

$$i) 2x - 3y + 7z = 5, \quad 3x + y - 3z = 13, \quad 2x + 9y - 4z = 32$$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$\stackrel{2}{\sim} \begin{bmatrix} 2 & -3 & 7 \\ 0 & 22 & -54 \\ 0 & 1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 27 \\ 13 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{3}{2} R_1$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

Rank of $A = 2$
Rank of $A:B = 3$

$\rho(A) \neq \rho(A:B)$ Inconsistency.

$$ii) 2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{array}{l} x = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix} \\ A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & \frac{-38}{5} \end{bmatrix} B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$\begin{aligned} R_3 &\rightarrow R_3 - \frac{1}{3}R_2 \\ &\rightarrow -1 - \frac{35}{3} \\ &\rightarrow -\frac{38}{3} \end{aligned}$$

$\delta(A) = \delta(A:B)$ consistency.

2(iii)

$$Q) 4x - y = 12, \quad -x + 5y - 2z = 0, \quad -2x + 4z = -8$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$\delta(A) = \delta(A:B)$ consistency.

b) For what values of λ and μ

$$x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$$

i) no solution, ii) a unique solution (iii) infinite no. of solutions.

$$Ax = B$$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda - 3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

i) No solution $S(A) \neq S(A:B)$

$$\lambda = 3, \mu \neq 10$$

(ii) a unique solution. $S(A) = S(A:B)$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \cancel{\mu - 10} \end{bmatrix}$$

$$\lambda \neq 3, \text{ many values}$$

iii) Infinite solution $S(A) = S(A:B) < 3n$

$$\lambda = 3, \mu = 10$$

$n \rightarrow$ no. of unknowns.
 x, y, z .

c) Find for what value of λ .

$$x+y+z=1$$

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

Have a solution. &

Find λ to solve for each value of λ .

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

What division do we:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda \\ 0 & 0 & 0 & \lambda^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$$f(A)=2$$

$$f(A:B)=2$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2 - 1 - 3\lambda + 3 \end{bmatrix}$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-1) - \lambda(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1, 2$$

$$f(A) = f(A:B) \neq n$$

3 unknowns

Case-1 $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$

$$y+3z=0$$

$$z=k$$

$$y=-3k$$

$$x = 1 - y - z$$

$$= 1 + 3k - k = 1 + 2k$$

Case-2 $\lambda = 2$

$$\text{Ans: } \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$

$$y+3z=1$$

$$y=1-3z \quad z=k$$

$$x = 1 - 1 + 3k - k = 2k$$

d) Find the sol' of the system of equations .
 $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$.
B.Karthikeya.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -1 & 4 \\ -1 & 4 & 1 \end{bmatrix} \quad x = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} - & & & \\ 0 & 0 & 3 & -2 \\ 0 & -7 & -\frac{1}{4} & \frac{8}{10} \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} - & & & \\ 0 & 0 & 3 & -2 \\ 0 & -7 & -\frac{1}{4} & \frac{8}{10} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

e) find for what values of λ the given equations $3x+4y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may possess non-trivial solution and solve them completely in each case

$$A = \begin{bmatrix} 3 & +1 & -x \\ 4 & -2 & -3 \\ 2x & 4 & x \end{bmatrix} \xrightarrow{\text{Row } 3 \leftarrow R_3 + R_1} \begin{bmatrix} 3 & +1 & -x \\ 4 & -2 & -3 \\ 2x+3 & 4+1 & 0 \end{bmatrix} \quad \text{Infinite}$$

$$\left[\begin{array}{ccc} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{array} \right] \xrightarrow{\substack{R_1 = R_1/4 \\ R_2 = R_2/3}} \left[\begin{array}{ccc} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{array} \right]$$

$$R_1 = R_{14} \quad / \quad R_2 = R_{23}$$

$$= \begin{bmatrix} 12 & 4 & -4x \\ 0 & -10 & -9+ \\ 2x+3 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} \left| \begin{array}{ccc|c} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 0 & -\frac{9+4\lambda}{2} & 0 \end{array} \right. \quad x(2\lambda+3) + 5y = 0 \\ \qquad \qquad \qquad -10y + (-9+4\lambda)\frac{z}{2} = 0 \\ 2x+3=0 \quad -\frac{9+4\lambda}{2}=0 \quad 2x+4y-4\lambda\frac{z}{2}=0 \\ x=-\frac{3}{2} \quad \lambda=\frac{9}{4} \quad 10y = (-9+4\lambda)\frac{z}{2} \end{array}$$

$$= 12x + 6y - 93 = 0$$

$$-10y + (-9+4x)3 = 0$$

$$\begin{array}{r} (2x+3)x + 5y = 0 \\ - \frac{(2x+3)x}{5} y, \end{array}$$

$$12x - 6y - 9z = 0$$

$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$

$$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{10^2}{(-9+4\lambda)}\left(\frac{2\lambda+3}{5}\right)x\right) = 0$$

$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(-9+4\lambda)} = 0$$

$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9) + 18(5)(2\lambda+3) = 0$$

(12\lambda+18)

$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 1620 = 0$$

$$48\lambda^2 + 384\lambda - 432 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda+9) + 9(\lambda-1) = 0$$

$$(\lambda+9)(\lambda-1) = 0 \Rightarrow \lambda = 1, \lambda = -9$$

$$\underline{\lambda=1}$$

$$-x = y, z = -2y$$

$$12x - 6y - 9z = 0$$

$$12x(-y) - 6y - 9(-2y) = 0$$

$$18y - 18y = 0$$

$y \neq 0$ non-trivial solution

$$\underline{\lambda=-9}$$

$$y = \left(-\frac{(2\lambda+3)x}{5}\right)$$

$$z = \frac{(-18+3)x}{5} = +\frac{3}{5}x$$

$$z = \frac{10y}{(-9+4\lambda)}$$

$$= \frac{10y}{-45} = \frac{2y}{-9}$$

$$12x - 6y - 9z = 0$$

$$x = \left(\frac{y}{2}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$$

$$4y - 6y + 2y = 0$$

$y = 0$ trivial solution

+ 1 solution

Assignment - 2

Q) 1) $[1, 0, 0], [1, 1, 0], [1, 1, 1]$

$$\begin{bmatrix} C_1 + C_2 + C_3 & C_2 + C_3 & C_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$C_3 = 0, C_2 = 0, C_1 = 0$$

unique solution
linearly independent.

