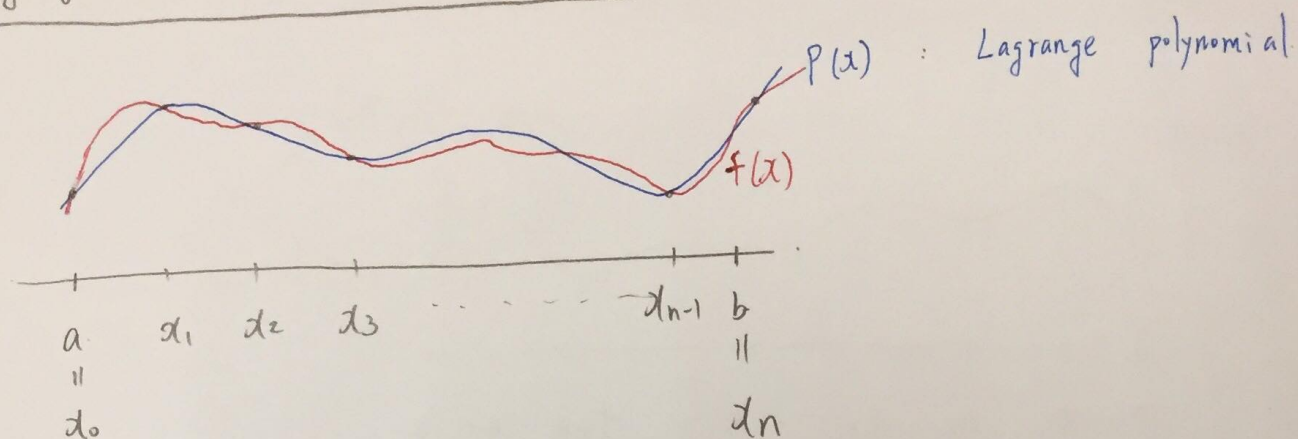


Lagrange Interpolating Polynomial



Find a polynomial P of degree at most n
s.t. $P(x_i) = f(x_i)$, for $i = 0, 1, \dots, n$.

* Lagrange cardinal functions

• 概念：第 k th 節點 函數值 = 1，其它 為 0

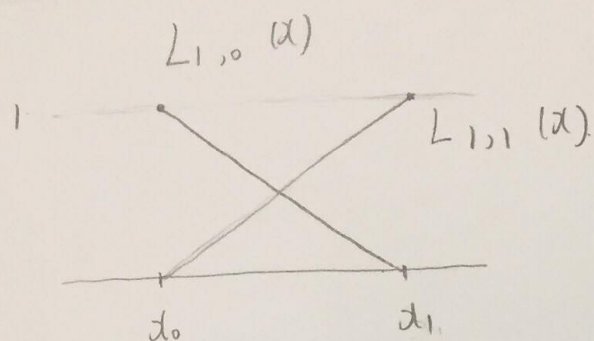
For $k = 0, 1, \dots, n$.

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

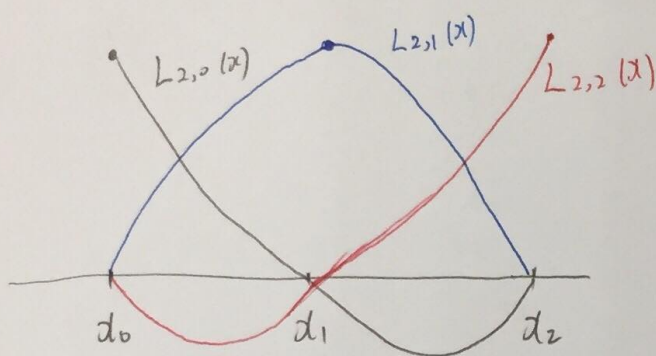
Then

$$L_{n,k}(x_i) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}$$

e.g. $n=1$



$n=2$



* $P(x)$

$$\begin{aligned} P(x) &= f(x_0) \cdot L_{n,0}(x) + f(x_1) \cdot L_{n,1}(x) + \dots + f(x_n) \cdot L_{n,n}(x) \\ &= \sum_{i=0}^n f(x_i) \cdot L_{n,i}(x) \end{aligned}$$

$P(x)$ 就是 $L_{n,k}(x)$ 做线性组合.