum to 3π. So, the sum of all the angles is $3\pi + \pi = 4\pi$.

14. C These 3 equation are

$$x(x+y+z) = 50$$
$$y(x+y+z) = 40$$
$$z(x+y+z) = 10$$

Since $x, y, z, x+y+z \neq 0$, we can safely divide our equations. Dividing the second equation by the first and the third equation by the first, we get

$$\frac{y}{x} = \frac{4}{5}$$
$$\frac{z}{x} = \frac{1}{5}$$

So, we get $y = \frac{4}{5}x$ and $z = \frac{1}{5}x$. Substituting this back into the first equation, we get $x\left(x + \frac{1}{5}x + \frac{4}{5}x\right) = x(2x) = 2x^2 = 50$. So, we get $x = \pm 5$. However, since the question states $x > 0$, we know that $x = 5$. So, we get $y = 4, z = 1$ and $x + y + z = 10$.

15. B Looking at the equations, we notice they look eerily like sum of roots, sum of roots two at a time, and product of roots. Say $z = -c, x = a, and\ z = b$ where a, b, c are the roots of some polynomial. Now, the equations are

$$-a - b - c = -3$$
$$ab + bc + ac = -24$$
$$-abc = 80$$

So, from this, we gather that the polynomial is $x^3 - 3x^2 - 24x + 80$. This factors into $(x-4)^2(x+5)$. So, the ordered triplets are (4,4,-5),(4,5,-4), and (5,4,-4) so the answer is 3.

16. B Looking at the equation, this the same as saying the sum of the distances from any point (x, y) to the points $(5,3)$ and $(-5,-7)$ is 15. Since 15 is greater than the distance between $(5,3)$ and $(-5,-7)$, this is the definition of an ellipse where the two foci are $(5,3)$ and $(-5,-7)$

17. A You could just solve for x and plug it in, however we want to avoid this because there are is a lot of multiplying with imaginary numbers which may get confusing. So, we think to use some algebraic manipulation to avoid arithmetic.
We can rewrite what we are finding as

$$(x+1)(x+7)(x+4)^2 = (x^2+8x+7)(x^2+8x+16)$$

From the first equation, we get $x^2 = -3x - 4$. Substituting this into what we are solving for, we get $(5x+3)(5x+12) = 25x^2 + 75x + 36$. Substituting once again, we get $25(-3x-4) + 75x + 36 = -64$

18. B Take the two vectors, $< a_1, a_2 \ldots, a_n >$ and $< 1,1,1 \ldots, 1 >$ where the second vector has n 1s. Now, by applying the Cauchy-Schwartz inequality we get
$a_1 + a_2 + \cdots a_n \leq \sqrt{a_1^2 + a_2^2 + \cdots a_n^2} \cdot \sqrt{n}$. We know that $a_1 + a_2 + \cdots a_n = 1$.
Substituting that back in and dividing both sides by $\sqrt{n}$, we get

$$\sqrt{a_1^2 + a_2^2 + \cdots a_n^2} \geq \frac{1}{\sqrt{n}}$$

Since we know everything is always positive, we square both sides and get

$$a_1^2 + a_2^2 + \cdots a_n^2 \geq \frac{1}{n}$$

So, the least value n where $a_1^2 + a_2^2 + \cdots a_n^2 = \frac{1}{50}$ is when $n = 50$.

19. B So, the first thing we should do is convert this to cis to make it easier to work with. So, we have

$$2^{\frac{p}{2}}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^p = 2^q\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^q$$

Which is

$$2^{\frac{p}{2}} cis\left(\frac{\pi p}{4}\right) = 2^q cis\left(\frac{\pi q}{6}\right)$$

We know the magnitudes have to be the same, so we know that $p = 2q$.
From this, we know that $cis\left(\frac{\pi q}{2}\right) = cis\left(\frac{\pi q}{6}\right)$. From this equation, you know that $3|q$ because q is an integer and you need the denominators of the angles to be the same.
Going through values of q, the smallest solution is $q = 6$ so $p = 12$ so $p + q = 18$

20. C For this question, the hardest part is setting up the equations. Once you set them up, the question is a pre-algebra question. The output of the Goods and Services sectors are both defined as p_g *and* p_s respectively. The input into Goods is 30% of its own output and 67% of the output of the Services sector. This gives the equation
$p_g = .3p_g + .67p_s$
Conversely, the input into Services is 33% of its own output and 70^ of the output of the Goods sector. This gives the equation
$p_s = .7p_g + .33p_s$
These equations are actually equivalent. This should make intuitive sense as this is a closed economy, and we are only trading within ourselves so excess money can't just appear. Simplifying the first equation, we get
$.7p_g = .67p_s$
So,
$p_g = \frac{6.7}{7}p_s$
Since we are given $\mathrm{p_s} = 700$, we have
$p_g = \frac{6.7}{7} * 700 = 6.7 * 100 = 670.$

