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M. Kathleen Heid
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Computer Algebra Systems: Revolution or Retrofit for Today's Mathematics Classrooms?

FOR THE PAST TWO DECADES, a revolutionary technology has been poised at the door of school mathematics classrooms, yet until recently it has been given little more than a nod to acknowledge its existence. Since the early 1980s, symbolic manipulation programs, computer-based mathematics packages that perform *exact* arithmetic and *symbolic algebra* calculations, have been available for use by secondary mathematics teachers and students. Symbolic manipulation packages, such as *DERIVE* (Soft Warehouse, 1995) and *Maple* (Waterloo Maple, 2000), are able to perform complicated mathematical tasks that are difficult, if not impossible, using only pencil-and-paper (by hand) methods (Heid, 1989). The introduction of computer-based manipulation utilities into secondary school classrooms opened the possibility of a shift from emphasis on the execution of traditional algebraic tasks such as equation solving and simplification of algebraic expressions to the development of deeper conceptual understanding and ability to apply algebra to real world settings.

By the early 1990s, symbolic manipulators were housed in a range of computer algebra systems (CAS's), programs that feature multiply-linked graphical, numerical, and symbolic manipulation utilities. Teachers and students in technologically-rich settings gained access to a full range of representations afforded by the CAS, which they could apply to a myriad of real world situations and problems. For instance, using a single technology, students could define algebraic rules to describe real-life situations, they could find approximate answers to problems by employing graphical or tabular methods, and they could then produce exact answers with automated symbolic algebra tools. Students could explore problem settings dynamically. They could investigate many versions of a problem at once by examining the effects of changing initial problem parameters.

Recognizing such potential benefits of CAS-based instruction as a deep understanding of algebra and richer environments for students' mathematical investigations, mathematics educators began to explore possible uses of the technology in secondary school classrooms. By the early 1990s, classrooms at the vanguard of the CAS movement were those beyond the borders of the United States. In 1990, Austria spearheaded a movement to incorporate computer algebra into secondary school classrooms when it purchased a nationwide site license of the *DERIVE* (Soft Warehouse, 1995) CAS for all of its high schools (Schneider, 2000). Meanwhile, studies of CAS in a variety of other countries—notably France and Great Britain—sprang up at the secondary school level at approximately the same time.

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By the late 1990s, versions of computer algebra systems with capabilities similar to those found in powerful computer-based systems such as *Mathematica* (Wolfram Research, 1997) or *DERIVE* (Soft Warehouse, 1995) were now available in user-friendly hand-held calculators. Now, at the dawn of the 21st century, schools, teachers, and education policy-makers are beginning to discuss possible roles of the CAS in United States classrooms. This article discusses theory, classroom trials, and empirical research that may serve as a springboard for such discussions.

Not Your Parents' Algebra

The past two decades in mathematics education have been characterized by calls for (and movement toward) fundamental changes in the nature and purpose of school algebra. In a technological world, algebra would no longer be centered on the by-hand symbolic manipulation procedures that have dominated school mathematics instruction for countless years. Algebraic representations (graphs, tables of values, and symbolic rules) have continually grown in importance as intellectual tools that aid workers in an increasingly technological, information-based society. For instance, algebraic representations have proved to be able assistants in the development of communication networks, statistical analysis, and population projections.

A richer approach

Classroom-accessible technological power has enabled teachers to provide their students with a richer approach to mathematics, with this new approach arguably affecting school algebra more profoundly than any other areas of school mathematics. The new face of algebra is multirepresentational instead of primarily symbolic, centered on applications instead of solely on theory, and focused on symbolic reasoning instead of primarily on symbolic manipulation. With the increasing availability of computer algebra systems equipped with symbolic manipulation capabilities, mathematics educators are beginning to take a closer look at the roles of symbolic representations in school algebra.

Prior to common classroom availability of computer algebra systems, the *raison d'être* of school algebra was, for many, the development of students' abilities to manipulate algebraic symbols according to a set of transformation rules. School algebra instruction consisted of solving equations and manipulating symbolic expressions. The CAS era, with its facility for automated manipulation of symbolic forms, however, has ushered in new roles for symbolic representation. Symbol sense, symbolic reasoning, and symbolic disposition assume greater importance in a world in which computer-based algebra exists.

