8.1

Consider to the 'Law of mass action'.

As for the reaction rate:

$$A \xrightarrow{k_1} B$$

$$r = \frac{-d[A]}{dt} = \frac{d[B]}{dt}$$

$$r = k_1[A]$$

where k_1 is a rate constant.

As for the eeversible rate:

$$A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} B$$

$$r = \frac{-d[A]}{dt} = \frac{d[B]}{dt}$$

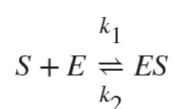$$r = k_1[A] - k_2[B]$$

When the reaction is in equilibrium:

$$r = 0$$

$$k_1[A] = k_2[B]$$

$$\frac{k_2}{k_1} = \frac{[A]}{[B]} = K_{eq}$$

Now, consider to the Enzyme-catalyzed reactions.

First step: E forms a complex with S to form an intermediate species ES in a reversible manner at the forward rate k1 and reverse rate k2.

$$S + E \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES$$

In the Law of Mass Action, we assume that the rate of forward reaction is linearly proportional to the concentrations of S and E, and the back reaction is linearly proportional to the concentration of ES:

$$r = k_1[S][E] - k_2[ES]$$

Second step: The intermediate ES then breaks down into the product P at a rate k3,thereby releasing E.
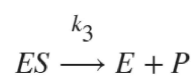
$$ES \xrightarrow{k_3} E + P$$

Then,

$$r = \frac{d[P]}{dt} = k_3[ES]$$

Since the concentration of ES affects both the first step reaction and the second step reaction, we have:

$$\frac{d[S]}{dt} = -k_1[E][S] + k_2[ES]$$

$$\frac{d[E]}{dt} = -k_1[E][S] + k_2[ES] + k_3[ES]$$

$$\frac{d[ES]}{dt} = k_1[E][S] - k_2[ES] - k_3[ES]$$

$$\frac{d[P]}{dt} = k_3[ES]$$

8.2

From the solution of 8.1, we have:

$$\frac{d[S]}{dt} = -k_1[S][E] + k_2[ES]$$

$$\frac{d[E]}{dt} = -k_1[S][E] + k_2[ES] + k_3[ES]$$

$$\frac{d[ES]}{dt} = k_1[S][E] - k_2[ES] - k_3[ES]$$

$$\frac{d[P]}{dt} = k_3[ES]$$

with the initial conditions:

$$[S]_0 = 10\mu M \quad [E]_0 = 1\mu M \quad [ES]_0 = 0 \quad [P]_0 = 0$$

and the constants:

