UNIVERSIDAD ESTATAL DE MILAGRO
UNEMI
Evolución Académica

SÍGUENOS

Formato para Recurso de Aprendizaje

TAREA

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UNIVERSIDAD ESTATAL DE MILAGRO U.N.E.M.I.

FACULTAD DE CIENCIAS A LA INGENIERÍA

PRIMER SEMESTRE DE INGENIERÍA DE SOFTWARE

ASIGNATURA:

-: CALCULO: -

AUTOR:

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TEMA:

APLICACION DE INTEGRALES

DOCENTE:

ING. ARISTIDES BASCARDI

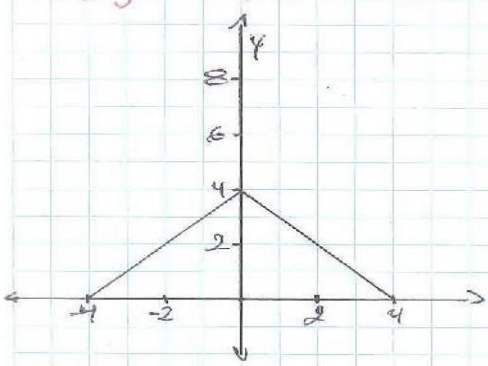
CURSO:

AULA B1

Fecha de entrega: 14/03/2021

1) Determina el área de la región que se indica

$f(x) = 4 - |x|$



$$\int_{-4}^{4} (4 - |x|)\, dx$$

$$\int 4\, dx - \int |x|\, dx = \left[4x - \frac{x|x|}{2} \right]_{-4}^{4}$$

$$\left[4(4) - \frac{4|4|}{2} \right] - \left[4(-4) - \frac{4|-4|}{2} \right]$$

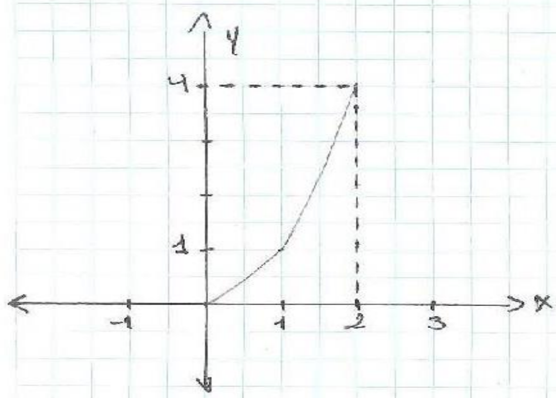
$$(16 - 8) - (-16 + 8)$$

$$8 + 8$$

$$16$$

R=// $A = 16\ u^2$

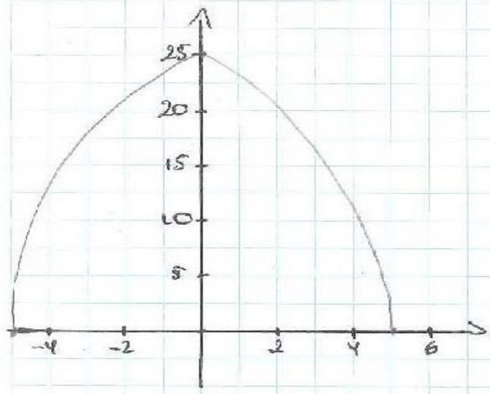
$f(x) = x^2$



$$\int_{0}^{2} x^2\, dx = \left[\frac{x^3}{3} \right]_{0}^{2} = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3} - 0 = \frac{8}{3}$$

R=// $A = \frac{8}{3}\ u^2$

$f(x) = 25 - x^2$



$$\int_{-5}^{5} (25 - x^2)\, dx$$

$$\int_{-5}^{5} 25\, dx - \int_{-5}^{5} x^2\, dx$$

$$\left[25x - \frac{x^3}{3} \right]_{-5}^{5}$$

$$\left(25(5) - \frac{5^3}{3} \right) - \left(25(-5) - \frac{(-5)^3}{3} \right)$$