2) $[6, 0, 3, 1, 4, 2], [0, -1, 2, 7, 0, 5], [1, 2, 3, 0, -1, 9, 8]$

$$\begin{bmatrix} 6C_1 + 12C_3 & -1C_2 + 3C_3 & 3C_1 + 2C_2 & C_1 + 7C_2 - 19C_3 \end{bmatrix}$$

$$\begin{bmatrix} 4C_1 + 8C_3 & 5C_2 - 11C_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1 + 2C_3 = 0$$

$$C_1 + 2C_3 = 0$$

$$3C_1 + 2C_2 = 0$$

$$-C_2 + 3C_3 = 0$$

$$C_1 + 7C_2 - 19C_3 = 0$$

$$5C_2 - 11C_3 = 0$$

$$C_2 = 3C_3$$

$$C_1 = 0$$

$$15C_3 - 11C_3 = 0$$

$$C_2 = 0$$

unique solution. linearly independent

$$4C_3 = 0$$

$$C_3 = 0$$

$$2) [7 \ -3 \ 11 \ -6], [-56 \ 24 \ -88 \ 48]$$

$$\begin{bmatrix} 7c_1 - 56c_2 & -3c_1 + 24c_2 & 11c_1 - 88c_2 & -6c_1 + 48c_2 \end{bmatrix}$$

$$= [0 \ 0 \ 0 \ 0]$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$7c_1 = 56c_2$$

$$c_1 = 8c_2$$

$$c_1 = 8k$$

linearly dependent

$$3) [-1 \ 5 \ 0], [16 \ 8 \ -3], [-64 \ 56 \ 9]$$

$$\begin{bmatrix} -1c_1 + 16c_2 - 64c_3 & 5c_1 + 8c_2 + 56c_3 & -3c_2 + 9c_3 \end{bmatrix} = [0 \ 0 \ 0]$$

$$-3c_2 + 9c_3 = 0$$

$$3. AC_3 = 3C_2$$

$$C_2 = 3C_3 \\ = 3K$$

$$-c_1 + 16c_2 - 64c_3 = 0$$

$$16c_2 - 64c_3 + c_1$$

$$16(3c_3) = 64c_3 + c_1$$

$$\Rightarrow c_1 = 0$$

$$\text{vector} = k \begin{bmatrix} 0 \\ 3K \\ K \end{bmatrix} = k \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

linearly dependent

$$4) [1 \ -1 \ 1]; [1 \ 1 \ -1]; [-1 \ 1 \ 1] \cdot [0 \ 1 \ 0]$$

$$\begin{bmatrix} c_1 + c_2 - c_3 & -c_1 + c_2 + c_3 + c_4 & c_1 - c_2 + c_3 \end{bmatrix}$$

$$c_1 + c_2 - c_3 = 0$$

$$c_1 + c_2 = c_3$$

$$c_1 + c_2 + c_3 = c_4$$

$$2c_1 = 0 \Rightarrow c_1 = 0.$$

$$-c_1 + c_2 + c_3 + c_4 = 0$$

$$c_1 = c_2 + c_3 + c_4$$

$$c_4 = -2k$$

$$c_1 - c_2 + c_3 = 0$$

$$\text{vector} = k \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$c_2 = c_1 + c_3.$$

$$c_2 = c_2 = k.$$

linearly dependent
dependent

5) $\begin{bmatrix} 2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 9 \end{bmatrix}, \begin{bmatrix} 3 & 5 \end{bmatrix}$

$$\begin{cases} 2C_1 + C_2 + 3C_3 \\ 2C_1 + C_2 + 3C_3 \\ 4C_1 + 2C_2 + 6C_3 \\ 9C_2 + 5C_3 + 2C_2 + 6C_3 \end{cases} \quad \begin{array}{l} -4C_1 + 9C_2 + 5C_3 = 0 \\ 2C_1 + 3C_3 = 0 \\ 4C_1 + 2C_2 + 6C_3 = 0 \\ 11C_2 + 11C_3 = 0 \\ C_2 = -C_3 = -K \end{array}$$

$$4C_1 = 9CK - 5K$$

$$C_1 = 4K$$

$$C_1 = K$$

vector = $\begin{bmatrix} K \\ 4K \\ -K \end{bmatrix} = k \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$, linearly dependent

6) $\begin{bmatrix} 3 & -2 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 & 1 \end{bmatrix}; \begin{bmatrix} -6 & 1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3C_1 + 5C_2 - 6C_3 + 2C_4 & -2C_1 + C_3 & 0 \\ 4C_1 + C_2 + C_3 + 3C_4 & & \\ \hline & [0 & 0 & 0 & 0] & \end{array}$$

$$-2C_1 + C_3$$

$$2C_1 = C_3$$

$$C_1 = K$$

$$C_3 = 2C_1 \\ = 2K$$

$$4C_1 + C_2 + C_3 + 3C_4 = 0$$

$$20C_1 + 5C_2 + 5C_3 + 15C_4 = 0$$

$$\underline{3C_1 + 5C_2 - 6C_3 + 2C_4 = 0}$$

$$\underline{\underline{17C_1 + 11C_3 - 13C_4 = 0}}$$

$$17K + 11(2K) = 13C_4$$

~~$$(17+22)K = 13C_4$$~~

$$4C_1 + C_2 + C_3 + 3C_4$$

$$4(K) + C_2 + 2K + 3(3K) = 0$$

$$C_2 + 15K = 0$$

$$C_2 = -15K$$

$$\frac{3}{17} 34K = C_4 - 3K$$

vector = $\begin{bmatrix} K \\ -15K \\ 2K \\ 3K \end{bmatrix} = K \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$