Symbolic manipulation includes the ability to change symbolic form both by paper and pencil and by computer. *Symbol sense* includes one's capacity to assess the reasonableness of particular symbolic results, be they generated by paper and pencil or by computer. *Symbolic reasoning* centers on the ability to reason about quantitative relationships through analysis of their symbolic representations. All of these symbolic capacities are influenced by *symbolic disposition*, the extent to which students turn to and rely on symbols in their quantitative reasoning.

The CAS offers specific opportunities for the development of symbolic understanding, affording students with the opportunity and motivation:

- to see that different symbolic expressions provide different information (a notion illustrated in the section that follows);
- to outsource routine work to the CAS so that they can focus on more conceptual ideas, on the "bigger picture," or on more general ideas;
- to reason with confidence about symbolic results (possibly reducing students' anxiety over "making mistakes");
- to develop their own symbolic procedures;
- to bridge the gap between concrete examples and abstract generalization;
- to interpret information gained through one representation in an equivalent one (to see the symbolic in the graphic, to see the graphic in the symbolic, to visualize a contextual situation symbolically);
- to develop generalized rules for problem solving; and
- to examine symbolic patterns (more concretely).

Equivalent expressions and CAS

In the NCTM publication, *Algebra in a Technological World*, Heid and colleagues (Heid, Choate, Sheets, & Zbiek, 1995) comment:

> Students too often leave their algebra experience with a modicum of ability to produce equivalent forms but very little understanding of the meaning of that equivalence. In a technological world in which students have access to computer-algebra utilities . . . the importance of producing equivalent forms no

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> longer overshadows the importance of understanding what the equivalent expressions mean. (p. 127)

Using CAS, Edwards (2000) and his students explored the importance of equivalent algebraic representations while investigating the algebraic properties of number systems. Expanding on the "box problem," a problem that has become a staple in technology-savvy high school mathematics classrooms, Edwards' algebra students examined various properties of a box they constructed by cutting square corners from a rectangular piece of material. Specifically, students cut x by x squares out of each corner of rectangular material that was 14 units wide and 8 units long (see Figure 1).

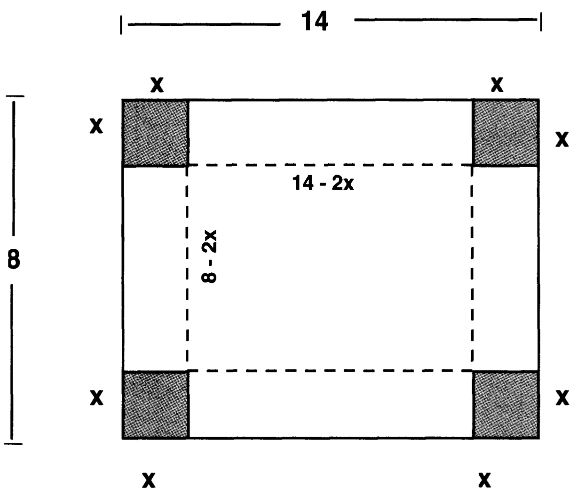

Figure 1. A diagram of a box constructed by cutting squares from a rectangular piece of cardboard. [The five sections of remaining cardboard consist of left and right "vertically positioned" rectangles each of area x(8-2x), top and bottom "horizontally positioned" rectangles each of area x(14-2x), and a middle rectangle of area (8-2x)(14-2x).]

Members of Edwards' class found the area of the remaining cardboard (the resulting net) by adding together the different areas. They concluded that the surface area (sa) of the box may be expressed in the following (equivalent) forms:

1. $sa = 2x \cdot (8-2x)+2x \cdot (14-2x)+(14-2x) \cdot (8-2x)$
2. $sa = 8 \cdot (14-2x)+2x \cdot (8-2x)$
3. $sa = 14 \cdot (8-2x)+2x(14-2x)$

In a series of classroom discussions, students generated and subsequently investigated the equivalence of these expressions while using a CAS-equipped calculator, Texas Instruments' TI-92. Each expression yielded $-4x^2+112$ when entered into the CAS. In a teacher journal documenting class activities, Edwards (2000) notes that the exercise motivated a rich discussion of the distributive property among the students.