21. E From the polynomial, we know that $a_1 + a_2 + a_3 + a_4 = 6$.
So, each time we sum 3 terms, we can instead write it out as $6 - a_i$ where i is the term missing from the sum. So, what we're looking for becomes

$$a_1^3(6-a_1)+a_2^3(6-a_2)+a_3^3(6-a_3)+a_4^3(6-a_4).$$

This becomes

$$6\sum_{i=1}^{4} a_i^3 - \sum_{i=1}^{4} a_i^4$$

However, finding the sum of the cubes of the roots and fourth of the roots is difficult. So, looking back at our polynomial, we know that for each root,

$$x^4-6x^3-4x^2+15x-20=0$$

So, for each root,

$$-4x^2+15x-20=6x^3-x^4$$

This means we can simplify our sum in lower terms, that being

$$-4\sum_{i=1}^{4} a_i^2 + 15\sum_{i=1}^{4} a_i - \sum_{i=1}^{4} 20$$

This means our final answer is $-4\left(6^2-2(-4)\right)+15(6)-4(20)=-166$

22. A By AM/GM, we know that $\frac{x^4+y^4+z^4+16}{4} \geq \sqrt[4]{16x^4y^4z^4}$. This simplifies to $x^4+y^4+z^4+16 \geq 8|xyz|$. Since our equation is when they are equal, we take the equality case, where $16=x^4=y^4=z^4$. This means that $2=|x|=|y|=|z|$. We know that xyz must be positive. So, either x, y, z are all positive or only one of them is positive. So, the solutions are $(2,2,2,), (2,-2,-2), (-2,2,-2), (-2,-2,2)$. Therefore, there are 4 solutions.

23. D Say $a=x+3$ and $b=y-4$. Then, the first equation turns into

$$\sqrt{a-1}+\sqrt{a}+\sqrt{a+1}=\sqrt{b-1}+\sqrt{b}+\sqrt{b+1}$$

From this, we can see this is true when $a=b$ or in other words when $x+3=y-4$. This gives us $y=x+7$. Plugging this into the second equation, we get

$$(x+7)^2-2(x+7)-x^2+x=126$$
$$x^2+14x+49-2x-14-x^2+x=126$$
$$13x+35=126$$
$$13x=91$$
$$x=7$$
$$y=x+7=7+7=14$$
$$x+y=7+14=21$$

24. B You don't actually have to solve for the exact point to do this question, but you can. Instead, let's take some arbitrary (a,b,c) on the line. Then, we get that $\frac{a-1}{2}=\frac{b-3}{-3}$. So, $-\frac{3(a-1)}{2}=(b-3)$ which gives us $b=-\frac{a+3}{2}+3=-\frac{3a+9}{2}$. Doing the same with c, we get $c=\frac{a-1}{2}-4=\frac{a-9}{2}$. So, $a+b+c=a-\frac{3a+9}{2}+\frac{a+9}{2}=\frac{2a-3a-9+a+9}{2}=0$. So, every point on this line's coordinates sum to zero, so $a+b+c=0$

25. E
   I. Triangle Inequality so it's always true
   II. Triangle Inequality so it's always true
   III. This is just an alternate way to write AM/GM so it's always true
   IV. From AM/GM, we have $\frac{a^4+a^4+a^4+b^4}{4} \geq \sqrt[4]{a^4\cdot a^4\cdot a^4\cdot b^4}=a^3b$.

Similarly, we have $\frac{b^4+b^4+b^4+c^4}{4} \geq b^3c$ and $\frac{c^4+c^4+c^4+a^4}{4} \geq c^3a$.
Adding up all these inequalities, we get our desired inequality, so it's true.

26. E There are two ways to approach this problem. One way is to say $v_1 =< a, b, c >$ and $v_2 =< 2d, -d, 2d >$.
From the information in the problem, we get a system of 4 variables and 4 equations:

$$a + 2d = 3$$
$$b - d = 5$$
$$c + 2d = 2$$
$$2a - b + 2c = 0$$

Solving this system for a, b, c, d will yield the correct answer.
The other way is to directly solve for v_2. If we know our properties of vectors, we can figure out that v_2 is just the projection of v onto u.
So, we have $\overrightarrow{v_2} = \frac{\vec{u}\cdot\vec{v}}{|u|} * \frac{\vec{u}}{|u|} = \frac{5}{3} * \frac{<2,-1,2>}{3} =< \frac{10}{9}, -\frac{5}{9}, \frac{10}{9} >$.
So, $\overrightarrow{v_1} = \vec{v} - \overrightarrow{v_2} =< 3,5,2 > - < \frac{10}{9}, -\frac{5}{9}, \frac{10}{9} > =< \frac{17}{9}, \frac{50}{9}, \frac{8}{9} >$. So, the sum of the elements of $\overrightarrow{v_1}$ is $\frac{17}{9} + \frac{50}{9} + \frac{8}{9} = \frac{75}{9} = \frac{25}{3}$.

27. C Applying AM/GM, we get $\frac{6x^2+y^2}{2} \geq \sqrt{6x^2y^2}$. Plugging in the ellipse and simplifying, we get $6 \geq xy\sqrt{6}$. Isolating xy, we get $xy \leq \sqrt{6}$. S. Squaring both sides, we get what we want, $x^2y^2 \leq 6$.

28. D These are 3 homogenous equations, so we know trivially that x=0,y=0,z=0 works. So, we know that the answer can't be A. With questions like these, it can only be 0,1, or infinity since we are working with equations of linear variables, so the only other possible answers are B or D. The second equation is the sum of the first and third equation. This means if we were to solve this by Cramer's rule, our coefficient matrix has determinant zero. This means that there is no unique solution, meaning there are no or infinite solutions. Since we know a solution exists, there must be infinitely many. Another way of thinking about it is that you can "eliminate" the second equation since it doesn't actually add any new information that is not said by the other two equations. Then, we have 3 variables and 2 equations, once again leading to having infinitely solutions.

29. A A knight and knave cannot make the statement "I am a knave". Therefore, we know that Jack is a spy. Now we know the knave is one of Andrew and Eddie. The knight cannot say "I am a spy" so Andrew must be the knave.

30. A Any factorial greater than 4 ends in a 0.