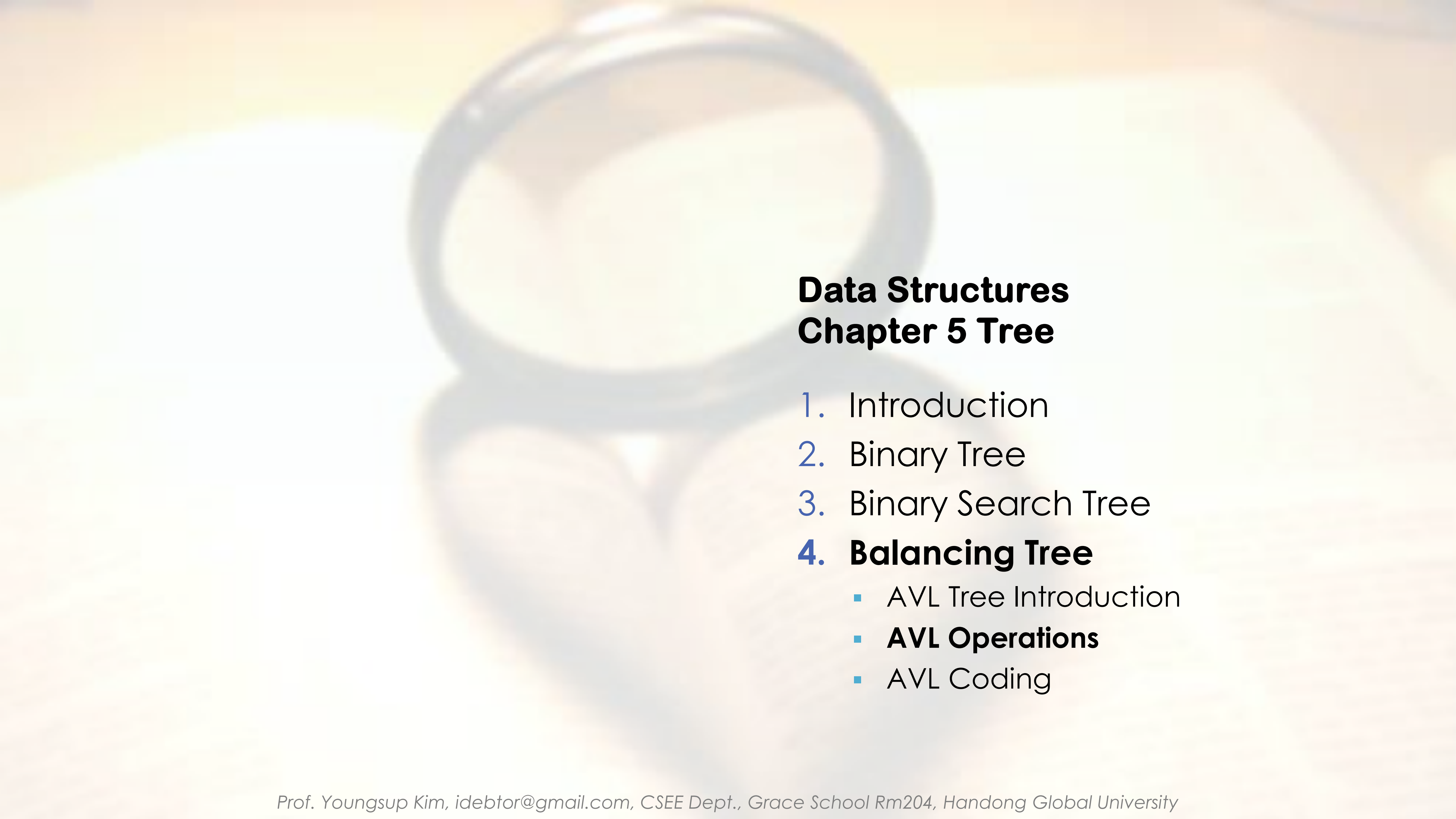
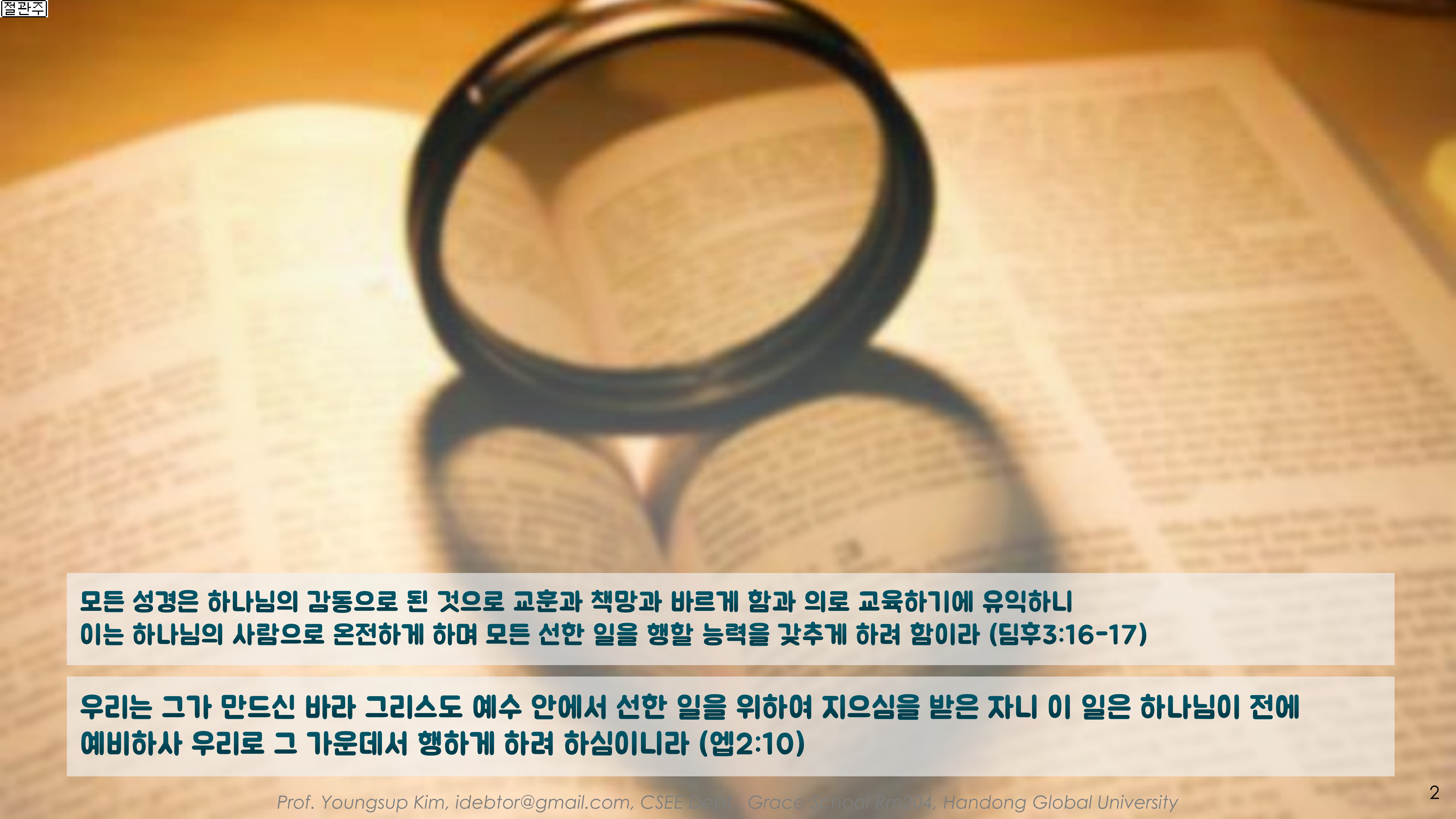


# Data Structures
# Chapter 5 Tree

1. Introduction
2. Binary Tree
3. Binary Search Tree
4. **Balancing Tree**
   - AVL Tree Introduction
   - **AVL Operations**
   - AVL Coding

*Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204, Handong Global University*

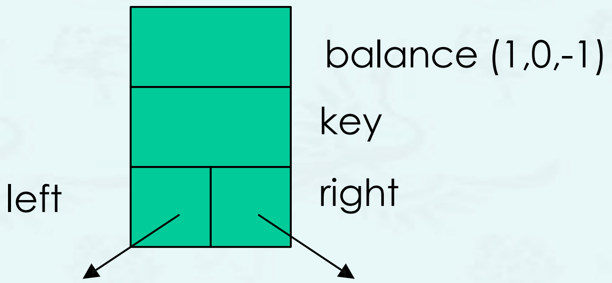


모든 성경은 하나님의 감동으로 된 것으로 교훈과 책망과 바르게 함과 의로 교육하기에 유익하니
이는 하나님의 사람으로 온전하게 하며 모든 선한 일을 행할 능력을 갖추게 하려 함이라 (딤후3:16-17)

우리는 그가 만드신 바라 그리스도 예수 안에서 선한 일을 위하여 지으심을 받은 자니 이 일은 하나님이 전에
예비하사 우리로 그 가운데서 행하게 하려 하심이니라 (엡2:10)

