Two-Way Analysis of Variance: ANOVA

Dr. J. Kyle Roberts

Southern Methodist University
Simmons School of Education and Human Development
Department of Teaching and Learning

Introduction to Two-Way ANOVA

- In a two-way analysis of variance we analyze the dependence of a continuous response on two, cross-classified factors.
- The factors can be experimental factors that are both of interest or they can be one experimental factor and one blocking factor.
- A blocking factor is a known source of variability, such as the "subject" or, more generally, the "experimental unit". We expect this factor to account for a substantial portion of the variability in the response. We wish to control for this variability but are not interested in comparing the particular levels of this factor.
- A cross-classified experiment is said to be *balanced* if every pair of conditions occurs the same number of times. You can check with the `xtabs` function.

Hypothesis Test in Two-Way ANOVA

- Recall that the null hypothesis for a one-way ANOVA can be written as:

$$H_0 : \overline{Y}_1 = \overline{Y}_2 = \overline{Y}_3$$

- For the two-way ANOVA, we have the potential to test three separate hypothesis tests: one for each way and one for the interaction effect. We do not have to test all 3, but if we do, we refer to this as a *full factorial* ANOVA.
- These hypotheses are:

$$H_{0:MainA} : \mu_{1.} = \mu_{1.} = \cdots = \mu_{j.}$$
$$H_{0:MainB} : \mu_{.1} = \mu_{.2} = \cdots = \mu_{.k}$$
$$H_{0:ABInteraction} : \mu_{11} = \mu_{12} = \mu_{21} = \mu_{22} = \cdots = \mu_{jk}$$

- where j are the "rows" and k are the "columns."

How the Data are Analyzed in a Two-Way

- For heuristic purposes, suppose that we have a dataset with two independent variables each with two levels.
- In a full factorial setting, we would view this data as:

		Levels of Second IV		
		b_1	b_2	Mean
Levels of First IV	a_1	$\bar{X}_{11}$	$\bar{X}_{12}$	$\bar{X}_{1.}$
	a_2	$\bar{X}_{21}$	$\bar{X}_{22}$	$\bar{X}_{2.}$
	Mean	$\bar{X}_{.1}$	$\bar{X}_{.2}$	$\bar{X}$

The Two-Way ANOVA Summary Table

Source	SS	df	MS	F	p	η^2
Main A		$k_A - 1$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_e}$		$\frac{SS_A}{SS_t}$
Main B		$k_B - 1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_e}$		$\frac{SS_B}{SS_t}$
$AB_{Inter.}$		$df_A * df_B$	$\frac{SS_{AB}}{df_{AB}}$	$\frac{MS_{AB}}{MS_e}$		$\frac{SS_{AB}}{SS_t}$
Error		$df_t - df_A - df_B - df_{AB}$	$\frac{SS_e}{df_e}$			
Total		$n - 1$				

- The statistical significance of F can be obtained by computing the F-critical value. Determining statistical significance follows the same pattern for the t test only we have two sources of df: between and within.
- For the two-way ANOVA, the df needed for testing that effect are the df due to that effect and the df_{error}. Therefore, the df numerator *could* be different for each effect.

Two-Way ANOVA Practice

- Fill in the Missing Values Below. Use `pf` to compute p.

Source	SS	df	MS	F	p	η^2
Main A		5				
Main B		4				
AB_{inter}						0.22
Error						
Total	600	752				

- Fill in the Missing Values Below. Use `pf` to compute p.