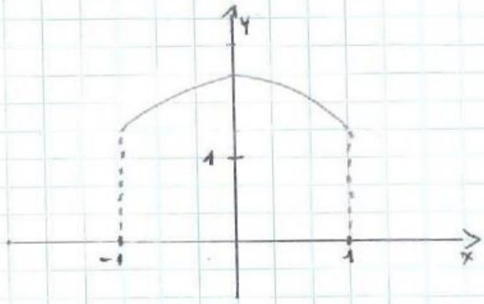
$$125 - \frac{125}{3} - \left(-125 + \frac{125}{3} \right)$$

$$125 - \frac{125}{3} + 125 - \frac{125}{3}$$

$$\frac{250}{1} - \frac{250}{3} = \frac{750 - 250}{3} = \frac{500}{3}$$

R=// $\frac{500}{3}\ u^2$

$$f(x) = \frac{4}{x^2+2}$$



$$\int_{-1}^{1} \frac{4}{x^2+2}\, dx$$

$$4\int_{-1}^{1} \frac{1}{x^2+2}\, dx$$

$$4\left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C\right]\Big|_{-1}^{1}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

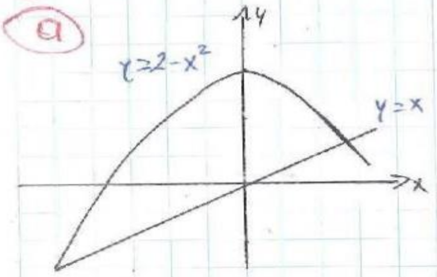
$$4\left[\frac{\sqrt{2}}{2} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) + \cancel{C} - \frac{\sqrt{2}}{2} \tan^{-1}\left(-\frac{1}{\sqrt{2}}\right) - \cancel{C}\right]$$

$$\overset{2}{\cancel{4}}\left\{\frac{\sqrt{2}}{\underset{1}{\cancel{2}}}\left[\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right]\right\}$$

$2\sqrt{2}\,(1.238959) = 3.48$ R// $A = 3.48\ u^2$

② Determinar El Área de la Región Acotada Por Los Sgt funciones

(a)



① $\begin{cases} y = 2 - x^2 \\ y = x \end{cases}$

② Hallamos las Intersecciones

$$2 - x^2 = x$$
$$0 = x^2 + x - 2$$
$$0 = (x+2)(x-1)$$

$x+2=0$, $x=-2$ $\qquad$ $x-1=0$, $x=1$

③ Definimos El elemento Diferencial (vertical)

④ La Integral definida Para El Área Será

$$A = \int_{-2}^{2}\left[(2-x^2)-x\right]dx = \int_{-2}^{1}(2-x^2-x)\,dx = \int 2\,dx - \int x^2\,dx - \int x\,dx$$

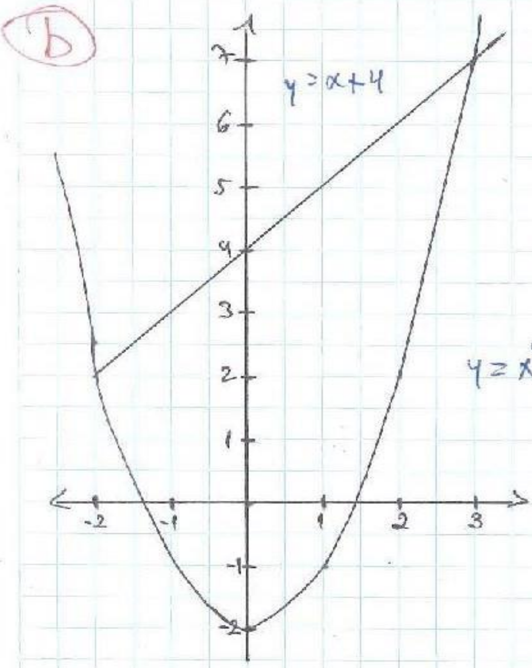
$$A = \left[2x - \frac{x^3}{3} - \frac{x^2}{2}\right]\Big|_{-2}^{1} = \left[2(1) - \frac{1^3}{3} - \frac{1^2}{2}\right] - \left[2(-2) - \frac{(-2)^3}{3} - \frac{(-2)^2}{2}\right]$$

$$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right)$$

R// $A = \frac{9}{2}\ u^2$

$$= \frac{7}{6} + \frac{10}{3} = \frac{7+20}{6} = \frac{27}{6} = \frac{9}{2}$$

(b)



(1) $\begin{cases} y = x+4 \\ y = x^2-2 \end{cases}$

(2) Hallamos las Intersecciones.