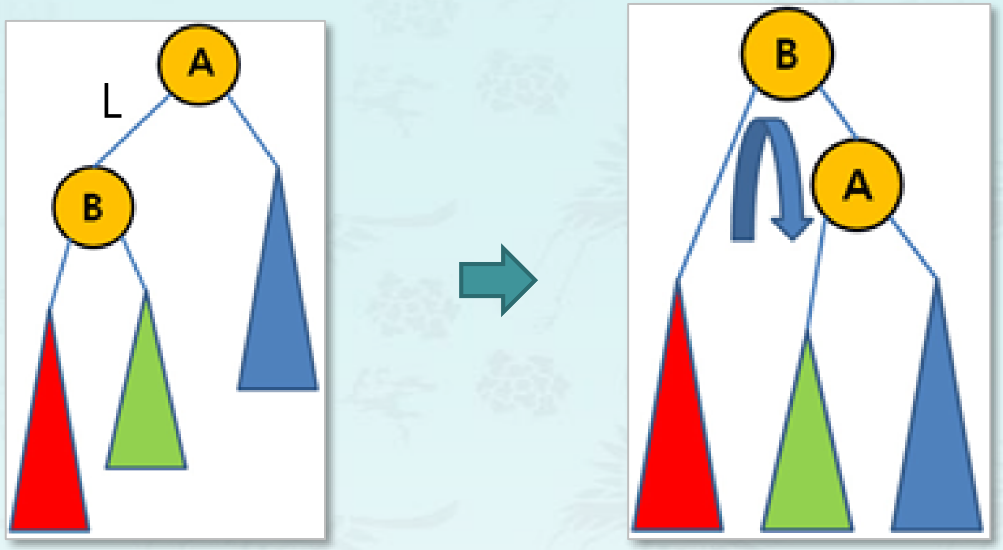
# Coding

- You can either keep the height or just the difference in height, i.e. the balance factor; this has to be modified on the path of  insertion even if you don’t perform rotations.
  - Once you have performed a rotation (single or double) you won’t  need to go back up the tree for the computation.
- You may compute the balance factor **on the fly** after the insert is done during the recursion.



# Single Rotation - LL case

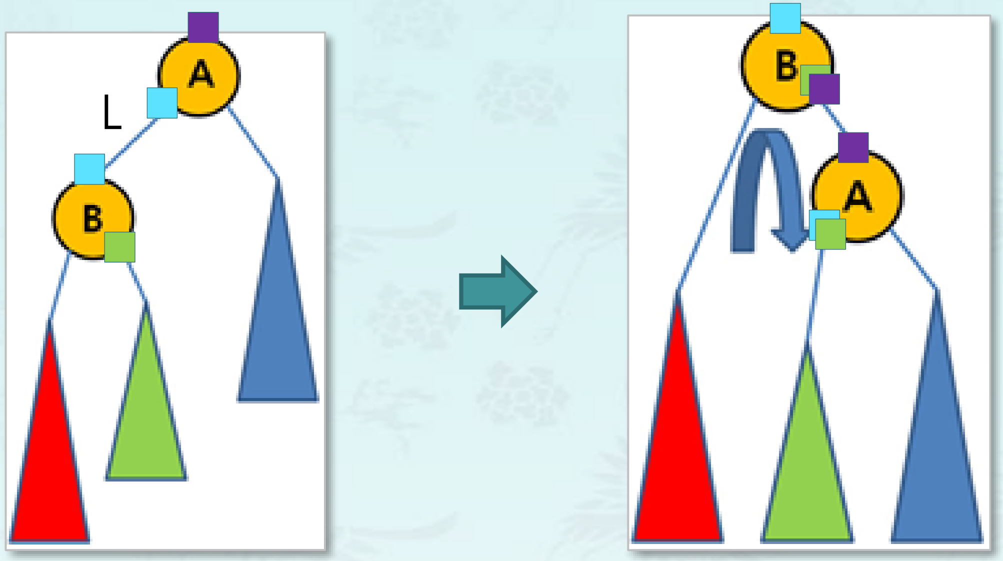
outside case



```
tree rotateLL(tree A)
{



  return
}
```

# Single Rotation - LL case

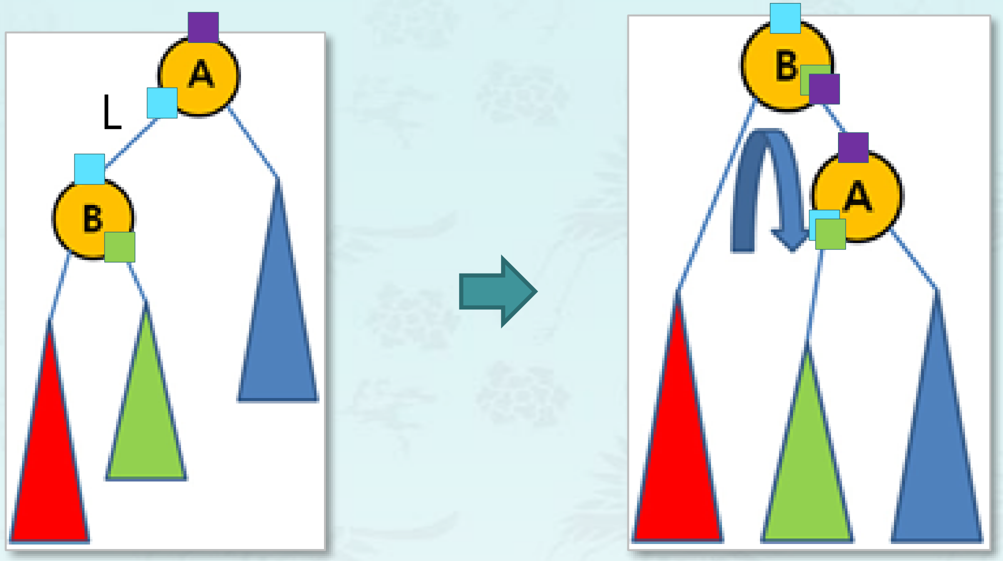
outside case



```
tree rotateLL(tree A)                              🟪
{
                  ?                                🟦
                                                   🟦🟩
                                                   🟩🟪
  return          ?                                🟦
}
```

# Single Rotation - LL case

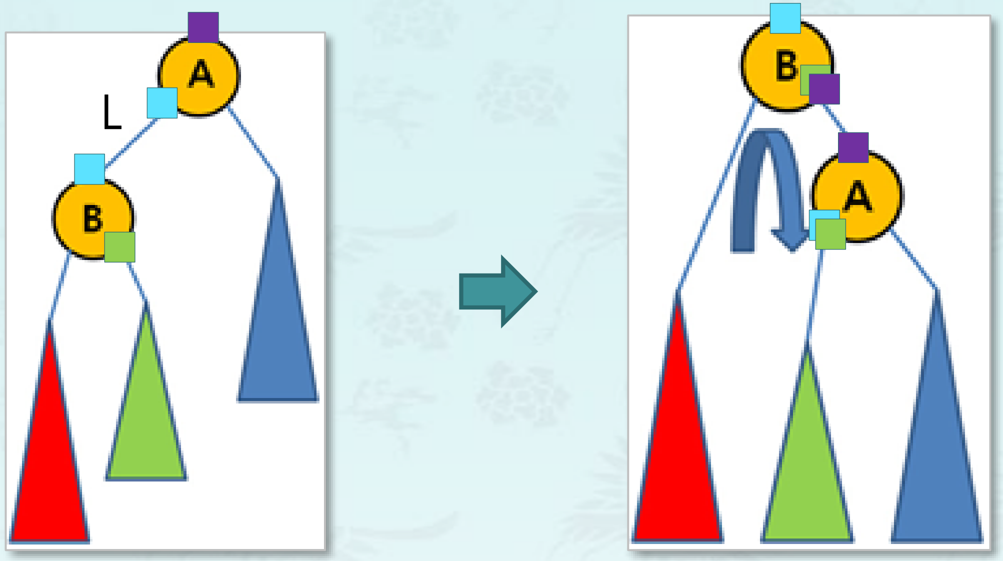
outside case



```
tree rotateLL(tree A)                 🟪
{
  tree B   = A->left;                 🟦
                                      🟦🟩
                                      🟩🟪
  return B;                           🟦
}
```

# Single Rotation - LL case

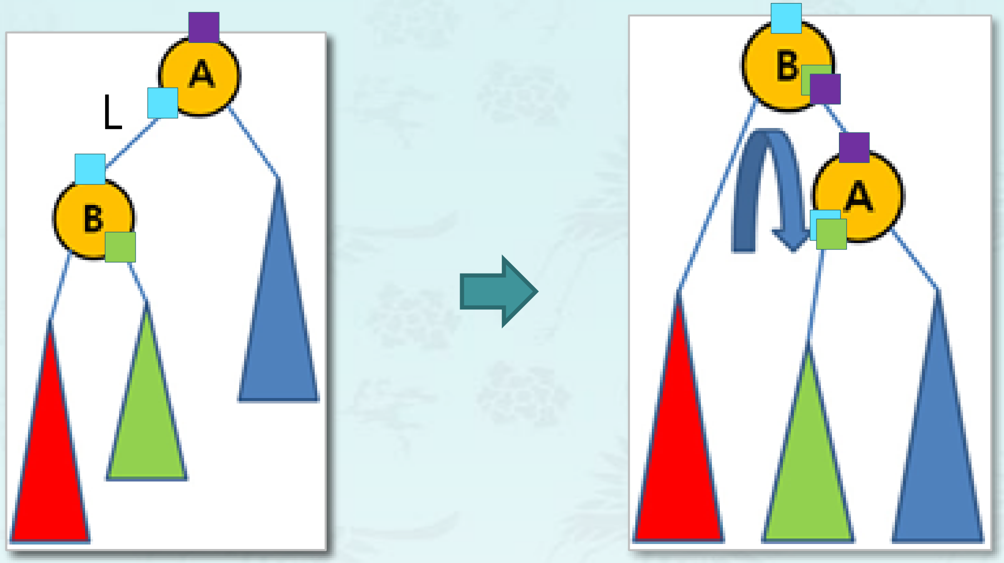
outside case



```
tree rotateLL(tree A)
{
  tree B    = A->left;
  A->left   = B->right;

  return B;
}
```