$$k_1 = 100(\mu M)^{-1}\,min^{-1} \approx 1.67(\mu M)^{-1}\,s^{-1}$$

$$k_2 = 600\,min^{-1} = 10\,s^{-1}$$

$$k_3 = 150\,min^{-1} = 2.5\,s^{-1}$$

```
clc;
clear all;
% Set the initial conditions, constants, & time span
yzero=[10, 1, 0, 0];
k1=1.67; k2=10; k3=2.5;
tspan=[0 30];

% Integrate the equations
[t,y]=ode45('E_K_equations',tspan,yzero,[],k1,k2,k3);

% Print out the results
n=length(y);
for i=1:n
    if y(i,1)<=0.001*yzero(1)
    fprintf('Reaction is 99.9 percent complete at time = %4.0f seconds',t(i));
    break
    end
end
```
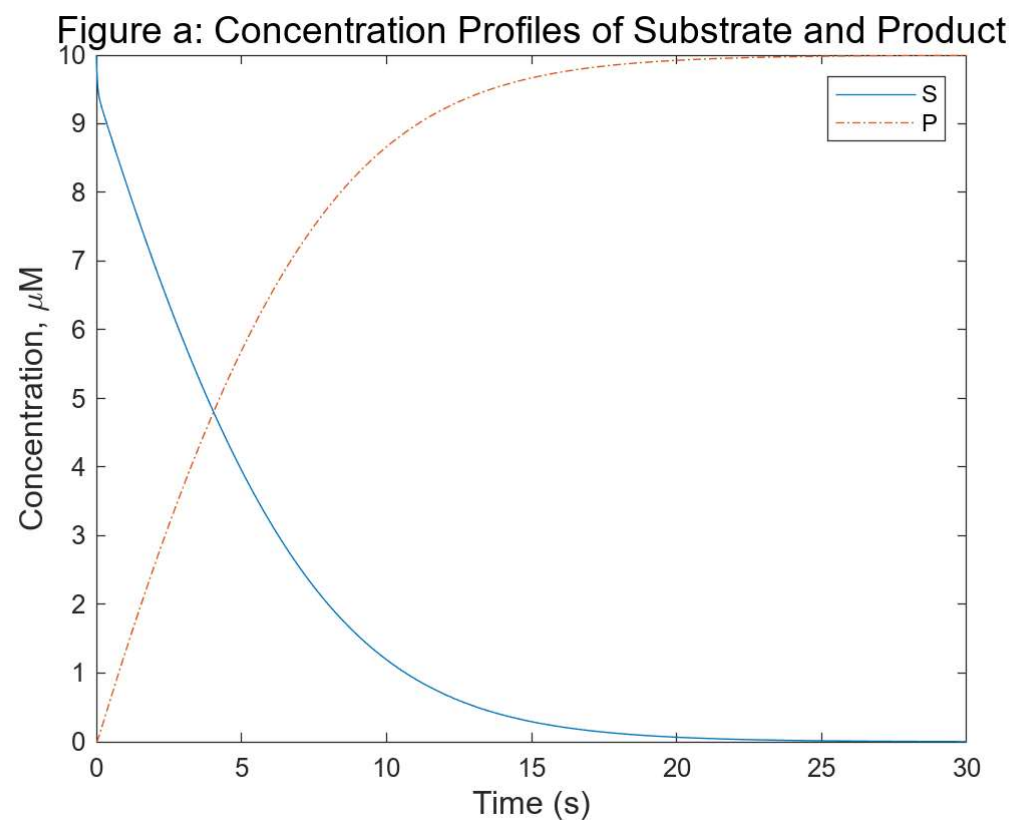
```
Reaction is 99.9 percent complete at time =   26 seconds
```

```
% Plot concentration profiles
clf; figure(1); plot(t,y(:,1),'-',t,y(:,4),'-.')
title('Figure a: Concentration Profiles of Substrate and Product', 'FontSize',14)
xlabel('Time (s)','FontSize',12);
ylabel('Concentration, \muM', 'FontSize',12);
legend('S','P');
```
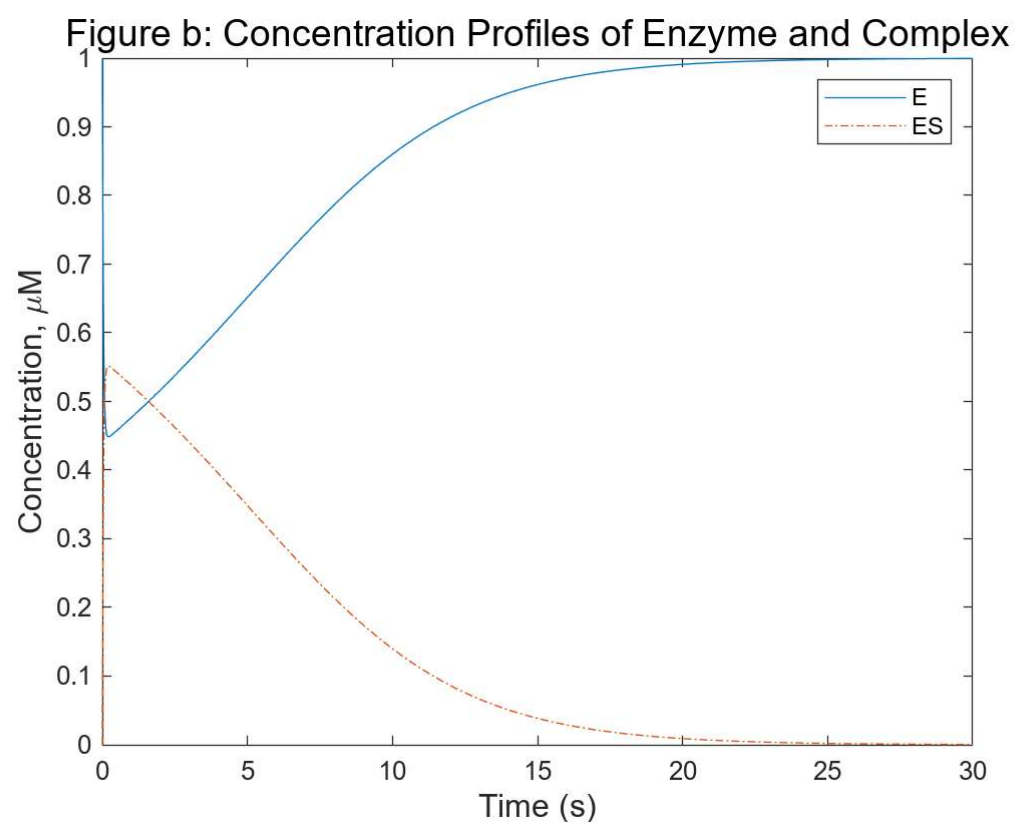