$x+4 = x^2-2.$

$0 = x^2 - x - 2 - 4$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x-3=0 \qquad x+2=0$

$x=3 \qquad x=-2$

(3) Definimos elementos diferenciales (Vertical)

(4) La Integral definida Para el Area Sería.

$$A = \int_{-2}^{3} \left[(x+4)-(x^2-2)\right]dx = \int_{-2}^{3}(x+4-x^2+2)\,dx = \int_{-2}^{3}(-x^2+x+6)\,dx$$

$$A = -\int_{-2}^{3} x^2 + \int_{-2}^{3} x + \int_{-2}^{3} 6 = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6\right]\Big/_{-2}^{3}$$

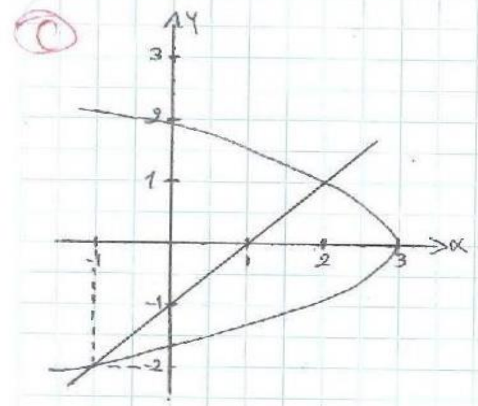
$$A = \left[-\frac{3^3}{3} + \frac{3^2}{2} + 6(3)\right] - \left[-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6(-2)\right]$$

$$A = \left(-9 + \frac{9}{2} + 18\right) - \left(+\frac{8}{3} + 2 - 12\right)$$

$$A = \frac{27}{2} + \frac{22}{3} = \frac{81+44}{6} = \frac{125}{6}$$

R:// $A = \frac{125}{6}\ u^2$

C



① $\begin{cases} x = 3 - y^2 \\ x = y + 1 \end{cases}$

② Hallamos las Intersecciones

$3 - y^2 = y + 1$

$0 = y + 1 + y^2 - 3$

$0 = y^2 + y - 2$

$0 = (y+2)(y-1)$

$y + 2 = 0 \qquad y - 1 = 0$

$y = -2 \qquad y = 1$

③ Definimos Elemento diferencial (Horizontal)

④ La Integral Definida Para El Area Seria:

$$A = \int_{-2}^{1} (3 - y^2) - (y+1)\, dy = \int_{-2}^{1} (3 - y^2 - y - 1)\, dy = \int_{-2}^{1} (-y^2 - y + 2)\, dy$$

$$A = -\int_{-2}^{1} y^2 dx - \int y\, dx + \int 2\, dy = \left[-\frac{y^3}{3} - \frac{y^2}{2} + 2y \right]_{-2}^{1}$$

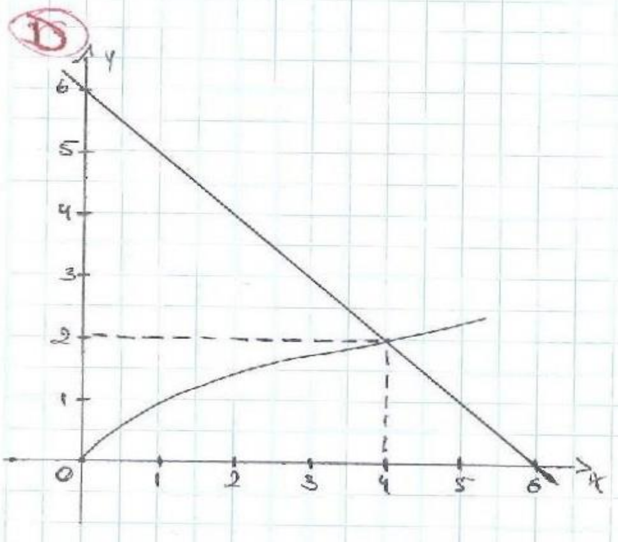
$$A = \left[-\frac{1^3}{3} - \frac{1^2}{2} + 2(1) \right] - \left[-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 2(-2) \right]$$

$$A = \left(-\frac{1}{3} - \frac{1}{2} + 2 \right) - \left(\frac{8}{3} - 2 - 4 \right)$$

$$A = \frac{7}{6} + \frac{10}{3} = \frac{7+20}{6} = \frac{\cancel{27}^{9}}{\cancel{6}_{2}} = \frac{9}{2}$$

R=// $A = \frac{9}{2}\ u^2$

D



① $\begin{cases} x = 6 - y \\ x = y^2 \end{cases}$

② Hallamos Los Intersecciones

$6 - y = y^2$

$0 = y^2 + y - 6$

$0 = (y+3)(y-2)$

$y + 3 = 0 \qquad y - 2 = 0$

$y = -3 \qquad y = 2$

③ Definimos El Elemento diferencial (Horizontal)