# Single Rotation - LL case

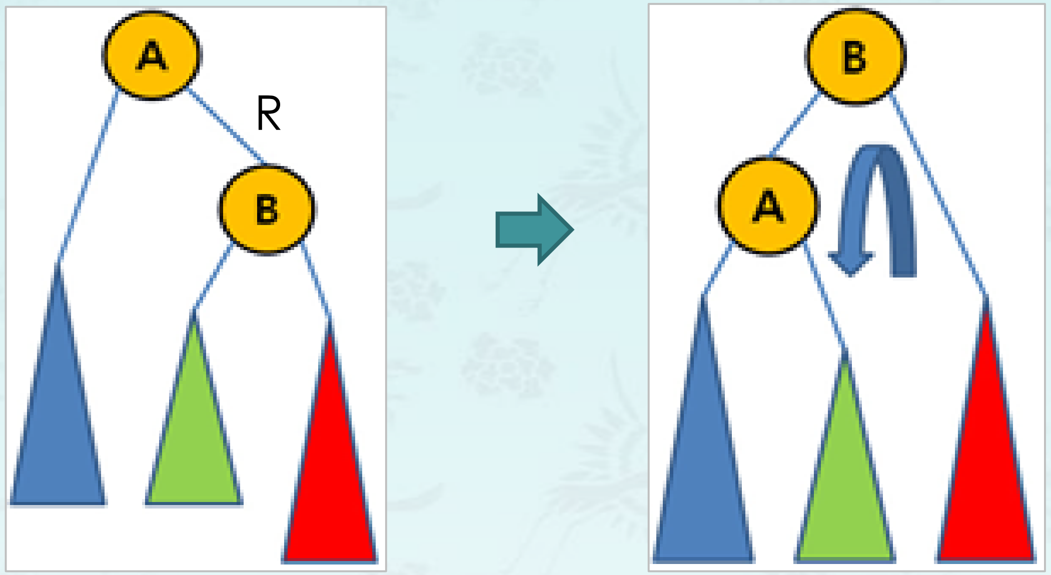
outside case



```
tree rotateLL(tree A)              🟪
{
  tree B    = A->left;             🟦
  A->left   = B->right;            🟦🟩
  B->right = A;                    🟩🟪
  return B;                        🟦
}
```

# Single Rotation – RR case

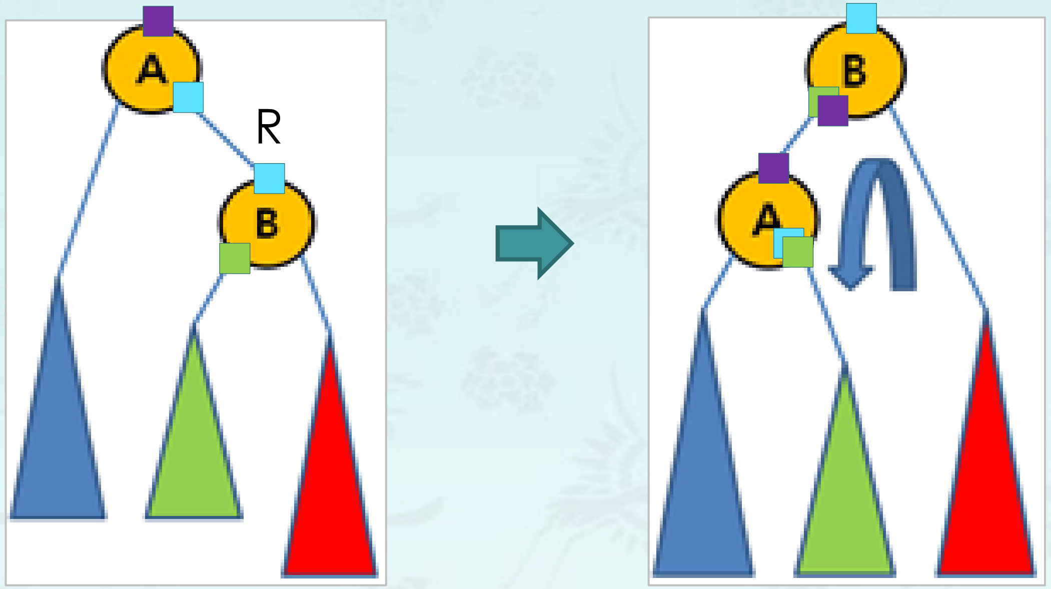
outside case



```
tree rotateRR(tree A)
{
               ?
                                      ;

  return        ?
}
```

# Single Rotation – RR case

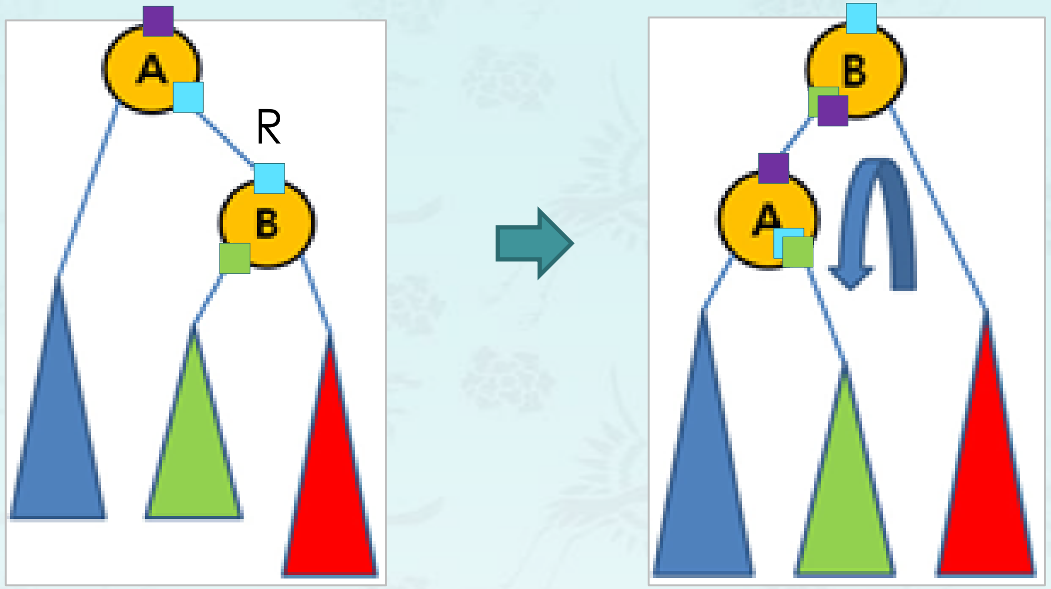
outside case



```
tree rotateRR(tree A)                 🟪
{
                 ?                    🟦
                                      🟦🟩
                                      🟩🟪
    return            ?               🟦
}
```

# Single Rotation – RR case

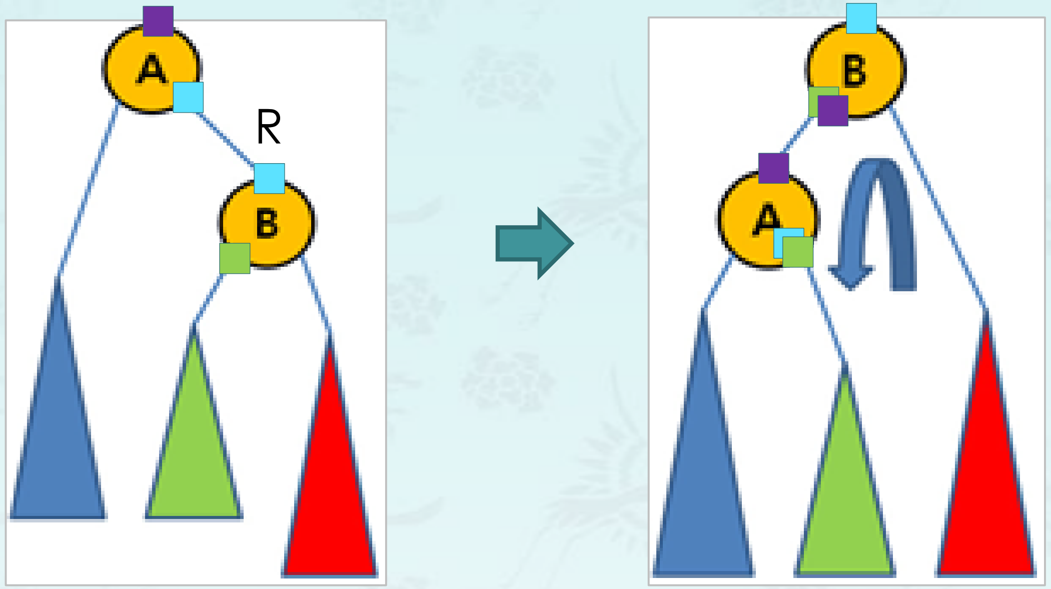
outside case



```
tree rotateRR(tree A)                  🟪
{
  tree B   = A->right;                 🟦
                                       🟦🟩
                                       🟩🟪
  return B;                            🟦
}
```

