UNIVERSIDAD ESTATAL DE MILAGRO
UNEMI
Evolución Académica

SÍGUENOS

Formato para Recurso de Aprendizaje

TAREA

UNIVERSIDAD ESTATAL DE MILAGRO U.N.E.M.I.

FACULTAD DE CIENCIAS A LA INGENIERÍA

PRIMER SEMESTRE DE INGENIERÍA DE SOFTAWRE

ASIGNATURA:

Calculo

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CURSO:

AULA B1

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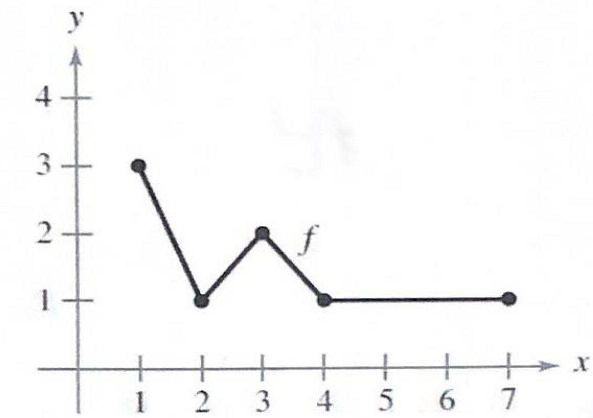
EJERCICIOS A DESARROLLAR

1.- Resolver los siguientes ejercicios relacionados al teorema fundamental del cálculo

a) La gráfica de *f(x)* se muestra a continuación



AB ; BC ; CD ; DE,

Calcular $\int_1^7 f(x)\,dx$

Ecuación AB

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$y - 3 = \left(\frac{1-3}{2-1}\right)(x-1)$

$y = -2(x-1)+3$

$y = -2x + 2 + 3$

$y = -2x + 5 \; ; \; [1,2]$

Ecuación BC

$y = -1\left(\frac{2-1}{3-2}\right)(x-2)$

$y = 1(x-2)+1$

$y = x - 2 + 1$

$y = x - 1 \; ; \; [2,3]$

Ecuación CD

$y - 2\left(\frac{1-2}{4-3}\right)(x-3)$

$y = -1(x-3)+2$

$y = -x + 3 + 2$

$y = -x + 5 \; ; \; [3,4]$

Ecuación DE

$y = 1 \; ; \; [4,7]$

Es constante

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2) Determinar el valor medio f(x) en el intervalo de x = 1 a x = 7

$$f(x)\begin{cases}5-2x & 1\le x\le 2\\ -1+x & 2\le x\le 3\\ 5-x & 3\le x\le 4\\ 1 & 4\le x\le 7\end{cases}$$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$[1, 3] \quad [2, 1]$

Pendiente

$m = \frac{1-3}{2-1} = -2$

Punto Pendiente

$y = 3+(-1)(x-1)$

$y = 5-2x$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$[2, 1] \quad [3, 2]$

$m = \frac{2-1}{3-2} = 1$

$y = 1+1(x-2)$

$y = 1+x-2$

$y = -1+x$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$[3, 2] \quad [4, 1]$