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④ la Integral Definida Para El Area Seria

$$A = \int_0^2 [(6-y)-y^2]\,dy = \int_0^2 (6-y-y^2)\,dx = \int_0^2 6\,dy - \int_0^2 y\,dy - \int_0^2 y^2\,dy$$

$$A = \left[6y - \frac{y^2}{2} - \frac{y^3}{3}\right]\Big/_0^2$$

$$A = \left[6(2) - \frac{2^2}{2} - \frac{2^3}{3}\right] - \left[6(0) - \frac{0^2}{2} - \frac{0^3}{3}\right]$$

$$A = \left(12 - 2 - \frac{8}{3}\right) - 0$$

R= // $A = \dfrac{22}{3}\ u^2$

③ En los Sgnts Ejercicios Determine la longitud de la Curva que Se indica

ⓐ $y = 4x^{3/2}$ Entre $x = 1/3$ y $x = 5$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_{1/3}^5 \sqrt{1 + (6x^{1/2})^2}\,dx$$

$$L = \int_{1/3}^5 \sqrt{1 + 36x}\,dx$$

$$L = \int_{1/3}^5 (1+36x)^{1/2}\,dx = \left[\frac{1}{54}(1+36x)^{3/2}\right]\Big/_{1/3}^5$$

$$L = \left[\frac{1}{54}(181)^{3/2}\right] - \left[\frac{1}{5}(13)^{3/2}\right]$$

$$\frac{1}{54}\left(181^{3/2} - 13^{3/2}\right) = 44.23$$

R= // 44.23 u

$$b)\quad y=\frac{2}{3}(x^2+1)^{3/2} \text{ Entre } x=1 \text{ y } x=2$$

$$y=\frac{2}{3}(x^2+1)^{3/2}$$

$$dy=\frac{2}{3}\left[\frac{3}{2}(x^2+1)^{1/2}(2x)\right]$$

$$dy=(x^2+1)^{1/2}(2x)$$

$$L=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_1^2\sqrt{1+(2x)^2\left(\sqrt{x^2+1}\right)^2}\,dx$$

$$L=\int_1^2\sqrt{1+4x^2(x^2+1)}\,dx$$

$$L=\int_1^2\sqrt{1+4x^4+4x^2}\,dx$$

$$L=\int_1^2(4x^4+4x^2+1)^{1/2}\,dx$$

$$L=\int_1^2\left[(2x^2+1)^2\right]^{\frac{1}{2}}dx$$

$$L=\int_1^2(2x^2+1)\,dx$$

$$L=\int_1^2 2x^2\,dx+\int_1^2 1\,dx$$

$$L=\left[\frac{2x^3}{3}+\right]\Big|_1^2$$

$$L=\left[\frac{2(2)^3}{3}+2\right]-\left[\frac{2(1)^3}{3}+1\right]$$

$$L=\left[\frac{16}{3}+2\right]-\left[\frac{2}{3}+1\right]$$

$$L=\frac{22}{3}-\frac{5}{3}$$

$$L=\frac{17}{3}$$

$$L=5{,}67\quad R//$$

(C) $y=(4-x^{2/3})^{3/2}$ Entre $x=1$ y $x=8$

$$L=\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_1^8 \sqrt{1+\left(-\frac{\sqrt{4-x^{2/3}}}{\sqrt[3]{x}}\right)^2}\,dx$$

$$L=\int_1^8 \sqrt{1+\frac{4-x^{2/3}}{x^{2/3}}}\,dx$$

$$L=\int_1^8 \sqrt{\cancel{1}+\frac{4}{x^{2/3}}-\cancel{1}}\,dx$$

$$L=\int_1^8 \sqrt{\frac{4}{x^{2/3}}}\,dx=\int_1^8 \frac{2}{\sqrt[3]{x}}\,dx=2\int_1^8 x^{-1/3}\,dx$$

$$L=2\left[\frac{3}{2}x^{2/3}\right]\Big|_1^8$$

$$L=2\left[\frac{3}{2}\left(\sqrt[3]{8}\right)^2-\frac{3}{2}\left(\sqrt[3]{1}\right)^2\right]$$

$$L=2\left[\frac{3}{\cancel{2}}(4)^2-\frac{3}{2}(1)\right]$$

$$L=2\left(6-\frac{3}{2}\right)$$

$$L=\cancel{2}\left(\frac{9}{\cancel{2}}\right)=9$$

R=// 9 u

(4) RESOLVER

Utilice una integracion en y para encontrar la longitud del segmento de recta $2y-2x+3=0$, entre $y=1$ y $y=3$. Verifique por medio de la formula de distancia