# Single Rotation – RR case

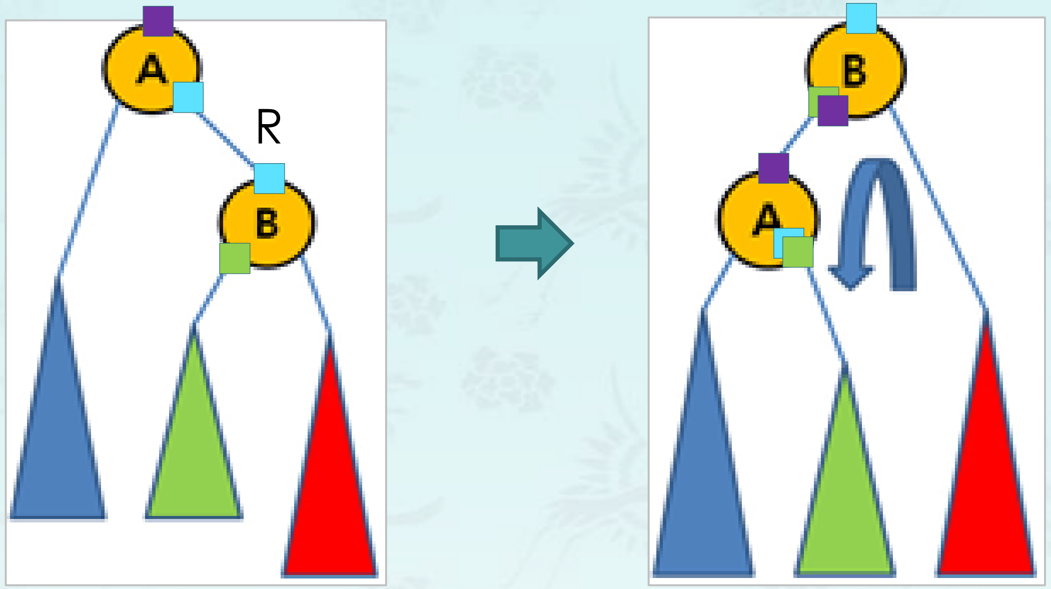
outside case



```
tree rotateRR(tree A)            🟪
{
  tree B    = A->right;          🟦
  A->right = B->left;            🟦🟩
                                 🟩🟪
  return B;                      🟦
}
```

# Single Rotation – RR case

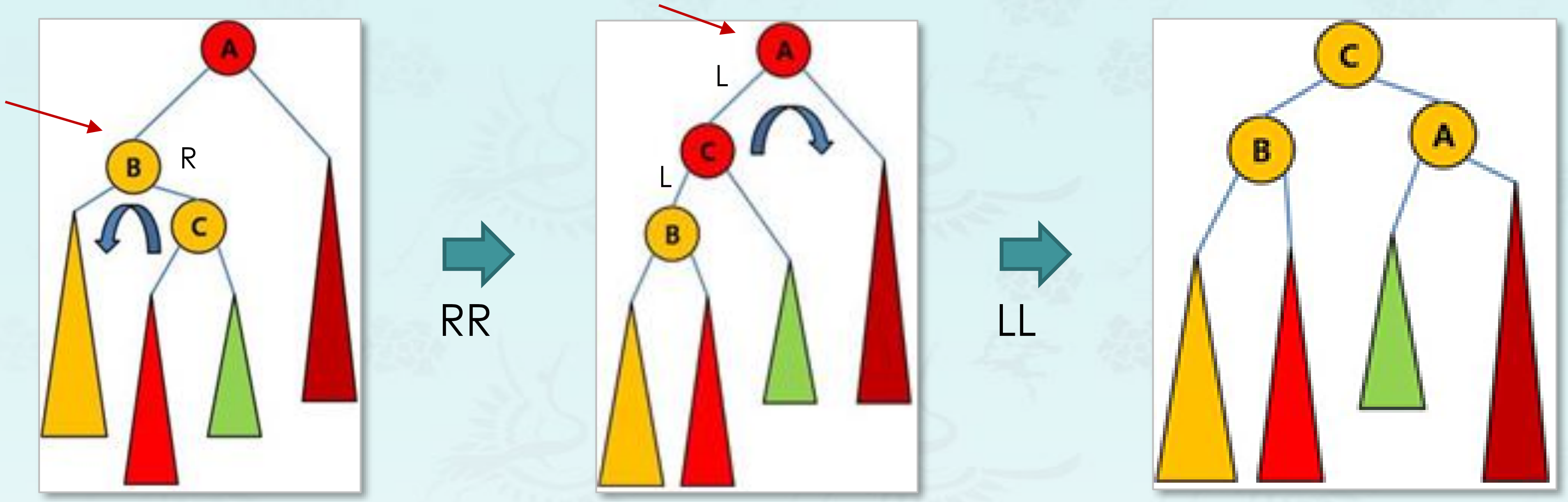
outside case



```
tree rotateRR(tree A)                🟪
{
  tree B    = A->right;              🟦
  A->right = B->left;                🟦🟩
  B->left  = A;                      🟩🟪
  return B;                          🟦
}
```

# Double Rotation - LR case

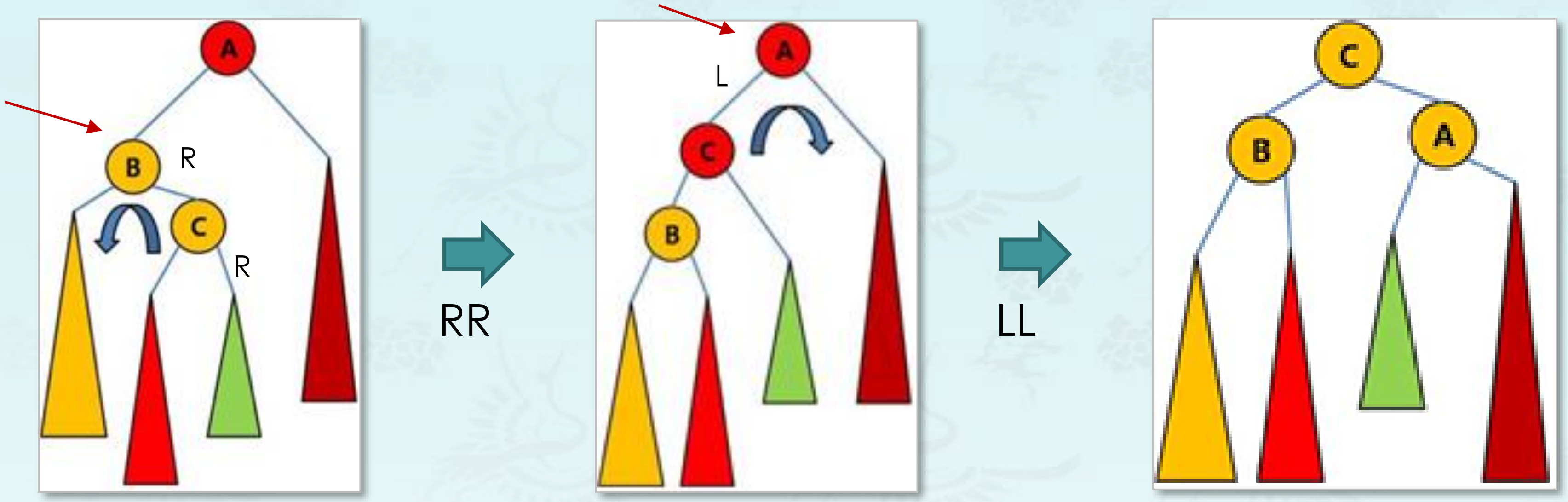
inside case



```
tree rotateLR(tree A) // RR and LL
{


}
```

# Double Rotation - LR case

inside case

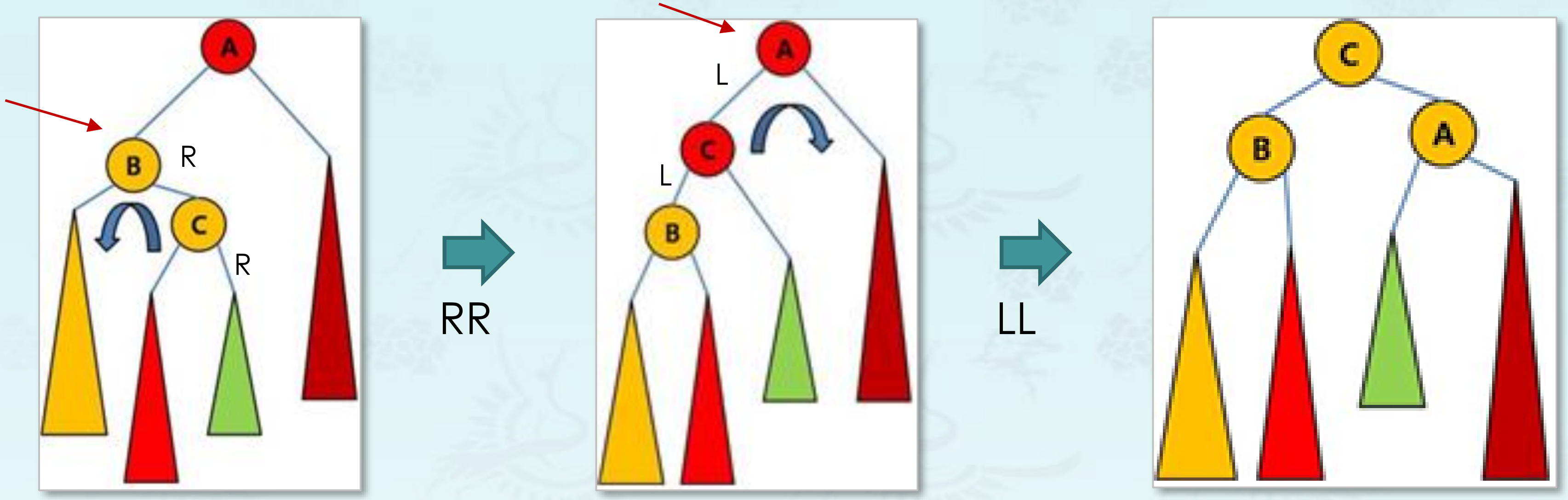


```
tree rotateLR(tree A) // RR and LL
{
  tree B  = A->left;


}
```

What will return eventually?