> Students that typically were reluctant to talk about manipulation expressed curiosity regarding the equivalency of various forms of surface area. Many genuinely wanted to know *why* the forms were equivalent. The activity fostered a rich discussion of distribution, with students and myself actively participating in the dialogue. (January 19, 2000)

Capitalizing on the multi-representational capabilities of the TI-92, Edwards' students used the graphing capabilities of the TI-92 to determine the maximal volume of the box, as well as the surface area of the box with maximal volume. The context of the problem afforded increased relevance to otherwise detached algebraic concepts, such as domain, range, and graphical representations.

Mathematics Curriculum and the CAS

Since the time of the early trials of computer algebra systems on classroom microcomputers, educators have wondered about what role the symbolic manipulation capabilities of the CAS might play in their classrooms (Heid, 1989). With the capability of performing all or any part of traditional symbolic manipulation, CAS can take on a range of possible roles. CAS can be used as a supplement to paper and pencil instruction (Mayes, 1995), as a catalyst for some new approaches and as a replacement for some traditional mathematics (Edwards, 2000), or as replacement for traditional symbolic manipulation (Heid, 1984, 1988).

Four possible roles

One possible role for the CAS is as the primary producer of symbolic results. This use of the CAS enables students to focus on other aspects of mathematics. The curriculum can then be restructured to prioritize the development of mathematical concepts or understanding of applications over the acquisition of paper and pencil symbolic manipulation skills. One introductory algebra curriculum (Fey

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et al., 1999) relies on the CAS in order to focus student attention on the mathematical concept of function (a mathematical relationship between quantities that vary) and its applications.

A second possible role for the CAS is to create and generate symbolic procedures, giving the user access to symbolic procedures of almost any "chunk size." For example, the CAS can perform traditional symbolic manipulations in a step-by-step fashion, with students issuing commands to transform an equation until it is solved (see Figures 2-4). In this manner, they can practice recognizing an appropriate sequence of symbolic manipulations without being distracted by paper and pencil errors.

We propose that computer symbolic algebra utilities may encourage weak students to examine algebraic expressions from a more conceptual point of view, enabling students to build more mature notions of equation solving. The solution of the equation 2x + 7 = 3 using the TI-89 CAS in Figures 2-4 illustrates this point. The TI-89 treats algebraic equations as objects that may be manipulated symbolically. For instance, students may combine various algebraic expressions with 2x+7=3. To isolate x, one can first subtract 7 from each side of the original equation, as shown in Figure 2.

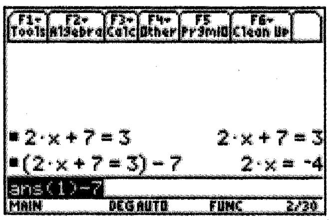

Figure 2. The TI-89 calculator executes the procedure of subtracting 7 from each side of the original equation, 2x+7 = 3.

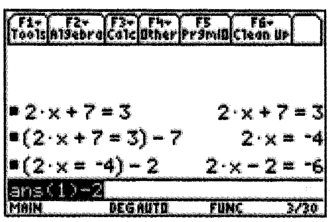

Figure 3. The TI-89 executes the procedure of subtracting 2 from each side of the equation 2x = -4.

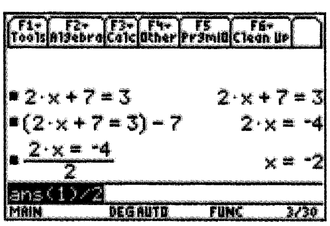

Figure 4. The TI-89 executes the procedure of dividing each side of the equation 2x = -4 by 2.

Even if students do not choose the optimally efficient route and, instead, subtract 2 from each side of the equation 2x=-4 to "get x by itself," the CAS ably executes the command it is given. Figure 3 shows that subtracting 2 from each side does not "cancel out" multiplication by 2. Using the delete key, a student is able to clear off this step and try again. On the other hand, the student may successfully solve the equation by continuing without clearing. Figure 4 shows the sequence of steps that follow the student clearing the subtraction of 2 and continuing by dividing each side of 2x=-4 by 2, isolating x on one side of the equation.

Such techniques illustrate the use of computer symbolic algebra as a pedagogical tool, assisting students in constructing sound conceptual understandings that underlie symbolic manipulation. The example clearly suggests that computer symbolic algebra may be utilized by novice algebra students learning to solve equations.