$$\Rightarrow [3 \ 4 \ 7], [2 \ 0 \ 3], [8 \ 2 \ 3], [5 \ 5 \ 6]$$

$$\begin{bmatrix} 3C_1 + 2C_2 + 8C_3 + 5C_4 \\ 4C_1 + 2C_3 + 5C_4 \end{bmatrix} = \begin{bmatrix} 7C_1 + 3C_2 + 3C_3 + 6C_4 \\ 14C_1 + 6C_2 + 6C_3 + 12C_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$3C_1 + 2C_2 + 8C_3 + 5C_4 = 0$$

$$\underline{\underline{4C_1 + 2C_3 + 5C_4 = 0}}$$

$$-C_1 + 2C_2 + 6C_3 = 0$$

$$C_1 = 2C_2 + 6C_3$$

$$5C_1 = 10C_2 + 30C_3$$

$$7C_1 + 3C_2 + 3C_3 + 6C_4 = 0$$

$$\begin{array}{r} 14C_1 + 6C_2 + 6C_3 + 12C_4 = 0 \\ -9C_1 + 6C_2 + 24C_3 + 15C_4 = 0 \\ \hline 5C_1 - 18C_3 - 3C_4 = 0 \end{array}$$

$$5C_1 = 18C_3 + 3C_4$$

$$10C_2 + 30C_3 = 18C_3 + 3C_4$$

$$12C_3 = 3C_4 - 10C_2$$

$$4C_1 + 2C_3 + 5C_4 = 0$$

$$36C_1 + 18C_3 + 45C_4 = 0$$

$$\underline{5C_1 - 18C_3 - 3C_4 = 0}$$

$$41C_1 + 42C_4 = 0$$

$$41C_1 = -42C_4$$

$$C_1 = -\frac{42}{41}C_4 = -\frac{42}{41}K$$

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = K \begin{bmatrix} -\frac{42}{41} \\ \frac{10543}{492} \\ 1 \\ \frac{1506}{216} \end{bmatrix}$$

$$18C_3 = 5C_1 - 3C_4$$

linearly dependent

$$C_3 = 5\left(-\frac{42}{41}\right)K - 3(K)\left(\frac{1}{\cancel{41}}\right)$$

$$= \frac{210(6) - 6(41)}{(41)(6)}K = \frac{(1260 + 216)K}{216}$$

$$\frac{-42}{41}K = 2C_2 + 8\left(\frac{251}{36}\right)K \Rightarrow \frac{1506K}{216}$$

$$2C_2 = -\frac{42K}{41} - \frac{251}{6}K$$

$$C_2 = -\left(\frac{252 + 10291}{2 \times 246}\right)K = \frac{10543K}{492}$$

$$\frac{6 \times 36 + 35}{36}$$

$$1) \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}, \quad A - \lambda I = 0$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = (-2-\lambda)(-\lambda(1-\lambda)+6(-2)) - 2(2+6(-1)) - 3(-4+1-\lambda)$$

$$= (-2-\lambda)(-\lambda+\lambda^2-12) + 2(+\lambda+6) + 3(+3+\lambda)$$

$$= +\underline{2\lambda} - 2\lambda^2 + \underline{24} + \lambda^2 - \underline{\lambda^3} + \underline{12\lambda} + 2\lambda + 12 + 9 + 3\lambda$$

$$\textcircled{Q} \quad 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\lambda^3 + \lambda^2 - 19\lambda - 45 = 0$$

$$2) \quad A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad A - \lambda I = 0$$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(+2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda) + 2)$$

$$= (4-\lambda - 4\lambda + \lambda^2 + 2)$$

$$\lambda = -1, 2, 3.$$

Eigenvalues.

$$\lambda = -1$$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$5x + z = 0$$

$$-2x + 2y = 0$$

$$-2x + 2z = 0$$

$$\underline{10x + 2z = 0}$$

$$-12x = 0$$

$$x = 0$$

Eigen vector = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\lambda = 2$$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$2x + z = 0$$

$$-2x - y = 0$$

$$-2x - z = 0$$

$$x = K$$

$$z = 2K$$

$$y = -2K$$

Eigen vector = $K \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$$\lambda = 3$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x + z = 0$$

$$-2x - 2y = 0$$

$$-2x - 2z = 0$$

$$x = -z = -K$$

$$z = K$$

$$y = -2x$$

$$y = K$$

Vector = $K \begin{bmatrix} -K \\ K \\ K \end{bmatrix}$
 $+ K \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$$3) \Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5$$

$$\lambda = 0$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\begin{aligned} 5x &= 0 \Rightarrow x = 0 \\ -x + 3z &= 0 \\ z &= 0 \end{aligned}$
 vector = $k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\lambda = 3$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\begin{aligned} 2x &= 0 \\ -3y &= 0 \\ -x &= 0 \end{aligned}$

$$\text{vector} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\lambda = 5$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\begin{aligned} -5y &= 0 \\ -x - 2z &= 0 \\ x &= 2z = -2x \end{aligned}$

$$\text{vector} = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$4) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

5.

$$\rightarrow \lambda(3-\lambda)(-2+\lambda) = 0$$

$$\lambda = 0, 3, -2$$

$$\lambda = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{cases} 3y + 4z = 0 \\ -2z = 0 \end{cases}$$

$$z = 0, y = 0$$

$$\text{vector} = k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{cases} 4z = 0 \\ -5z = 0 \end{cases}$$

$$\text{vector} = k \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5y + 4z = 0$$

$$5y - 4z = -4k$$

$$y = -\frac{4}{5}k$$

$$k \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

$$i) 2x - 3y + 7z = 5, \quad 3x + y - 3z = 13, \quad 2x + 9y - 4z = 32$$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{3}{2}R_1$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 15 & -\frac{27}{2} \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 15 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & 15 & -\frac{27}{2} \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 15 \\ 0 \end{bmatrix}$$

Rank of $A = 2$ Rank of $A:B = 3$

$\rho(A) \neq \rho(A:B)$ Inconsistency.

$$ii) 2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & \frac{-38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$\begin{aligned} R_3 &\rightarrow R_3 - \frac{7}{3}R_2 \\ &\rightarrow -1 - \frac{7}{3} \cdot \frac{5}{3} \\ &\rightarrow -\frac{38}{3} \end{aligned}$$

$\delta(A) = \delta(A:B)$ consistency.