# Double Rotation - LR case

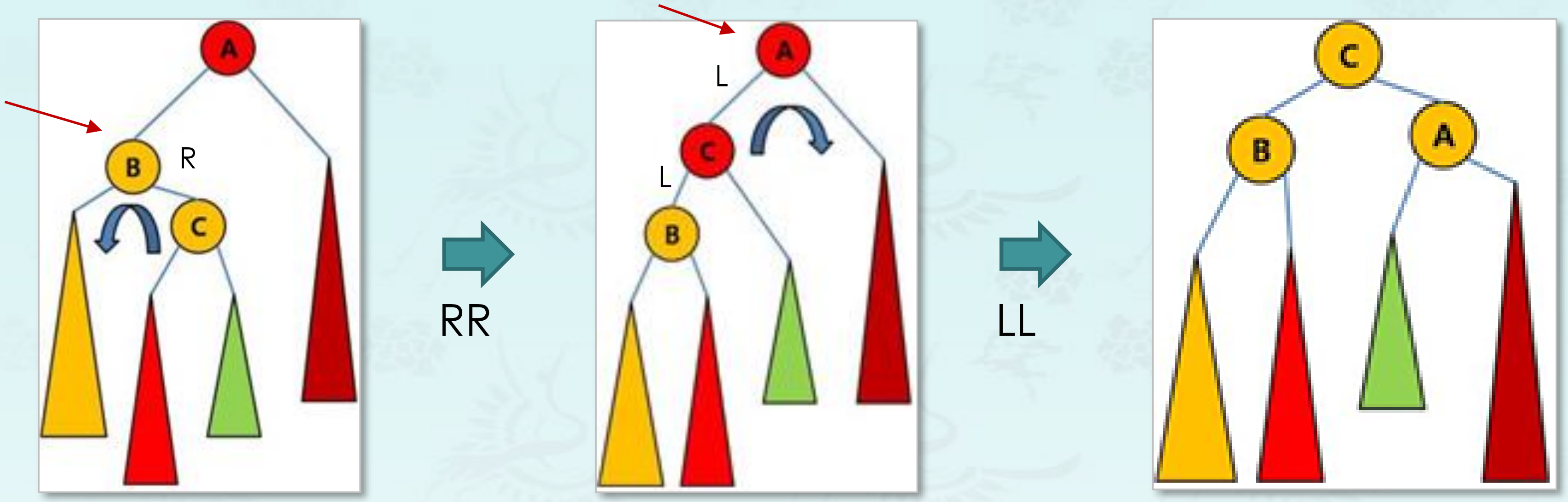
inside case



```
tree rotateLR(tree A) // RR and LL
{
  tree B  = A->left;
  A->left = rotateRR(B);

}  What will return eventually?
```

# Double Rotation - LR case

inside case

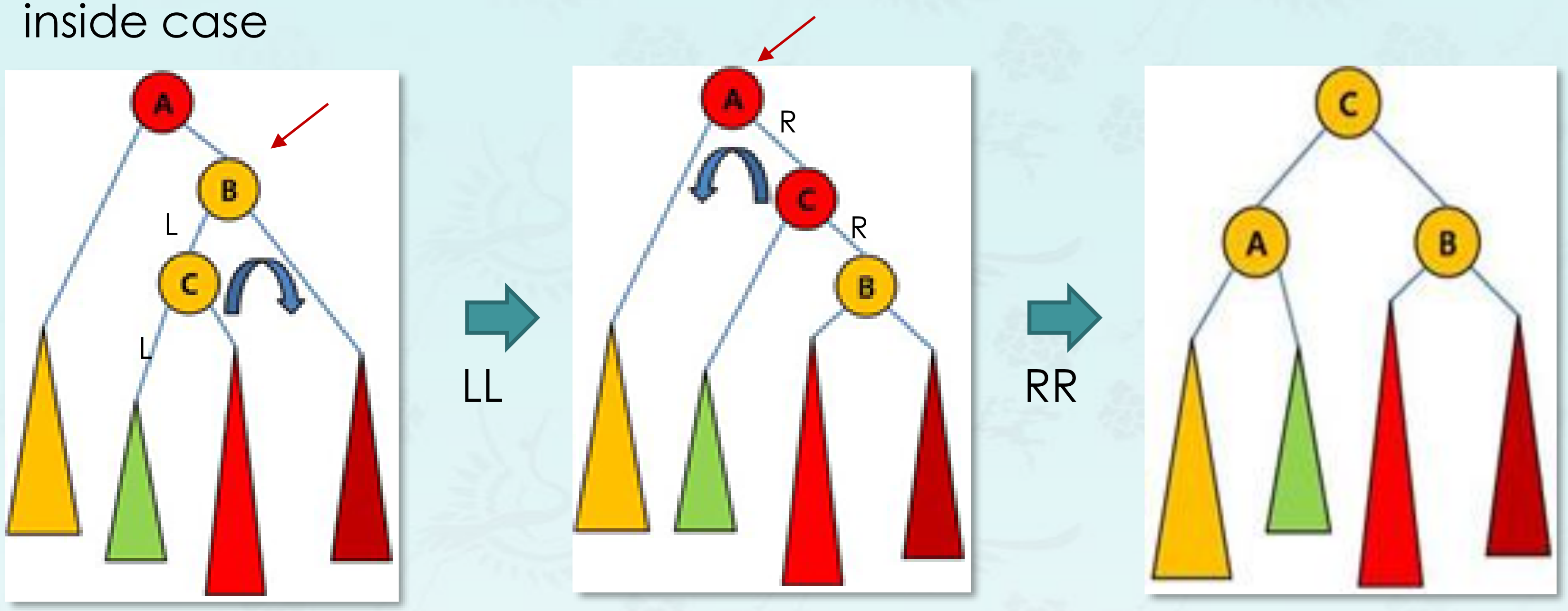


```
tree rotateLR(tree A) // RR and LL
{
  tree B  = A->left;
  A->left = rotateRR(B);
  return rotateLL(A);
}
```

What will return eventually?

# Double Rotation - RL case

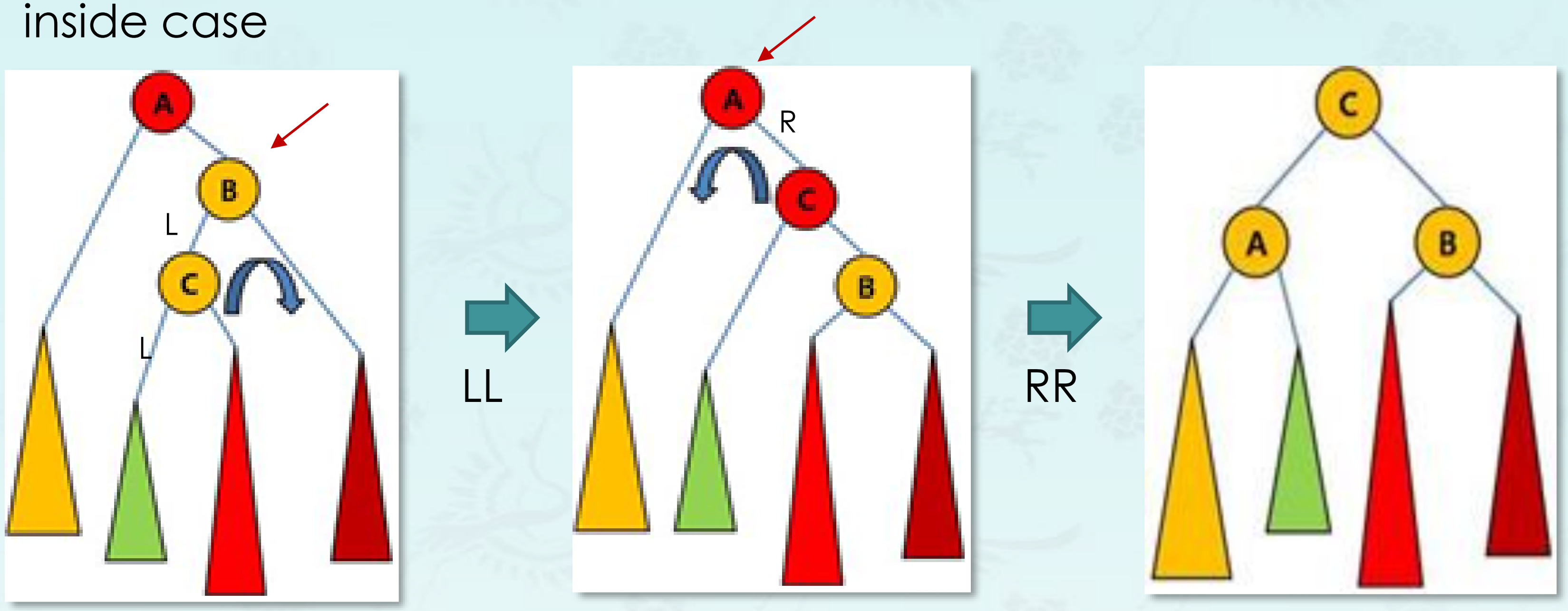
inside case



```
tree rotateRL(tree A) { // LL and RR
{


}
```