$$\begin{cases} 2y-2x+3=0 \\ x=\frac{-2y-3}{-2} \\ x=y+\frac{3}{2} \end{cases}$$ entre $y=1$ y $y=3$

$$\begin{cases} f(y)=y+\frac{3}{2} \\ f(y)=1 \end{cases} \qquad L=\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dx$$

$$L=\int_1^3 \sqrt{1+(1)^2}\,dx$$

$L = \int_1^3 \sqrt{2}\, dy = \sqrt{2} \int_1^3 dy = \left[\sqrt{2}y\right]\Big/_1^3$

$L = \sqrt{2}(3) - \sqrt{2}(1)$

$L = 3\sqrt{2} - \sqrt{2}$

$L = 2\sqrt{2}$ $\quad$ R=// $2\sqrt{2}$ M

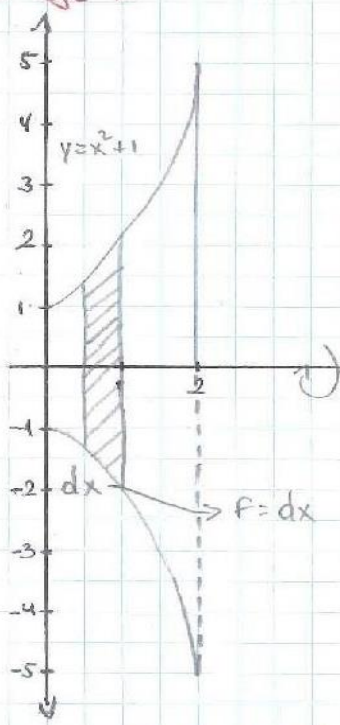
Comparación con la formula de la distancia, como ya tengo las coordenadas $y_1 = 1$; $y_2 = 3$ busco x_1 y x_2 $\quad x_1 = 1+3-5$

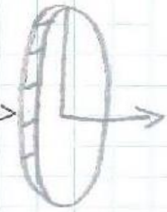
$x_2 = 3 + \frac{3}{2} = \frac{9}{2}$

(3) Determinar el volumen de la región definida en cada caso y aplicar el método más adecuado.

En los problemas del 1 al 4 encuentre el volumen del sólido generado cuando la región que se indica se hace girar alrededor del eje especificado; rebane, aproxime, integre.

(1) Eje X





$R = y = x^2+1$

dv (Volumen del cilindro)

$dv = \pi R^2 h$

$dv = \pi (x^2+1)^2 dx$

$dv = \pi (x^4 + 2x^2 + 1) dx$

$dv = \pi \int_0^3 (x^4 + 2x^2 + 1) dx$

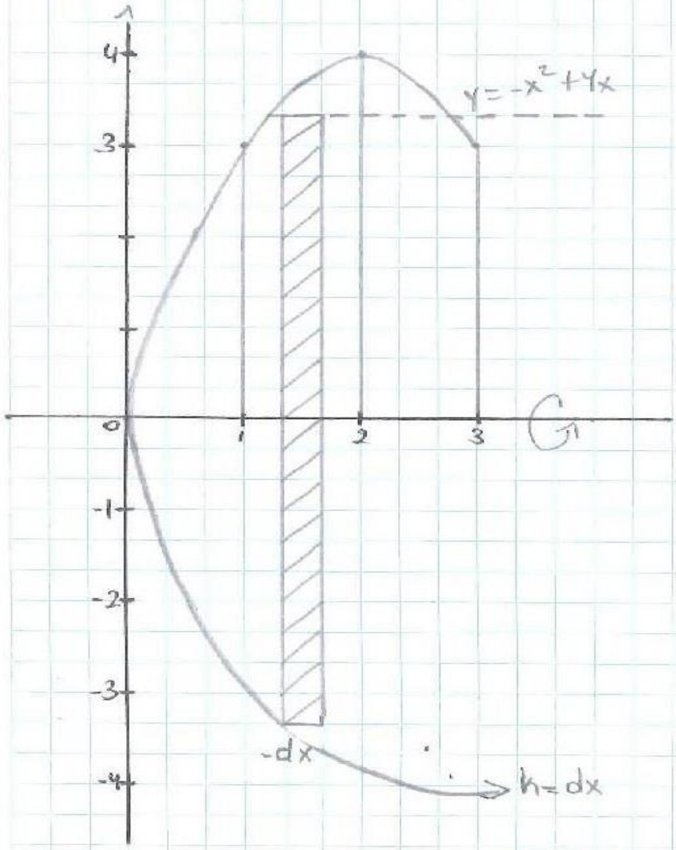
$v = \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]\Big/_0^2$