Figure a: Concentration Profiles of Substrate and Product

```
figure(2); plot(t,y(:,2),'-',t,y(:,3),'-.')
title('Figure b: Concentration Profiles of Enzyme and Complex', 'FontSize',14)
xlabel('Time (s)','FontSize',12);
ylabel('Concentration, \muM', 'FontSize',12);
legend('E','ES')
```


Figure b: Concentration Profiles of Enzyme and Complex

8.3

Since the velocity, V, of the enzymatic reaction to be the rate of change of the product P. From 8.1 we know:

$$V = \frac{d[P]}{dt} = k_3[ES]$$

At steady state:

$$k_1[E][S] = k_2[ES] + k_3[ES]$$

Hence we have:

$$[ES] = [E][S]\frac{k_1}{k_2 + k_3}$$

Define:

$$K_M = \frac{k_2 + k_3}{k_1}$$

At the beginning of the reaction, the concentration of E is the highest, so the reaction rate is also the largest. In the process of subsequent reaction, the sum of the concentration of E and ES is a certain value, we define as:

$$[E]_t = [E] + [ES]$$

$$V_{\max} = k_3[E]_t$$

$$[ES] = [E]_t\frac{[S]}{K_M + [S]}$$

Thus, we can conclude:

$$V = k_3[ES] = k_3[E]_t\frac{[S]}{K_M + [S]} = V_{\max}\frac{[S]}{K_M + [S]}$$

```
KM=(k2+k3)/k1;
Vmax=k3*yzero(2);
V=zeros(1,n);
S=linspace(1,250,n)
```

```
S = 1×649
    1.0000    1.3843    1.7685    2.1528    2.5370    2.9213    3.3056    3.6898    4.0741    4.4
```

```
for i=1:n
    V(1,i)=Vmax*S(i)/(KM+S(i));
end
```

```
clf; figure(3); plot(S(:),V(1,:),'-')
title('Figure c: velocity V as the concentration of the substrate S changes', 'FontSize',14)
xlabel('concentration of the S (\muM)','FontSize',12);
ylabel('V \muM/s', 'FontSize',12);
```


Figure c: velocity V as the concentration of the substrate S change
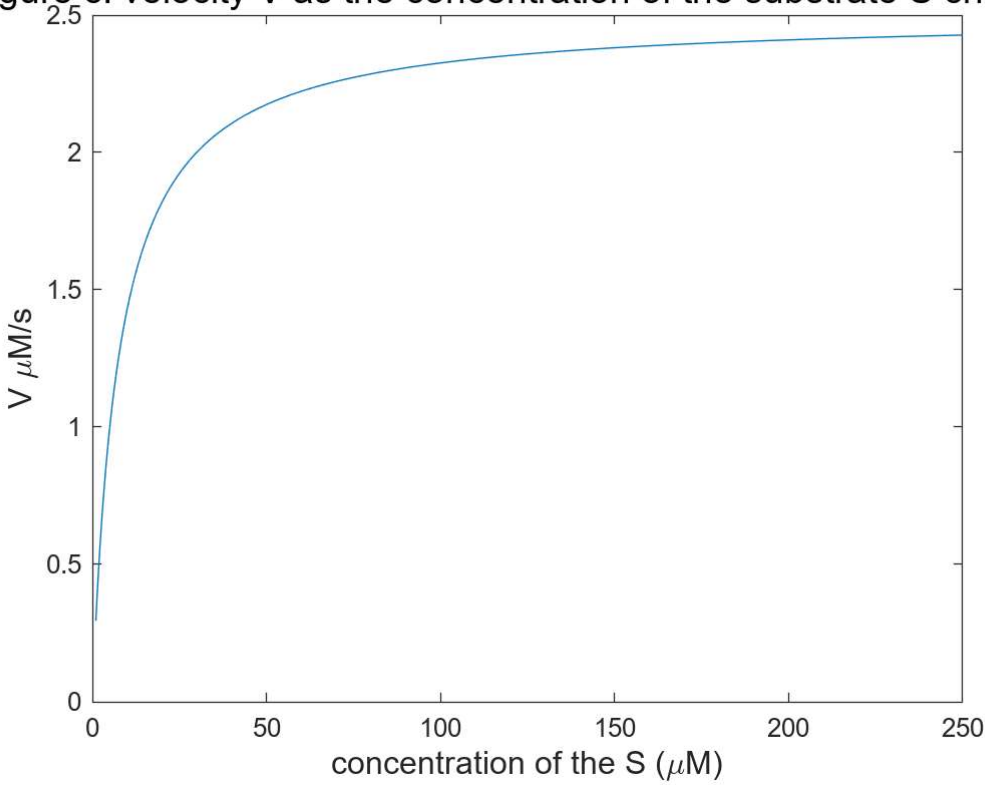
```
fprintf('The maximum value of velocity V is %0.2f muM/s.',Vmax);
```

```
The maximum value of velocity V is 2.50 muM/s
```