A third possible use of the CAS is to assist students in generating many examples from which they can search for symbolic patterns. For example, Figure 5 suggests a possible way of using CAS to help students generate a rule for cubing a binomial of the form x + n.

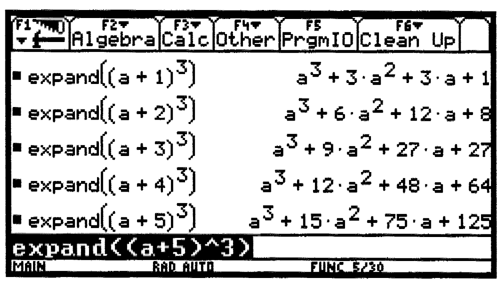

Figure 5. A familiar exponent rule may be suggested to students by means of CAS-generated examples.

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Fourth, the CAS can be used to generate results for problems posed in abstract form. For example, a student might examine the following problem: "Find the roots of any quadratic equation." The student may use the "solve" feature of the CAS, issuing a "solve ($ax^2 + bx + c = 0,x$)" command and getting a CAS result of

$$x = \frac{\sqrt{b^2-4ac}-b}{2a} \quad \text{or} \quad x = \frac{-(\sqrt{b^2-4ac}+b)}{2a}$$

to investigate solutions to the problem.

White box/black box

Metaphors for characterizing these different roles for the CAS in algebra instruction have been advanced by Kutzler (1996). He draws on Buchberger's (1989) initiation of the white box/black box metaphor—the white box referring to a procedure in which the steps to the procedure are not hidden and the black box referring to a procedure whose steps are entirely hidden.

Macintyre (2000) discusses possible configurations of black box (BB) and white box (WB) techniques for use in algebra instruction. He prefers a teaching strategy (WB then BB) in which students use the CAS to produce results only after they have learned the procedures by paper and pencil, attributing a better understanding of the procedures to this sequence. He explains that when teachers implement the opposite instructional strategy (BB first), they are using the technology as an explorative tool. Tonisson (2000) and Macintyre (2000) both refer to a procedure in which the CAS is used to execute procedures step by step, suggesting an intermediate strategy.

Assessment

Just as the CAS can assume a variety of roles in instructional plans, there are also several possible policies for CAS technologies in assessment (Drijvers, 1998). It may be the case that CAS is allowed on parts of exams and not allowed on other parts. It may be that CAS is allowed, but test items are designed so that using the machine is not fruitful. Or, CAS may be recommended and useful, but students are required to show paper and pencil substantiation of their work, so that use of the CAS is not rewarded. Finally, CAS may be both required and rewarded. Each of these technological stances on testing carries certain implications for curriculum.

Research on CAS in the Curriculum

Depending on its configuration, the incorporation of CAS in school mathematics can foster significant changes in content. A CAS-present curriculum can result in: more realistic problems, deeper exploration of mathematical concepts, increased opportunities to develop connections among mathematical ideas, a wider range of examples, more abstraction, a more complete set of examples and non-examples in a shorter period of time, and new ways to understand traditional procedures. Not only can it transform the content of the mathematics classroom, but it has effects on teacher roles as well as on student behavior and understanding.

Transforming classroom activity

Technology transforms mathematics classrooms, and CAS technology is no exception. Researchers report work in the context of a CAS computer lab that seemed to foster the growth of a community centered on interpreting and using CAS results (Pierce & Stacey, 2001; Sheets & Heid, 1990). In many cases, the CAS is another player in the classroom, generating results that demand student interpretation. The CAS can become an additional "expert" for the classroom.

Essential to school mathematics instruction is the nature of classroom interaction. In today's classrooms, the teacher-student and student-student interaction is supplemented by the interaction of both teacher and students with the technology present in that classroom. Technology can enhance the communication among teachers and students (Noss & Hoyles, 1996). Technological tools such as the CAS provide an arena within which teachers and students can attempt to externalize their understandings and in which teachers and students can express, alter, and investigate their emerging mathematical knowledge (Noss & Hoyles, 1996; Sheets & Heid, 1990).

The CAS provides immediate nonjudgmental feedback. Computer or calculator output can serve as a "conversation piece" (Thornton, 1992) around which teachers and students can center their mathematical discussion. Pierce and Stacey (in press) corroborated this observation in their study of CAS

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use in undergraduate mathematics courses: "When sharing a computer to work on exploratory exercises, they usually talked about mathematics rather than social events" (p. 93).