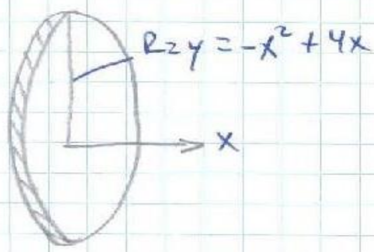
$V = \pi \left[\left(\frac{2^5}{5} + \frac{2(2^3)}{3} + 2\right) - \left(\frac{0^5}{5} + \frac{2(0)}{3} + 0\right)\right]$

$V = \pi \left(\frac{32}{5} + \frac{16}{3} + 2\right)$

$V = \pi \left(\frac{96 + 80 + 30}{15}\right)$ $\quad V = \pi \left(\frac{206}{15}\right) \Rightarrow$ $V = \frac{206}{15}\pi M^3$ R//

② Eje x





$dv = $ (volumen del disco)

$dv = \pi R^2 h$

$dv = \pi(-x^2+4x)^2\,dx$

$dv = \pi(x^4 - 8x^3 + 16x^2)\,dx$

$\int dv = \pi \int_0^3 (x^4 - 8x^3 + 16x^2)\,dx$

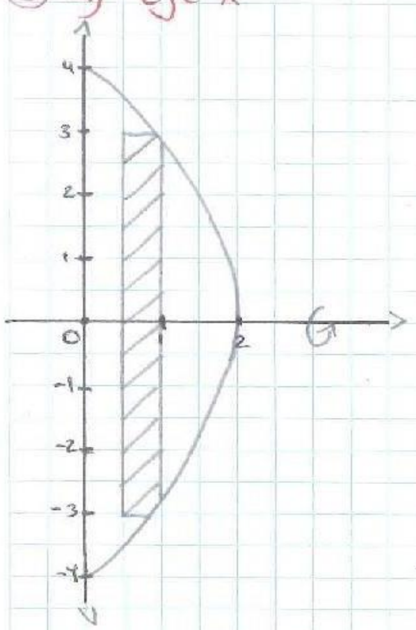
$V = \pi \left[\frac{x^5}{5} - 2x^4 + 16x^3\right]\Big|_0^3$

$V = \pi \left[\left(\frac{3^5}{5} - 2(3^4) + \frac{16(3^3)}{3}\right) - \left(\frac{0^5}{5} - 2(0^4) + \frac{16(0^3)}{3}\right)^0\right]$

$V = \pi \left(\frac{243}{5} - 162 + 144\right)$

$V = \pi \left(\frac{153}{5}\right) \Rightarrow$ R// $V = \frac{153}{5}\,u^3$

③ a) Eje X



dv (volumen del disco)

$dv = \pi R^2 h$

$dv = \pi (4 - x^2)^2 dx$

$dv = \pi (16 - 8x^2 + x^4) dx$

$\int dv = \pi \int_0^2 (16 - 8x^2 + x^4) dx$

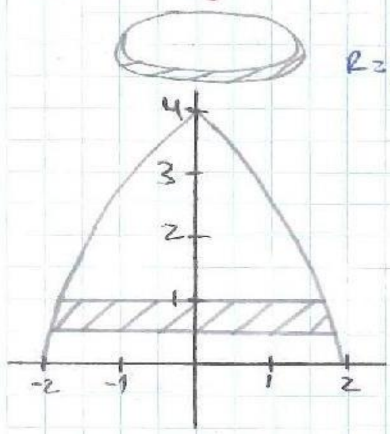
$v = \pi \left(16x - \frac{8x^2}{3} + \frac{x}{5}\right)\Big|_0^2$

$V = \pi \left(16(2) - \frac{8(2)^2}{3} + \frac{2^5}{5}\right) - \left(16(0) - \frac{8(0)}{3} +\right.$

$V = \pi \left(36 - \frac{64}{3} + \frac{32}{5}\right)$

$V = \pi \left(\frac{256}{15}\right) \Rightarrow V = \frac{256}{15} \pi u^3$ R//

③ b) Eje Y



$R = y = \sqrt{4-y}$

dv (volumen del disco)