Source	SS	df	MS	F	p	η^2
Main A						0.11
Main B	80		40			0.08
AB_{inter}		22				0.04
Error		437				
Total						

Partitioning the Sum of Squares

$$SS_A = nK \sum_{j=1}^{J} (\bar{X}_{j.} - \bar{X})^2$$

$$SS_B = nJ \sum_{k=1}^{K} (\bar{X}_{.k} - \bar{X})^2$$

$$SS_{AB} = n \sum_{k=1}^{K} \sum_{j=1}^{J} (\bar{X}_{jk} - \bar{X}_{j.} - \bar{X}_{.k} - \bar{X})^2$$

$$SS_{Error} = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{n} (X_{ijk} - \bar{X}_{jk})^2$$

$$SS_{Total} = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{i=1}^{n} (X_{ijk} - \bar{X})^2$$

Two-Way ANOVA Data

- Read in a table using `read.table` which resides on this website at http://faculty.smu.edu/kyler/courses/7311/twoway1.txt
- Make sure that you include `header=T`

```
> head(twoway)

  gender program gre
1      2       1  24
2      2       1  27
3      2       1  33
4      2       1  25
5      2       1  26
6      2       1  30

> str(twoway)

'data.frame': 48 obs. of  3 variables:
 $ gender : int  2 2 2 2 2 2 2 2 1 1 ...
 $ program: int  1 1 1 1 1 1 1 1 1 1 ...
 $ gre    : int  24 27 33 25 26 30 22 29 33 26 ...
```

Structuring and viewing data

```
> twoway$gender <- factor(twoway$gender)
> twoway$program <- factor(twoway$program)
> str(twoway)
```

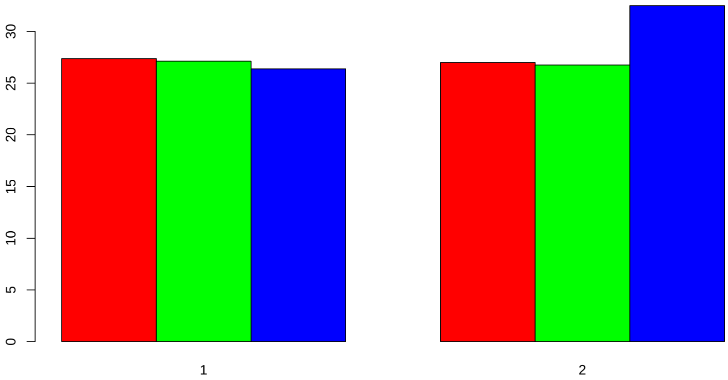
```
'data.frame': 48 obs. of  3 variables:
 $ gender : Factor w/ 2 levels "1","2": 2 2 2 2 2 2 2 2 1 1 ...
 $ program: Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
 $ gre    : int  24 27 33 25 26 30 22 29 33 26 ...
```

```
> table(twoway$gender, twoway$program)
```

```
    1 2 3
  1 8 8 8
  2 8 8 8
```

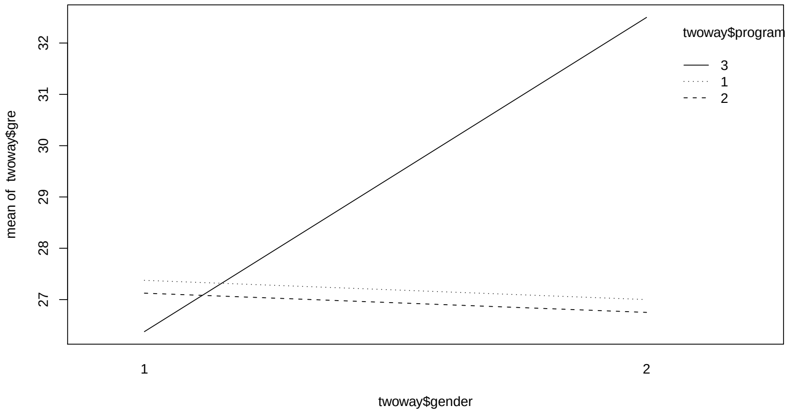
Program vs. Gender

```
> barplot(tapply(twoway$gre, list(twoway$program, twoway$gender)
+       mean), beside = T, col = rainbow(3))
```



Interaction Plot

```
> interaction.plot(twoway$gender, twoway$program, twoway$gre)
```