Changing student behavior

The integration of a CAS in students' mathematical experiences has been predicted to foster a range of positive changes in student behavior and strategies. Many, but not all, of those predictions have been evidenced in research studies. Several researchers (Lagrange, 1999a, 1999b; Zehavi & Mann, 1999) have observed that although it might be predicted that by relieving students of the heavy technical load, CAS would allow them to focus on applications and concepts, easier symbolic manipulation did not automatically enhance their reflection and understanding. Reflection, however, often results from enhanced communication, and Pierce and Stacey (2001) observed that the CAS was a catalyst for certain aspects of communication—negotiating meaning with peers and teachers, including the computer in the negotiating process.

A second category of learning behaviors that researchers have studied in the context of the integration of CAS has to do with the ways in which students approach problem solving. In addition to being more communicative, students in the CAS groups were often more persistent and flexible. Tynan and Asp (1998) observed the effects of making the TI-92 available to ninth-grade students on a range of symbol manipulation tasks and found that the TI-92 students were more inclined than their peers, who used a non-CAS graphics calculator, to persist with algebraic methods for equation-solving tasks.

When CAS was available to first-year undergraduate mathematics students, more students seemed to attempt more parts of the questions (Pierce & Stacey, in press). In a study of business calculus students using a CAS, the class was more interested, participated more actively, and spent more time preparing for class (Vlachos & Kehagias, 2000). In this study, when students failed in their use of one particular approach, they were able to select and apply a different one.

CAS does not always promote positive behavior, however. When the CAS was available to first-year undergraduate mathematics students, the students did not necessarily check answers (strong students allowed simple numerical and algebraic errors to remain) (Pierce & Stacey, in press).

Changing role of the teacher

An understanding of the changing role of the teacher must be tempered with an understanding of the complex nature of change in schools and teaching. With the entry of the CAS into the mathematics classroom, and the intricate interactions of the teacher, student, and technology, the role of the teacher becomes more complex. In one example, Sara, a beginning algebra teacher who engaged her students in the learning of algebra with the use of a CAS, was the subject of an in-depth study (Heid, 1996). Heid describes how Sara's shaky mathematical understanding, her reluctance to use technology, the use of a CAS with unfortunate default values, and her classroom teaching style interacted to produce a pattern of misconceptions about symbolic notation in her students. That pattern was still in place 6 months after Sara's class had ended.

On the other hand, in their study of the use of CAS in teaching story problems, Zehavi and Mann (1999) observed: "Our experience with this unit (and others) indicated that using CAS technology increased teachers' awareness of the cognitive aspects of learning. The teachers were challenged to rethink curricular and didactic aspects of mathematics learning" (p. 66).

Even if the challenges CAS teachers face are apparent to them, appropriate responses to those challenges may not be as apparent. Some new CAS-oriented curriculums pose considerable challenges for teachers. Schneider (2000) observes:

> We learned that teachers do not possess sufficient didactical knowledge and experience to make use of the "free time" generated by the outsourcing of operative activities to the computer in a way intended by us. In particular, they searched for possibilities of substituting the operative activities lost by introducing other operative activities. (p. 138)

This observation of the tendency to stay the traditional course in teaching is corroborated in other studies. Kendal and Stacey (1999), for example, observed two teachers teaching CAS-oriented calculus classes for 2 consecutive years. They noted: "In fact, there was little difference in teaching over the 2 years—both teachers used CAS alongside their previous teaching styles and practices, without fundamental change, in ways consistent with

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their fundamental conceptions" (p. 65). Similarly, Heid and her colleagues (Heid, Blume, Zbiek, & Edwards, 1998/1999) observed the ways in which teachers' perceptions of the nature of learning influenced their interactions with students in assessment settings for CAS-intensive algebra courses.

As teachers engage students in learning to capitalize on new technologies, they need to be aware of the extent to which their behavior shapes the strategies students learn to use. In a study of three CAS-active introductory calculus classes of 11th grade students, researchers (Kendal & Stacey, 1999; McCrae, Asp, & Kendal, 1999) observed that the ways students used the CAS was influenced by teachers' cognitive "privileging." The researchers draw on a definition of "privileging" they attribute to Wertsch (1990), who defined privileging as "the social setting and values that may elevate one form of mental functioning over another" (p. 119).