# Double Rotation - RL case

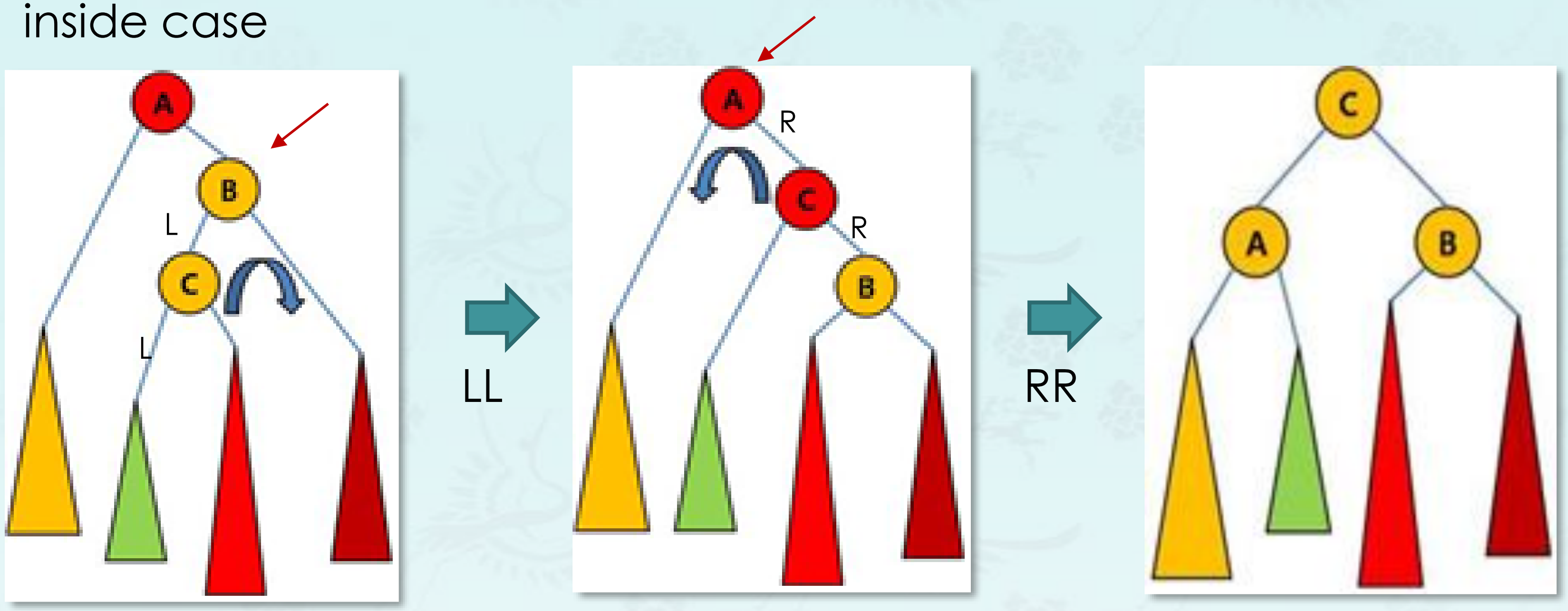
inside case



```
tree rotateRL(tree A) { // LL and RR
{
   tree B  = A->right;

}
```

# Double Rotation - RL case

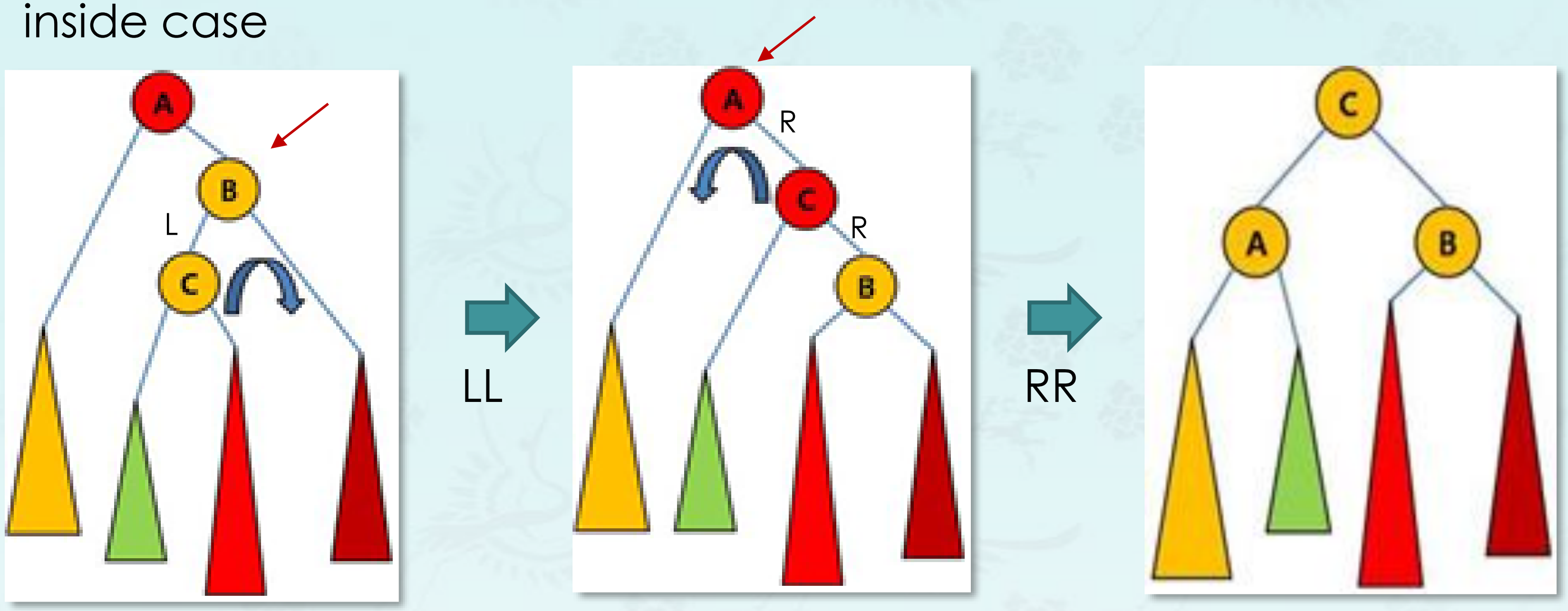
inside case



```
tree rotateRL(tree A) { // LL and RR
{
  tree B  = A->right;
  A->right = rotateLL(B);

}
```

# Double Rotation - RL case

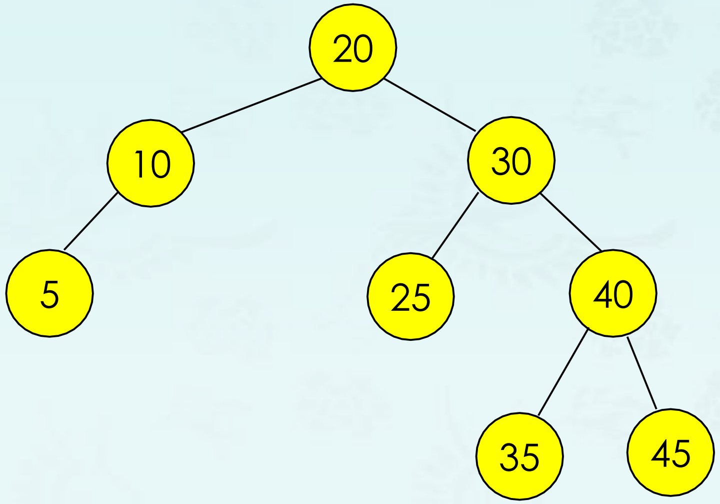
inside case



```
tree rotateRL(tree A) { // LL and RR
{
  tree B  = A->right;
  A->right = rotateLL(B);
  return rotateRR(A);
}
```

# Double Rotation -??case

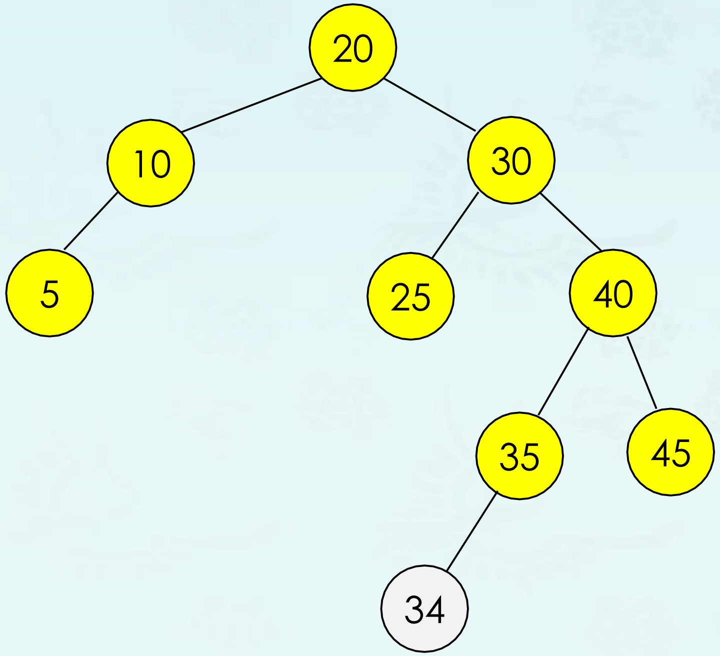
- **Insertion of 34**
- Imbalance at ?
- Balance factor ??
- Rotation ___??___ case