2(iii)

$$Q) 4x - y = 12, \quad -x + 5y - 2z = 0, \quad -2x + 4z = -8$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$$

$\delta(A) = \delta(A:B)$ consistency.

b) For what values of λ and μ

$$x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$$

i) no solution, ii) a unique solution iii) infinite no. of solutions.

$$Ax = B$$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda - 3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

i) No solution $S(A) \neq S(A:B)$

$$\lambda = 3, \mu \neq 10$$

(ii) a unique solution. $S(A) = S(A:B) = \boxed{B \text{ n}}$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \cancel{\mu - 10} \end{bmatrix}$$

$$\lambda \neq 3, \mu \text{ any value}$$

iii) Infinite solution $S(A) = S(A:B) < \boxed{B \text{ n}}$

$$\lambda = 3, \mu = 10$$

$n \rightarrow$ no. of unknowns.
 x, y, z .

c) Find for what value of λ .

$$x+y+z=1$$

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

Have a solution. &

Find λ to solve for each value of λ .

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

With unknowns x, y, z :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$$f(A) = 2$$

$$f(A:B) = 2$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-1) - \lambda(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1, 2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2 - 1 - 3\lambda + 3 \end{bmatrix}$$

$$f(A) = f(A:B) \neq n$$

3 unknowns

Case-1 $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$

$$y+3z=0$$

$$z=k$$

$$y=-3k$$

$$x = 1 - y - z$$

$$= 1 + 3k - k = 1 + 2k$$

Case-2 $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$

$$y+3z=1$$

$$y=1-3z, z=k$$

$$x = 1 - 1 + 3k - k = 2k$$

d) Find the sol' of the system of equations.

$$x+3y-2z=0, 2x-y+4z=0, x-11y+14z=0.$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\xrightarrow{\cdot 2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \quad \vdots$$

e) find for what values of λ the given equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may posse non-trivial solution and solve them completely in each case

$$\begin{aligned} 12x &= 0 & S(A) = S(A+E) \\ 2 &\neq 0 & -2 \neq n \\ -7y &= 0 & \text{Infinite solutions} \\ y &= 0 \\ x &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2x+3 & 4+1 & 0 \end{bmatrix} \quad \text{Infinite}$$

$R_3 = R_3 + R_1$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$R_1 = R_1/4, R_2 = R_2/3$

$$= \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -10 & -9+4\lambda \\ 2x+3 & 5 & 0 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$$\begin{aligned} 12-6-9 &= 0 & 2(2\lambda+3)+5y=0 \\ 0 &= 0 & -10y+(-9+4\lambda)z=0 \\ 2\lambda+3 &= 0 & 2x+4y-4\lambda z=0 \\ 2x+3 &= 0 & -9+4\lambda = 0 \\ x &= -\frac{3}{2} & x = \frac{9}{2} \\ z &= \frac{9}{4\lambda} & y = \frac{-9+4\lambda}{2} \\ 3 &= 10y/(-9+4\lambda) & \end{aligned}$$

$$\begin{aligned} 12x+6y-9z &= 0 \\ -10y+(-9+4\lambda)z &= 0 \end{aligned}$$

$$\begin{aligned} (2\lambda+3)x+5y &= 0 \\ -(2x+3)y &= 9+4\lambda \end{aligned}$$

$$12x - 6y - 9z = 0$$

$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$

$$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{10^2}{(-9+4\lambda)}\left(\frac{2\lambda+3}{5}\right)x\right) = 0$$

$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(-9+4\lambda)} = 0$$

$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9) + 18(5)(2\lambda+3) = 0$$

(12\lambda+18)

$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$$

$$48\lambda^2 + 384\lambda - 432 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda+1) + 9(\lambda-1) = 0$$

$$(\lambda+9)(\lambda-1) = 0 \Rightarrow \lambda = 1, \lambda = -9$$

$$\underline{\lambda=1}$$

$$-x = y, z = -2y$$

$$12x - 6y - 9z = 0$$

$$12x(-y) - 6y - 9(-2y) = 0$$

$$18y - 18y = 0$$

$y \neq 0$ non-trivial solution

$$\underline{\lambda=-9}$$

$$y = \left(-\frac{(2\lambda+3)x}{5}\right)$$

$$z = \frac{(-18+3)x}{5} = +\frac{3}{5}x$$

$$z = \frac{10y}{(-9+4\lambda)}$$

$$= \frac{10y}{-45} = \frac{2y}{-9}$$

$$12x - 6y - 9z = 0$$

$$x = \left(\frac{y}{8}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$$

$$4y - 6y + 2y = 0$$

$y = 0$ trivial solution

+ 1st solution

Assignment - 2

Q) 1) $[1, 0, 0], [1, 1, 0], [1, 1, 1]$

$$\begin{bmatrix} C_1 + C_2 + C_3 & C_2 + C_3 & C_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$C_3 = 0, C_2 = 0, C_1 = 0$$

unique solution
linearly independent.