$m = \frac{1-2}{4-3} = -1$

$y = 2+(-1)(x-3)$

$y = 2-x+3$

$y = 5-x$

Intervalo

1) $[1,2]$; $f(x) = 5-2x\,dx$

$$\int_1^2 5-2x\,dx$$

$$5x-2\int x\,dx$$

$$[5x-x^2]_1^2$$

$$(5(2)-(2)^2)-(5(1)-(1)^2)$$

$$(10-4)-(5-1)$$

$$10-4-5+1 = \boxed{2}$$

2) $[2,3]$ $f(x) = -1+x$

$$\int_2^3 -1+x\,dx$$

$$\int -1\,dx + \int x\,dx$$

$$-x+\frac{x^2}{2} = \left[-x+\frac{x^2}{2}\right]_2^3$$

$$\left(-(3)+\left(\frac{3^2}{2}\right)\right)-\left(2+\frac{(2)^2}{2}\right)$$

$$\left(3+\frac{9}{2}\right)-\left(-2+\frac{4}{2}\right)$$

$$-3+\frac{9}{2}-(-2+2)$$

$$-3+\frac{9}{2} = \frac{3}{2} = \boxed{1,5}$$

3) $[3, 4]$, $f(x) = 5 - x$

$$\int_3^4 5 - x\,dx$$

$$\int 5dx - \int x dx$$

$$\left[5x - \frac{x^2}{2}\right]_3^4$$

$$\left(5(4) - \frac{(4)^2}{2}\right) - \left(5(3) - \frac{(3)^2}{2}\right)$$

$$\left(20 - \frac{16}{2}\right) - \left(15 - \frac{9}{2}\right)$$

$$(20 - 8) - \left(\frac{21}{2}\right)$$

$$12 - \frac{21}{2} = \frac{3}{2} = \boxed{1,5}$$

4) $[4, 7]$; $f(x) = 1$

$$\int_4^7 x\,dx$$

$$[x]_4^7$$

$$((7)) - ((4))$$

$$7 - 4 = \boxed{3}$$

Suma de integrales

$$\int_1^7 f(x) = 8$$

$$\frac{1}{7-1}$$

$$\frac{1}{6}\,8 \rightarrow \frac{1}{3}\cdot 4 \qquad \frac{4}{3} = \boxed{1,33}$$

Valor medio

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

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b) Determinar el valor medio de la siguiente función

$$f(x) = 9 - x^2 \quad , \quad [-3,\ 3]$$

$f(x) = 9 - x^2 \quad , \quad [-3, 3]$

$\frac{1}{b-a} \int_a^b f(x)\,dx$

$\boxed{a = -3}$ $\boxed{b = 3}$

- Sustituyo

$\frac{1}{3-(-3)} \int_{3}^{3} (9 - x^2)\,dx = \left(\frac{1}{3+3}\right)\left[\left(9x - \frac{x^3}{3}\right)\right]_3^3$

$\frac{1}{6}\left[\left(9x - \frac{x^3}{3}\right|_3^3\right] = \frac{1}{6}\left[9(3) - \frac{3^3}{3}\right) - \left(9(-3) - \frac{(3)^3}{3}\right)\right]$

$\frac{1}{6}(27 - 9 + 27 - 9)$

$\frac{1}{\cancel{6}}\left(\overset{6}{\cancel{36}}\right) = \boxed{6}$ Valor medio

2.- En los siguientes ejercicios resolver las integrales (aplique método de sustitución)

a) $\int x\sqrt{25 - x^2}\,dx$

$\int x \quad 25 - x^2\,dx$

$u = 25 - x^2 \quad = \int -\frac{\sqrt{u}}{2}\,du$

$\int a \cdot f(x)\,dx = a \cdot \int f(x)\,dx =$

$-\frac{1}{2} \cdot \int \sqrt{u}\,du$

$\sqrt{a} = a\frac{1}{2} \quad = -\frac{1}{2} \int u^{\frac{1}{2}}\,du$

$$\int x^a\,dx = \frac{x^{a+1}}{a+1}, \quad a \neq -1$$

$$-\frac{1}{2} \cdot \frac{u^{1/2+1}}{1/2+1}$$

$$u = 25 - x^2 \qquad = -\frac{1}{2}\,\frac{(25-x^2)^{1/2+1}}{\frac{1}{2}+1}$$

$$-\frac{1}{2}\,\frac{(25-x^2)^{1/2+1}}{\frac{1}{2}+1} \qquad = -\frac{1}{3}\,(25-x^2)^{3/2}$$

$$\boxed{= -\frac{1}{3}\,(25-x^2)^{3/2} + C}$$

b) $\int \frac{x^2}{(1+x^3)^2}\,dx$

$$u = 1 + x^3 \qquad = \int \frac{1}{3u^2}\,du$$

$$\frac{1}{3} \cdot \int \frac{1}{u^2}\,du \quad \rightarrow \quad \frac{1}{u^2} = u^{-2}$$

$$= \frac{1}{3} \cdot \frac{u^{-2+1}}{-2+1}$$

$$= \frac{1}{3}\,\frac{1+x^{3\;-2+1}}{-2+1}$$

$$= -\frac{1}{3(1+x^3)} \qquad = \boxed{-\frac{1}{3(1+x^3)} + C}$$

c) $\int \sqrt{\tan x}\ \sec^2 x\, dx$

$\sqrt{u} = u^{1/2}$

$u = \tan x$

$du = \sec^2 x$

$\int u^{1/2} \cdot du = \frac{u^{1/2+1}}{3/2} = \frac{2}{3} u \sqrt{u}$

$$\boxed{= \frac{2}{3} \tan x \sqrt{\tan x} + C}$$

3) En los siguientes ejercicios resolver las integrales (aplicando integración por partes)