AVL balanced tree

# Double Rotation -??case

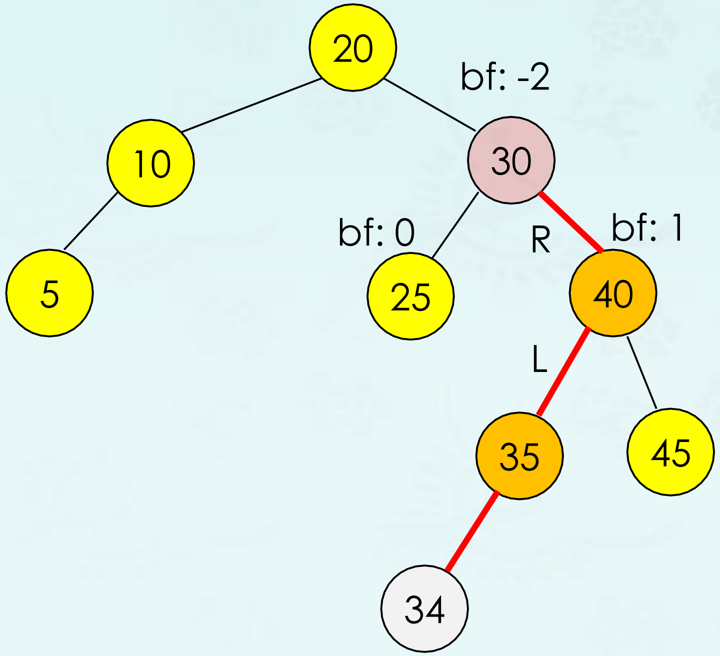
- Insertion of 34
- Imbalance at ?
- Balance factor ??
- Rotation ___??___ case



After insertion, AVL imbalanced tree

# Double Rotation -??case

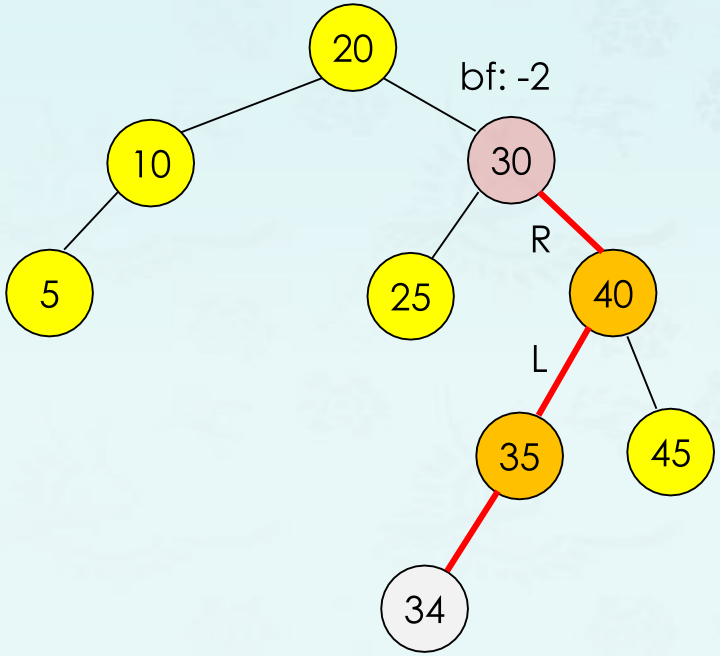
- Insertion of 34
- Imbalance at 30
- Balance factor **-2**
- Rotation ___??___ case



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

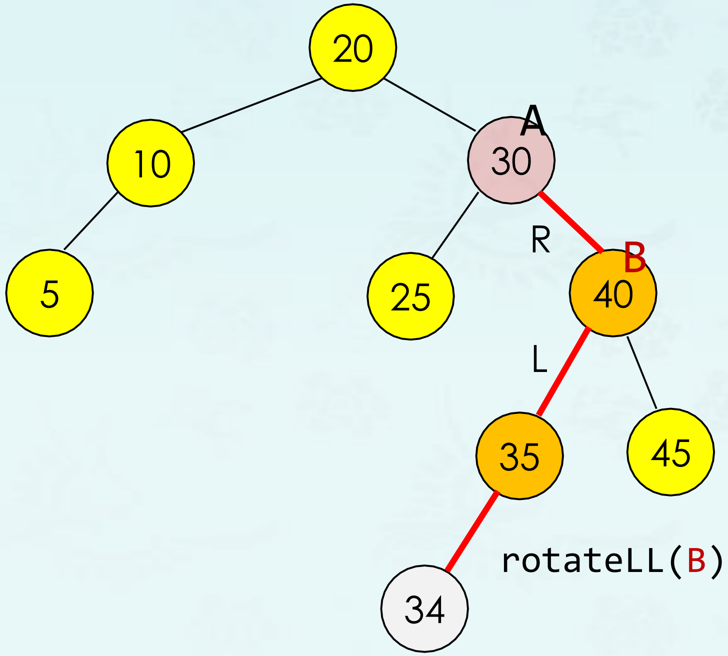
```
tree rotateRL(tree A) {
  tree B  = A->right;
  A->right = rotateLL(B);
  return rotateRR(A);
}
```



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

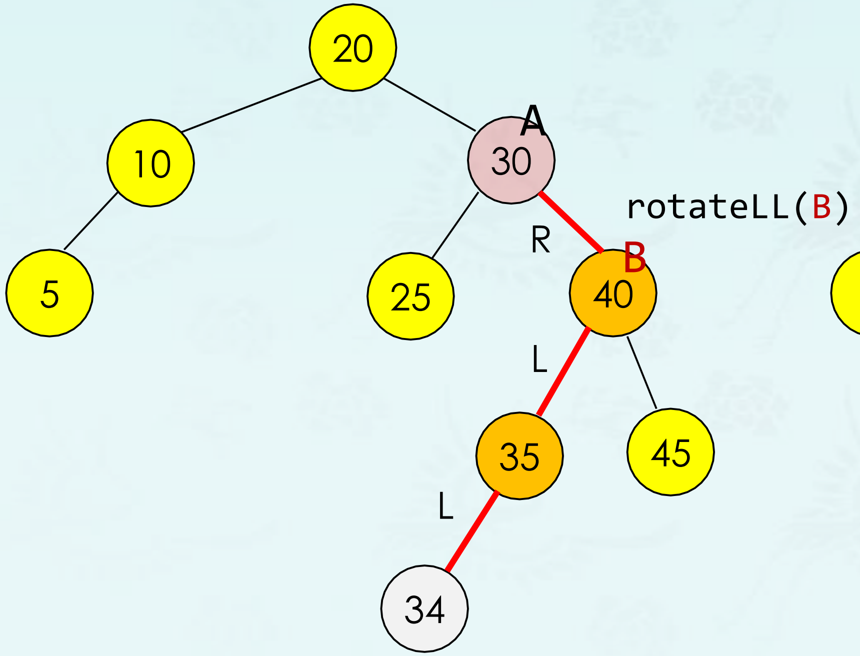
```
tree rotateRL(tree A) {
  tree B  = A->right;
  A->right = rotateLL(B);
  return rotateRR(A);
}
```

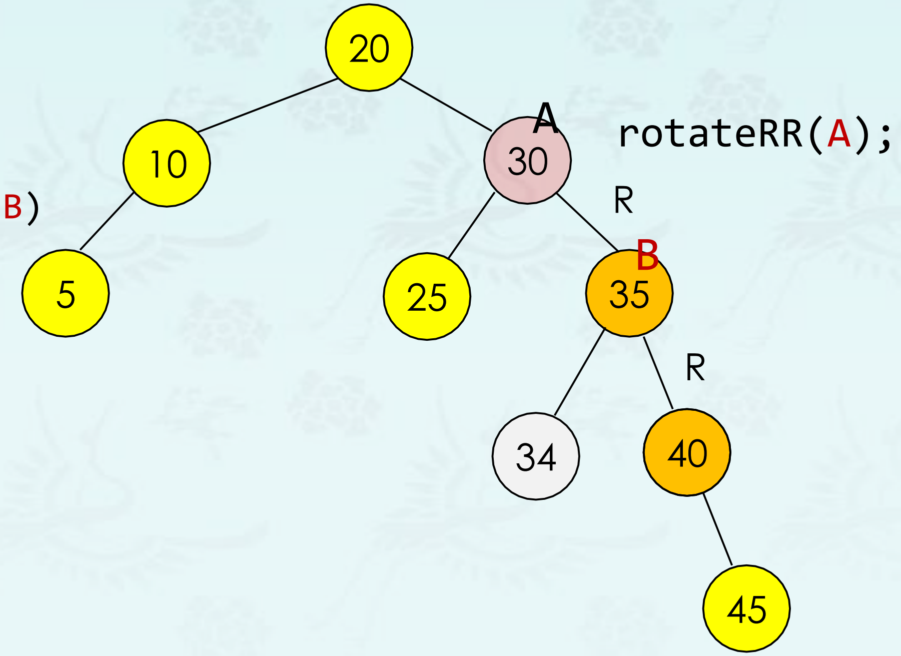


# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

```
tree rotateRL(tree A) {
  tree B  = A->right;
  A->right = rotateLL(B);
  return rotateRR(A);
}
```

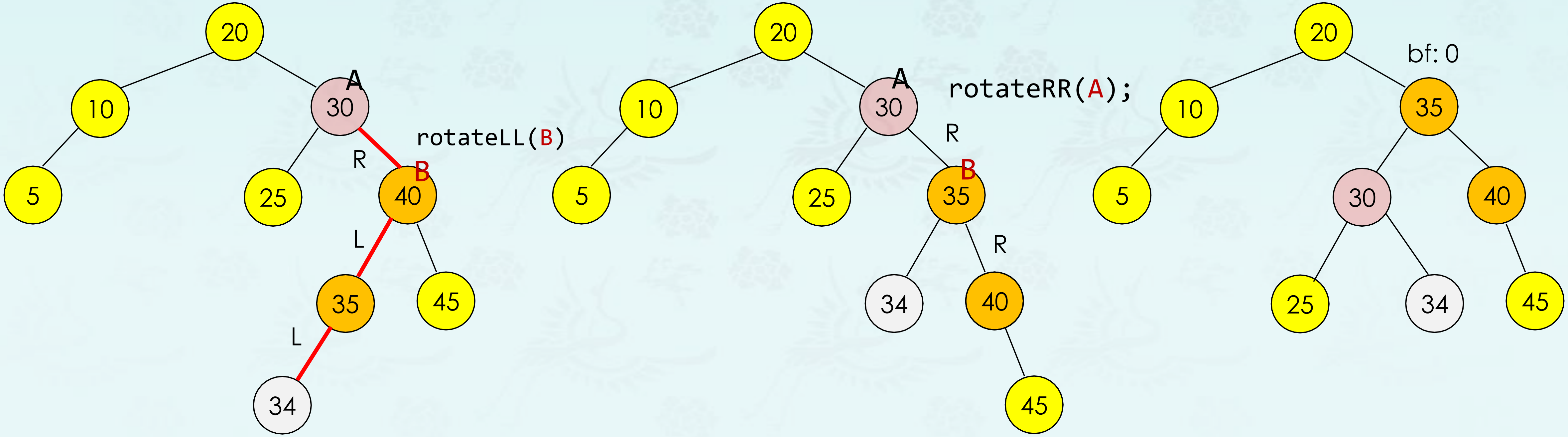




# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

```
tree rotateRL(tree A) {
  tree B  = A->right;
  A->right = rotateLL(B);
  return rotateRR(A);
}
```