Q) $[6, 0, 3, 1, 4, 2], [0, -1, 2, 7, 0, 5], [1, 2, 3, 0, -1, 9, 8]$

$$\begin{bmatrix} 6C_1 + 12C_3 & -1C_2 + 3C_3 & 3C_1 + 2C_2 & C_1 + 7C_2 - 19C_3 \end{bmatrix}$$

$$\begin{bmatrix} 4C_1 + 8C_3 & 5C_2 - 11C_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1 + 2C_3 = 0$$

$$C_1 + 2C_3 = 0$$

$$3C_1 + 2C_2 = 0$$

$$-C_2 + 3C_3 = 0$$

$$C_1 + 7C_2 - 19C_3 = 0$$

$$5C_2 - 11C_3 = 0$$

$$C_2 = 3C_3$$

$$C_1 = 0$$

$$15C_3 - 11C_3 = 0$$

$$C_2 = 0$$

unique solution. linearly independent

$$4C_3 = 0$$

$$C_3 = 0$$

$$2) [7 \ -3 \ 11 \ -6], [-56 \ 24 \ -88 \ 48]$$

$$\begin{bmatrix} 7c_1 - 56c_2 & -3c_1 + 24c_2 & 11c_1 - 88c_2 & -6c_1 + 48c_2 \end{bmatrix}$$

$$= [0 \ 0 \ 0 \ 0]$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$. 7c_1 = 56c_2$$

$$c_1 = 8c_2$$

$$c_1 = 8k$$

linearly dependent

$$3) [-1 \ 5 \ 0], [16 \ 8 \ -3], [-64 \ 56 \ 9]$$

$$\begin{bmatrix} -1c_1 + 16c_2 - 64c_3 & 5c_1 + 8c_2 + 56c_3 & -3c_2 + 9c_3 \end{bmatrix} = [0 \ 0 \ 0]$$

$$-3c_2 + 9c_3 = 0$$

$$3. AC_3 = 3C_2$$

$$C_2 = 3C_3 \\ = 3K$$

$$-c_1 + 16c_2 - 64c_3 = 0$$

$$16c_2 - 64c_3 + c_1$$

$$16(3c_3) = 64c_3 + c_1$$

$$\Rightarrow c_1 = 0$$

$$\text{vector} = k \begin{bmatrix} 0 \\ 3K \\ K \end{bmatrix} = k \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

linearly dependent

$$4) [1 \ -1 \ 1]; [1 \ 1 \ -1]; [-1 \ 1 \ 1] \cdot [0 \ 1 \ 0]$$

$$\begin{bmatrix} c_1 + c_2 - c_3 & -c_1 + c_2 + c_3 + c_4 & c_1 - c_2 + c_3 \end{bmatrix}$$

$$c_1 + c_2 - c_3 = 0$$

$$c_1 + c_2 = c_3$$

$$c_1 + c_2 + c_3 = c_4$$

$$2c_1 = 0 \Rightarrow c_1 = 0.$$

$$-c_1 + c_2 + c_3 + c_4 = 0$$

$$c_1 = c_2 + c_3 + c_4$$

$$c_4 = -2k$$

$$c_1 - c_2 + c_3 = 0$$

$$\text{vector} = k \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$c_2 = c_1 + c_3.$$

$$c_2 = c_2 = k.$$

linearly dependent

5) $\begin{bmatrix} 2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 9 \end{bmatrix}, \begin{bmatrix} 3 & 5 \end{bmatrix}$

$$\begin{cases} 2C_1 + C_2 + 3C_3 \\ 2C_1 + C_2 + 3C_3 \\ 4C_1 + 2C_2 + 6C_3 \\ 9C_2 + 5C_3 + 2C_2 + 6C_3 \end{cases} \quad \begin{array}{l} -4C_1 + 9C_2 + 5C_3 = 0 \\ 2C_1 + C_2 + 3C_3 = 0 \\ 4C_1 + 2C_2 + 6C_3 = 0 \\ 11C_2 + 11C_3 = 0 \\ C_2 = -C_3 = -K \end{array}$$

$$4C_1 = 9CK - 5K$$

$$C_1 = 4K$$

$$C_1 = K$$

vector = $\begin{bmatrix} K \\ 4K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$, linearly dependent

6) $\begin{bmatrix} 3 & -2 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 & 1 \end{bmatrix}; \begin{bmatrix} -6 & 1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{cases} 3C_1 + 5C_2 - 6C_3 + 2C_4 \\ -2C_1 + C_3 \\ 4C_1 + C_2 + C_3 + 3C_4 \end{cases} \quad \begin{array}{l} 0 \\ 0 \\ -[0 \ 0 \ 0 \ 0] \end{array}$$

$$-2C_1 + C_3$$

$$2C_1 = C_3$$

$$C_1 = K$$

$$C_3 = 2C_1 \\ = 2K$$

$$4C_1 + C_2 + C_3 + 3C_4 = 0$$

$$20C_1 + 5C_2 + 5C_3 + 15C_4 = 0$$

$$\underline{3C_1 + 5C_2 - 6C_3 + 2C_4 = 0}$$

$$\underline{17C_1 + 11C_3 - 13C_4 = 0}$$

$$17K + 11(2K) = 13C_4$$

~~$$(17+22)K = 13C_4$$~~

$$\frac{39K}{13} = C_4 - 3K$$

$$4C_1 + C_2 + C_3 + 3C_4$$

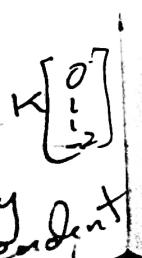
$$4(K) + C_2 + 2K + 3(3K) = 0$$

$$C_2 + 15K = 0$$

$$C_2 = -15K$$

$$\text{vector} = \begin{bmatrix} K \\ -15K \\ 2K \\ 3K \end{bmatrix} = K \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$$

linearly dependent



$$\Rightarrow [3 \ 4 \ 7], [2 \ 0 \ 3], [8 \ 2 \ 3], [5 \ 5 \ 6]$$

$$\begin{bmatrix} 3C_1 + 2C_2 + 8C_3 + 5C_4 \\ 4C_1 + 2C_3 + 5C_4 \end{bmatrix} = \begin{bmatrix} 7C_1 + 3C_2 + 3C_3 + 6C_4 \\ 0 \ 0 \end{bmatrix}$$