$dv = \pi R^2 h$

$dv = \pi (\sqrt{4-y})^{\cancel{2}} dy$

$dv = \pi (4-y) dy$

$\int dv = \pi \int_0^4 (4-y) dy$

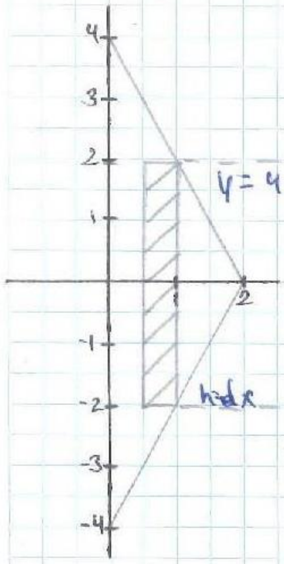
$v = \pi \left[4y - \frac{y^2}{2}\right]\Big|_0^4$

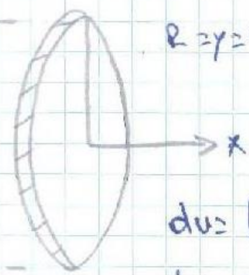
$v = \pi \left[\left(4(4) - \frac{4^2}{2}\right) - \left(4(0) - \frac{0^2}{2}\right)\right]^0$

$v = \pi (16 - 8)$

$v = \pi (8) \Rightarrow V = 8\pi u^3$ R//

(4) a) Eje x





$R = y = 4-2x$

$dv =$ (volumen del disco)

$dv = \pi R^2 h$

$dv = \pi (4-2x)^2 dx$

$\int dv = \pi \int_0^2 (4-2x)^2 dx$

$v = \pi (16 - 16x + 4x^2) dx$

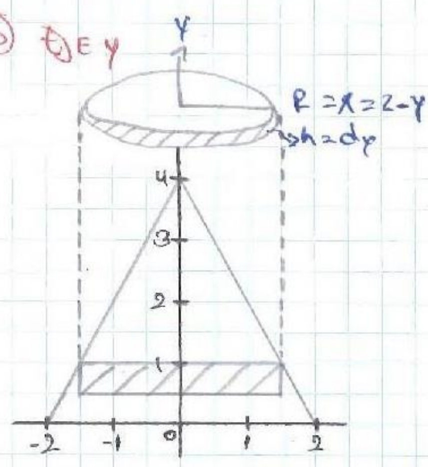
$$v = \pi \left[16x - 8x^2 + \frac{4x^3}{3}\right]\Big|_0^2$$

$$v = \pi \left[\left(16(2) - 8(2)^2 + \frac{4(2^3)}{3}\right) - \left(16(0) - 8(0^2) + \frac{4(0)^3}{3}\right)\right]$$

$$v = \pi \left[32 - 32 + \frac{32}{3}\right]$$

$$v = \pi \left(\frac{32}{3}\right) \Rightarrow \quad R// \quad v = \frac{32}{3}\pi$$

(b) Eje y



$dv =$ (volumen del Cilindro)

$dv = \pi R^2 h$

$dv = \pi \left(2 - \frac{y}{2}\right)^2 dy$

$\int dv = \pi \left(4 - 2y + \frac{y^2}{4}\right) dy$

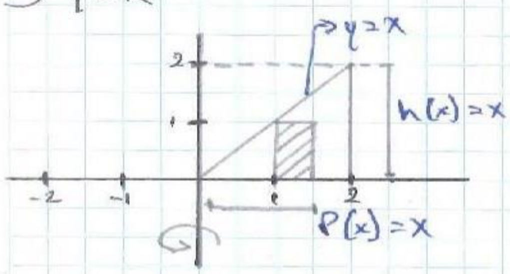
$$v = \pi \left[4y - y^2 + \frac{y^3}{12}\right]\Big|_0^4$$

$$v = \pi \left\{\left(4(4) - 4^2 + \frac{4^3}{12}\right) - \left(4(0) - 0^2 + \frac{0^3}{12}\right)^0\right\}$$

$$v = \pi \left(\cancel{16} - \cancel{16} + \frac{\cancel{64}^{16}}{\cancel{12}_3}\right) \Rightarrow \quad R// \quad \frac{16}{3}\pi u^3$$

(5) (2) 1) En los Ejercicios Usar El Metodo de las capas Para formular y Evaluar la Integral Que Da El Volumen del Solido Generado Al Girar la Región Plana Alrededor del Eje y