a) $\int x^2 \cos x\, dx$

$u = x^2 \qquad v = \cos(x)$

$x^2 \sin(x) - \int 2x \sin(x)\, dx =$

$\int 2x \sin(x)\, dx = 2(-x\cos(x) + \sin(x))$

$= x^2 \sin(x) - 2(x\cos(x) + \sin(x))$

$\int x^2 \cos x\, dx = x^2 \sin(x) - 2(-x\cos(x) + \sin(x)) + C$ ✓

b) $\int e^{2x} \sin x\, dx$

$u = e^{2x} \qquad v' = \sin(x)$

$-e^{2x}\cos(x) - \int -2e^{2x} \cos(x)\, dx$

$-e^{2x}\cos(x) - (-2) \cdot \int e^{2x} \cos(x)\, dx =$

$-e^{2x}\cos(x) - (-2(e^{2x}\sin(x)) - \int e^{2x} \cdot 2\sin(x)\, dx$

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$-e^{2x}\cos(x) - (-2(e^{2x}\sin(x) - 2 \cdot \int e^{2x}\sin(x)dx))$

$\int e^{2x}\sin(x)\,dx =$

$-\frac{e^{2x}\cos(x)}{5} + \frac{2e^{2x}\sin(x)}{5}$

$= -\frac{e^{2x}\cos x}{5} + \frac{2e^{2x}\sin(x)}{5} + C \rightarrow \int e^{2x}\sin x\,dx$

c) $\int x \ln x\,dx$

$u = \ln x \qquad v' = x$

$\frac{1}{2}x^2\ln x - \int \frac{x}{2}\,dx$

$\int \frac{x}{2}\,dx = \frac{x^2}{4}$

$\frac{1}{2}x^2\ln(x) - \frac{x^2}{4}$

$\int x\ln x\,dx = \frac{1}{2}x^2\ln(x) - \frac{x^2}{4} + C$

4) En los sig ejercicios resolver las integrales (aplicando integración por fracción parcial)

a $\int \frac{5}{x^2+3x-4}\,dx$

$\int F(x)\,dx = 5 \cdot \int \frac{1}{x^2+3x-4}\,dx$

$\frac{1}{x^2+3x-4}, \quad \frac{1}{5x-1} - \frac{1}{5x+4}\,dx$

$$5\int \frac{1}{5x-1} - \frac{1}{5x+4}\,dx =$$

$$5\left(\int \frac{1}{5(x-1)}\,dx - \int \frac{1}{5(x+4)}\,dx\right)$$

$$\int \frac{1}{5(x-1)}\,dx = \frac{1}{5}\ln|x-1|$$

$$\int \frac{1}{5(x+4)}\,dx = \frac{1}{5}\ln|x+4|$$

$$5\left(\frac{1}{5}\ln|x-1| - \frac{1}{5}\ln|x+4|\right)$$

$$\boxed{\ln|x-1| - \ln|x+4| + C}$$

b) $\int \frac{x^2-1}{x^3+x}\,dx$

$$\frac{x^2-1}{x^3+x} ; \quad -\frac{1}{x} + \frac{2x}{x^2+1}$$

$$\int \frac{1}{x} + \frac{2x}{x^2+1}\,dx$$

$$-\int \frac{1}{x}\,dx + \int \frac{2x}{x^2+1}\,dx =$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int \frac{2x}{x^2+1}\,dx = \ln|x^2+1|$$

$$\boxed{-\ln|x| + \ln|x^2+1| + C}$$

c) $\int \frac{x^2+3x-4}{x^3-4x^2+4x}\,dx$

$$\frac{x^2+3x-4}{x^3-4x^2+4x} \rightarrow -\frac{1}{x} + \frac{2}{x-2} + \frac{3}{(x-2)^2} =$$

$$\int -\frac{1}{x} + \frac{2}{x-2} + \frac{3}{(x-2)^2}\,dx$$

$$-\int \frac{1}{x}\,dx + \frac{2}{x-2}\,dx \int \frac{3}{(x-2)^2}\,dx$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int \frac{2}{x-2}\,dx = 2\ln|x-2|$$