$$3C_1 + 2C_2 + 8C_3 + 5C_4 = 0$$

$$\underline{\underline{4C_1 + 2C_3 + 5C_4 = 0}}$$

$$\underline{-C_1 + 2C_2 + 6C_3 = 0}$$

$$C_1 = 2C_2 + 6C_3$$

$$5C_1 = 10C_2 + 30C_3$$

$$7C_1 + 3C_2 + 3C_3 + 6C_4 = 0$$

$$\underline{\underline{14C_1 + 6C_2 + 6C_3 + 12C_4 = 0}}$$

$$\underline{\underline{-9C_1 + 6C_2 + 24C_3 + 15C_4 = 0}}$$

$$\underline{\underline{5C_1 - 18C_3 - 3C_4 = 0}}$$

$$5C_1 = 18C_3 + 3C_4$$

$$10C_2 + 30C_3 = 18C_3 + 3C_4$$

$$12C_3 = 3C_4 - 10C_2$$

$$4C_1 + 2C_3 + 5C_4 = 0$$

$$\underline{\underline{36C_1 + 18C_3 + 45C_4 = 0}}$$

$$\underline{\underline{5C_1 - 18C_3 - 3C_4 = 0}}$$

$$41C_1 + 42C_4 = 0$$

$$41C_1 = -42C_4$$

$$C_1 = -\frac{42}{41}C_4 = -\frac{42}{41}K.$$

$$= \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = K \begin{bmatrix} -\frac{42}{41} \\ \frac{10543}{492} \\ 1 \\ \frac{1506}{216} \end{bmatrix}$$

$$18C_3 = 5C_1 - 3C_4$$

linearly dependent

$$C_3 = 5\left(-\frac{42}{41}\right)K - 3(K)\left(\frac{1}{\cancel{41}}\right)$$

$$= \frac{(210(6) - 6(41))}{(41)(6)}K = \frac{(1260 + 216)K}{216}$$

$$\frac{-42}{41}K = 2C_2 + 8\left(\frac{251}{38}\right)K \stackrel{=} \frac{1506K}{216}$$

$$2C_2 = -\frac{42K}{41} - \frac{251}{6}K$$

$$C_2 = -\left(\frac{252 + 10291}{2 \times 246}\right)K = \frac{10543K}{492}$$

$$\frac{6 \times 36 + 35}{36}$$

$$1) \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}, \quad A - \lambda I = 0$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = (-2-\lambda)(-\lambda(1-\lambda)+6(-2)) - 2(2+6(-1)) - 3(-4+1-\lambda)$$

$$= (-2-\lambda)(-\lambda+\lambda^2-12) + 2(+\lambda+6) + 3(+3+\lambda)$$

$$= +\underline{2\lambda} - 2\lambda^2 + \underline{24} + \lambda^2 - \underline{\lambda^3} + \underline{12\lambda} + 2\lambda + 12 + 9 + 3\lambda$$

$$\textcircled{Q} \quad 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\lambda^3 + \lambda^2 - 19\lambda - 45 = 0$$

$$2) \quad A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad A - \lambda I = 0$$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(+2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda) + 2)$$

$$= (4-\lambda - 4\lambda + \lambda^2 + 2)$$

$$\lambda = -1, 2, 3.$$

Eigenvalues.

$$= (\lambda^2 - 6\lambda + 6)$$

$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$$\lambda = -1$$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$5x + z = 0$$

$$-2x + 2y = 0$$

$$-2x + 2z = 0$$

$$\underline{10x + 2z = 0}$$

$$-12x = 0$$

$$x = 0$$

Eigen vector.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 2$$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$2x + z = 0$$

$$-2x - y = 0$$

$$-2x - z = 0$$

$$x = k$$

$$z = 2k$$

$$y = -2k$$

$$\text{Eigen vector} = k \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$\lambda = 3$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$x + z = 0$$

$$-2x - 2y = 0$$

$$-2x - 2z = 0$$

$$x = -z = -k$$

$$z = k$$

$$\begin{aligned} y &= -x \\ y &= k \end{aligned}$$

$$\begin{aligned} \text{Vector} &= k \begin{bmatrix} -k \\ k \\ k \end{bmatrix} \\ &= +k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \end{aligned}$$

$$3) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5$$

$$\lambda = 0$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore 5x=0 \Rightarrow x=0$
 $-x+3z=0$
 $\therefore z=0$
 $\text{vector}_1 = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\lambda = 3$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore 2x=0$
 $-3y=0$
 $-x=0$

$$\text{vector}_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\lambda = 5$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore -5y=0$
 $-x-2z=0$
 $x=2z = -2x$

$$\text{vector}_3 = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$4) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 \rightarrow & 0 & 0 \\ 0 & 3 \rightarrow & 4 \\ 0 & 0 & -2 \rightarrow \end{bmatrix}$$

$$\rightarrow \lambda(3-\lambda)(-2+\lambda) = 0$$

$$\lambda = 0, 3, -2$$

$$\lambda = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{array}{l} 3y + 4z = 0 \\ -2z = 0 \end{array}$$

$\left| \begin{array}{l} z=0, y=0 \end{array} \right.$

$$\text{vector}_2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \begin{array}{l} 4z = 0 \\ -5z = 0 \end{array}$$

$$\text{vector}_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \begin{array}{l} 5y + 4z = 0 \\ 5y - 4z = -4k \end{array}$$

$$y = -\frac{4}{5}k$$

$$k \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

Assignments

1)

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$ $R_4 \rightarrow R_4 - 4R_1$ $R_3 \rightarrow R_3 - R_1$ $R_4 \rightarrow R_4 + R_3$
 $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

no. of non zero rows.
rank = 3

2)

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow \frac{1}{4-1} \begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & +\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} = \frac{(\frac{2}{3}-\lambda)^2 - (\frac{1}{3})^2}{a^2-b^2} = 0$$

$$(\frac{1}{3}-\lambda)(1-\lambda) = 0$$

$$\lambda = 1$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Eigenvalues $\lambda = 1, \frac{1}{3}$

$$x \frac{1}{3} + y \left(\frac{1}{3} \right) = 0$$

Eigen vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$x = y = K$$

$$\lambda = \frac{1}{3}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x = -y$$

