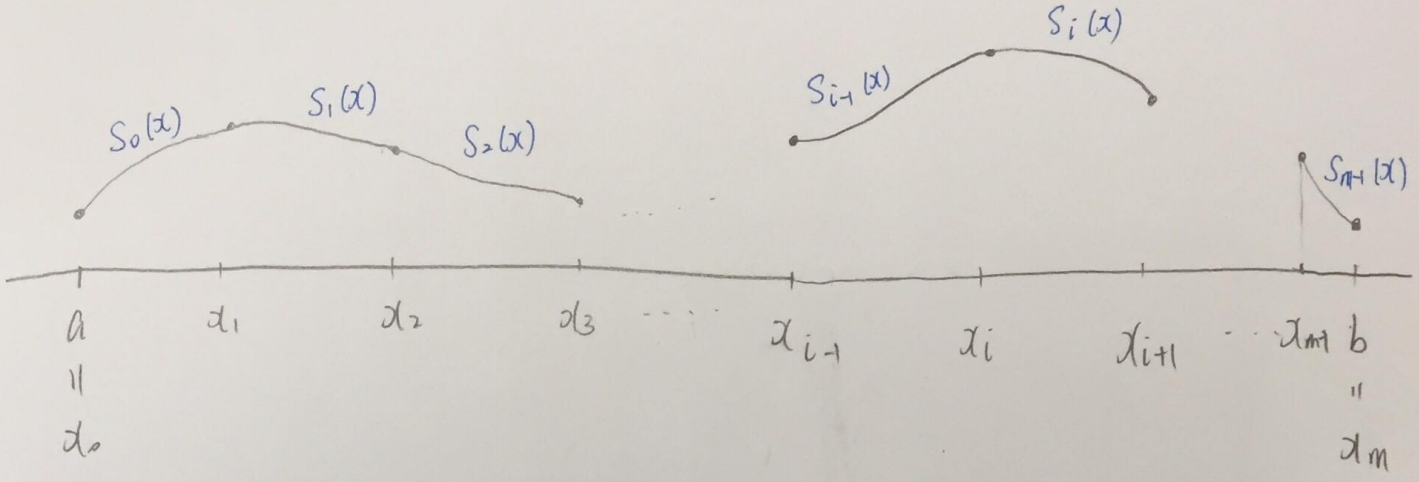
# Spline (樣條)




$S : [a,b] \to \mathbb{R}$ 參數曲線

$$S(x) = \begin{cases} S_0(x), & \text{if } x_0 \le x \le x_1 \\ S_1(x), & \text{if } x_1 \le x \le x_2 \\ \vdots & \\ S_{n-1}(x), & \text{if } x_{n-1} \le x \le x_n \end{cases}, \quad S \in C^{n-1}(a,b)$$

For $i = 0, 1, \dots, n-1$.

$S_i \in \Pi_n$, $\Pi_n = \{ p(x) : \text{polynomial of degree at most } n. \}$

For $i = 0, 1, 2, \dots, n-2$

$S_i(x_{i+1}) = S_{i+1}(x_{i+1}) = f(x_{i+1})$

$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$

$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$

$\vdots$

$S_i^{(n-1)}(x_{i+1}) = S_{i+1}^{(n-1)}(x_{i+1})$

$S_0(x_0) = f(x_0)$

$S_{n-1}(x_n) = f(x_n)$

## Cubic Spline : 三次樣條 ($n=3$)

$$S(x) = \begin{cases} S_0(x), & \text{if } x_0 \le x \le x_1 \\ S_1(x), & \text{if } x_1 \le x \le x_2 \\ \vdots & \vdots \\ S_{m-1}(x), & \text{if } x_{m-1} \le x \le x_m \end{cases}$$

For $i = 0, 1, \cdots, m-2$.

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}) = f(x_{i+1})$$

$$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$$

$$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$$

$$S_0(x_0) = f(x_0)$$

$$S_{m-1}(x_m) = f(x_m)$$