After insertion, AVL imbalanced tree

After insertion, AVL balanced tree

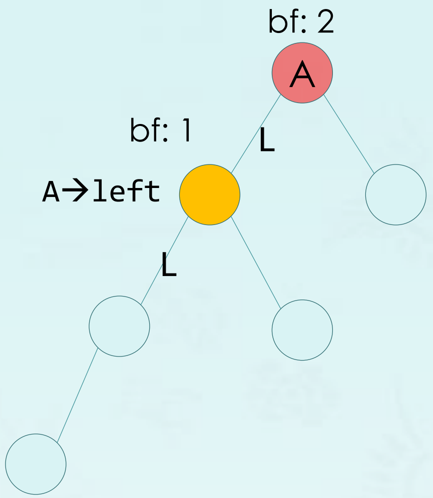
# Balance Factor and Height

```
int height(tree node) {
  if (empty(node)) return -1;
  int left = height(node->left);
  int right = height(node->right);
  return max(left, right) + 1;
}
```

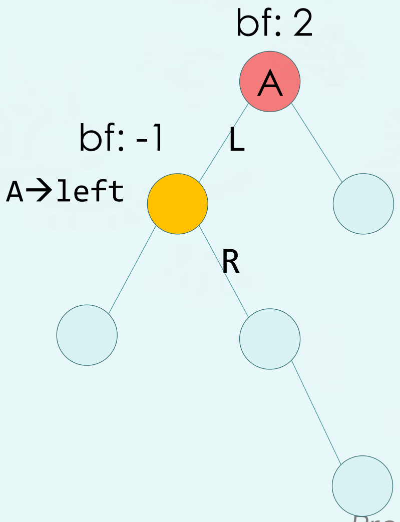
```
int balanceFactor(tree node) {
  if (node == NULL) return 0;
  int left  = height(node->left);
  int right = height(node->right);
  return left – right;
}
```

# Rebalance

## outside case



## inside case



```
tree rebalance(tree A) {
  int bf = balanceFactor(A);
  if (bf == 2) {
    if (balanceFactor(A->left) == -1)



  }
```

checking single or double rotation

inside case

outside case

**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.
**If the bf of A's child is 0, treat it like the same sign of A.**

# Rebalance

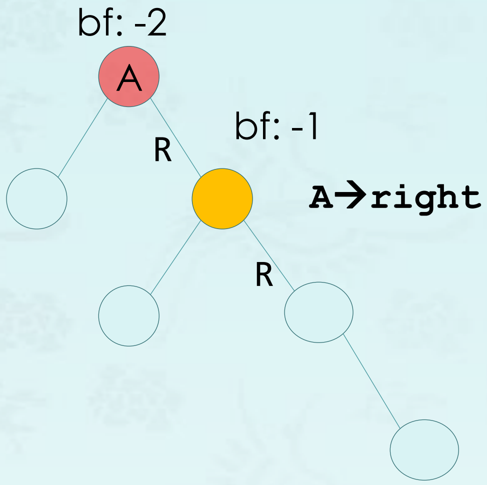
```
tree rebalance(tree A) {
  int bf = balanceFactor(A);
  if (bf == 2) {






  else if (bf == -2) {
    if (balanceFactor(A->right) == 1)




  }
  return A;
}
```

checking single or double rotation

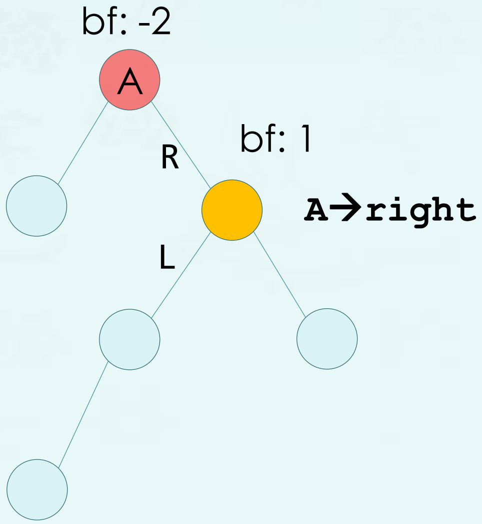
inside case

outside case

outside case



inside case

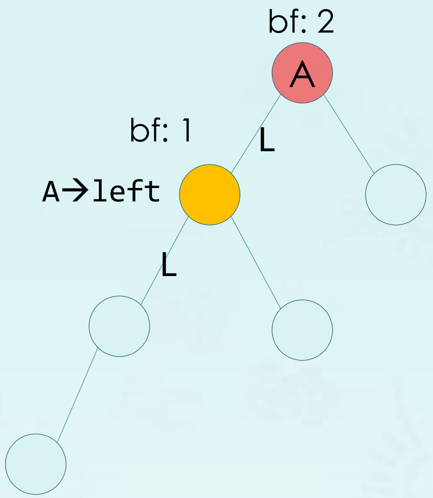


**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.

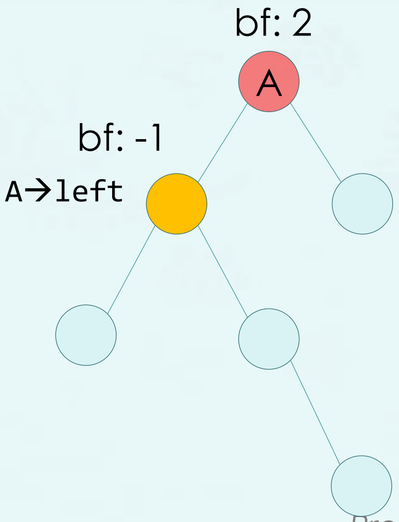
**If the bf of A's child is 0, treat it like the same sign of A.**

# Rebalance

outside case



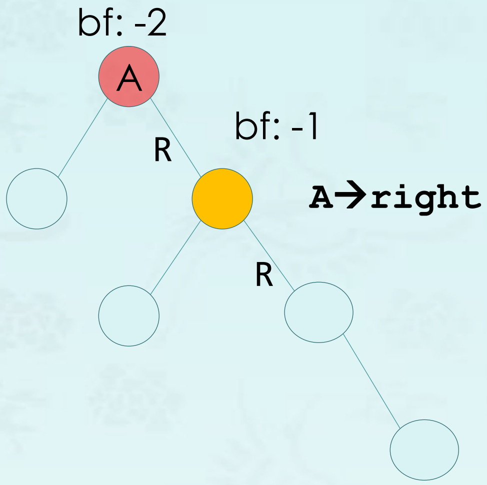
inside case



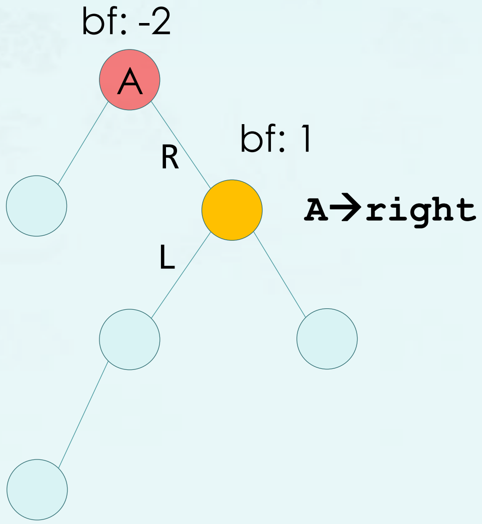
```
tree rebalance(tree A) {
  int bf = balanceFactor(A);
  if (bf == 2) {
    if (balanceFactor(A->left) == -1)
                                inside case

                                outside case
  }
  else if (bf == -2) {
    if (balanceFactor(A->right) == 1)
                                inside case

                                outside case
  }
  return A;  // no rebalanced needed
}
```

**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.
**If the bf of A's child is 0, treat it like the same sign of A.**

outside case



inside case



# growAVL() & trimAVL()

```
// inserts a key into the AVL tree and rebalance it.
tree growAVL(tree node, int key) {
  if (node == nullptr) return new TreeNode(key);

  // your code here

  return rebalance(node);      // O(log n)
}
```

almost same as grow()

AVL rotation if necessary

```
// deletes a key into the AVL tree and rebalance it.
tree trimAVL(tree node, int key) {
  if (node == nullptr) return new TreeNode(key);

  // your code here

  return rebalance(node);      // O(log n)
}
```

almost same as trim()

AVL rotation if necessary

## Data Structures
## Chapter 5 Tree

1. Introduction
2. Binary Tree
3. Binary Search Tree
4. **Balancing Tree**
   - AVL Tree Introduction
   - **AVL Operations**
   - AVL Coding