Eigen vector $\begin{bmatrix} K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$$(a^2 - b^2) = (a-b)(a+b)$$

$$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$$

$$= (5-\lambda)(7-\lambda) = 0$$

Eigenvalues $\lambda = 5, 7$

$$\lambda = 5$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{aligned} x-y &= 0 \\ x &= y \end{aligned}$$

Eigen Vector $= k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\lambda = 7$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x = y$$

Eigen vector $= k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$2) T: W \rightarrow P_2$$

$$T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)x + (b-c)x^2 + (c-d)x^3.$$

find the rank and nullity of T.

$$\rightarrow T \begin{pmatrix} a & b \\ b & c \end{pmatrix} = (a-b)x + (b-c)x^2 + (c-a)x^3.$$

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$\begin{aligned} T(A) &= (a-b)x + (b-c)x^2 + (c-a)x^3 \\ &= a - bx + c(x^2 - x + 1) \end{aligned}$$

\therefore The image of T is the set of all polynomials
of degree at most 2, denoted as P_2 .

Rank of T:

The rank of T is the dimension of its image
since P_2 has a dimension of 3 (coefficients for
 x^0, x^1 , and x^2) the rank of T is 3.

The Null Space of symmetric matrix

$$T(A) = 0$$

this leads to the system of equations

$$\begin{aligned} a-b &= 0 & b-c &= 0 & c-a &= 0 \end{aligned}$$

$$\therefore a = b = c$$

$$4) 3x - 0.1y + 0.2z = 7.85 \Rightarrow x = \frac{7.85 - 0.2z + 0.1y}{3}$$

$$0.1x + 7y - 0.3z = -19.3 \Rightarrow y = \frac{-19.3 + 0.3z - 0.1x}{7}$$

$$0.3x - 0.2y + 10z = 71.4 \Rightarrow z = \frac{71.4 + 0.2y - 0.3x}{10}$$

$$x(0) = 0, y(0) = 0, z(0) = 0$$

Iteration-1

$$x(1) = \frac{7.85}{3} = 2.616.$$

$$y(1) = \frac{-19.3 + 0.1(2.616)}{7} = \frac{-19.5616}{7} = -2.794$$

$$z(1) = \frac{71.4 + 0.2(-2.794) - 0.3(2.616)}{10}$$

Iteration-2

$$= 7.0056.$$

$$x(2) = \frac{7.85 - 0.2(7.0056) + 0.1(-2.794)}{3} = \frac{6.675}{3} = 2.057$$

$$y(2) = \frac{-19.3 + 0.3(7.0056) - 0.1(-2.794)}{7} = -2.487$$

$$z(2) = \frac{71.4 + 0.2(-2.487) - 0.3(2.057)}{10} = 7.023$$

Iteration-3

$$x(3) = \frac{7.85 - 0.2(7.023) + 0.1(-2.487)}{3} = 2.064$$

$$\Rightarrow x.$$

$$y(3) = \frac{-19.3 + 0.3(7.023) - 0.1(-2.487)}{7} = -2.544.$$

$$z(3) = \frac{71.4 + 0.2(-2.544) - 0.3(2.064)}{10} = 7.027$$

5)

Define Consistent and inconsistent system of equations hence
Solve the following system of equations if consistent.

$$x+3y+2z=0, 2x-y+3z=0, 3x-5y+4z=0, x+17y+4z=0$$

Sol: Consistent: A system of equations is consistent if it has at least one solution, meaning the equations have a common solution.

Inconsistent: A system of equations is inconsistent if it has no solution, meaning the equations do not intersect at any point or are contradictory.

$$\begin{aligned} x+3y+2z &= 0 \\ 2x-y+3z &= 0 \\ 3x-5y+4z &= 0 \\ x+17y+4z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & x \\ 2 & -1 & 3 & y \\ 3 & -5 & 4 & z \\ 1 & 17 & 4 & 0 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} \right]$$

$$C \rightarrow [A:B]$$

$$\left[\begin{array}{cccc} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array} \right] = \left[\begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array} \right] = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1, R_4 \rightarrow R_4 - R_1 \quad R_4 \rightarrow R_4 + R_3$$

$$\left[\begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 2R_2$$

Rank(C) = Rank(A) \neq no. of unknowns
infinite Solutions.

$\alpha X = B$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

let $\boxed{z = k}$

$$x + 3\left(-\frac{k}{7}\right) + 2k = 0$$

$$7x + (-3k) + 14k = 0$$

$$\boxed{x = -\frac{11k}{7}}$$

$$x + 3y + 2z = 0$$

$$-7y - z = 0$$

$$-7y = z$$

$$\boxed{y = -\frac{z}{7}}$$

6) $T: P_2 \rightarrow P_2$ is linear transformation.

$$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$$

$$\rightarrow T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$$

is a linear transformation, we need to check two properties.

1) Additivity. $T(u+v) = T(u)+T(v)$

2) homogeneity of degree 1:

$T(ku) = kT(u)$ for all u in the domain

of T and all scalars k .

$$\begin{aligned} 1) T(u+v) &= T((a_1+b_1x+c_1)+(a_2+b_2x+c_2)) \\ &= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)) \\ &= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2 \\ &= (a_1+1)+(b_1+1)x+(c_1+1)x^2 + (a_2+1)+(b_2+1)x^2 + (c_2+1)x^2 \end{aligned}$$

$$T(a_1 + b_1 x + c_1) + T(a_2 + b_2 x + c_2)$$

so function is additive.

\therefore Homogeneity of Degree 1:

$$T(Kv) = T(K(a+bx+c))$$

$$= T(Ka+Kbx+Kc) = (Ka+1)x + (Kb+1)x^2 + (Kc+1)$$

$$= K(a+1) + K(b+Dx) + K(c+Dx^2)$$

$$= KT(a+bx+c)$$

So, the function is homogeneous of degree 1.

