# Recursion

Module 12

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## Outline

# Recursion Concepts

## Introduction

# What is Recursion?

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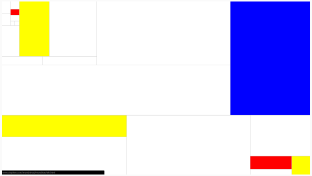

**Figure 1:** My Attempt at Recursive Mondrian Art

# Recursion Analogies

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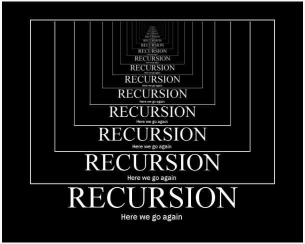

# Recursion Concepts

## Divide and Conquer

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### Steps for Recursive Problem

1. Identify the base case.

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2. Determine the recursive call.

# Divide and Conquer

**Steps for Recursive Problem**

1. Identify the base case.
2. Determine the recursive call.
3. Break the program down into small pieces to form a whole.

# Recursive Factorials

- Factorials are a great way to understand divide and conquer.

### Steps for Recursive Factorial

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- Factorials are a great way to understand divide and conquer.
- Since $n!$ can be defined as $n \cdot (n-1)!$, and $0! = 1$, factorial problems can easily be solved with recursion.

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### Steps for Recursive Factorial

1. 5!

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2. $5 \cdot 4!$

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2. $5 \cdot 4!$
3. $5 \cdot 4 \cdot 3!$
4. $5 \cdot 4 \cdot 3 \cdot 2!$
5. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1!$
6. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 0!$ (0! is 1)
7. $= 120$

# Piecewise Functions

## Introduction

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- For a piecewise function to not be infinite, it must have a "base case", and the other cases must approach it.

**Example**

$$f(x) = \begin{cases} f(x-1)+2 & \text{if } x > 10 \\ 8 & \text{if } x \leq 10 \end{cases}$$

# Solving Piecewise Functions

- Solving piecewise functions involves a simple strategy called Simplify-Substitute-Solve, or S-S-S.

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**S-S-S Strategy**

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1. Simplify - Plug and chug until you reach the base case.
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# Piecewise Functions

## Examples

# Factorials Revisited

## Factorial Piecewise Function

$$f(x) = \begin{cases} f(x-1) \cdot x & \text{if } x > 0 \\ 1 & \text{if } x = 0 \end{cases}$$

- This piecewise equation should seem very familiar; it uses the exact same concept as the recursive factorial shown earlier!

# Factorials Revisited

## Factorial Piecewise Function

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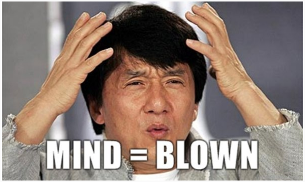

# Working Through a Piecewise Function

## Piecewise Function

$$f(x) = \begin{cases} f(x-1)+2 & \text{if } x > 10 \\ 8 & \text{if } x \leq 10 \end{cases}$$

- Here is the example from a few slides ago.

## Simplify

# Working Through a Piecewise Function

## Piecewise Function

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- If you wanted to solve for $x = 14$, you could do as follows . . .

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- If you wanted to solve for $x = 14$, you could do as follows ...

## Simplify

1. $f(14) : 14 > 10 : f(14-1)+2$

# Working Through a Piecewise Function

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- Here is the example from a few slides ago.
- If you wanted to solve for $x = 14$, you could do as follows ...

## Simplify

1. $f(14) : 14 > 10 : f(14-1)+2$
2. $f(13) : 13 > 10 : f(13-1)+2$

# Working Through a Piecewise Function

## Piecewise Function

$$f(x) = \begin{cases} f(x-1)+2 & \text{if } x > 10 \\ 8 & \text{if } x \leq 10 \end{cases}$$

- Here is the example from a few slides ago.
- If you wanted to solve for $x = 14$, you could do as follows ...

## Simplify

1. $f(14) : 14 > 10 : f(14-1)+2$
2. $f(13) : 13 > 10 : f(13-1)+2$
3. $f(12) : 12 > 10 : f(12-1)+2$

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11 - 1) + 2$

## Substitute

## Solve

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11-1)+2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

## Solve

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11-1)+2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

1. $f(10) = 8$

## Solve

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11-1) + 2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

1. $f(10) = 8$

## Solve

1. $f(11) = 8 + 2 \quad = 10$

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11-1) + 2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

1. $f(10) = 8$

## Solve

1. $f(11) = 8 + 2 \quad = 10$
2. $f(12) = 10 + 2 = 12$

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11-1)+2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

1. $f(10) = 8$

## Solve

1. $f(11) = 8+2 \quad = 10$
2. $f(12) = 10+2 = 12$
3. $f(13) = 12+2 = 14$

# Working through a Piecewise Function (Continued)

## Simplify

4. $f(11) : 11 > 10 : f(11 - 1) + 2$
5. $f(10) : 10 \leq 10 : 8$

## Substitute

1. $f(10) = 8$

## Solve

1. $f(11) = 8 + 2 \quad = 10$
2. $f(12) = 10 + 2 = 12$
3. $f(13) = 12 + 2 = 14$
4. $f(14) = 14 + 2 = \mathbf{16}$