Running the Data

```
> m1 <- aov(gre ~ gender + program, twoway)
> summary(m1)
```

```
            Df Sum Sq Mean Sq F value Pr(>F)
gender       1  38.52  38.521  2.2942 0.1370
program      2  60.67  30.333  1.8066 0.1762
Residuals   44 738.79  16.791
```

```
> m2 <- aov(gre ~ gender * program, twoway)
> summary(m2)
```

```
               Df Sum Sq Mean Sq F value  Pr(>F)
gender          1  38.52  38.521  2.5839 0.11544
program         2  60.67  30.333  2.0347 0.14340
gender:program  2 112.67  56.333  3.7788 0.03097
Residuals      42 626.12  14.908
```

Testing Assumptions

```
> bartlett.test(gre ~ gender * program, twoway)

Bartlett test of homogeneity of variances
data:  gre by gender by program
Bartlett's K-squared = 3.865, df = 1, p-value = 0.0493

> bartlett.test(gre ~ program * gender, twoway)

Bartlett test of homogeneity of variances
data:  gre by program by gender
Bartlett's K-squared = 6.5532, df = 2, p-value = 0.03776

> fligner.test(gre ~ gender * program, twoway)

Fligner-Killeen test of homogeneity of variances
data:  gre by gender by program
Fligner-Killeen:med chi-squared = 1.6152, df = 1, p-value
= 0.2038
```

Investigation of Means

```
> model.tables(m2, "means")

Tables of means
Grand mean
27.85417

 gender
gender
     1      2
26.958 28.750

 program
program
     1      2      3
27.187 26.938 29.437

 gender:program
      program
gender 1     2     3
     1 27.38 27.12 26.37
     2 27.00 26.75 32.50
```

Post Hoc Tests

```
> TukeyHSD(m2)$"gender:program"
          diff         lwr       upr      p adj
2:1-1:1 -0.375 -6.13810348  5.388103 0.99995875
1:2-1:1 -0.250 -6.01310348  5.513103 0.99999450
2:2-1:1 -0.625 -6.38810348  5.138103 0.99949105
1:3-1:1 -1.000 -6.76310348  4.763103 0.99516934
2:3-1:1  5.125 -0.63810348 10.888103 0.10654536
1:2-2:1  0.125 -5.63810348  5.888103 0.99999983
2:2-2:1 -0.250 -6.01310348  5.513103 0.99999450
1:3-2:1 -0.625 -6.38810348  5.138103 0.99949105
2:3-2:1  5.500 -0.26310348 11.263103 0.06898988
2:2-1:2 -0.375 -6.13810348  5.388103 0.99995875
1:3-1:2 -0.750 -6.51310348  5.013103 0.99876887
2:3-1:2  5.375 -0.38810348 11.138103 0.08000676
1:3-2:2 -0.375 -6.13810348  5.388103 0.99995875
2:3-2:2  5.750 -0.01310348 11.513103 0.05082404
2:3-1:3  6.125  0.36189652 11.888103 0.03144884
```

In-Class Assignment

- Run a two-way ANOVA with the following dataset

	Teaching Method		
	1	2	3
GT	40	32	30
	36	29	35
	38	36	33
Regular	24	20	18
	21	29	26
	20	25	26
Special Ed.	22	20	18
	18	18	14
	16	19	19

Take-Home Assignment

1. Create a dataset of no less than 20 people with two categorical/grouping/independent variables and one dependent variable. Make the dataset a 4x2 design. Create the dataset in such a way that the main effect for the first way is *not* statistically significant, the main effect for the second way is *not* statistically significant, but the interaction effect *is* statistically significant.
2. Create a dataset of no less than 20 people with two categorical/grouping/independent variables and one dependent variable. Make the dataset a 4x2 design. Create the dataset in such a way that the main effect for the first way *is* statistically significant, the main effect for the second way *is* statistically significant, but the interaction effect is *not* statistically significant.