$$\int \frac{3}{(x-2)^2}\,dx = \frac{-3}{x-2}$$

$$\boxed{-\ln|x| + 2\ln|x-2| - \frac{3}{x-2} + C}$$

5) En los sig ejercicios resolver integrales (aplique sustitucion trigonométrica)

a) $\int \frac{1}{\sqrt{x^2-25}}\,dx$

$x = 5\sec(u)$ $\quad \int \sec(u)\,du =$

$\int \sec(u)\,du = \ln|\tan(u) + \sec(u)|$

$\ln|\tan|(u) + \sec(u)|$

$u = \operatorname{arc\,sec}\left(\frac{1}{5}x\right)$

$\ln\left|\tan\left(\operatorname{arcsec}\left(\frac{1}{5}x\right)\right| + \sec\left(\operatorname{arcsec}\left(\frac{1}{5}x\right)\right)\right|$

$$\boxed{\ln\left(\frac{1}{5}\left|\sqrt{x^2-25}+x\right|\right)+c}$$

b) $\int \frac{4}{x^2\sqrt{16-x^2}}\,dx$

$\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$

$x = 4\,\text{Sen}(u) \quad = 4 \cdot \int \frac{\cot(u)}{8\,\text{sen}(2u)}\,du$

$4 \cdot \frac{1}{8}\int \frac{\cot(u)}{\text{sen}(2u)}\,du =$

$\text{Sen}(2x) = 2\cos(x)\,\text{Sin}(x) =$

$4 \cdot \frac{1}{8}\int \frac{\cot(u)}{2\cos(u)\,\text{Sin}(u)}\,du =$

$\frac{4}{1} \cdot \frac{1}{8} \cdot \frac{1}{2}\int \frac{\cot(u)}{\cos(u)\,\text{sen}(u)}\,du =$

$\frac{\cot(u)}{\cos(u)\,\text{sen}(u)} = \csc^2(u)$

$4 \cdot \frac{1}{8} \cdot \frac{1}{2}\int \csc^2(u)\,du =$

$\csc^2(u)\,du = -\cot(u) =$

$4 \cdot \frac{1}{8} \cdot \frac{1}{2}\,(-\cos(u))$

$u = \text{arcsen}\left(\frac{1}{4}x\right) = 4 \cdot \frac{1}{8} \cdot \frac{1}{2}\left(-\cot\left(\text{arcsen}\left(\frac{1}{4}x\right)\right)\right)$

$4\,\frac{1}{8} \cdot \frac{1}{2}\left(-\cot\left(\text{arcsen}\left(\frac{1}{4}x\right)\right)\right) - \frac{\sqrt{16-x^2}}{4x}$

$$\boxed{= -\frac{\sqrt{16-x^2}}{4x} + C}$$

c) $\int \frac{9x^3}{\sqrt{1+x^2}}\, dx$

$9\int \frac{x^3}{\sqrt{1+x^2}}\, dx$

$\cos\theta \quad \frac{1}{\sqrt{1+x^2}} \quad \sqrt{1+x^2},\ \frac{1}{\cos\theta} \quad \sqrt{1+x^2} = \sec\theta$

$\sqrt{1+x^2} = \sec\theta$

$\tan\theta = \frac{x}{1} + \tan\theta\ x \rightarrow \tan^2\theta = x^3$

$dx = \sec^2\theta\, d\theta$

$9\int \frac{\tan^2\theta \sec^2\theta}{\sec\theta}\, d\theta \rightarrow 9\int \tan^3\theta \sec\theta\, d\theta$

$9\int \tan^3\theta \quad \sec\theta \tan\theta\, d\theta$

$9\int (\sec^2\theta - 1) \sec\theta \tan\theta$

$9\int (u^2-1)\, du$

$9\left[\int u^2 du - \int du \rightarrow 9\left[\frac{u^3}{3} - u\right]\right.$

$\frac{9u^3}{3} - 9u + C \rightarrow 3se^3\theta - 9\sec\theta + C$

$\sec\theta - 1 + x^2$

$$\boxed{= 3\left(\sqrt{1+x^2}\right)^3 - 9\left(\sqrt{1+x^2} + C\right.}$$