\therefore it indeed linear transformation

7)

$S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ is a basis.

of $V_3(\mathbb{R})$. In case S is not a basis determine

sub space - spanned by $-S$.

$$\rightarrow S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$$

can be arranged as matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now, let's perform row reduction to obtain the Echelon form

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - 2R_1 \text{ and } R_3 \leftarrow R_3 - 3R_1$$

$$\left[\begin{array}{ccc} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + \frac{9}{5}R_1} \left[\begin{array}{ccc} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{array} \right]$$

\therefore Third row of zeros indicates that the vectors in S are linearly dependent.

for basis of the subspace spanned by S .

$\left[\begin{array}{ccc} 1 & 3 & -2 \\ 0 & -5 & 5 \end{array} \right]$ The vectors $(1, 3, -2)$ and $(0, -5, 5)$, form a basis for the subspace spanned by S .

\therefore Dimension of subspace spanned by $S = 2$.

\therefore Set S is not a basis of \mathbb{R}^3 because the row-reduced form has a row of zeros.

\therefore The basis for the subspace spanned by S is $\{(1, 3, -2), (0, -5, 5)\}$

\therefore The dimension of the subspace is 2

8) using Jacobi's method (perform 3 iterations) solve.

$$3x - 6y + 2z = 23, \quad -4x + y - z = -15, \quad x - 3y + 7z = 16,$$

with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

$$\rightarrow 3x - 6y + 2z = 23$$

$$-4x + y - z = -15$$

$$x - 3y + 7z = 16$$

with initial values

$$x = 1, y = 1, z = 1$$

Iteration-1.

$$x^{(1)} = \frac{23 + 6y^{(0)} - 2z^{(0)}}{3} \approx 9.0$$

$$y^{(1)} = \frac{-15 + 4(x^{(1)}) + z^{(0)}}{1} = -9.0$$

$$z^{(1)} = \frac{16 - x^{(1)} + 3(y^{(1)})}{7} = 2.0$$

Iteration-2

$$x^{(2)} = \frac{23 + 6y^{(0)} - 2z^{(1)}}{3} = 5.0$$

$$y^{(2)} = \frac{-15 + 4(x^{(2)}) + z^{(1)}}{1} = -5.0$$

$$z^{(2)} = \frac{16 - x^{(2)} + 3(y^{(1)})}{7} \approx 3.0$$

Iteration-3

$$x^{(3)} = \frac{23 + 6y^{(2)} - 2z^{(2)}}{3} = 6.0$$

$$y^{(3)} = \frac{-15 + 4(x^{(2)}) + z^{(2)}}{1} \approx -6.0$$

$$z^{(3)} = \frac{16 - x^{(2)} + 3(y^{(2)})}{7} \approx 2.0$$

Q) Explain one application of matrix operations in image processing with example.

Sol: Matrix Magic in Image Processing: Convoluting Takes Center Stage.

Digital images are essentially grids of tiny squares called pixels, each holding an intensity value (grayscale or color information (RGB)). To manipulate these images effectively, image processing relies heavily on the power of matrices.

1) Image as a Matrix

We represent the image as a matrix, where each pixel's value becomes an element in the grid.

2) Kernel: The Tiny Powerhouse:

We define a small matrix called a Kernel or filter. Different Kernel designs produce different results.

3) The Convolution Slide:
We slide the Kernel across the image matrix, one element at a time.

4) Summing Up for a New Value Pixel
The multiplied values from the element-wise multiplication are then summed up.

5) Iterating Across the Image
We repeat this process (sliding, multiplication, summation) for every position of the Kernel within the image matrix.

Here's an example of how convolution works using a simple image filtering task:

→ Let's consider a small 3×3 grayscale image matrix:

$$\begin{bmatrix} 100 & 150 & 200 \\ 120 & 180 & 220 \\ 110 & 160 & 190 \end{bmatrix}$$

→ Now, we want to apply a simple 3×3 filter/Kernel to this image. Let's take a blur filter.

Kernel: $\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$

→ To apply this filter, we perform element-wise multiplication of the filter kernel with the corresponding pixels in the image matrix and then sum up the results.

This sum becomes the new value for the ~~filter kernel~~
central pixel in the output image.

$$(100 \times \frac{1}{9}) + (150 \times \frac{1}{9}) + (200 \times \frac{1}{9}) + (20 \times \frac{1}{9}) + (180 \times \frac{1}{9}) + (220 \times \frac{1}{9}) \\ + (110 \times \frac{1}{9}) + (160 \times \frac{1}{9}) + (190 \times \frac{1}{9}) = 159$$

→ Each pixel value is weighted average of its neighbouring pixels effectively creating a blurred version of the original image.

→ Convolution allows for the application of various filters to image, enabling tasks such as blurring, sharpening, edge detection and more.

Q10) Give a brief description of linear transformation for Computer Vision for rotating 2D image

Sol: Linear transformations are a powerful tool in Computer Vision for tasks like image rotation. There's a breakdown.

1) Image as a Vector:
we can represent a 2D image as a collection of points, where each point corresponds to a pixel's location (x,y coordinates).

2) Rotation Matrix:

To rotate the image, we use a specific rotation matrix.
This matrix encodes the rotation angle and how it affects the x and y coordinates of each point (pixel) in the image.

3) Transformation Magic:
By multiplying the image vector (representing all pixel locations) with the rotation matrix, we perform a linear transformation on all the points simultaneously.

Affine transformation include operations such as rotation, translation, scaling and shearing. When rotating a 2D image the affine transformation involves applying a rotation matrix to coordinates of each pixel in the image.

For rotating a 2D image clockwise by an angle θ , the rotation matrix R is typically used.

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Each pixel coordinate (x, y) in the original image is multiplied by this rotation matrix to obtain the new coordinates (x', y') in the rotated image. The new pixel value at (x', y') is then determined by interpolating the values of neighbouring pixels in the original image.