In this study, one teacher (Teacher A) favored technological and algebraic approaches, did not take advantage of the graphical capabilities of the calculator, and only rarely made connections between symbolic and graphical ideas. Teacher A's students frequently used CAS for symbolic solutions, rarely used graphs, and had a higher conceptual error rate. Teacher B emphasized by-hand symbolic approaches, and Teacher B's students showed higher proficiency in by-hand algebra than students in either of the other two classes. Teachers B and C privileged conceptual understanding built from illustrating symbolic ideas graphically. Their students compensated for weak algebra skills by substituting a graphical for a symbolic approach. Conclusions about the effects of CAS use on the behavior of students must be viewed in light of the potential effects of different teaching strategies on student behavior.

Developing skills, understanding

Most mathematics instruction is driven by goals centering on the development of core conceptual and procedural understanding, and the history of CAS research in mathematics education is replete with studies examining the effects of CAS-oriented instruction on the development of such understandings. Typically, a study employs the CAS in one of the previously described fashions, and then compares the conceptual and/or procedural understandings or skills of students in the experimental curriculum with those of students in a more traditional curriculum. The experimental curriculums are designed to produce deeper conceptual or procedural understanding or to help students develop facility in creating and using mathematical models for realistic situations. The early experiments typically lasted a minimum of one semester, and most found that CAS-curriculum students developed superior understanding of concepts with no significant loss of computational skill (Crocker, 1991; Heid, 1984, 1988; Heid, Sheets, Matras, & Menasian, 1988; Palmiter, 1991; Park, 1993; Schrock, 1989; Vlachos & Kehagias, 2000).

A growing number of studies have veered from relying solely on the experimental/comparison group design and have examined the nature of student learning and understanding in the context of a variety of CAS-oriented curriculums. Tynan and Asp (1998) observed that ninth-grade students using the TI-92 were more inclined than their peers, who used a non-CAS graphics calculator, to persist with algebraic methods for equation-solving tasks. In addition, there was no significant difference in their abilities to perform by-hand symbolic manipulation. Researchers whose curricular approach centered on CAS-assisted discussions emphasizing a sense-making approach to calculus found that the CAS group was more successful than the peer group at symbolically solving problems (Keller & Russell, 1997; Keller, Russell, & Thompson, 1999). The researchers observed that although students were frequently over-reliant on technology when they first began to use it, "over time, students' desire to make sense of symbolic manipulations grew with their desire to make sense of mathematics" (p. 93).

Conclusion

Despite their relative absence in U.S. secondary school mathematics classrooms, symbolic manipulation utilities are not a new technology. Computer-based tools capable of performing exact arithmetic have existed commercially for roughly 20 years. Although an increasingly large body of evidence—both experimental and anecdotal—supports serious consideration of CAS usage with secondary school students in the United States, teachers and policymakers have been slow to act on the power of computer-based algebraic manipulation devices. As

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Schneider (2000) suggests, teachers in this country may not be adequately prepared to deal with the ramifications of the use of such powerful tools in their classrooms. The sheer power of CAS may have slowed their incorporation into mathematics instruction.

Utilities now exist that allow students with little or no knowledge of symbolic manipulation to solve equations and simplify expressions. In this context, how important is it for students to acquire refined skills in by-hand symbolic manipulation? If by-hand procedures are less important than they were 20 years ago, what should algebra teachers be teaching now?

Regardless of one's views of by-hand manipulation, it seems clear that symbolic manipulation utilities have the potential to play a meaningful role in school mathematics programs. This article provides examples of how the CAS may be used to supplement students' by-hand manipulation skills by highlighting the conceptual ideas underlying equation solving. On the other hand, the CAS may be used to accelerate the elimination of significant portions of mathematics content dealing with by-hand manipulation—allowing greater time for students to dedicate to the development of conceptual understanding and the study of mathematical applications.

Mathematics educators must now come to terms with the existence of symbolic manipulations technologies in their classrooms. The mathematics education community must confront the existence of CAS and plan a course of action based on sound research that best enables educators to teach students meaningful, significant mathematics.

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