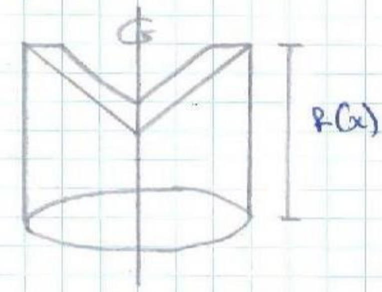
1) $y = x$



Eje de Rotacion: Eje y $(x=0)$

Rectangulo tipico: Vertical

Bosquejo del Solido



$$v = 2\pi \int_a^b p(x)\, h(x)\, dx$$

$$v = 2\pi \int_0^2 x(x)\, dx$$

$$v = 2\pi \int_0^2 x^2\, dx$$

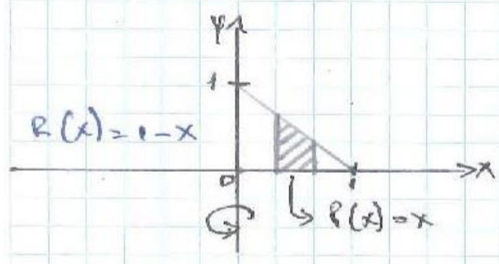
$$v = 2\pi \left[\frac{x^3}{3}\right]_0^2$$

$$v = 2\pi \left(\frac{2}{3} - \cancel{\frac{0^3}{3}}\right)^0$$

$$v = 2\pi \left(\frac{8}{3}\right) \Rightarrow \quad R/\!/ \; v = \frac{16}{3}\pi u^3$$

Metodo a Utilizar: Capas Cilindricas

② $y = 1 - x$



$$V = 2\pi \int_a^b P(x)\, h(x)\, dx$$

$$V = 2\pi \int_0^1 x\,(1-x)\, dx$$

$$V = 2\pi \int_0^1 x - x^2\, dx$$

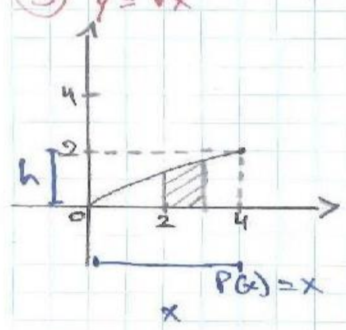
$$V = 2\pi \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1$$

$$V = 2\pi \left\{\left(\frac{1^2}{2} - \frac{1^3}{3}\right) - \cancel{\left(\frac{0^2}{2} - \frac{0^3}{3}\right)}^{0}\right\}$$

$$V = 2\pi \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$V = \cancel{2}\pi \left(\frac{1}{\cancel{6}_3}\right) \Rightarrow R//\ V = \frac{1}{3}\pi u^3$$

③ $y = \sqrt{x}$



$$V = 2\pi \int_0^4 x\,(\sqrt{x})\, dx$$

$$V = 2\pi \int_0^4 x^{3/2}\, dx$$

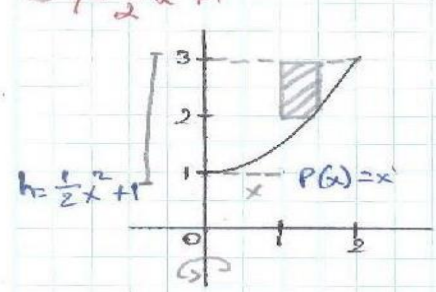
$$V = 2\pi \left[\frac{2x^{3/2}}{3}\right]\Big|_0^4$$

$$V = 2\pi \left[\frac{2}{5}\left(\sqrt{4}\right)^5 - \cancel{\frac{2}{5}\left(\sqrt{0}\right)^5}\right]$$

$$V = 2\pi \left(\frac{2}{5}(32)\right)$$

$$V = 2\pi \left(\frac{64}{5}\right) = R//\ V = \frac{128}{5}\pi u^3$$

④ $y = \frac{1}{2}x^2 + 1$



$$V = 2\pi \int_a^b P(x)\, h(x)\, dx$$

$$V = 2\pi \int_0^2 x\left(\frac{1}{2}x^2 + 1\right) dx$$

$$V = 2\pi \int_0^2 \left(\frac{1}{2}x^3 + x\right) dx$$

$$V = 2\pi \left[\frac{x^4}{8} + \frac{x^2}{2}\right]\Big|_0^2$$

$$V = \pi \left[\frac{x^4}{4} + x^2\right]\Big|_0^2$$

$$V = \pi \left[\left(\frac{2^4}{4} + 2^2\right) - \cancel{\left(\frac{0^4}{4} + 0^2\right)}^{0}\right]$$

$$V = \pi\,(4 + 4)$$

$R = //\ V = 8\pi u^3$